

SCARA Robot

الموضوع:

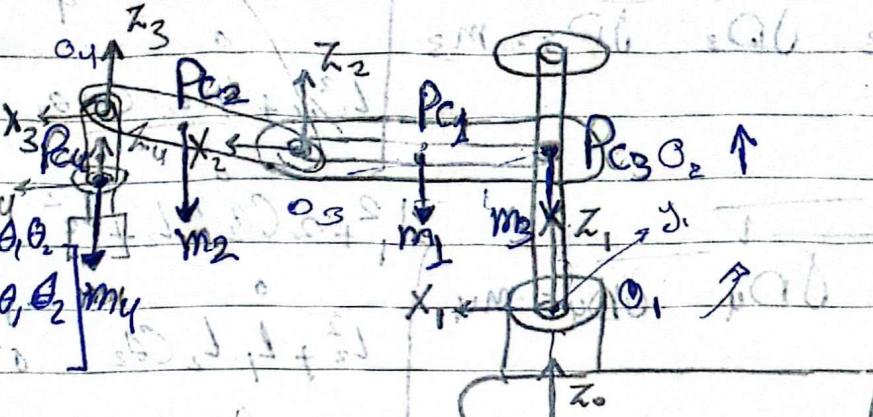
التعريف

$$P_{C_1} = \begin{bmatrix} \frac{1}{2} L_1 C\theta_1 \\ \frac{1}{2} L_1 S\theta_1 \\ d_1 + d_2 \end{bmatrix}$$

$$P_{C_2} = \begin{bmatrix} L_1 C\theta_1 + \frac{1}{2} L_2 C\theta_1 \theta_2 \\ L_1 S\theta_1 + \frac{1}{2} L_2 S\theta_1 \theta_2 \\ m_2 \end{bmatrix}$$

$$P_{C_3} = \begin{bmatrix} 0 \\ 0 \\ d_1 + d_2 \end{bmatrix}$$

$$P_{C_{all}} = \begin{bmatrix} L_1 C\theta_1 + L_2 C\theta_1 \theta_2 \\ L_1 S\theta_1 + L_2 S\theta_1 \theta_2 \\ d_1 + d_2 - d_3 - d_4 \end{bmatrix}$$



$$\overline{J}D = \begin{bmatrix} \frac{dP_{C_1}}{d\theta_1} & \frac{dP_{C_1}}{d\theta_2} & \frac{dP_{C_1}}{d\theta_3} & \frac{dP_{C_1}}{d\theta_4} \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

$$\overline{J}D_1 = \begin{bmatrix} \frac{dP_{C_1}}{d\theta_1} & \frac{dP_{C_1}}{d\theta_2} & \frac{dP_{C_1}}{d\theta_3} & \frac{dP_{C_1}}{d\theta_4} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} L_1 S\theta_1 & 0 & 0 & 0 \\ \frac{1}{2} L_1 C\theta_1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\overline{J}D_2 = \begin{bmatrix} \frac{dP_{C_2}}{d\theta_1} & \frac{dP_{C_2}}{d\theta_2} & \frac{dP_{C_2}}{d\theta_3} & \frac{dP_{C_2}}{d\theta_4} \end{bmatrix} = \begin{bmatrix} -L_1 S\theta_1 - \frac{1}{2} L_2 S\theta_1 \theta_2 & -L_2 S\theta_1 \theta_2 & 0 & 0 \\ L_1 C\theta_1 + \frac{1}{2} L_2 C\theta_1 \theta_2 & \frac{1}{2} L_2 C\theta_1 \theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{J}D_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\overline{J}D_4 = \begin{bmatrix} -L_1 S\theta_1 - L_2 S\theta_1 \theta_2 & -L_2 S\theta_1 \theta_2 & 0 & 0 \\ L_1 C\theta_1 + L_2 C\theta_1 \theta_2 & L_2 C\theta_1 \theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$l = K - PG$$

التاريخ:

