

Test

ENGR 210 / CSCI B441

Addition / Subtraction II

Andrew Lukefahr

Announcements

- P2 is out! Due **next Friday**

- C and V bits updates ↪

carry

overflow

$$Add = 1 + 1 = \underline{2}$$

$$\begin{array}{r} A = 0000\ 0001 \\ b = 0000\ 0001 \\ \hline s = \underline{1010} \end{array} \quad r = 0000\ 0000$$

C = 0 for everything but
Addition & Subtraction

V = defined for Addition & Sub
→ undefined for all other operations

→ AG will use V=0 for
all cases other than
Addition & Subtraction

wire vs logic

- **wire** ↪

- Only used with 'assign' and module outputs
- Boolean combination of inputs
- Can never hold state

- **logic**

- Used with 'always' and module outputs
- Can be Boolean combination of inputs
- Can also hold state

wire w_i
assign $w = 'h0;$

// $w = 1$ if $x == 1$
 0 if $x == 2$
 1 if $x == 3$

always_comb^{combinational} Blocks

```
wire foo;  
assign foo = x & y | z;
```

... is equivalent to ...

```
wire foo = x & y | z;
```

... is equivalent to ...

```
logic foo;
```

```
always_comb //combinational  
foo = x & y | z;
```

logic foo;

```
always-comb begin  
if (x & y | z)  
    foo = 1'h1;  
else  
    foo = 1'h0;
```

end

wrong!

logic foo = x & y |

works { logic foo;
 assign foo = x & y |

always_comb blocks with if

```
module decoder (
    input [1:0] sel,
    output logic [3:0] out
);
    always_comb begin
        if (sel == 2'b00) begin
            out = 4'b0001;
        end else if (sel == 2'b01) begin
            out = 4'b0010;
        end else if (sel == 2'b10) begin
            out = 4'b0100;
        end else if (sel == 2'b11) begin
            out = 4'b1000;
        end
    end
endmodule
```

always_comb with case

```
module decoder (
    input [1:0] sel,
    output logic [3:0] out
);
```

→ **always_comb begin**

case(sel)

- 2'b00: out=4'b0001;
- 2'b01: out=4'b0010;
- 2'b10: out=4'b0100;
- 2'b11: out=4'b1000;

endcase default: out = 4'b0000;

end

```
endmodule
```

"switch" in C

always_comb with case

```
module decoder (
    input [1:0] sel,
    output logic [3:0] out
);
```

```
    always_comb begin
        case(sel)
            2'b00: out=4'b0001; ←
            2'b01: out=4'b0010; ←
            2'b10: out=4'b0100; ←
        endcase
    end

endmodule
```

Annotations:

- A red arrow points to the `always_comb` keyword.
- A red box highlights the `case(sel)` statement.
- Blue arrows point from the three case statements to their corresponding assignments.
- An orange annotation `out can be whatever!` is written next to the question mark in the code, with a blue arrow pointing to it.

* add logic after class

always_ff

next time

always_comb with case

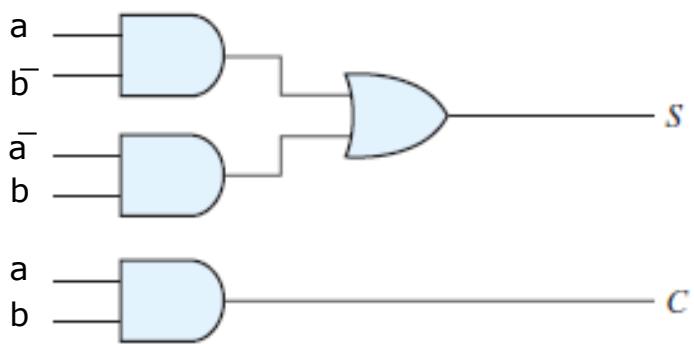
```
module decoder (
    input [1:0] sel,
    output logic [3:0] out
);
    always_comb begin
        out = 4'b0000; //default
        case(sel)
            2'b00: out=4'b0001;
            2'b01: out=4'b0010;
            2'b10: out=4'b0100;
            ✓ defalt: out=4'b0000 // what about sel==2'b11?
        endcase
    end
endmodule
```

Always specify
defaults for
always_comb!

"Worris: Inferred
Latch"

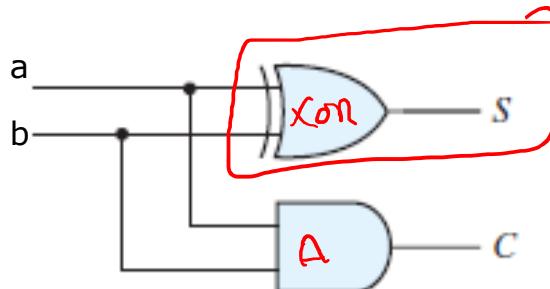
Half Adder

i	a	b	C	S
0	0	0	0	0
1	0	1	0	1
2	1	0	0	1
3	1	1	1	0



$$S = a \bar{b} + \bar{a} b = a \oplus b$$

$$C = a b$$



$$\begin{array}{r}
 & + \\
 & | \\
 & | \\
 \hline
 10
 \end{array}$$

$$\begin{array}{r}
 & | \\
 & | \\
 & | \\
 \hline
 100 \leftarrow 4
 \end{array}$$

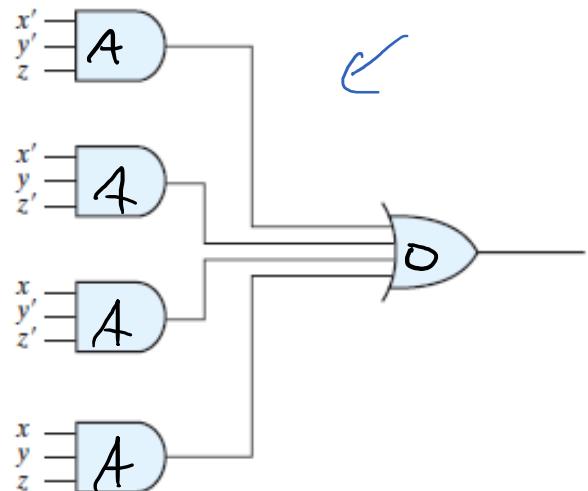
Handwritten addition example: $1 + 1 = 10$ and $1 + 1 + 1 = 100 \leftarrow 4$. The result 100 is circled in green.

