Surfing a Dark Gravitational Wave

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1. Introduction

Gravitational Waves (GWs) are solutions of General Relativity (GR) not only at linear level, but also in full nonlinear regime. However, GR suffers from pathologies as:

- it lacks a consistent high energy version;
- it requires two unknown dark components to match astrophysical and cosmological data;
- it presents singularities, …

Therefore, alternative theories of gravity have been proposed in the literature. One of such models generalises f(R) theories by introducing a non-minimal coupling between the matter Lagrangian and the scalar curvature (NMC) [1]:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}f_1(R) + f_2(R)\mathcal{L}\right] ,$$

from which follows a covariant non-conservation of the energy-momentum tensor of matter fields:

$$\nabla^\mu T_{\mu\nu} = \frac{F_2}{f_2}\left[g_{\mu\nu}\mathcal{L} - T_{\mu\nu}\right]\nabla^\nu R ,$$

which for a perfect fluid gives rise to an extra force term that allows for the mimicking of dark matter profiles at galaxies and clusters of galaxies, and accounts for the dark energy mechanism.

2. GWs in matter

In GR (and theories where only the gravitational sector is modified) we can extend the analysis by resorting to Green functions' method.

Other approaches:

- The Campbell-Morgan formalism of GR [2];
- Cyclotron damping and Faraday rotation in collisionless magnetised plasmas [3,4];
- presence of a cosmological constant (field equations lose their residual gauge freedom) [5,6].

Hence, matter matters when considering gravitational waves in alternative scenarios.

3. Nonminimally coupled GWs

3.1. Linearisation formalism [7]:

Expansion around a Minkowskian spacetime.

3.1.1. Cosmological constant case:

The fluctuation of the metric obeys the wave equation (in the Λ gauge, hence the ¼ factor):

$$\Box\left(h_{\mu\nu} - \frac{1}{4}h\eta_{\mu\nu}\right) = \frac{f_1 - 2f_2\Lambda}{F_1 - 2F_2\Lambda}h_{\mu\nu}$$

whilst the trace of the field equations lead to a wave equation for the scalar mode:

$$\Box\Omega = m_\Omega^2\Omega ,$$ where $$\Omega := \frac{\delta f'}{F_1 - 2F_2\Lambda} = \frac{F_1' - 2F_2'\Lambda}{F_1 - 2F_2\Lambda}\delta R ,$$

