# Large Scale Structure: tessellation statistics as a probe of Cosmology

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## Introduction

The spatial and angular distribution of galaxies and galaxy clusters are sensitive probes of Cosmology and large-scale structure (LSS) formation mechanisms. The ultimate challenge is to understand the accelerated expansion of the Universe and distinguish between alternative Cosmology models (see Fig. 1). These probes often involve the use of galaxy surveys and the application of spatial/angular N-point correlation estimators to constrain the underlaying cosmology parameters.

Here we investigate a complementary approach based on tessellation statistics of the angular distribution of galaxy clusters in the sky, form light-cone simulations of the future Euclid satellite (see Fig. 2 below) survey. Our first objective is to quantify the dependence of tessellation statistics with redshift. At a latter stage, we will study the dependence of these statistics with cosmology by applying a similar analysis to simulations with different cosmological parameters (work underway).

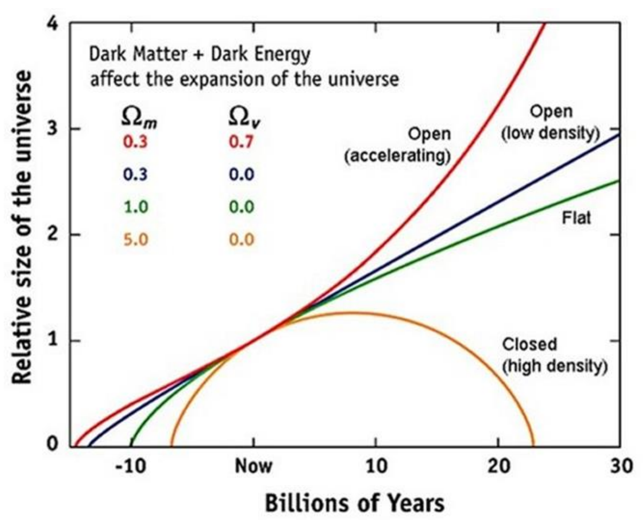


Fig. 1 – impact of cosmological parameters on the relative size of the Universe

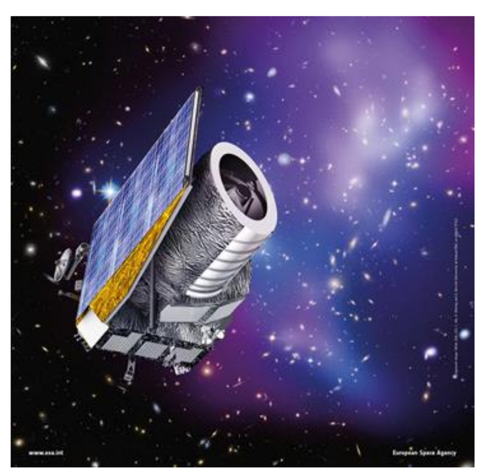

Fig. 2 – ESA / Euclid satellite

## Voronoi Tessellation (Dirichlet's Mosaic)

The Voronoi tessellation (see Fig. 3) is constructed from a set of seed points that we identify with the central positions of galaxy clusters. The region around each point, the Voronoi cell, is formed by all points that are closer to the seed point than to all the other seed points. This means that if we choose a random point inside a given cell then there is no closer point than the galaxy cluster used as seed for that cell. The areas formed by the collection of cells obtained in this way provide a tessellation pattern that is sensitive to the angular clustering of sources and to cosmological effects that depend on the nature and dynamics of dark energy and dark matter.

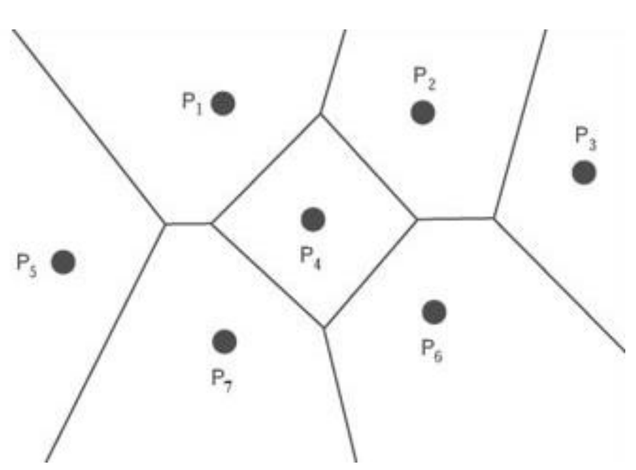

Fig. 3 – Tessellation pattern obtained from the set of seed points P1, P2, …, P7.

