Building next-Generation Tools for the Exploitation of Space-Based Seismic Data

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1. Introduction:

As a star enters the subgiant phase, the physical and chemical changes in its interior allows the coupling between acoustic and gravity modes. Because the frequencies of these mixed-modes have a fast evolution with age, we are able to determine the stellar properties with great precision. However, due to the lack of resolution of the stellar grids commonly used in today's forward modelling techniques, solutions such as interpolation processes have been integrated, in order to better describe the space between the profiles of a discrete grid.

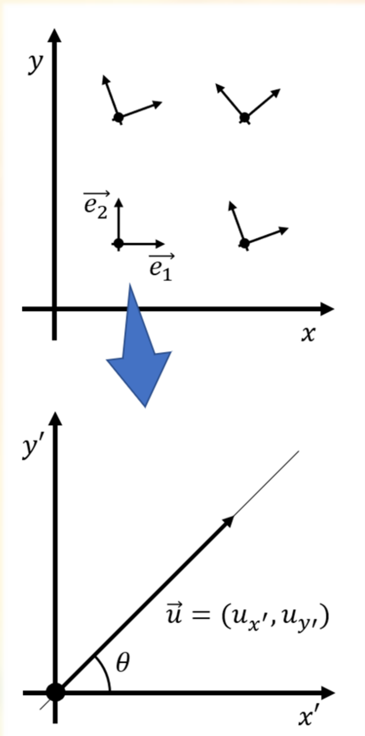
In this poster, we focused on setting up the conditions to improve on existing interpolation algorithms, and even develop a new module for interpolation, by:

- quantifying the adequacy of common model grids to linear interpolation;
- testing the improvement on linear interpolation brought about by the choice of well-motivated grid parameters.

2. Properties of the Hessian Matrix in a Stellar Grid: are our grids good enough for linear interpolation?

We started by characterizing the interpolation problem by studying the properties of the Hessian matrix applied to an analytic function $f(x, y)$, defined as

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$



This matrix is useful as it quantifies the second order terms of the Taylor expansion of the grid function. If these terms are small enough to be neglected, then we know that a direction of least variation of the first derivative exists in which linear interpolation is expected to work.

For each grid model, we can calculate the Hessian matrix and the associated eigenparameters. If we focus on one model, we can write the Hessian as a function of its eigenvalues λ_1 and λ_2, and use the eigenvectors to define a local coordinate system (x', y'). This allows us to write the 2nd derivative of the grid function as

$$\left|\frac{d^2 f}{dr^2}\right| = |\lambda_1 u_{x'}^2 + \lambda_2 u_{y'}^2|$$

where $u_{x'}$ and $u_{y'}$ are the components of a unit vector $\vec{u}$ (see illustration above). To determine the direction of least variation of the first derivative for each point, we minimize this equation, which results in solutions under three conditions:

- If $\lambda_1 = \lambda_2$, then all directions are directions of least variation of the first derivative.
- If $\mathrm{sign}(\lambda_1) = \mathrm{sign}(\lambda_2)$, then the direction of least variation of the first derivative is the one corresponding to the eigenvector with the smallest abs(eigenvalue).
- If $\mathrm{sign}(\lambda_1) \neq \mathrm{sign}(\lambda_2)$, then there are two directions of least variation of the first derivative.

We applied this theory to a stellar grid built by A. Serenelli, that considered the input parameters listed in the table to the right. We started by selecting one model per evolutionary track that best met the condition, $\tau \sim 4000 \pm 200\mathrm{Myr}$, to reduce the grid to a two-parameters plane, varying in mass and metallicity only. We then considered the function f to be the frequency of a given oscillation mode $\nu_{1,10}$ (for illustration, considered here to be a mode of degree $\ell = 1$ and radial order $n = 10$) and followed the procedure above to find the eigenvalues and the direction(s) of least variation of the first derivative of the mode frequency. The results are shown in Fig.1.

Inputs	(Range, Step)
Mass M	$([0.7,1.6], 0.01)\ \mathrm{M}_\odot$
$[Fe/H]$	$([-0.95, +0.60], 0.05)$ dex
Age	(SG Phase, 1) Myr

Fig.1 shows that the preferential direction for interpolation changes significantly from point to point, being impossible to identify a particular direction in which the grid can be improved. Also, by comparing the terms of the 1st and 2nd derivatives in the Taylor expansion, we conclude that the 2nd derivative term cannot in general be neglected, and, thus, that this grid is not well-adapted for linear interpolation.