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$$\overline{JR_1} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \dot{\theta}_1, \quad \overline{JR_2} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix} = \overline{JR_2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\overline{JR_3} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ d_2^x \end{bmatrix}$$

$$\overline{JR_4} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ d_2^x \end{bmatrix} \quad 3 \times 4 \quad 4 \times 1$$

$$\sum_{j=1}^n D_{ij}(q) \ddot{q}_j + f_{ikm} q_k q_m + g_i = Q_i$$

$$D = \sum_{i=1}^n (\overline{JR_i} m_i \overline{JR_i} + k \overline{JR_i} I_i \overline{JR_i})$$

if $n = 1$

$$m_i \overline{JR_i} \cdot \overline{JR_i} = \begin{bmatrix} \frac{l_1^2 \cdot m_1}{2} & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$k I_i \overline{JR_i} \cdot \overline{JR_i} = \begin{bmatrix} k I_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

if $n=2$

$$m_2 \overline{J} \overline{D}_2 \cdot \overline{D} \overline{D}_2 = \begin{bmatrix} (m_2(4L_1^2 + 4C\theta_2 \cdot L_1 \cdot L_2 + L_2^2))/4 & (L_2 m_2(L_2 + 2L_1 C\theta_2))/4 \\ (L_2 \cdot m_2(L_2 + 2L_1 C\theta_2))/4 & (L_2^2 \cdot m_2)/4 \end{bmatrix}$$

0 0
0 0
 m_2 0
0 0

$$0,5 L_2 \overline{J} \overline{R}_2 \cdot \overline{R} \overline{R}_2 = \begin{bmatrix} I_{2/2} & I_{2/2} & 0 & 0 \\ I_{2/2} & I_{2/2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

if $n=3$

$$m_3 \overline{J} \overline{D}_3 \cdot \overline{D} \overline{D}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

0 0 0 0
0 0 0 0
 m_3 0
0 0 0 0

$$0,5 I_3 \overline{J} \overline{R}_3 \cdot \overline{R} \overline{R}_3 = \begin{bmatrix} I_{3/2} & I_{3/2} & 0 & 0 \\ I_{3/2} & I_{3/2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

if $n=4$

$$m_4 \overline{J} \overline{D}_4 \cdot \overline{D} \overline{D}_4 = \begin{bmatrix} m_4(L_1^2 + 2 \cdot C\theta_2 \cdot L_1 \cdot L_2 + L_2^2) & L_2 \cdot m_4(L_2 + L_1 C\theta_2) & 0 & 0 \\ L_2 \cdot m_4(L_2 + L_1 C\theta_2) & L_2^2 \cdot m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

0,5 I_4.

$$\overline{J} \overline{R}_4 \cdot \overline{R} \overline{R}_4 = \begin{bmatrix} I_{4/2} & I_{4/2} & 0 & I_{4/2} \\ I_{4/2} & I_{4/2} & 0 & I_{4/2} \\ 0 & 0 & 0 & 0 \\ I_{4/2} & I_{4/2} & 0 & I_{4/2} \end{bmatrix}$$

$$D = \sum_{i=1}^n (\overline{J_D} \cdot m_i \overline{T_D}_i + \frac{1}{2} \overline{J_R}_i \cdot I_i \overline{T_R}_i) =$$

$$\rightarrow D = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix}$$

$$D_{11} = \frac{l_1^2 \cdot m_1}{4} + \frac{1}{2} (m_2 (4l_1^2 + 4(C\theta_2 l_1 \cdot l_2 + l_2^2)) / 4 + m_4 (l_1^2 + 2C\theta_2 l_1 \cdot l_2 + l_2^2) + \frac{1}{2} (I_1 + I_2 + I_3 + I_4))$$

$$D_{12} = (l_2 \cdot m_2 (l_2 + 2l_1 C\theta_2)) / 4 + l_2 \cdot m_4 (l_2 + l_1 C\theta_2) + \frac{1}{2} (I_2 + I_3 + I_4)$$

