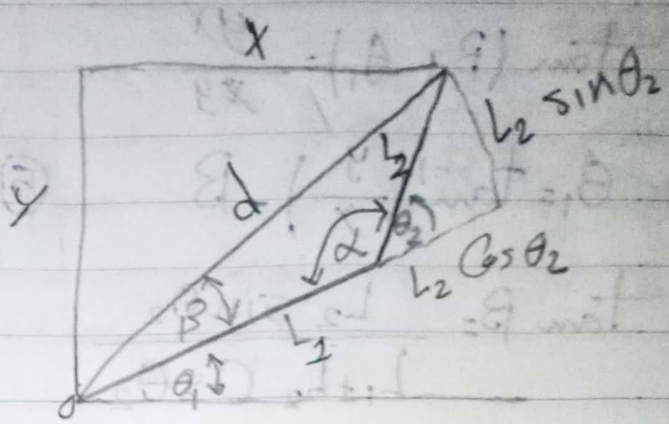
method (Scara Robot inverse kinematics) :-



$$d = \sqrt{x^2 + y^2}$$

$$\rightarrow d^2 = x^2 + y^2 \qquad ①$$

$$d^2 = L_1^2 + L_2^2 - 2L_1L_2\cos\alpha$$

$$\cos\alpha = \frac{L_1^2 + L_2^2 - d^2}{2L_1L_2} \qquad \boxed{2}$$

- from figure

$$\theta_2 = 180 - \alpha \text{ or } \pi - \alpha$$

$$\cos\theta_2 = \cos(\pi - \alpha)$$

$$\therefore \cos\theta_2 = \cos\pi \cdot \cos\alpha + \sin\pi \cdot \sin\alpha$$

$$\because \cos\pi = -1 \ , \ \cancel{\cos\pi} \ \sin\pi = 0$$

$$\therefore \cos\theta_2 = -1\cos\alpha \qquad ③$$

$$\cos\theta_2 = -\left(\frac{L_1^2 + L_2^2 - d^2}{2L_1L_2}\right)$$

$$\therefore \theta_2 = \cos^{-1}\left(\frac{d^2 - L_1^2 - L_2^2}{2L_1L_2}\right) \qquad ④$$

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From figure:

$$\tan(\beta + \theta_1) = \frac{\cancel{y}\,X}{\cancel{x}\,y}$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) - \beta \qquad (5)$$

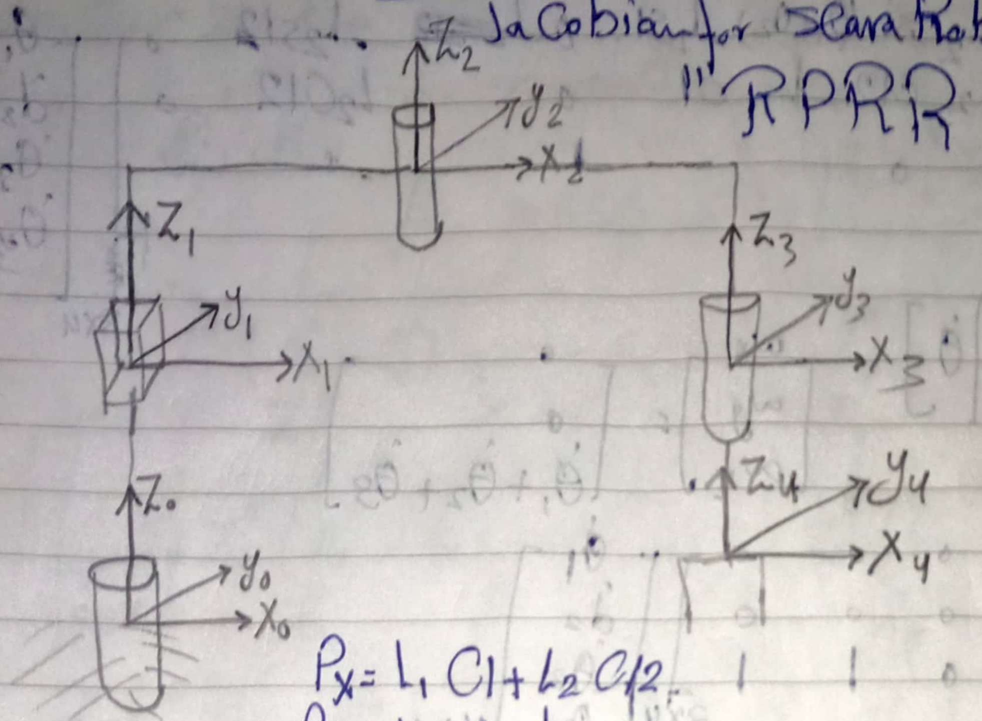
$$\tan\beta = \frac{L_2 \sin\theta_2}{L_1 + L_2 \cos\theta_2}$$