P2: use "+" in Verilog

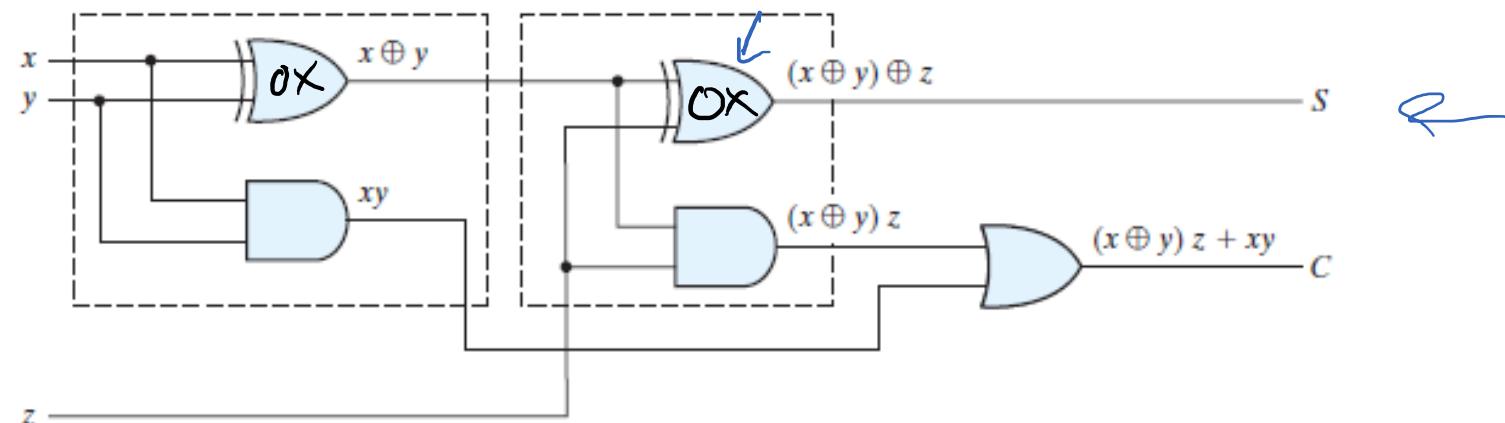
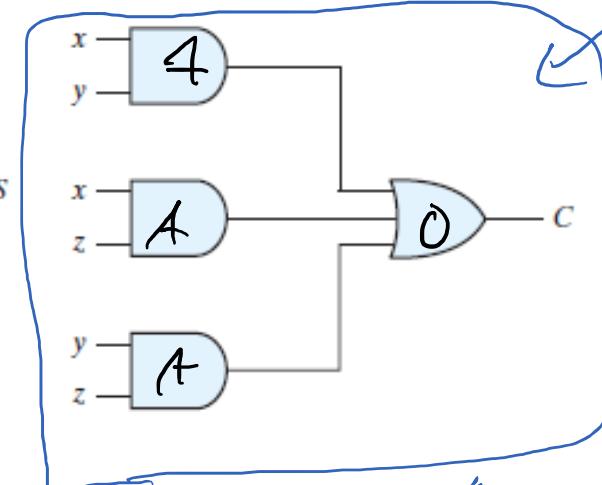
Full Adder

i	x	y	z	<u>C</u>	<u>S</u>
0	0	0	0	0	0
1	0	0	1	0	1
2	0	1	0	0	1
3	0	1	1	1	0
4	1	0	0	0	1
5	1	0	1	1	0
6	1	1	0	1	0
7	1	1	1	1	1

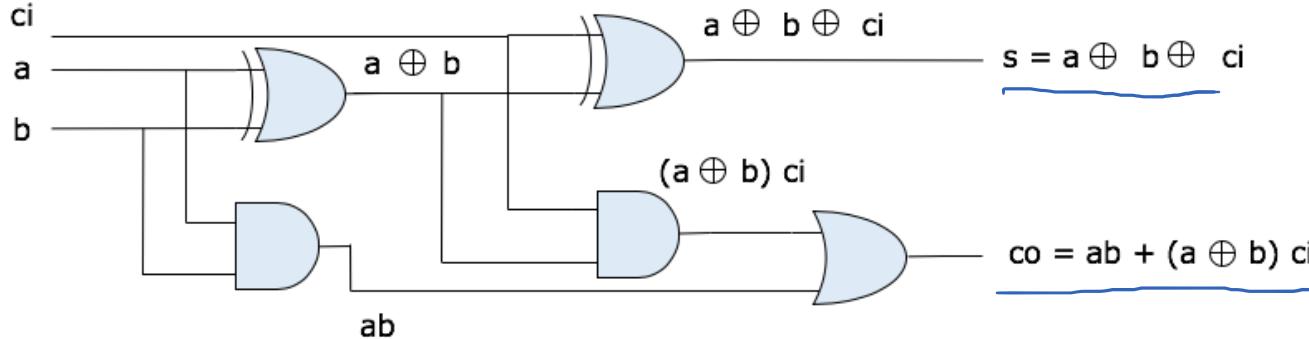
Sum



Carry



1-Bit “Full” Adder

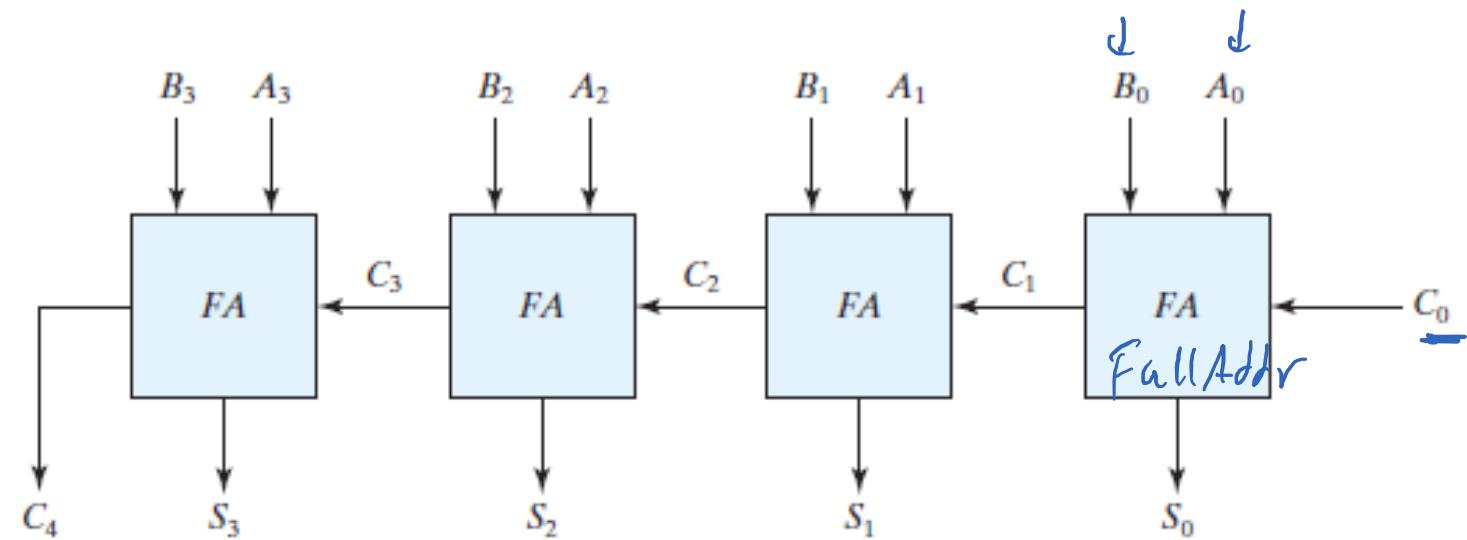


```
module FullAddr (
    input a,b,ci,
    output s, co
);
```

→ assign s = a ^ b ^ ci;
→ assign co = (a & b) | ((a ^ b) & ci);