## Methodology

In this work we use galaxy clusters catalogues from N-body simulations of a Lambda Cold Dark Matter (LCDM) model with Planck 2018 cosmology. The cluster catalogues were constructed to reproduce a light-cone along the line-of-sight, with an angular size equal to one octant in the celestial sphere. Each catalogue corresponds to a redshift slice, centered around the observer, with fixed comoving thickness (see Fig. 4) equal to 200 Mpc/h (h=0.67), and contains several cluster properties that include cluster sky coordinates, (ra, dec), and redshifts, z.

Fig. 4 – Catalogues were constructed from simulations using concentric shells of fixed comoving size 200 Mpc/h

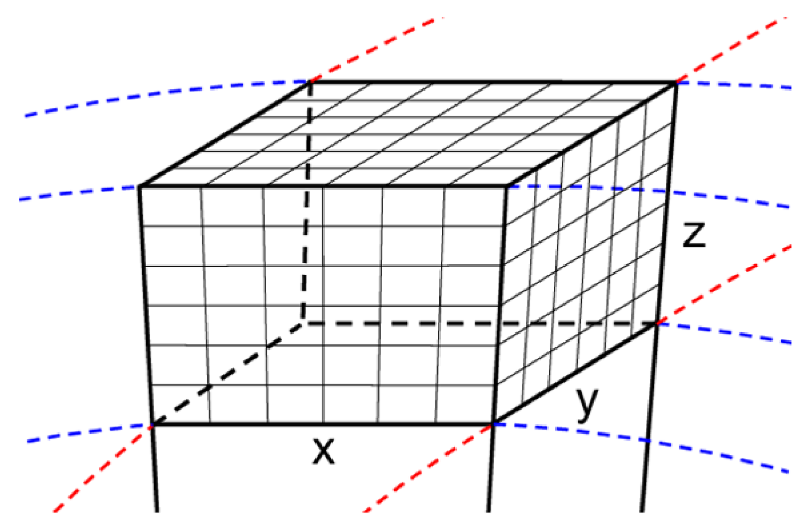


Fig. 5 – Sketch of the comoving volume contribution to a given sky patch projection of 10x10 square degrees.

We then produce sky tessellations, using the position of galaxy clusters as seeds, for sky patches of angular size 10x10 square degrees at each redshift, with the R package `deldir` [1] that implements the Voronoi Dirichlet/Delaunay method for flat surfaces. In this way our sky patches are approximately flat (flat sky approximation) and include the angular projection of all clusters inside cosmological volumes as sketched in Fig. 5. In Fig. 6 we see a typical tessellation pattern (built around cluster positions) from one of these (10x10 square degree) patches with an average redshift, z=1.27.

From the tessellations we then compute the distribution of cell areas and perimeters for several patches at each redshift. In total we used 14 redshift slices and average distributions for 9 sky patches at each z.

## Results

To avoid being exhaustive, we decided to report only results for the dependence of the distribution of Voronoi cell areas with redshift. The dependence of Voronoi cell perimeters with z was also analyzed and lead to results consistent with those reported here for the Voronoi cell areas statistics.

Fig. 7 shows the distribution of cell areas from clusters at z=1.27. As mentioned earlier the curve is obtained by averaging over 9 non-overlapping sky patches. We repeat this procedure for the 14 redshift bins considered in this work and compute the moments (mean, standard deviation, skewness and kurtosis) of the Voronoi cell area distributions at each redshift with the R package `moments`[2].

