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+ Degree of freedom

- the number of independent Parameters or inputs needed to specify the Configuration of the mechanism Completely.

- $Dof = 3(n-1) - 2j - H$

$n \longrightarrow$ number of links

$j \longrightarrow$ number of joints

$H \longrightarrow$ higher Pairs

- for the scara Robot

$n = 5$, $j = 4$, $H = 0$

$Dof = 3(5-1) - 2 \times 4 = 12 - 8 = 4$

$\therefore$ The SCARA robot has four degrees of freedom (Dof).

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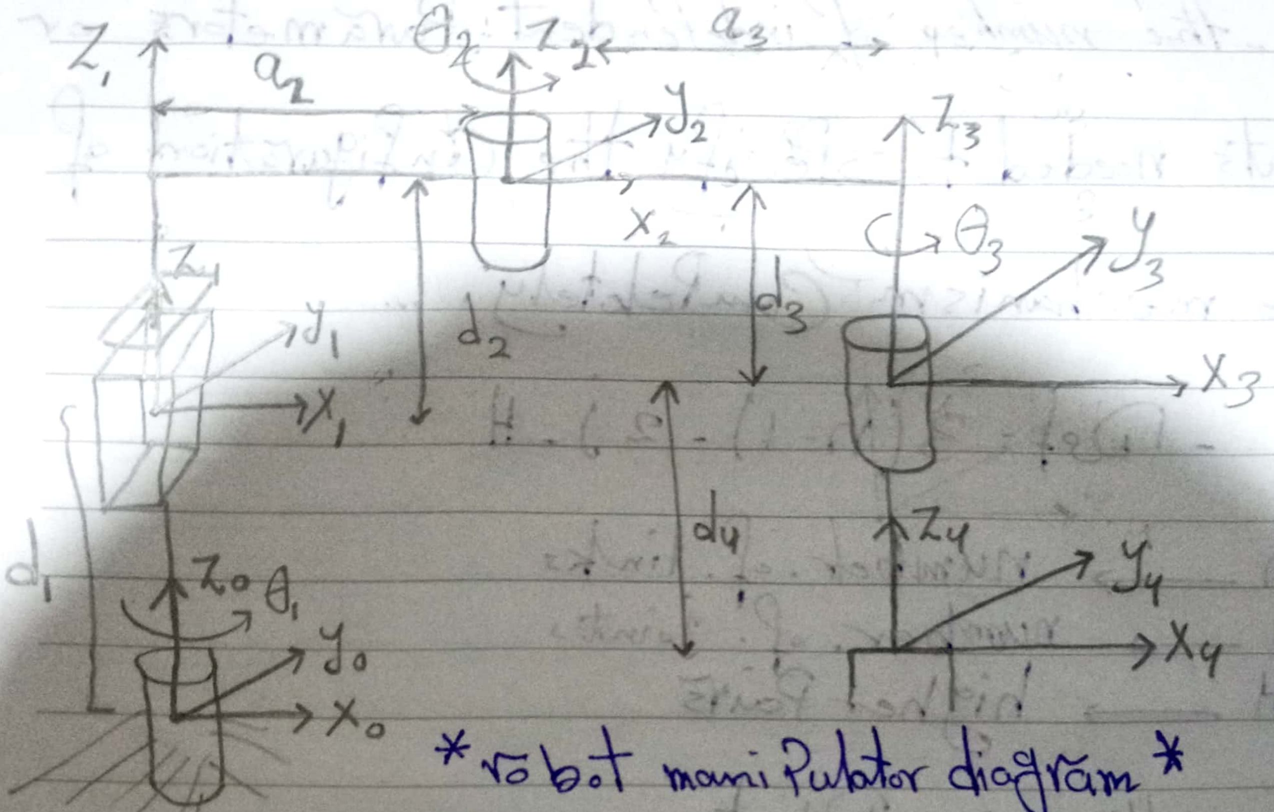
Forward kinematics :



* robot manipulator diagram *

* Denavit-Hartenberg (DH) Convention: —

- DH Parameters, it is obtaining by this away:

1. link offset d_i
   - distance along Z_{i-1} from X_{i-1} to X_i.
2. Joint angle θ_i
   - angle about Z_{i-1} from X_{i-1} to X_i.
3. link Length a_i
   - distance along X_i from Z_{i-1} to Z_i.
4. link Twist α_i
   - angle about X_i from Z_{i-1} to Z_i.