$$\therefore \beta = \tan^{-1}\left(\frac{L_2 \sin\theta_2}{L_1 + L_2 \cos\theta_2}\right) \qquad (6)$$

$$\therefore \theta_1 = \tan^{-1}\left(\frac{\cancel{y}\,X}{y\,\cancel{x}}\right) - \tan^{-1}\left(\frac{L_2 \sin\theta_2}{L_1 + L_2 \cos\theta_2}\right) \qquad (7)$$

$$\boxed{\begin{aligned} \therefore \theta_1 &= \tan^{-1}\left(\frac{X}{y}\right) - \tan^{-1}\left(\frac{L_2 \sin\theta_2}{L_1 + L_2 \cos\theta_2}\right) \\ \theta_2 &= \cos^{-1}\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2 L_1 L_2}\right) \end{aligned}}$$

Jacobian for Scara Robet "RPRR"



$P_x = L_1 C1 + L_2 C12$

$P_y = L_1 S1 + L_2 S12$

$P_z = d_1 + d_2 + d_3 + d_4$

$$\begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = J_P(\dot{\theta}) \qquad \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} L_1 C\theta_1 + L_2 C12 \\ L_1 S1 + L_2 S12 \\ d_1 + d_2^* + d_3 + d_4 \end{bmatrix}$$

$$\begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix} = \begin{bmatrix} (-L_2 \sin 12 - L_1 S1)\dot{\theta}_1 + (-L_2 \sin 12)\dot{\theta}_2 \\ (L_2 \cos(\theta_1+\theta_2) + L_1 \cos\theta_1)\dot{\theta}_1 + (L_2 \cos(\theta_1+\theta_2))\dot{\theta}_2 \\ \dot{d}_2^* \end{bmatrix}$$

$$\begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix} = \begin{bmatrix} \frac{dP_x}{d\theta_1} & \cancel{\frac{dP_x}{d}} \end{bmatrix}$$

$$\begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix} = \begin{bmatrix} \frac{dP_x}{d\theta_1} & \frac{dP_x}{d\cdot d_2^*} & \frac{dP_x}{d\theta_2} & \frac{dP_x}{d\theta_3} \\ \frac{dP_y}{d\theta_1} & \frac{dP_y}{d\cdot d_2^*} & \frac{dP_y}{d\theta_2} & \frac{dP_y}{d\theta_3} \\ \frac{dP_z}{d\theta_1} & \frac{dP_z}{d\cdot d_2^*} & \frac{dP_z}{d\theta_2} & \frac{dP_z}{d\theta_3} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2^* \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

$$\begin{bmatrix} \dot{P}_x \\ \dot{P}_y \\ \dot{P}_z \end{bmatrix} = \begin{bmatrix} -L_2 \sin(\theta_1+\theta_2) - L_1 S1 & 0 & -L_2 S12 & 0 \\ L_2 C12 - L_1 C1 & 0 & L_2 C12 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}_{3\times4} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}_{4\times1}$$

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J_o \left[\dot{\theta} \right]$$