$$D_{13} = \text{Zero} = 0$$

$$D_{14} = \frac{1}{2} I_4$$

$$D_{21} = (l_2 \cdot m_2 (l_2 + 2l_1 C\theta_2)) / 4 + l_2 \cdot m_4 (l_2 + l_1 C\theta_2) + \frac{1}{2} (I_2 + I_3 + I_4)$$

$$D_{22} = (l_2^2 \cdot m_2) / 4 + l_2^2 \cdot m_4 + \frac{1}{2} (I_2 + I_3 + I_4)$$

$$D_{23} = \text{Zero} = 0$$

$$D_{24} = \frac{1}{2} I_4$$

$$D_{31} = \text{Zero} = 0$$

$$D_{32} = \text{Zero} = 0$$

$$H_{ijk} = \sum_{j=1}^n \sum_{k=1}^n \left(\frac{dD_j}{dq_k} - \frac{1}{2} \frac{dD_{jk}}{dq_i} \right)$$

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$$D_{33} = m_1 + m_2 + m_3 + m_4$$

$$D_{34} = \text{Zero} = 0$$

$$D_{41} = \frac{1}{2} I_4$$

$$D_{42} = \frac{1}{2} I_4$$

$$D_{43} = \text{Zero} = 0$$

$$D_{44} = \frac{1}{2} I_4$$

$$\boxed{D_{13} = D_{23} = D_{31} = D_{32} = 0}$$

$$= D_{34} = D_{43} = \text{Zero}$$

$$\boxed{D_{14} = D_{24} = D_{41} = D_{42} = 0}$$

$$= D_{44} = \frac{1}{2} I_4$$

$$H_1 = H_{11} q_1 q_1 + H_{12} q_1 q_2 + H_{13} q_1 q_3 + H_{14} q_1 q_4 +$$

$$H_{12} q_2 q_1 + H_{13} q_2 q_3 + H_{14} q_2 q_4 + H_{13} q_3 q_1 + H_{13} q_3 q_2$$

$$+ H_{14} q_3 q_4 + H_{11} q_1 q_2 + H_{13} q_1 q_3 + H_{14} q_1 q_4 +$$

$$H_{14} q_4 q_1 + H_{14} q_4 q_2 + H_{14} q_4 q_3 + H_{14} q_4 q_4 +$$

$$H_{ij} = \sum_{j=1}^n \sum_{k=1}^n \left(\frac{dD_{ij}}{dq_k} - \frac{1}{2} \frac{dD_{jk}}{dq_i} \right)$$

$$H_{11} = \frac{dD_{11}}{dq_1} - \frac{1}{2} \frac{dD_{11}}{dq_1}$$

$$= 0 - \frac{1}{2} \cdot 0 = \text{Zero} = 0$$

$$H_{12} = \frac{dD_{12}}{dq_2} - \frac{1}{2} \frac{dD_{22}}{dq_1}$$

$$\theta_1 = q_1$$

$$\theta_2 = q_2$$

$$\theta_3 = q_3$$

$$\theta_4 = q_4$$

$$D_{11} \quad D_{12} \quad D_{21}$$

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$$\cancel{\frac{dD_{12}}{d\theta_2}} = -\frac{1}{2} L_1 L_2 m_2 \sin\theta_2 - L_1 L_2 m_2 S\theta_2$$

$$\therefore H_{122} = L_1 L_2 \sin\theta_2 (-\frac{1}{2} m_2 - m_1)$$

$$H_{133} = \frac{dD_{13}}{d\theta_2} - \frac{1}{2} \frac{dD_{33}}{d\theta_1} = \text{zero} - \frac{1}{2} 0 = \text{zero}$$

$$H_{144} = \frac{dD_{14}}{d\theta_3} - \frac{1}{2} \frac{dD_{44}}{d\theta_1} = 0 - \frac{1}{2} 0 = \text{zero} = 0$$

$$H_{121} = \frac{dD_{12}}{d\theta_1} - \frac{1}{2} \frac{dD_{21}}{d\theta_1} = \text{zero} - \frac{1}{2} 0 = 0$$