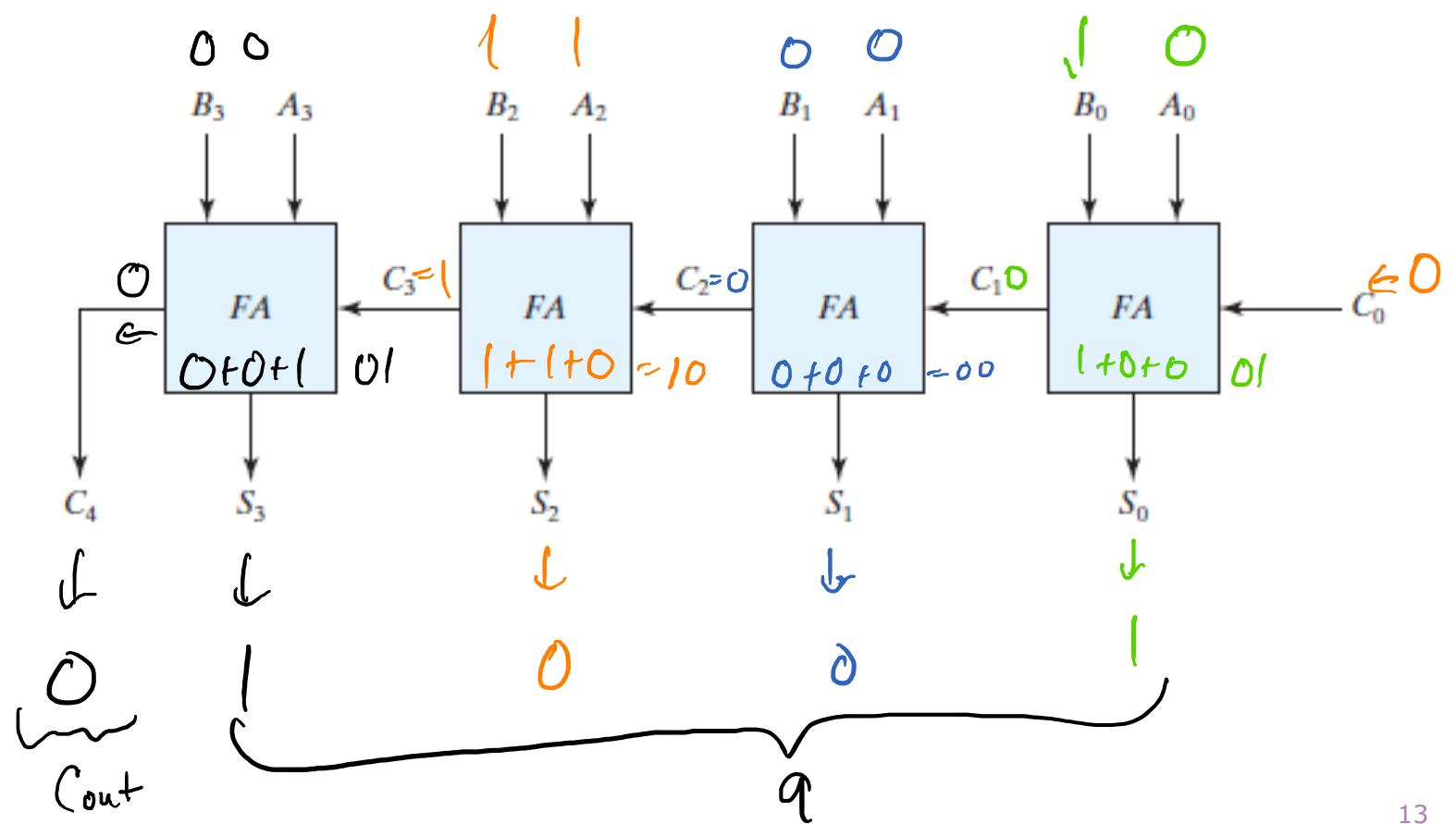
```
endmodule
```

Ripple-Carry Adder



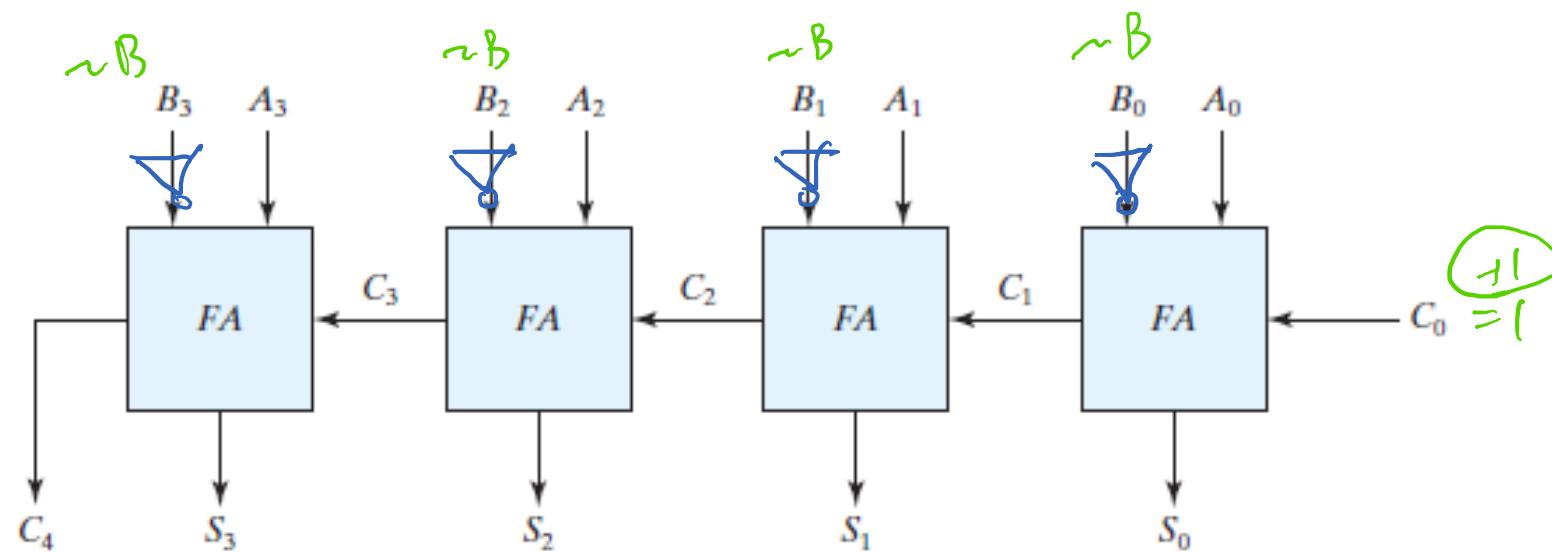
Ripple-Carry Adder

$$\begin{array}{r}
 a = 4 = \begin{array}{|c|c|c|c|} \hline 0 & 1 & 0 & 0 \\ \hline \end{array} \\
 + b = 5 = + \begin{array}{|c|c|c|c|} \hline 0 & 1 & 0 & 1 \\ \hline \end{array} \\
 \hline
 q = 1001
 \end{array}$$



$$A - B = A + (-B) = A + (\boxed{\sim B} + 1)$$

Subtraction with Adders?



Adder/Subtractor

- Mode input:
 - If $M = 0$, then $S = A + B$, the circuit performs addition
 - If $M = 1$, then $S = A + \bar{B} + 1$, the circuit performs subtraction