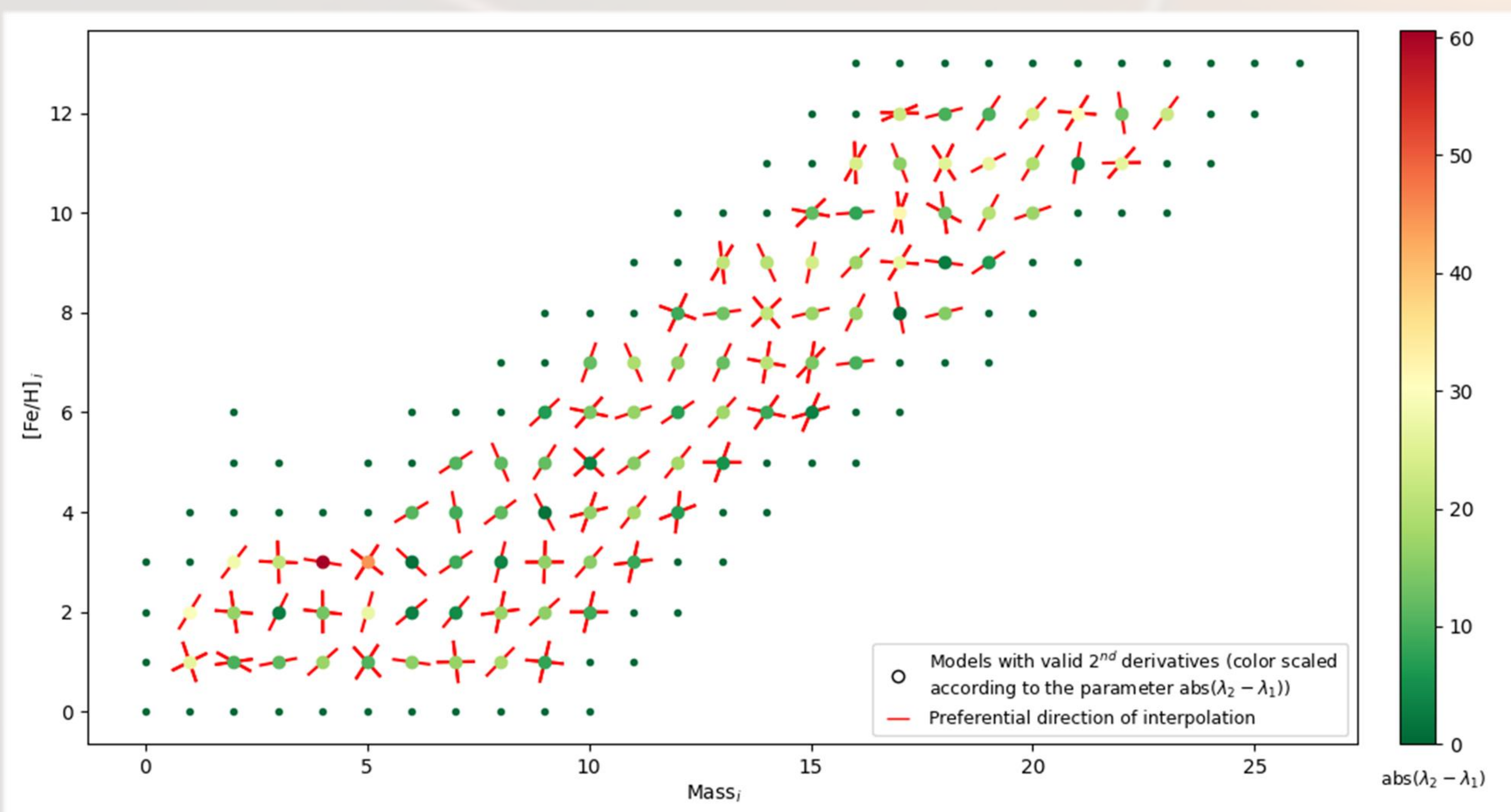


Fig.1: Directions of least variation of the first derivative of the frequency of the mixed mode $\nu_{1,10}$. The absolute difference between eigenvalues is depicted by the colour scale.

3. Search for Well-Motivated Parameters for the Grid:

If a stellar grid is not adequate for linear interpolation of the mode frequencies, we may want to: (i) implement of a new interpolation algorithm and/or (ii) choose better motivated physical parameters with which to build the stellar grid.

Here we focus on option (ii). To generate a well-motivated grid, we need to start by identifying a region of a subgiant grid where the current interpolation process leads to excessive errors when compared with the observational errors.

To this end, we looked at the evolution of the frequencies of dipolar modes with stellar age. We determined the oscillation frequencies for models in theoretical evolutionary tracks of masses $1.254M_\odot$ and $1.256M_\odot$, and used them to interpolate for the frequencies of models of mass $1.255M_\odot$, then comparing these frequencies with the actual frequencies derived for that evolutionary track. The results are shown in Fig. 2, where we see that, around the avoided crossings, the interpolation is clearly inadequate, resulting in errors significantly larger than the typical observational errors on the frequencies.