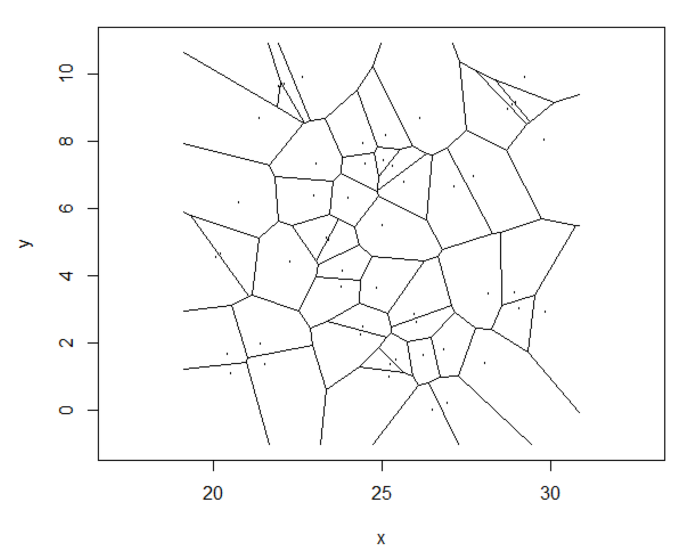

Fig. 6 – Voronoi / Dirichlet tessellation of clusters with 10x10 square degrees at z=1.27

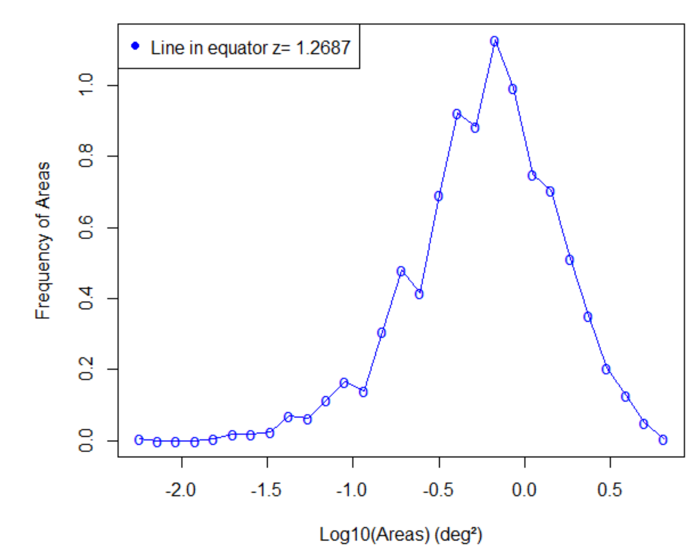


Fig. 7 – Density distribution of Voronoi cell areas (in logarithm base 10) from a sky patch with 10x10 square degrees at z=1.27

Fig. 8 shows our findings for the dependence of the moments of the Voronoi cell area distributions as a function of redshift. The panels show the mean (top left), standard deviation (top right), skewness (bottom left) and kurtosis (bottom right) of cell areas (in logarithm base 10 and square degree) .