$$H_{123} = \frac{dD_{12}}{d\theta_3} - \frac{1}{2} \cdot \frac{dD_{23}}{d\theta_1} = 0$$

$$H_{124} = \frac{dD_{12}}{d\theta_3} - \frac{1}{2} \frac{dD_{24}}{d\theta_1} = \text{zero}$$

$$H_{131} = \frac{dD_{13}}{d\theta_1} - \frac{1}{2} \frac{dD_{31}}{d\theta_1} = \text{zero} \quad q_1, q_2$$

$$H_{132} = \frac{dD_{13}}{d\theta_2} - \frac{1}{2} \frac{dD_{32}}{d\theta_1} = \text{zero} \quad \dot{\theta}_1, \dot{\theta}_2$$

$$H_{134} = \frac{dD_{13}}{d\theta_3} - \frac{1}{2} \frac{dD_{34}}{d\theta_1} = \text{zero}$$

$$H_{112} = \frac{dD_{11}}{d\theta_2} - \frac{1}{2} \frac{dD_{12}}{d\theta_1} = -m_2 \cdot L_1 \cdot L_2 \cdot \sin\theta_2 - 2m_2 \cdot L_1 \cdot L_2 \cdot S\theta_2$$

$$H_{113} = \frac{dD_{11}}{d\theta_3} - \frac{1}{2} \frac{dD_{13}}{d\theta_1} = \text{zero} \quad L_1 L_2 S\theta_2 (-m_2 - 2m_1)$$

$$D_{11} \quad D_{12} \quad D_{21} \rightarrow \theta_2$$

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$$H_{114} = \frac{dD_{14}}{d\theta_3} - \frac{1}{2} \frac{dD_{14}}{d\theta_1} = \text{zero}$$

$$H_{21} H_{14} = \frac{dD_{14}}{d\theta_1} - \frac{1}{2} \frac{dD_{14}}{d\theta_1} = \text{zero}$$

$$H_{142} = \frac{dD_{14}}{d\theta_2} - \frac{1}{2} \frac{dD_{14}}{d\theta_1} = \text{zero}$$

$$H_{143} = \frac{dD_{14}}{d\theta_3} - \frac{1}{2} \frac{dD_{14}}{d\theta_1} = \text{zero}$$

$$\therefore H_1 = -L_1 L_2 \sin \theta_2 \left(\frac{1}{2} m_2 - m_1 \right) \dot{\theta}_2^2 + L_1 L_2 \sin \theta_2 \left(-m_2 - 2m_1 \right) \dot{\theta}_1 \dot{\theta}_2$$

$$H_{211} = \frac{dD_{21}}{d\theta_1} - \frac{1}{2} \frac{dD_{21}}{d\theta_2} = 0 - \frac{1}{2} \left(-m_2 L_1 L_2 \sin \theta_2 - 2m_1 L_1 L_2 \sin \theta_2 \right) \\ = \frac{1}{2} L_1 L_2 \sin \theta_2 (m_2 + 2m_1)$$

$$H_{222} = \frac{dD_{22}}{d\theta_2} - \frac{1}{2} \frac{dD_{22}}{d\theta_2} = \text{zero}$$

$$H_{233} = \frac{dD_{23}}{d\theta_2} - \frac{1}{2} \frac{dD_{23}}{d\theta_2} = \text{zero}$$

$$H_{244} = \frac{dD_{24}}{d\theta_3} - \frac{1}{2} \frac{dD_{24}}{d\theta_2} = \text{zero}$$

$$H_{221} = \frac{dD_{22}}{d\theta_1} - \frac{1}{2} \frac{dD_{21}}{d\theta_2} = 0 - \frac{1}{2} \left(\frac{1}{2} L_1 L_2 \sin \theta_2 - L_1 L_2 m_1 \sin \theta_2 \right) \\ = \frac{1}{2} L_1 L_2 \sin \theta_2 \left(\frac{1}{2} m_2 + m_1 \right)$$