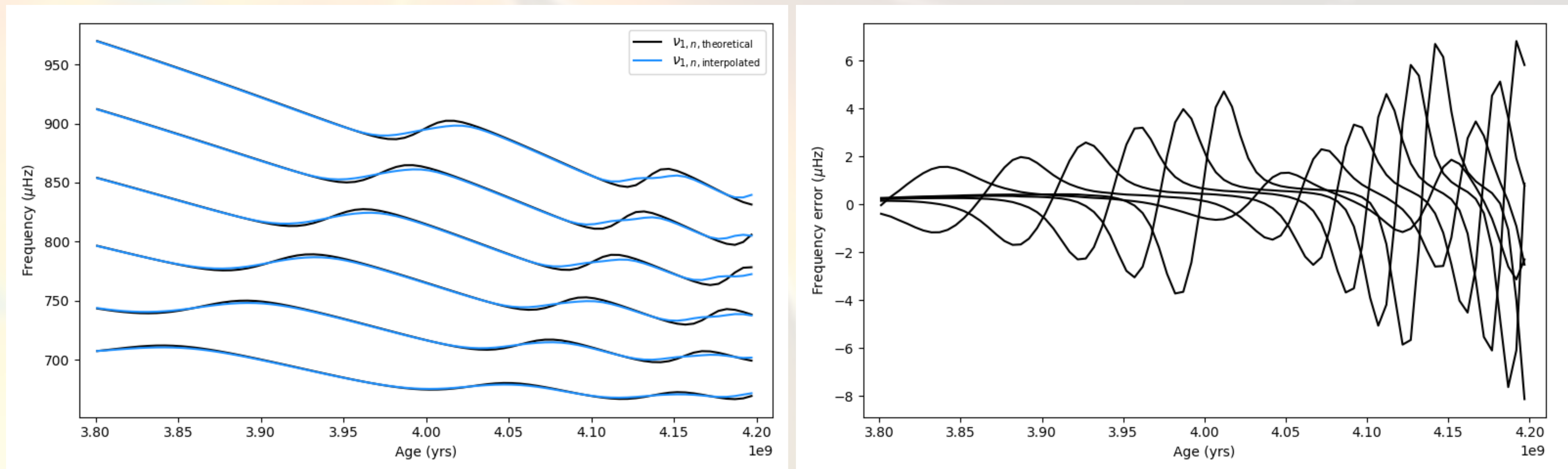


Fig.2: (Left) Comparison between theoretical (black) and interpolated frequencies (blue) of $\ell = 1$ modes, when interpolation is performed at fixed age for $M = 1.255M_\odot$ and [Fe/H]=-0.10. (Right) Differences between the theoretical and the interpolated frequencies of the model.

As argued by S. Deheuvels (private communication), the accuracy of the interpolation can be improved if we fix the central density, ρ_c, instead of age, when interpolating across evolutionary tracks. To test this, we compared the results of theoretical and interpolated frequencies when interpolation is performed at fixed ρ_c. While for interpolation at fixed age we had errors as high as 6 μHz, for interpolation at fixed ρ_c we find errors of 0.5 μHz (cf. Fig. 3), meaning that ρ_c works better as the fixed parameter for linear interpolation across evolutionary tracks.