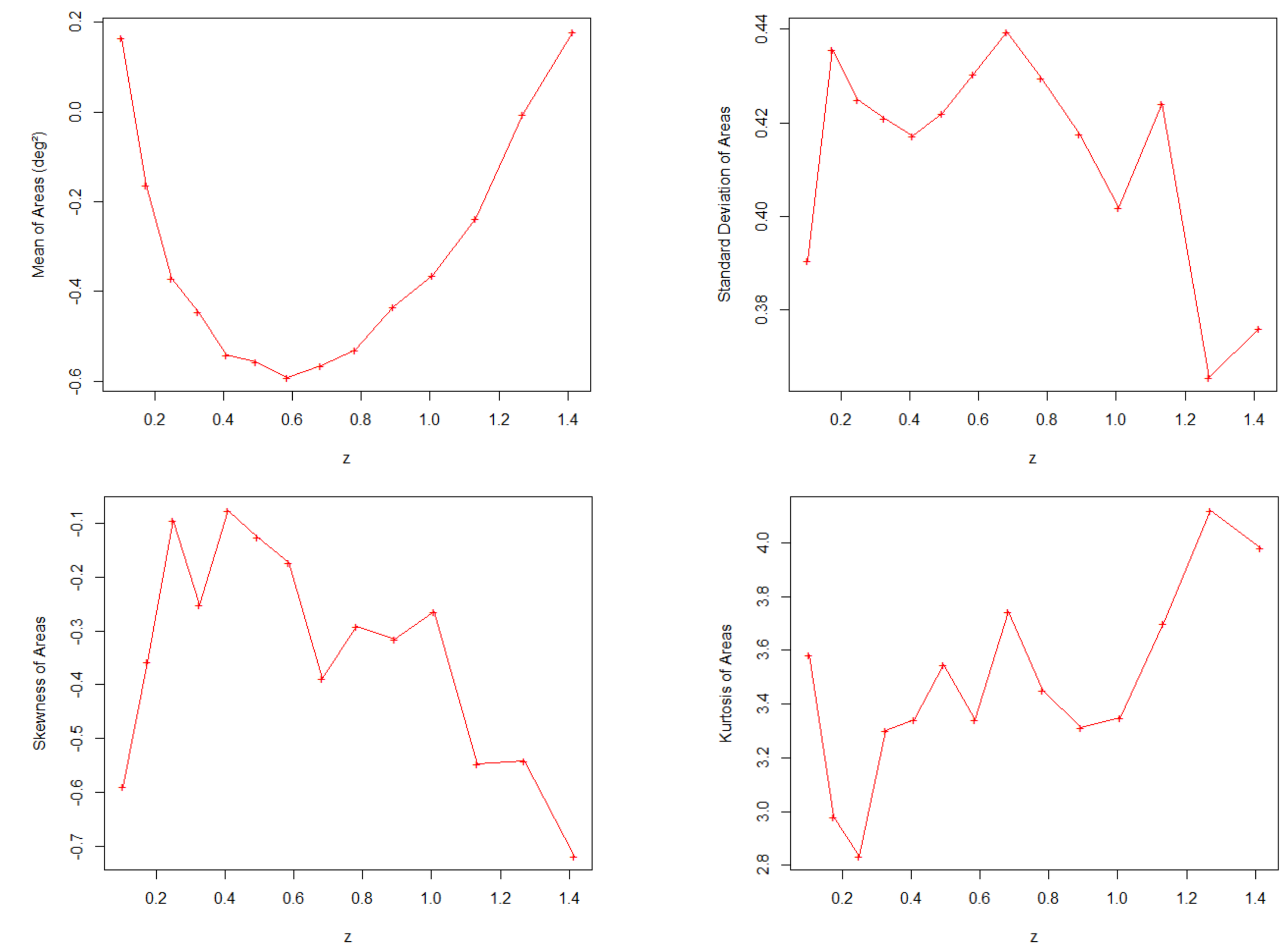


Fig. 8 – Moments of the Voronoi cell area distributions as a function of redshift

## Discussion and Future Developments

Our results clearly indicate that tessellation areas start by decreasing to a minimum and then increase with z. This is consistent with expectations, because the number of clusters from patches with fixed volume widths varies in an inverse way (i.e., increases to a maximum and then decreases with z, due to the variation of the cosmological volume element and mass function of clusters). The standard deviation and skewness plots display a tendency to peak at intermediate redshifts, an effect that is more evident in the skewness case. The Kurtosis plot is consistent with a monotonic increase with redshift.

Our results indicate that Voronoi tessellation statistics are a very promising tool for cosmology. We plan to continue this study by running our method for simulations with different models to quantify their dependence with cosmology. Ultimately, we plan to apply our methodology to the future Euclid galaxy cluster survey and to other existing surveys to try to impose further constraints on cosmology models.

## Acknowledgments and References

The authors acknowledge FCUL, IA and funding from FCT Projects No. UID/FIS/04434/2020, CERN/FIS-PAR/0037/2019, PTDC/FIS-OUT/29048/2017 and IF/01135/2015

[1] https://cran.r-project.org/web/packages/deldir/deldir.pdf

[2] https://cran.r-project.org/web/packages/moments/moments.pdf