BİM122 Discrete Computational Structures

Homework 1

1. **(10 pts)** You are eligible to be President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country. Express your answer in terms of e: "You are eligible to be President of the U.S.A.," a: "You are at least 35 years old," b: "You were born in the U.S.A," p: "At the time of your birth, both of your parents where citizens," and r: "You have lived at least 14 years in the U.S.A."

2. **(15 pts)** Show that each of these conditional statements is a tautology by using truth tables.

   a) $(p \wedge q) \rightarrow p$ **b)** $p \rightarrow (p \vee q)$
   c) $\neg p \rightarrow (p \rightarrow q)$ **d)** $(p \wedge q) \rightarrow (p \rightarrow q)$
   e) $\neg(p \rightarrow q) \rightarrow p$ **f)** $\neg(p \rightarrow q) \rightarrow \neg q$

3. **(20 pts)** Show that each conditional statement is a tautology without using truth tables.

   a) $[\neg p \wedge (p \vee q)] \rightarrow q$
   b) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$
   c) $[p \wedge (p \rightarrow q)] \rightarrow q$
   d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

4. **(15 pts)** Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

   a) Someone in your class can speak Hindi.
   b) There is a person in your class who was not born in California.
   c) No student in your class has taken a course in logic programming.

5. **(20 pts)** For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

   a) "If I take the day off, it either rains or snows." "I took Tuesday off or I took Thursday off." "It was sunny on Tuesday." "It did not snow on Thursday."
   b) "If I eat spicy foods, then I have strange dreams." "I have strange dreams if there is thunder while I sleep." "I did not have strange dreams."
   c) "I am either clever or lucky." "I am not lucky." "If I am lucky, then I will win the lottery."
   d) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."

6. **(20 pts)** Prove that if n is an integer and $3n + 2$ is even, then n is even using
   a) a proof by contraposition.
   b) a proof by contradiction.