- DH- table:

links	angle twist (α_i)	link length a_i	joint angle θ_i	link offset d_i
1	0	0	θ_1	d_1
2	0	a_2	0	d_2
3	0	a_3	θ_2	d_3
4	0	0	θ_3	d_4

$$A_i^{i-1} = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & a_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^0 = \begin{bmatrix} C1 & -S1\,C1 & 0 & 0 \\ S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_2^1 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} C3 & -S3\,C3 & 0 & a_3 C3 \\ S3 & C3 & 0 & a_3 S3 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3 = \begin{bmatrix} C4 & -S4\,C4 & 0 & 0 \\ S4 & C4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$T_4^0 = A_1^0 \cdot A_2^1 \cdot A_3^2 \cdot A_4^3$$

$$A_3^2 \cdot A_4^3 = \begin{bmatrix} C3 & -S3\,C3 & 0 & a_3\,C3 \\ S3 & C3 & 0 & a_3\,S3 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C4 & -S4\,C4 & 0 & 0 \\ S4 & C4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C3\,C4 - S3\,S4\,C3 & -C3\,C4\,S4 - S3\,C3\,C4 & 0 & a_3\,C3 \\ S3\,C4 + C3\,S4 & -S4\,C4\,S3 + C4\,C3 & 0 & a_3\,S3 \\ 0 & 0 & 1 & d_4 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 \cdot A_3^2 \cdot A_4^3 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \; \end{bmatrix}$$

$$= \begin{bmatrix} C3\,C4 - S3\,S4\,C3 & -C3\,C4\,S4 - S3\,C3\,C4 & 0 & a_3\,C3 + a_2 \\ S3\,C4 + C3\,S4 & -S4\,C4\,S3 + C4\,C3 & 0 & a_3\,S3 \\ 0 & 0 & 1 & d_4 + d_3 + d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^0 \cdot A_2^1 \cdot A_3^2 \cdot A_4^3 = \begin{bmatrix} C1 & -S1\,C1 & 0 & 0 \\ S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \; \end{bmatrix}$$

$$
\therefore T_4^0 = \begin{bmatrix} C1C3C4 - S3S4C3C1 - S1C1S3C4 - S1C1C3S4 & C1C3C4S4 - C4S3C3C4 + S1C1S4C4S3 - S1C1C4C3 & 0 & a_3C3C1 + a_2C1 - a_3S3S1C1 \\ C3C4 - S3S4C3S1 + C1S3C4 + C1C3S4 & C3C4S4S1 - S3C3C4S1 - C1S4C4S3 + C4C3C1 & 0 & a_3C3S1 + a_2S1 + a_3S3C1 \\ 0 & 0 & 1 & d_4+d_3+d_2+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

By using:

$$\sin(\alpha \pm B) = \sin\alpha \cos B \pm \cos\alpha \sin B$$
$$\cos(\alpha \pm B) = \cos\alpha \cos B \mp \sin\alpha \sin B$$

$$
T_4^0 = \begin{bmatrix} C123 & -S123 & 0 & a_2 C1 + a_3 C12 \\ S123 & C123 & 0 & a_2 S1 + a_3 S12 \\ 0 & 0 & 1 & d_1+d_2+d_3+d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

حيث أن:

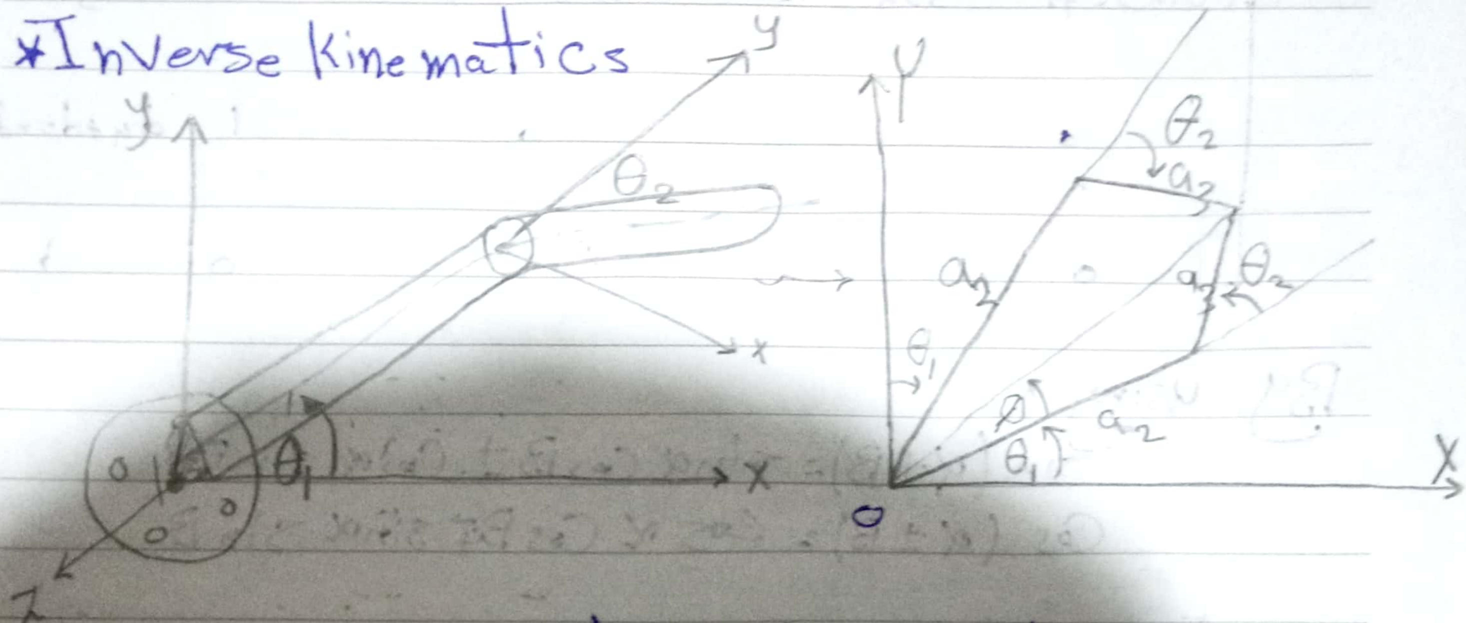
$C1 = \cos\theta_1$, $C2 = \cos\theta_2$, $C123 = \cos(\theta_1+\theta_2+\theta_3)$