B	M	XOR
0	0	0
0	1	1
1	0	1
1	1	0

if $M=0$

$$XOR = B$$

if $m=1$

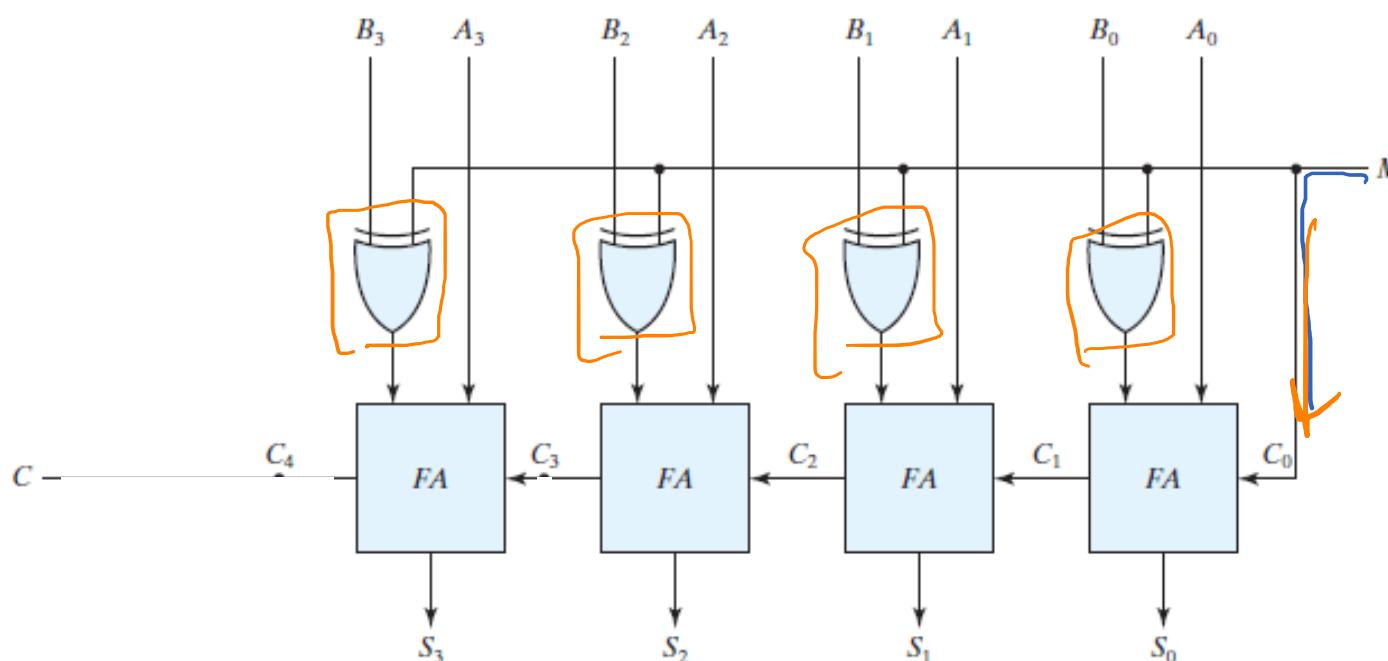
$$XOR = \sim B$$

$$m = 0$$

$$S = A + B + 0$$

$$m = 1$$

$$S = A + (\sim B) + 1$$



if $M=0$

$$C_0 = 0$$

if $m=1$

$$C_0 = 1$$

Overflow for signed numbers?

- Unsigned →

$$\begin{array}{r} 10 \\ + 8 \\ \hline 18 \end{array} = 8_{12} = \begin{array}{r} 1010 \\ + 1000 \\ \hline 10010 \end{array} \quad \begin{array}{l} \leftarrow \\ \leftarrow \end{array}$$

$\frac{10}{+ 8} = 2$ error if
Carry is set

- Signed

$$\begin{array}{r} 5 \\ + 6 \\ \hline 11 \end{array} = \begin{array}{r} 0101 \\ 0110 \\ \hline 01011 \end{array} = 0100 + 1 = 0101 = 5$$

carry bits \Rightarrow different \Rightarrow error

Carry won't tell you if something went wrong! \Rightarrow overflow

Overflow for signed numbers?

$$\begin{array}{r} -2 \\ + -1 \\ \hline -3 \end{array}$$

Overflow for signed numbers

$$\begin{array}{r} c_4 \quad c_3 \quad c_2 \quad c_1 \\ \cancel{x_3} \quad \cancel{x_2} \quad \cancel{x_1} \quad x_0 \\ + \quad y_3 \quad y_2 \quad y_1 \quad y_0 \\ \hline z_4 \quad z_3 \quad z_2 \quad z_1 \quad z_0 \end{array}$$

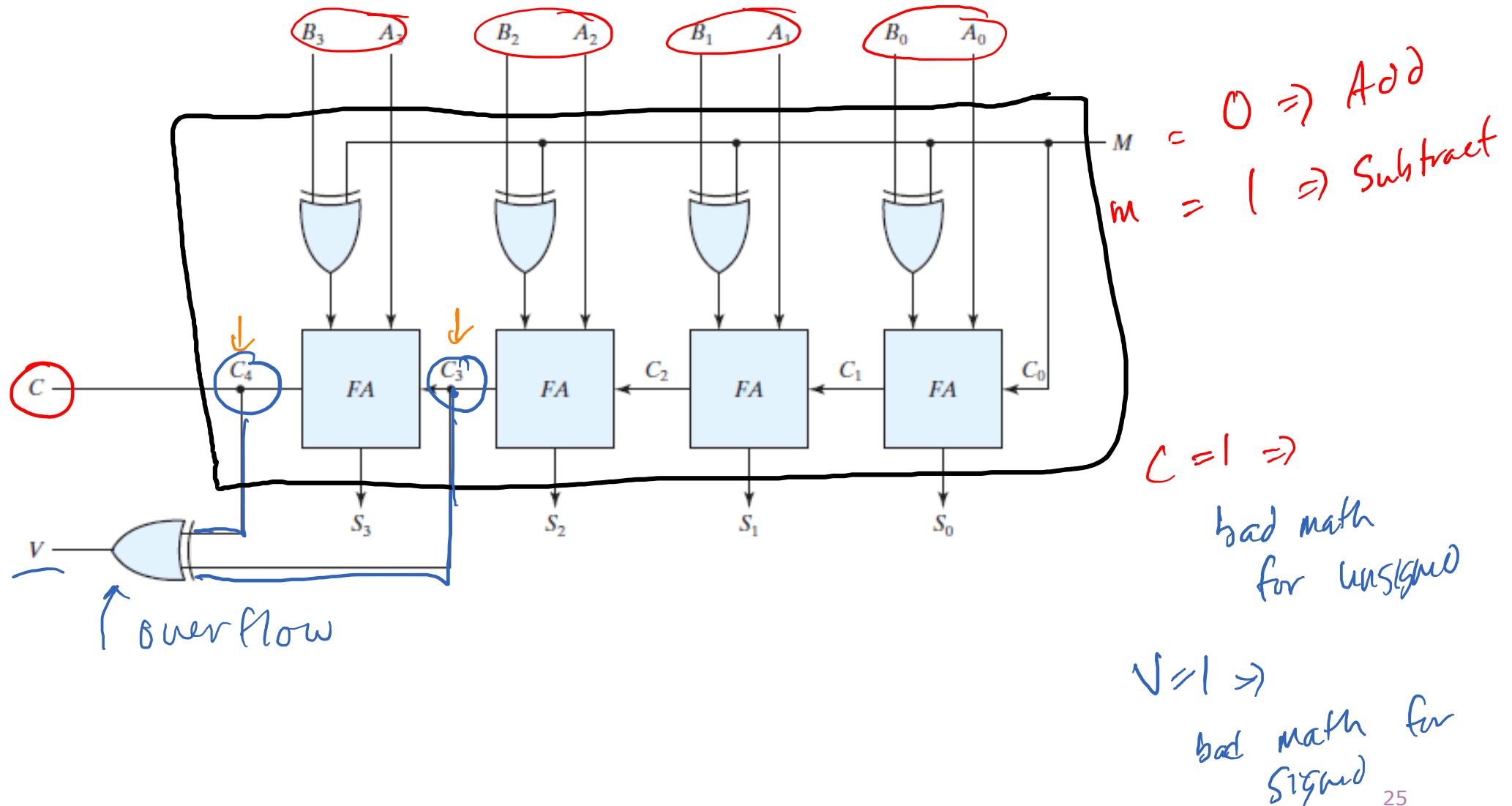
Overflow detection

- When two numbers with n digits each are added and the sum is a number occupying $n + 1$ digits, we say that an overflow occurred.
- The detection of an overflow after the addition of two binary numbers depends on whether the numbers are considered to be signed or unsigned.
- When two unsigned numbers are added, an overflow is detected from the end carry out of the most significant position.
- In case of signed numbers, two details are important:
 - the leftmost bit always represents the sign,
 - negative numbers are in 2's-complement form.
- When two signed numbers are added:
 - the sign bit is treated as part of the number
 - the end carry does not indicate an overflow.

Overflow detection

- An overflow cannot occur after an addition if one number is positive and the other is negative, since adding a positive number to a negative number produces a result whose magnitude is smaller than the larger of the two original numbers.
- An overflow may occur if the two numbers added are both positive or both negative.
- An overflow condition can be detected by observing the carry into the sign bit position and the carry out of the sign bit position.
 - If these two carries are equal, there was no overflow.
 - If these two carries are not equal, an overflow has occurred.
- If the two carries are applied to an exclusive-OR gate, an overflow is detected when the output of the gate is equal to 1.