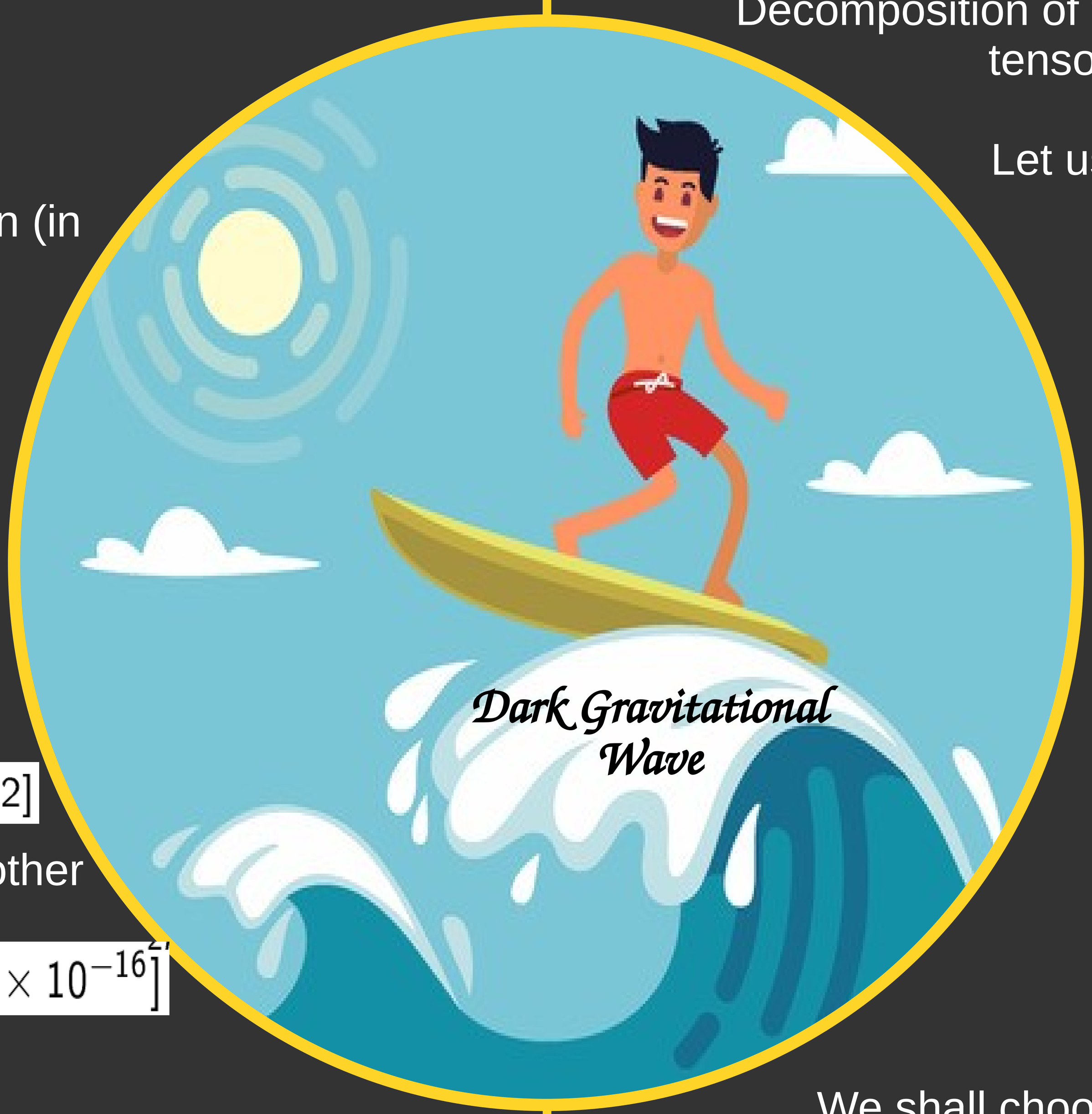
Theories which have rotation invariance can be parametrised by the dispersion relation:

$$\omega^2 = m_g^2 + c_{gw}^2k^2 + a\frac{k^4}{\Delta}) \text{ is constrained to } c_{gw} \in [0.55, 1.42]$$

that for this model $c_{gw}=1$, matching the GW data. Another quantity is the group velocity:

$$v_g \equiv \frac{\partial\omega}{\partial k} \approx 1 - \frac{m_{gw}^2}{2k^2}$$ such that $$v_g \in [1 - 3\times10^{-15}, 1 + 7\times10^{-16}]$$

and this model can be arbitrarily be fit to $v_g=1$.



3.1.2. Dark energy-like case:

Analogously, for this case we have: $$\Box\left(h_{\mu\nu} - \frac{1}{4}h\eta_{\mu\nu}\right) = \frac{f_1 - 2f_2\rho}{F_1 - 2F_2\rho}h_{\mu\nu}$$

And a nontrivial scalar mode which can be decoupled into two fundamental ones:

$$\omega_f \equiv \frac{\delta f'}{F_1 - 2F_2\rho} = \frac{F_1' - 2F_2'\rho}{F_1 - 2F_2\rho}\delta R \qquad \omega_h \equiv \frac{\delta h'}{F_1 - 2F_2\rho} = \frac{-2F_2}{F_1 - 2F_2\rho}\delta\rho$$

3.2. Newman-Penrose formalism [7]:

Decomposition of the Riemann tensor into its irreducible parts: Weyl tensor, Ricci tensor and scalar curvature.