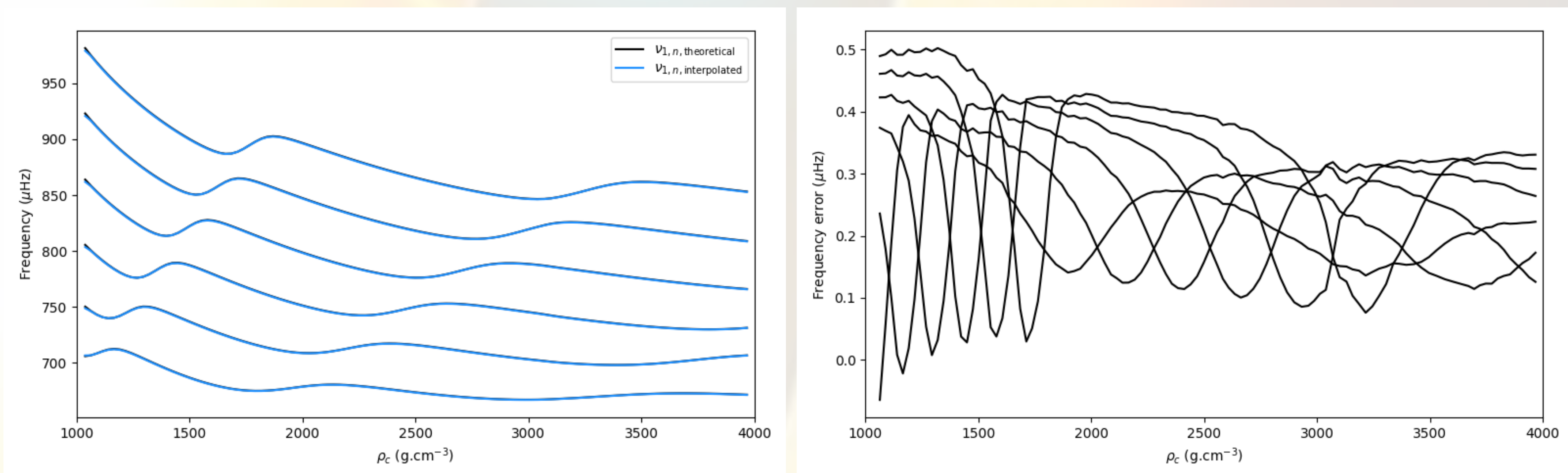


Fig.3: (Left) Comparison between theoretical (black) and interpolated frequencies (blue) of $\ell = 1$ modes, when interpolation is performed at fixed ρ_c for $M = 1.255M_\odot$ and [Fe/H]=-0.10. (Right) Differences between the theoretical and the interpolated frequencies of the model.

The underlying reason for this improvement is that the avoided crossings for different masses are more aligned when the evolution of the frequencies is considered as a function of ρ_c than when it is considered as a function of age (cf. Fig. 4).

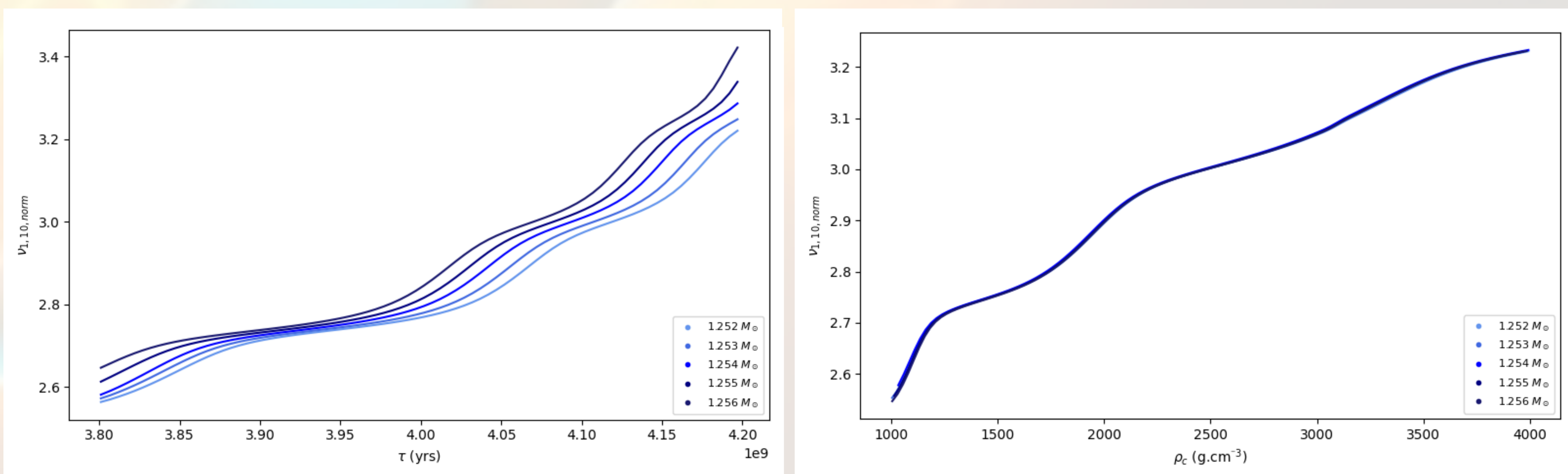


Fig.4: Evolution of the frequency of the mode $\nu_{1,10}$ with age (left) and ρ_c (right) during the subgiant phase, for the indicated stellar masses. Frequencies have been normalized by a factor of $\sqrt{GMR^{-3}}$.

4. Conclusions:

- With this work, we have found a way of testing whether our grids are adequate for linear interpolation. Not only that, the method we tested also provides information on how we can restructure not well-adapted stellar grids in order to improve the results of linear interpolation.
- We also tested the improvement on linear interpolation brought about by the choice of well-motivated grid parameters. We found that fixing ρ_c rather than age, when interpolating across evolutionary tracks leads to more accurate results.

References:

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