Adder with overflow detection



WARNING: MAJOR TOPIC SHIFT

$$\begin{array}{r}
 \begin{array}{r}
 1000 \quad 0000 \\
 + 1000 \quad 0001 \\
 \hline
 0000 \quad 0001
 \end{array}
 \quad
 \begin{array}{r}
 128 \\
 + 129 \\
 \hline
 257
 \end{array}
 \end{array}$$

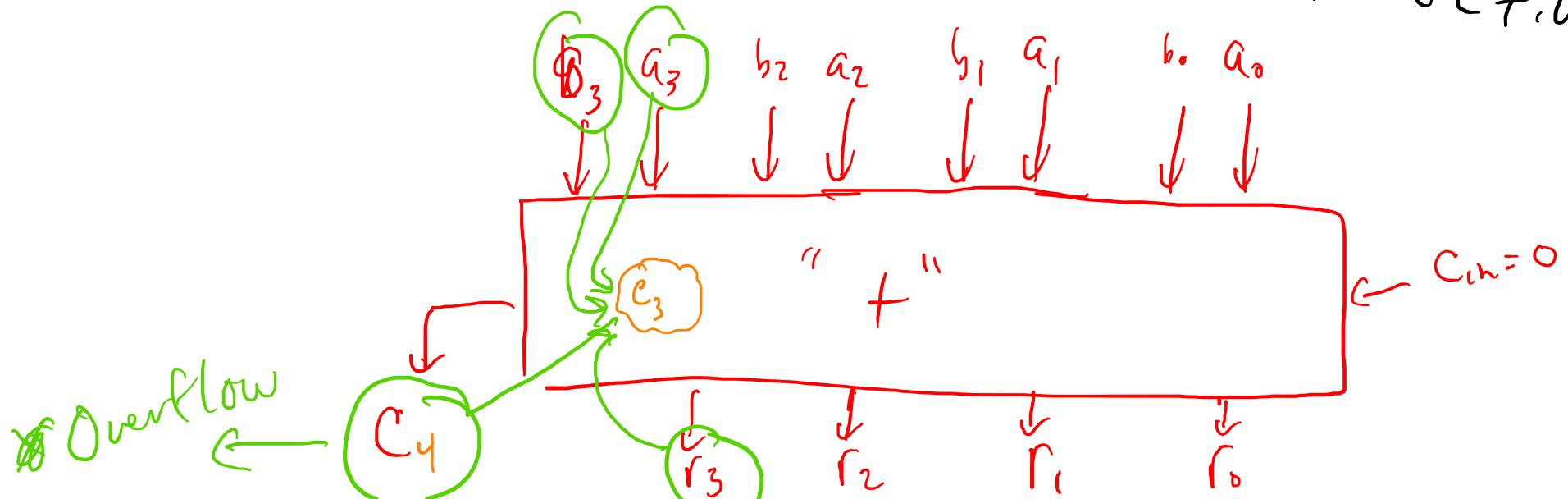
↑

carry

$\log_{12} [7:0] a_i$
 $\log_{12} [7:0] b_i$

$$\begin{aligned}
 & \log_{12} [7:0] z_j \Rightarrow \log_{12} [8:0] z; \\
 & z = a + b; \\
 & z = \{1'b0, a\} + \\
 & \quad \{1'b0, b\};
 \end{aligned}$$

SEQUENTIAL LOGIC

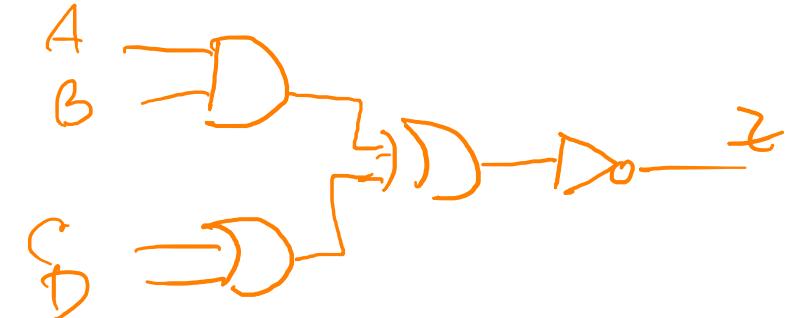


Stopped here!

Sequential vs. Combinational

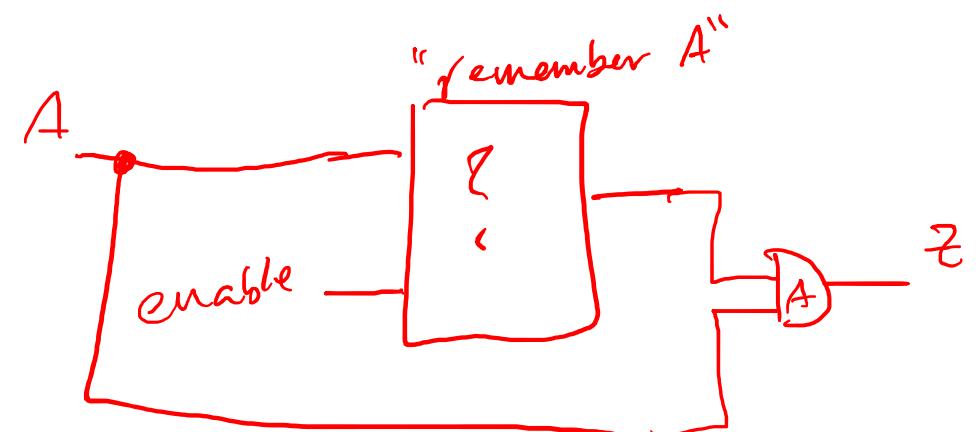
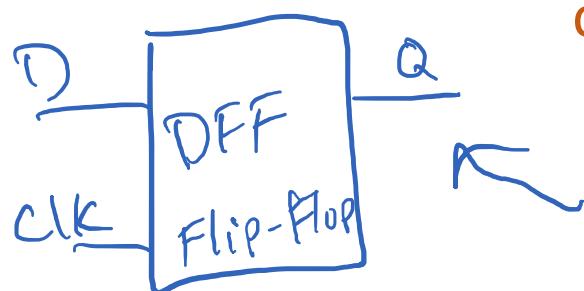
- Combinational Logic

- The output is a combination of the **current inputs only**



- Sequential Logic

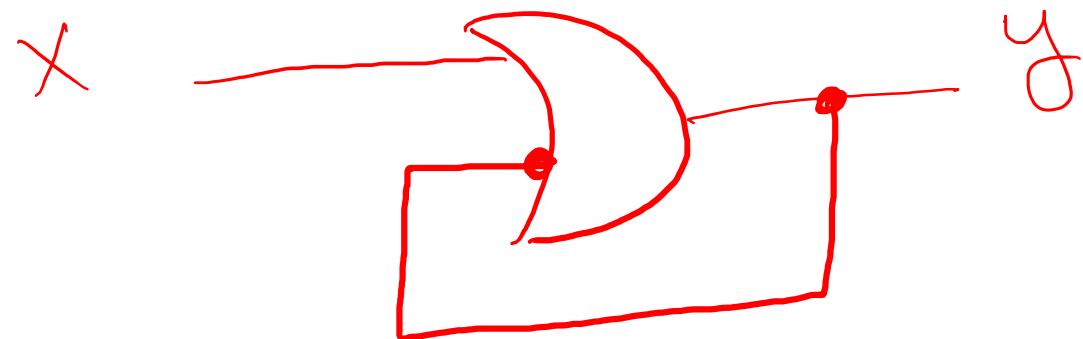
- The output is a combination of the **current and past inputs**



How do I store data?