Let us introduce the null complex tetrad:

$$\mathbf{k} = \frac{1}{\sqrt{2}}(\mathbf{e}_t + \mathbf{e}_z) , \quad \mathbf{l} = \frac{1}{\sqrt{2}}(\mathbf{e}_t - \mathbf{e}_z) ,$$
$$\mathbf{m} = \frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y) , \quad \bar{\mathbf{m}} = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y) ,$$

which obey the the following conditions:

$$-k\cdot l = m\cdot\bar{m} = 1 \wedge k\cdot m = k\cdot\bar{m} = l\cdot m = l\cdot\bar{m} = 0$$

From this tetrad, we can write the Newman-Penrose quantities:

$$\begin{aligned}
\Psi_0 &\equiv C_{kmkm} = R_{kmkm}\\
\Psi_1 &\equiv C_{klkm} = R_{klkm} - \frac{R_{km}}{2}\\
\Psi_2 &\equiv C_{km\bar{m}l} = R_{km\bar{m}l} + \frac{R}{12}\\
\Psi_3 &\equiv C_{kl\bar{m}l} = R_{kl\bar{m}l} + \frac{R_{l\bar{m}}}{2}\\
\Psi_4 &\equiv C_{l\bar{m}l\bar{m}} = R_{l\bar{m}l\bar{m}}\\
\Phi_{00} &\equiv \frac{R_{kk}}{2}
\end{aligned}$$

$$\begin{aligned}
\Phi_{11} &\equiv \frac{R_{kl} + R_{m\bar{m}}}{4}\\
\Phi_{22} &\equiv \frac{R_{ll}}{2}\\
\Phi_{01} &\equiv \frac{R_{km}}{2} = \Phi_{10}^* \equiv \left(\frac{R_{k\bar{m}}}{2}\right)^*\\
\Phi_{02} &\equiv \frac{R_{mm}}{2} = \Phi_{20}^* \equiv \left(\frac{R_{\bar{m}\bar{m}}}{2}\right)^*\\
\Phi_{12} &\equiv \frac{R_{lm}}{2} = \Phi_{21}^* \equiv \left(\frac{R_{l\bar{m}}}{2}\right)^*\\
\tilde{\Lambda} &\equiv \frac{R}{24} ,
\end{aligned}$$

We shall choose the following forms for the functions:

$$f_1(R) = R - \alpha R^{-\beta} \qquad f_2(R) = 1 + \gamma R^n$$

In GR, only Ψ_4 Is nonzero (plus and cross polarisations).

In NMC, with cosmological constant, other polarisation states are possible as Φ_{00} , Φ_{11} , Φ_{22} , $\tilde{\Lambda}$ are nonzero, but full characterisation only when the full solution is known (needed for the Ψ_i).

4. Conclusions

Surfing a gravitational wave in alternative theories of gravity (dark GW) is not an easy task as one has to face the possibility of additional scalar or vector modes on top of the tensor ones [8]. Moreover, when matter is included new physical implications appear. Hence, discriminating between different scenarios is mandatory.

In particular, gravitational waves solutions from the NMC gravity model have been analysed. In the far-field (no matter), the NMC becomes pure f(R); however when matter is relevant, as in the Λ and DE-like fluid cases, where extra longitudinal modes appear: $\omega = \omega(\delta R, \delta L)$. Beyond linear level, one has to implement the Newman-Penrose formalism and extra polarisation modes appear.

Moreover, in GR, both metric and "Palatini" approaches lead to the same field equations, and polarisation modes. However, for alternative theories of gravity, this is not the case. For instance:
- metric f(R) theories may present up to six polarisation modes.
- "Palatini" f(R) only exhibits two tensor modes, likewise GR.

Another question arises when matter is taken into account: do matter fields feel the connection built from metric field or the independent connection (e.g. fermions)? For this, we can have three approaches, namely the metric, Palatini and metric-affine ones.

Take home message: matter matters!

5. References

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6. Acknowledgements

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