EuroCC National Competence Centre Sweden

ROYAL INSTITUTE OF TECHNOLOGY

# Introductory OpenFOAM Workshop

## *Overview of the math inside*


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# Overview

- Modest goal: set the stage so that one can understand what some of the options in OpenFOAM stand for.
- A bird-eye view of what is happening in the solver.
- By no means an exhaustive coverage of the topic, not even a comprehensive introduction.

# Two things we need to look at

- Transforming a PDE to a linear system of equations using the Finite Volume method.

- Solution procedure for the Navier-Stokes equations (pressure-velocity coupling).

# FVM bird-eye view

PDE in differential form
$\nabla \cdot (\rho \boldsymbol{u} \phi)$

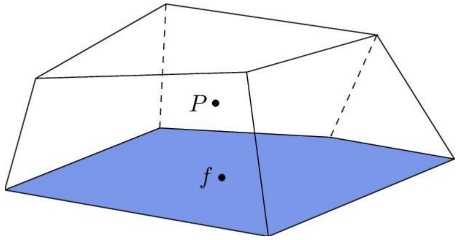
Integrate →

PDE in integrated form
$\int \nabla \cdot (\rho \boldsymbol{u} \phi) \mathrm{d}V$

Gauss theorem →

Volume integrals are now surface integrals
$\oint (\rho \boldsymbol{u} \phi) \cdot \boldsymbol{n} \mathrm{d}s$

The volume is a convex polyhedron




Surface integrals can be split across the volume faces

$$\sum_f \oint (\rho \boldsymbol{u} \phi) \cdot \boldsymbol{n} \mathrm{d}s_f$$

Face values are approximated at face centres

Surface integrals replaced by evaluation at face centres

$$\sum_f (\rho \phi \boldsymbol{u} \cdot \boldsymbol{n})_f S_f$$

Face values computed from volume centroid values

Discrete system in space

Select time integration scheme

Fully discrete

# Obtaining the integral form

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\rho \boldsymbol{u} \phi) = 0$$

$$\int \frac{\partial \phi}{\partial t} \mathrm{d}V + \int \nabla \cdot (\rho \boldsymbol{u} \phi) \mathrm{d}V = 0$$

$$\int \frac{\partial \phi}{\partial t} \mathrm{d}V + \oint \rho \phi (\boldsymbol{u} \cdot \boldsymbol{n}) \mathrm{d}S = 0$$

**Gauss-Otrogradsky theorem**

$$\int \nabla \cdot \mathbf{u} \mathrm{d}V = \oint \boldsymbol{u} \cdot \boldsymbol{n} \mathrm{d}S$$

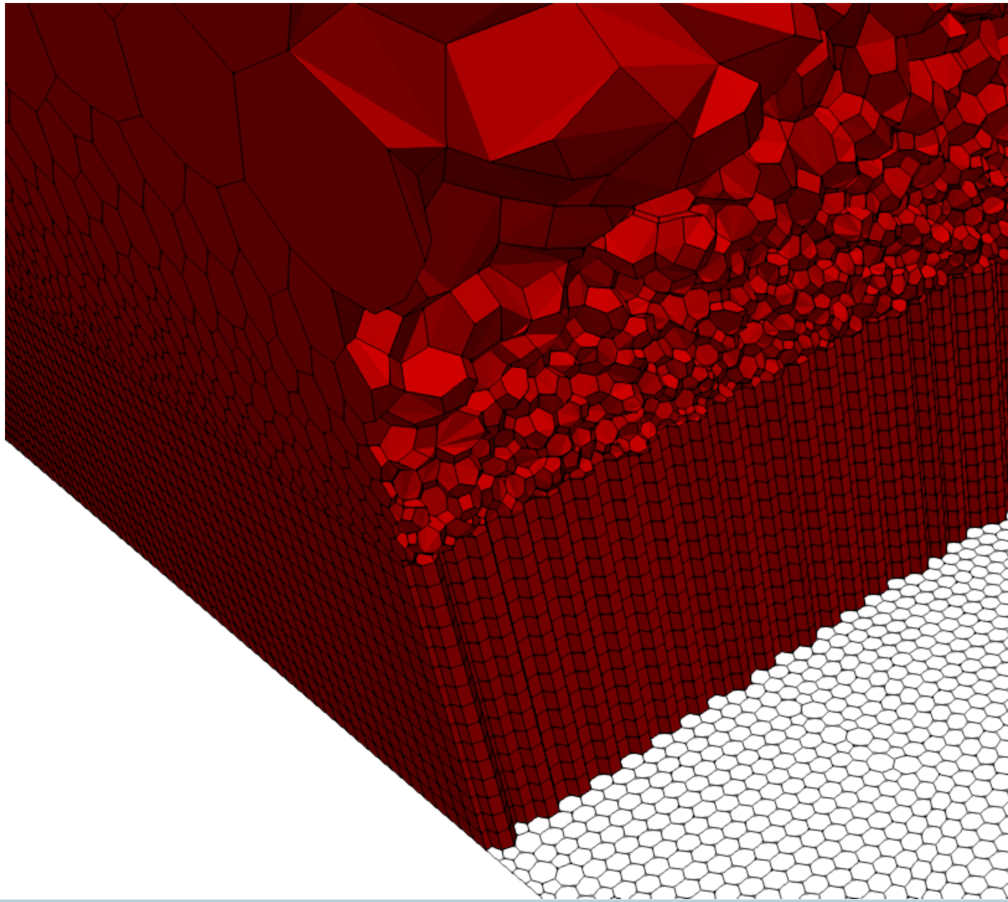
- Can be applied to gradient and diffusion terms as well.
- We replace the differential operators with the evaluation of the field at the surface of the volume.