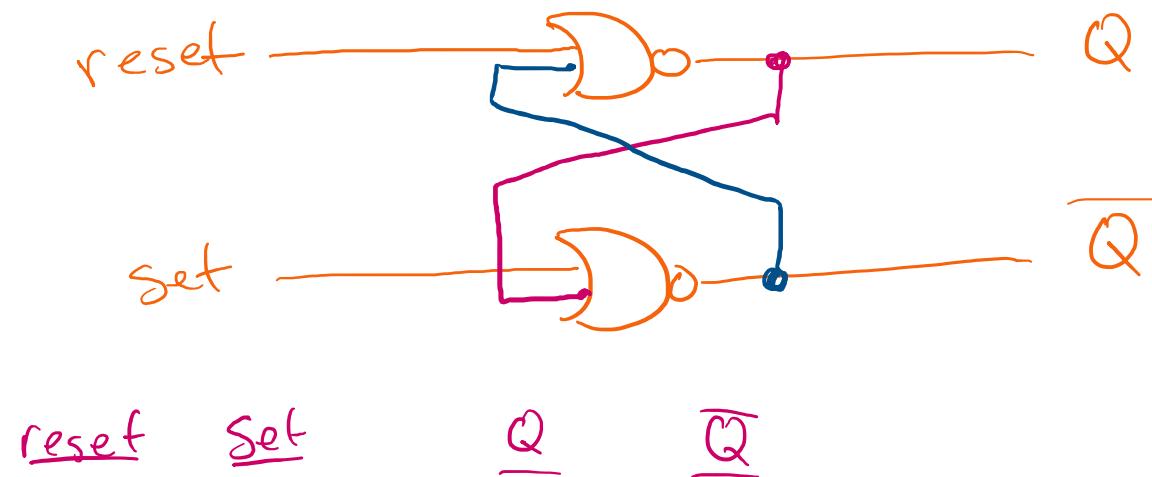
 → "1" or "true" or "hot"

 → "0" or "false" or "cold"