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3 \end{bmatrix}$$

$$\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}_{3\times4} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}_{4\times1}$$

$$\therefore J = \begin{bmatrix} J_P \\ J_o \end{bmatrix} = \begin{bmatrix} -L_2 S12 - L_1 S1 & 0 & -L_2 S12 & 0 \\ L_2 C12 - L_1 C1 & 0 & L_2 C12 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

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another solution

$z_{i-1}^0 = R_{i-1}^0 \cdot k$

for Prismatic $\rightarrow J_i = \begin{bmatrix} z_{i-1}^0 \\ 0 \end{bmatrix}$

for revolute $\rightarrow J_i = \begin{bmatrix} z_{i-1}^0 (O_n^0 - O_{i-1}^0) \\ z_{i-1}^0 \end{bmatrix}$

$J_1 = \begin{bmatrix} Z_0^0 (O_4^0 - O_0^0) \\ Z_0^0 \end{bmatrix}$, $J_2 = \begin{bmatrix} Z_1^0 \\ 0 \end{bmatrix}$

$J_3 = \begin{bmatrix} Z_2^0 (O_4^0 - O_2^0) \\ Z_2^0 \end{bmatrix}$, $J_4 = \begin{bmatrix} Z_3^0 (O_4^0 - O_3^0) \\ Z_3^0 \end{bmatrix}$

$Z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $Z_1^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $Z_2^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $Z_3^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$O_0^0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $O_1^0 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix}$, $O_2^0 = \begin{bmatrix} L_1 C1 \\ L_1 S1 \\ d_1 + d_2^* \end{bmatrix}$, $O_3^0 = \begin{bmatrix} L_1 C1 + L_2 C12 \\ L_1 S1 + L_2 S12 \\ d_1 + d_2^* - d_3 \end{bmatrix}$

$O_4^0 = \begin{bmatrix} L_1 C1 + L_2 C12 \\ L_1 S1 + L_2 S12 \\ d_1 + d_2^* - d_3 - d_4 \end{bmatrix}$

$Z_0^0 (O_4^0 - O_0^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} L_1 C1 + L_2 C12 \\ L_1 S1 + L_2 S12 \\ d_1 + d_2^* - d_3 - d_4 \end{bmatrix} = \begin{bmatrix} +i & -j & +k \\ 0 & 0 & 1 \\ L_1 C1 + L_2 C12 & L_1 S1 + L_2 S12 & d_1 + d_2^* - d_3 - d_4 \end{bmatrix}$

$= \begin{bmatrix} -L_1 S1 - L_2 S12 \\ L_1 C1 + L_2 C12 \\ 0 \end{bmatrix}$

$$Z_2^0(O_4^0 - O_2^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} L_2 C12 \\ L_2 S12 \\ -d_3 - d_4 \end{bmatrix} = \begin{bmatrix} +i & -j & +K \\ 0 & 0 & 1 \\ L_2 C12 & L_2 S12 & -d_3 - d_4 \end{bmatrix}$$

$$= \begin{bmatrix} -L_2 S12 \\ L_2 C12 \\ 0 \end{bmatrix}$$

$$Z_3^0(O_4^0 - O_3^0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ -d_4 \end{bmatrix} = \begin{bmatrix} +i & -j & +K \\ 0 & 0 & 1 \\ 0 & 0 & -d_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} J_v \\ J_w \end{bmatrix} = \begin{bmatrix} -L_1 S1 - L_2 S12 & 0 & -L_2 S12 & 0 \\ L_1 C1 + L_2 C12 & 0 & L_2 C12 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{X} \\ \dot{y} \\ \dot{Z} \\ \omega_X \\ \omega_y \\ \omega_Z \end{bmatrix}_{6\times 1} = \begin{bmatrix} -L_1 S1 - L_2 S12 & 0 & -L_2 S12 & 0 \\ L_1 C1 + L_2 C12 & 0 & L_2 C12 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}_{6\times 4} \begin{bmatrix} \dot{\theta}_1 \\ \dot{d}_2 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}_{4\times 1}$$