# Connection to meshing

- The integral form is valid for an arbitrary volume.

- When we mesh, we split the domain into many volumes, for each of them the integral equations are valid.

- In principle, as long as we are consistent in how we compute $\boldsymbol{u}_f$ and $\phi_f$, our method is conservative.

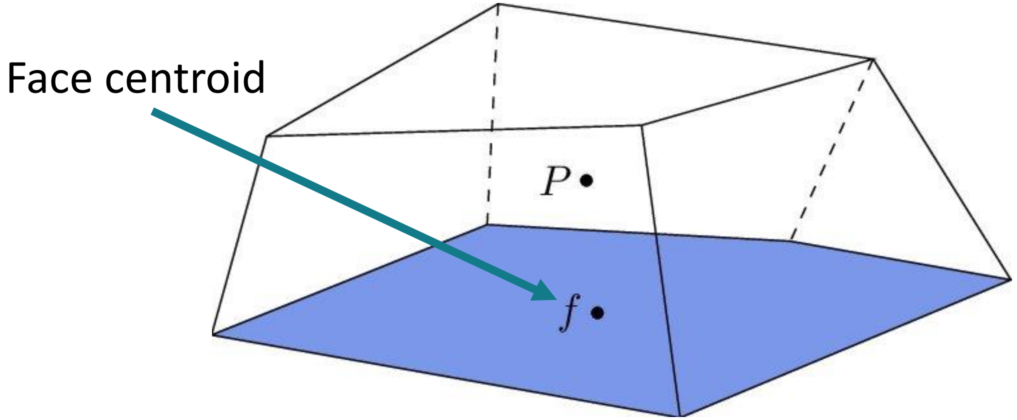


# Towards a semi-discrete formulation

$$\int \frac{\partial \phi}{\partial t} \mathrm{d}V + \int \nabla \cdot (\rho \boldsymbol{u} \phi) \mathrm{d}V = 0$$

$$\int \frac{\partial \phi}{\partial t} \mathrm{d}V + \sum_f \oint \rho \phi (\boldsymbol{u} \cdot \boldsymbol{n}) \mathrm{d}S_f = 0$$

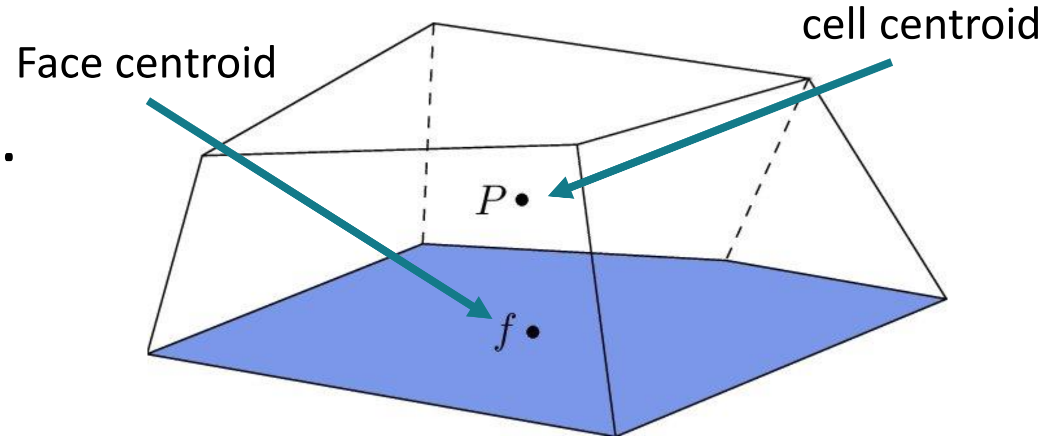
$$\int \frac{\partial \phi}{\partial t} \mathrm{d}V + \sum_f \rho_f \phi_f (\boldsymbol{u}_f \cdot \boldsymbol{n}_f) S_f = 0$$




- Assume the volume is a convex poly.
- We replace the integral with a 2nd order accurate approximation.

# Computing the face centroid values

- At this point, we need a recipe for computing the unknowns at the faces.
- In collocated FVM, the unknowns we solve for are located at the *cell* centroids.

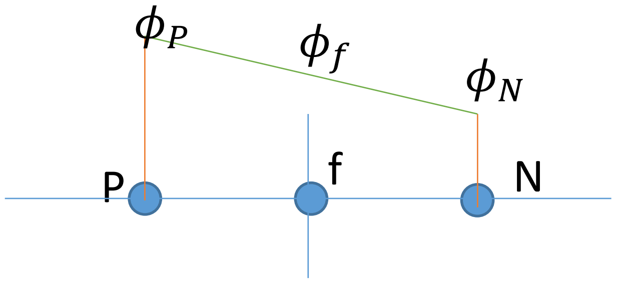



- The task is then to compute the values at the *face* centroid given the values at the centroids of the two cells sharing the face!

# Computing the face centroid values

- The most obvious option is linear interpolation, 2nd order accurate.

- However, for convective fluxes it introduces oscillations.




- Many options for interpolating the convective flux exist. Often the most important numerical choice in the simulation!