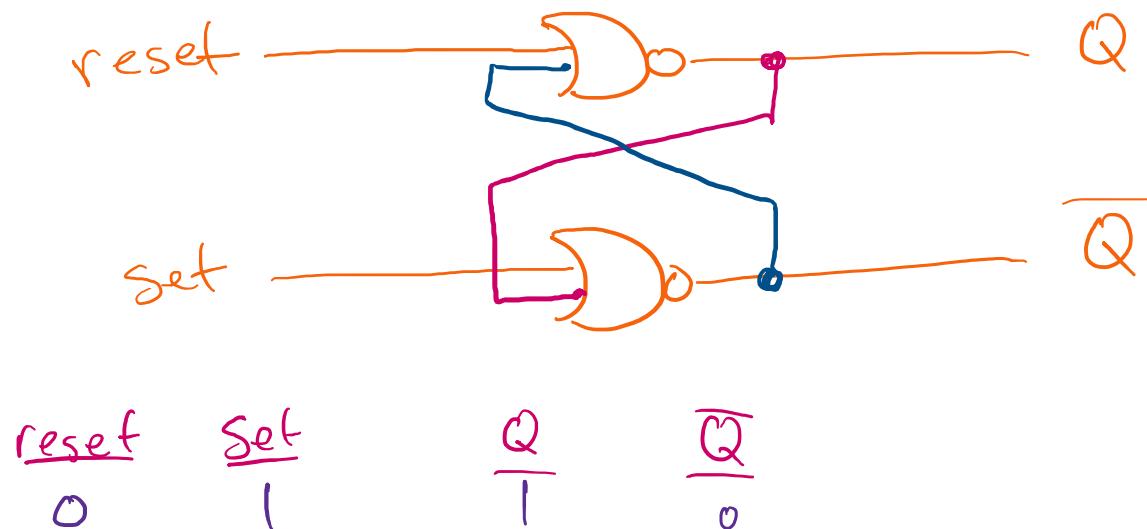
SR (Set-Reset) Latch

 = 1
 = 0



SR (Set-Reset) Latch

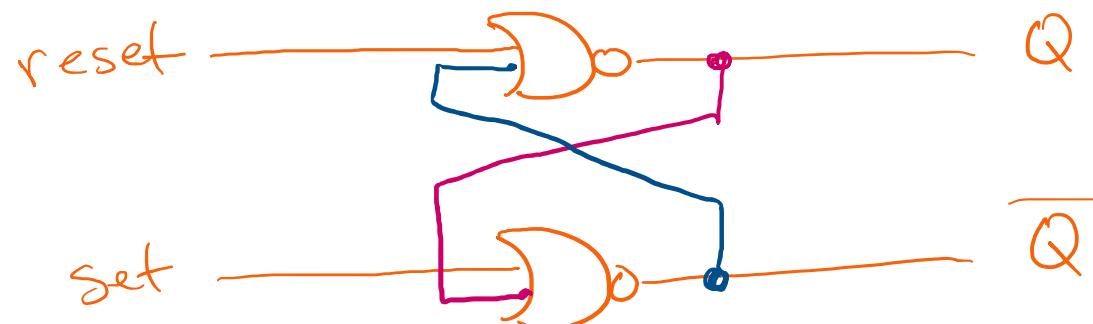
 = 1
 = 0



<u>reset</u>	<u>Set</u>	$\frac{Q}{1}$	$\frac{\bar{Q}}{0}$
0	1		

SR Latch

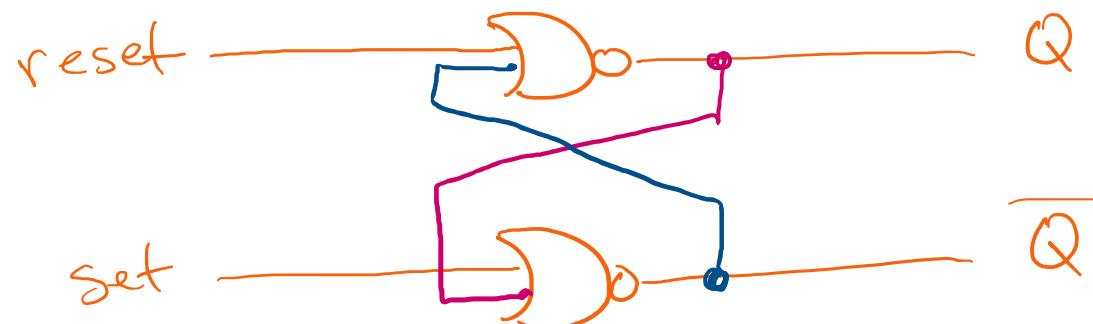
 = 1
 = 0



<u>reset</u>	<u>Set</u>	<u>Q</u>	<u>\bar{Q}</u>
0	1	1	0
0	0	0	1

SR Latch

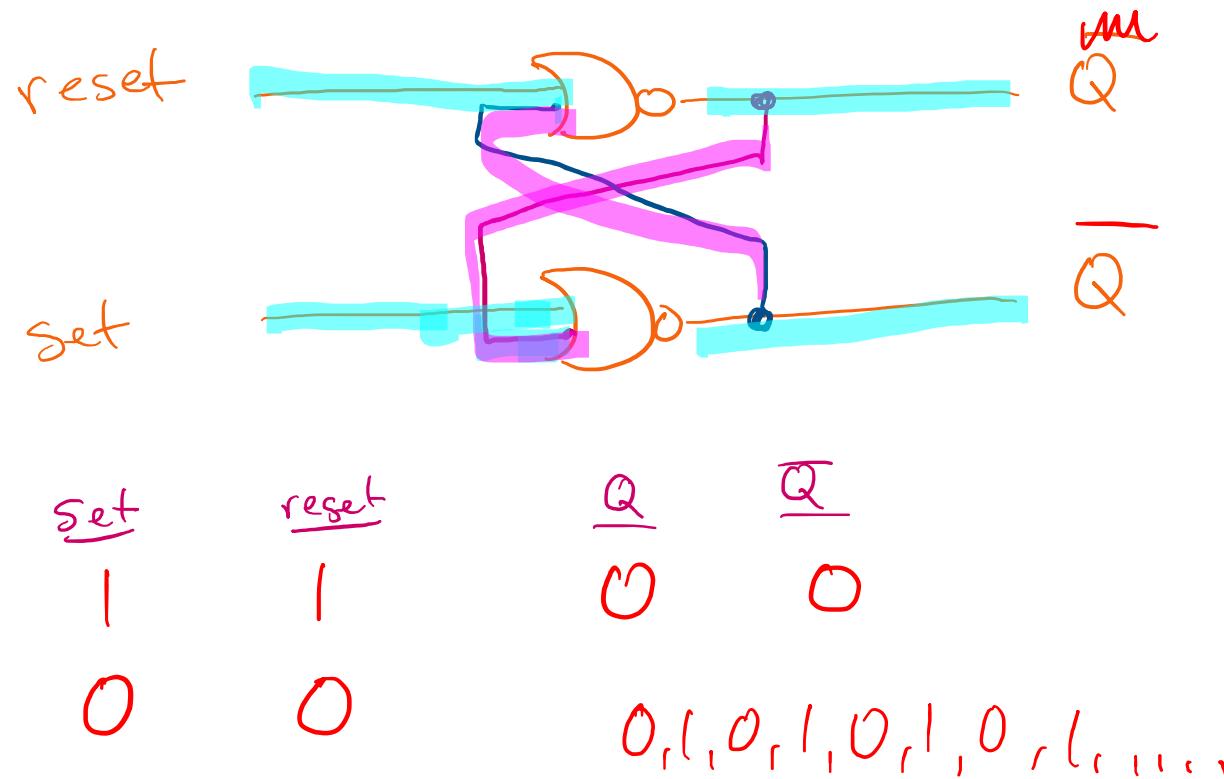
 = 1
 = 0



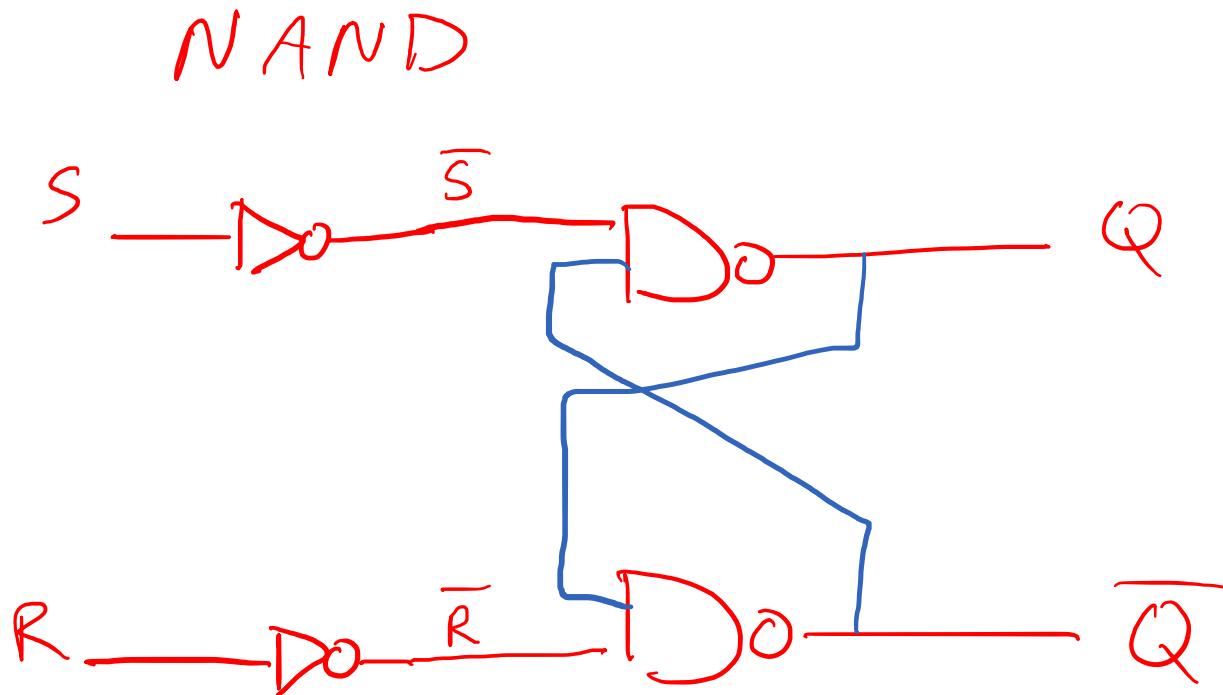
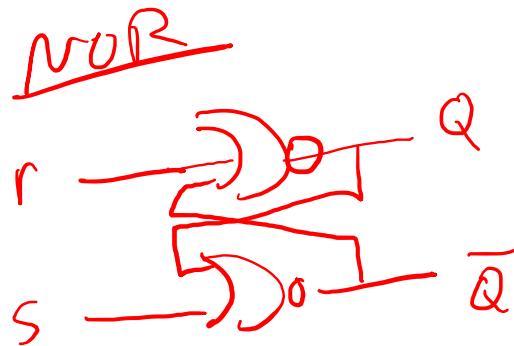
<u>reset</u>	<u>Set</u>	<u>Q</u>	<u>\bar{Q}</u>
0	1	1	0
0	0	1	0
1	0	0	1