$C12 = \cos(\theta_1+\theta_2)$, $S1 = \sin\theta_1$, $S2 = \sin\theta_2$

$S123 = \sin(\theta_1+\theta_2+\theta_3)$, $S12 = \sin(\theta_1+\theta_2)$

$$T_4^0 = \left[\begin{array}{c|c} \text{orientation matrix} & \text{Position Vector} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right]$$

*Inverse Kinematics



From the Forward kinematics matrix T_4^0 the Position of the robot arm determined by this equations:

$$X = a_2 C1 + a_3 C12 \longrightarrow ①$$

$$Y = a_2 S1 + a_3 S12 \longrightarrow ②$$

بتربيع ① و ② والجمع نحصل على ...

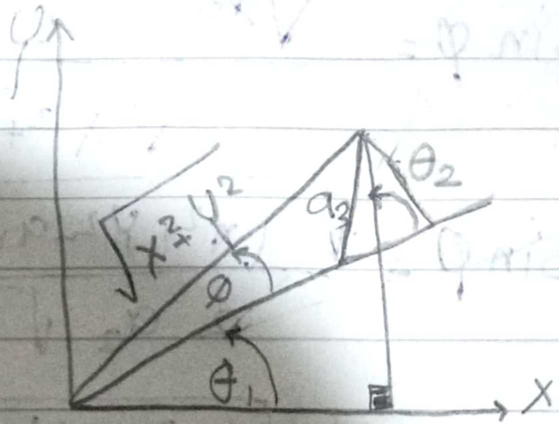
$$X^2 + Y^2 = a_2^2 C1^2 + a_3^2 C12^2 + 2a_3 a_2 C1 \; C12 +$$

$$a_2^2 S1^2 + a_3^2 S12^2 + 2a_3 a_2 S1 \; S12$$

$$\therefore X^2 + Y^2 = a_2^2 + a_3^2 + 2 a_2 a_3 C2$$

$$\therefore C2 = \frac{X^2 + Y^2 - a_2^2 - a_3^2}{2 a_2 a_3}$$

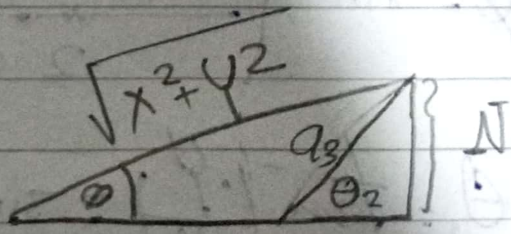
$$\therefore \theta_2 = \pm \cos^{-1}\left(\frac{X^2 + Y^2 - a_2^2 - a_3^2}{2 a_2 a_3}\right)$$



من الشكل المقابل :-

$$\tan(\theta_1 + \phi) = \frac{Y}{X}$$

$$\theta_1 + \phi = \tan^{-1}\left(\frac{Y}{X}\right)$$



(1)

من الشكل (1)

$$\sin\theta_2 = \frac{N}{a_3}$$

$$\therefore N = \sin\theta_2 \cdot a_3$$

$$\therefore \sin\phi = \frac{N}{\sqrt{X^2+Y^2}} = \frac{a_3 \sin\theta_2}{\sqrt{X^2+Y^2}}$$

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$$S2 = \sqrt{\cos^2\theta_2 - 1} \longrightarrow \cos\theta_2 = \frac{x^2 + y^2 - a_2^2 - a_3^2}{2a_2a_3}$$

$$\sin\theta_2 = \sqrt{\left(\frac{x^2 + y^2 - a_2^2 - a_3^2}{2a_2a_3}\right)^2 - 1}$$

$$\sin\phi = \frac{\sqrt{a_3^2 \frac{(x^2 + y^2 - a_2^2 - a_3)^2}{4a_2^2a_3^2} - a_3^2}}{\sqrt{x^2 + y^2}}$$

$$\sin\phi = \frac{\sqrt{(x^2 + y^2 - a_2^2 - a_3^2)^2 - 4a_2^2a_3^2}}{2a_2\sqrt{x^2 + y^2}}$$

$$\phi = \sin^{-1}\left(\frac{\sqrt{(x^2 + y^2 - a_2^2 - a_3^2)^2 - 4a_2^2a_3^2}}{2a_2\sqrt{x^2 + y^2}}\right)$$

$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) \pm \sin^{-1}\left(\frac{\sqrt{(x^2 + y^2 - a_2^2 - a_3^2)^2 - 4a_2^2a_3^2}}{2a_2\sqrt{x^2 + y^2}}\right)$$

AL-Haramien