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التاريخ

الموسم:

$$H_{223} = \text{Zero} \quad H_{224} = \text{Zero} \quad H_{232} = \text{Zero}$$

$$H_{231} = \text{Zero} \quad H_{234} = \text{Zero} \quad H_{241}$$

$$H_{212} = \frac{dD_{21}}{d\theta_2} - \frac{1}{2} \frac{dD_{12}}{d\theta_2} = -m_2 L_1 L_2 S\theta_2 - 2m_4 L_1 L_2 S\theta_2$$

$$H_{212} = -\frac{1}{2} m_2 L_1 L_2 S\theta_2 - m_4 L_1 L_2 S\theta_2 + \frac{1}{2} m_2 L_1 L_2 S\theta_2 + m_4 L_1 L_2 S\theta_2$$

$$= L_1 L_2 S\theta_2 (-\frac{1}{2} m_2 - m_4)$$

$$H_{213} = \frac{dD_{21}}{d\theta_3} - \frac{1}{2} \frac{dD_{13}}{d\theta_2} = \text{Zero} = 0$$

$$H_{214} = \frac{dD_{21}}{d\theta_3} - \frac{1}{2} \frac{dD_{41}}{d\theta_2} = \text{Zero} = 0$$

$$H_{241} = \text{Zero} \quad H_{242} = \text{Zero} \quad H_{243} = \text{Zero}$$

$$\therefore H_2 = \frac{1}{2} L_1 L_2 S\theta_2 (m_2 + m_4) \dot{\theta}_1^2 + \frac{1}{2} L_1 L_2 S\theta_2 \sqrt{\frac{1}{2} m_2 + m_4}$$

$$\dot{\theta}_2 \cdot \dot{\theta}_1 + L_1 L_2 S\theta_2 (\frac{1}{2} m_2 + m_4) \dot{\theta}_1 \dot{\theta}_2$$

$$H_2 = \frac{1}{2} L_1 L_2 S\theta_2 (m_2 + 2m_4) \dot{\theta}_1^2 + \frac{1}{2} L_1 L_2 S\theta_2 \sqrt{\frac{1}{2} m_2 + m_4} \dot{\theta}_1 \dot{\theta}_2$$

$$H_{311} = \frac{dD_{31}}{d\theta_1} - \frac{1}{2} \frac{dD_{11}}{d \cdot d\theta_2} = \text{Zero}$$

$$H_{322} = \text{Zero} \quad (H_{344} = \text{Zero}) \quad H_{321} = \frac{dD_{32}}{d\theta_1} - \frac{1}{2} \frac{dD_{21}}{d \cdot d\theta_2} = 0$$

$$H_{323} = \text{Zero} \quad (H_{324} = \text{Zero}) \quad H_{331} = \text{Zero}$$

$$H_{332} = \text{Zero} \quad (H_{334} = \text{Zero}) \quad H_{312} = \frac{dD_{31}}{d\theta_2} - \frac{1}{2} \frac{dD_{12}}{d \cdot d\theta_2} = \text{Zero}$$

$$H_{313} = \text{Zero} \quad (H_{314} = \text{Zero}) \quad (H_{341} = \text{Zero}) \quad H_{342} = \text{Zero}$$

$$H_{343} = \text{Zero}$$

$$\therefore H_3 = \text{Zero} = 0$$

$$H_4 = \text{Zero} = 0$$

$$H = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} L_1 L_2 S\theta_2 (-m_2 + 2m_4) \dot{\theta}_2 \\ \frac{1}{2} L_1 L_2 S\theta_2 (m_2 + 2m_4) \dot{\theta}_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} L_1 L_2 S\theta_2 (-\frac{1}{2} m_2 - m_4) \dot{\theta}_2 \\ -\frac{1}{2} L_1 L_2 S\theta_2 (\frac{1}{2} m_2 + m_4) \dot{\theta}_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix}$$

$$G(q) = \sum_{j=1}^n m_j q^j \bar{J} D_j$$

$$G_1 = m_1 \bar{g} \bar{J} \bar{D}_1 + m_2 \bar{g} \bar{J} \bar{D}_2 + m_3 \bar{g} \bar{J} \bar{D}_3 + m_4 \bar{g} \bar{J} \bar{D}_4$$

$$G_1 = m_1 [0 \ -g \ 0] \begin{bmatrix} L_1 \sin \theta_1 \\ L_2 \sin \theta_2 \\ L_3 \sin \theta_3 \end{bmatrix} + m_2 [0 \ -g \ 0] \begin{bmatrix} L_1 \cos \theta_1 + \frac{1}{2} L_2 \cos \theta_2 \\ L_2 \cos \theta_2 \\ L_3 \cos \theta_3 \end{bmatrix}$$

$$+ m_3 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_4 [0 \ -g \ 0] \begin{bmatrix} L_1 \cos \theta_1 + \frac{1}{2} L_2 \cos \theta_2 \\ L_2 \cos \theta_2 \\ L_3 \cos \theta_3 \end{bmatrix}$$

$$G_1 = -\frac{1}{2} m_2 g L_2 C\theta_1 - (m_2 g (L_1 C\theta_1 + \frac{1}{2} L_2 C\theta_2)) - (m_4 g (L_1 C\theta_1 + L_2 C\theta_2))$$

$$G_2 = m_1 \bar{g} \bar{J} \bar{D}_1^2 + m_2 \bar{g} \bar{J} \bar{D}_2^2 + m_3 \bar{g} \bar{J} \bar{D}_3^2 + m_4 \bar{g} \bar{J} \bar{D}_4^2$$

$$= m_1 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_2 [0 \ -g \ 0] \begin{bmatrix} -\frac{1}{2} L_2 \sin \theta_2 \\ L_2 \cos \theta_2 \\ 0 \end{bmatrix} +$$

$$m_3 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_4 [0 \ -g \ 0] \begin{bmatrix} -L_2 \sin \theta_2 \\ L_2 \cos \theta_2 \\ 0 \end{bmatrix}$$

$$G_2 = -\frac{1}{2} m_2 g L_2 C\theta_1 \theta_2 - m_4 g L_2 C\theta_1 \theta_2$$

$$G_3 = m_1 \bar{g} \bar{J} \bar{D}_1^3 + m_2 \bar{g} \bar{J} \bar{D}_2^3 + m_3 \bar{g} \bar{J} \bar{D}_3^3 + m_4 \bar{g} \bar{J} \bar{D}_4^3$$

$$= m_1 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_2 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_3 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$+ m_4 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = -m_1 g - m_2 g - m_3 g - m_4 g$$

$$= -g (m_1 + m_2 + m_3 + m_4)$$

$$\begin{aligned}
 G_4 &= m_1 \vec{g}^T \vec{J} D_1 + m_2 \vec{g}^T \vec{J} D_2 + m_3 \vec{g}^T \vec{J} D_3 + m_4 \vec{g}^T \vec{J} D_4 \\
 &= m_1 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_2 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + m_3 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
 &\quad + m_4 [0 \ -g \ 0] \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = 0 \\
 \therefore \dot{G}_4 &= 0
 \end{aligned}$$

$$\therefore T_1 = D_{11} \ddot{\theta}_1 + D_{12} \ddot{\theta}_2 + D_{14} \ddot{\theta}_3 + H_{11} \dot{\theta}_1 + H_{12} \dot{\theta}_2 + G_1$$

$$\therefore T_2 = D_{21} \ddot{\theta}_1 + D_{22} \ddot{\theta}_2 + D_{24} \ddot{\theta}_3 + H_{21} \dot{\theta}_1 + H_{22} \dot{\theta}_2 + G_2$$

$$\therefore f = D_{33} \cdot d_2 + f_3$$

$$\therefore T_3 = D_{41} \ddot{\theta}_1 + D_{42} \ddot{\theta}_2 + D_{44} \ddot{\theta}_3$$

Block Diagram for Control