SR Latch w/ S=1 & R=1

 = 1
 = 0



SR Latch w/NAND gates



→ better setup for ~~⇒~~ Flip-Flops

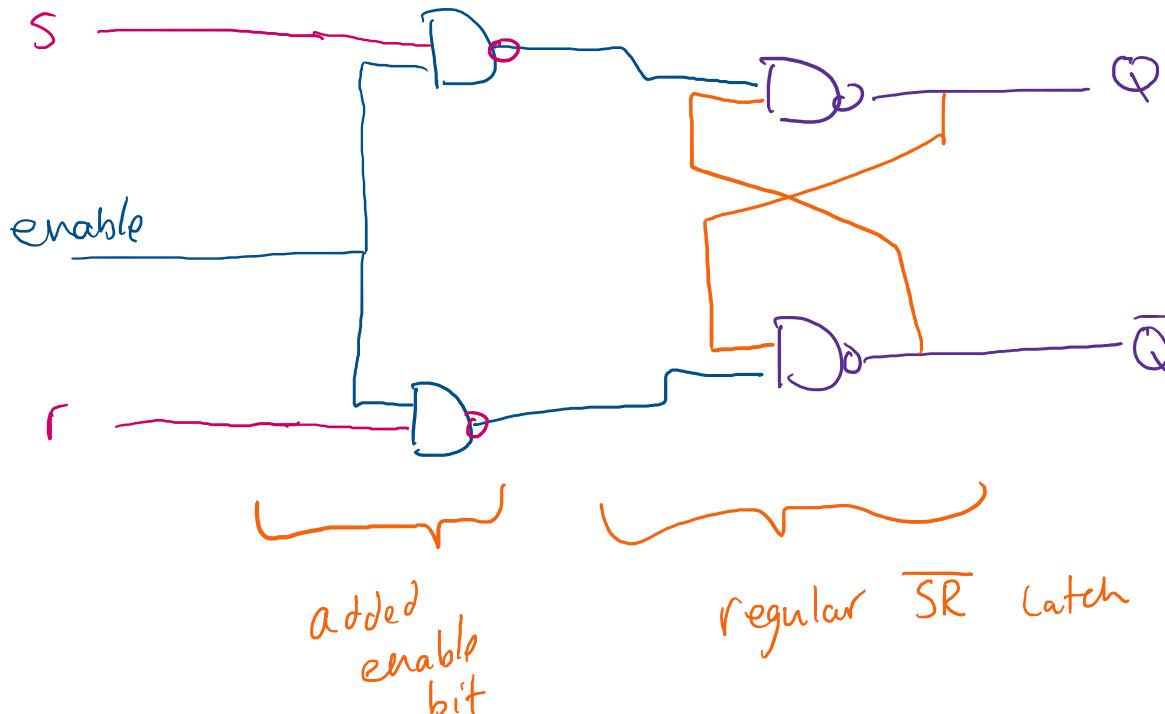
→ easier for me to draw

SR Latch w/NAND gates

SR Latch with Enable

SR Latch with Enable

Prevent changes in S & R from changing circuit output

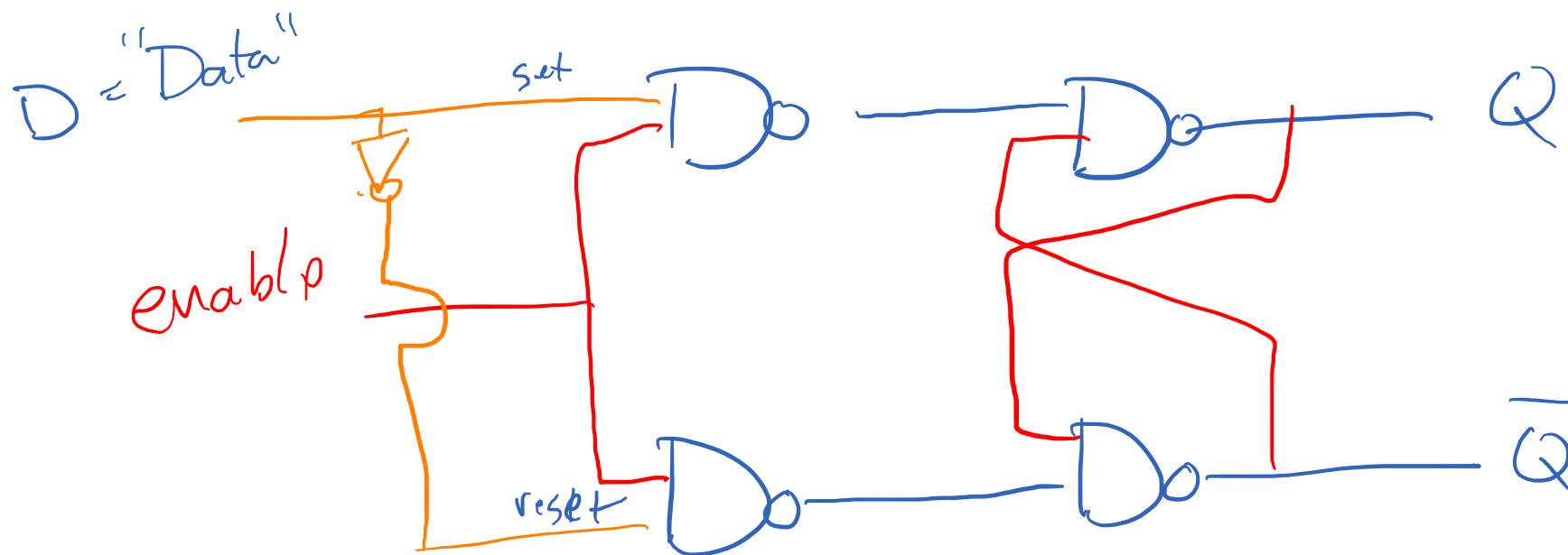


S	R	E	Q	\bar{Q}
x	x	0	Q	\bar{Q}
1	0	1	1	0
0	1	1	0	1

* assume
no $S=1$
 $r=1$

D-Latch

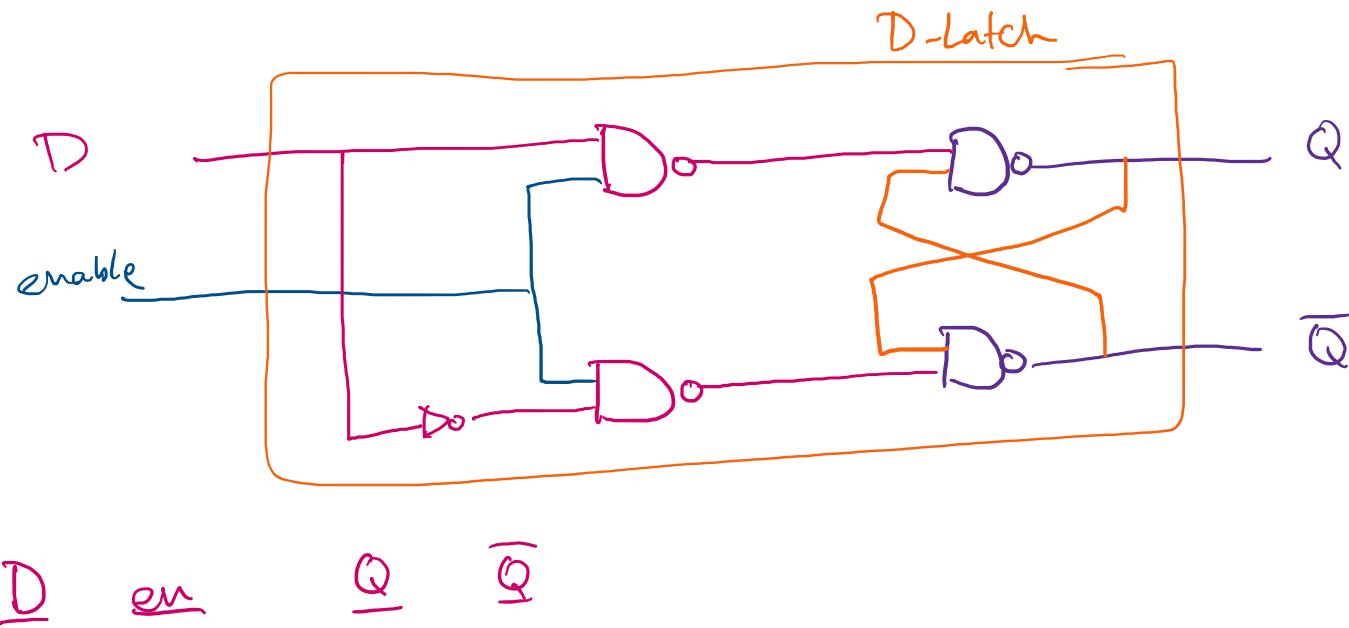
“Data” Latch



D-Latch

 = 1

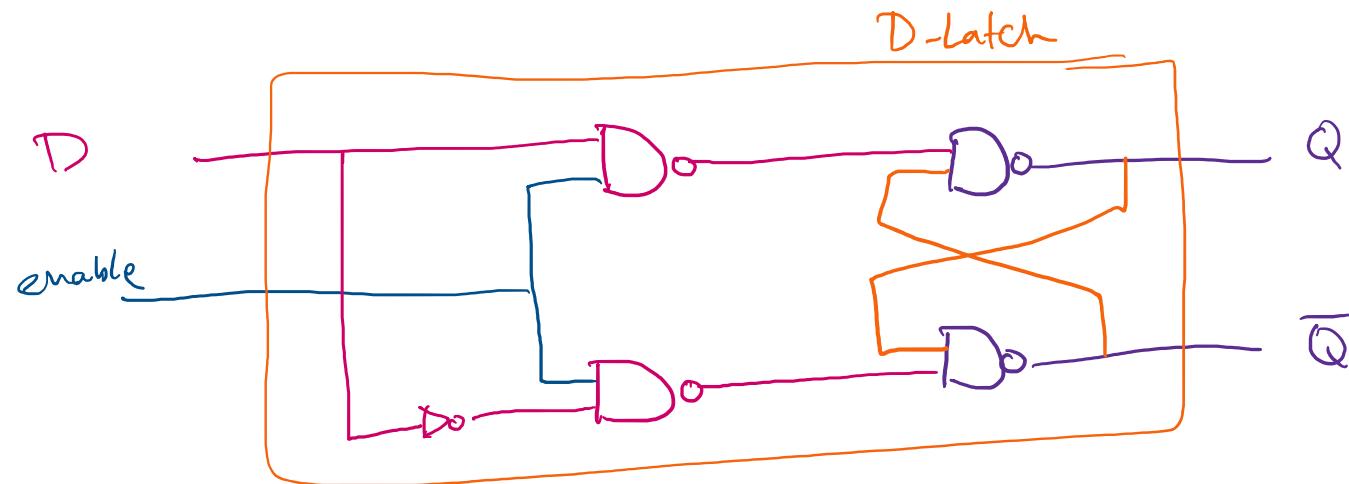
 = 0



D-Latch

 = 1

 = 0

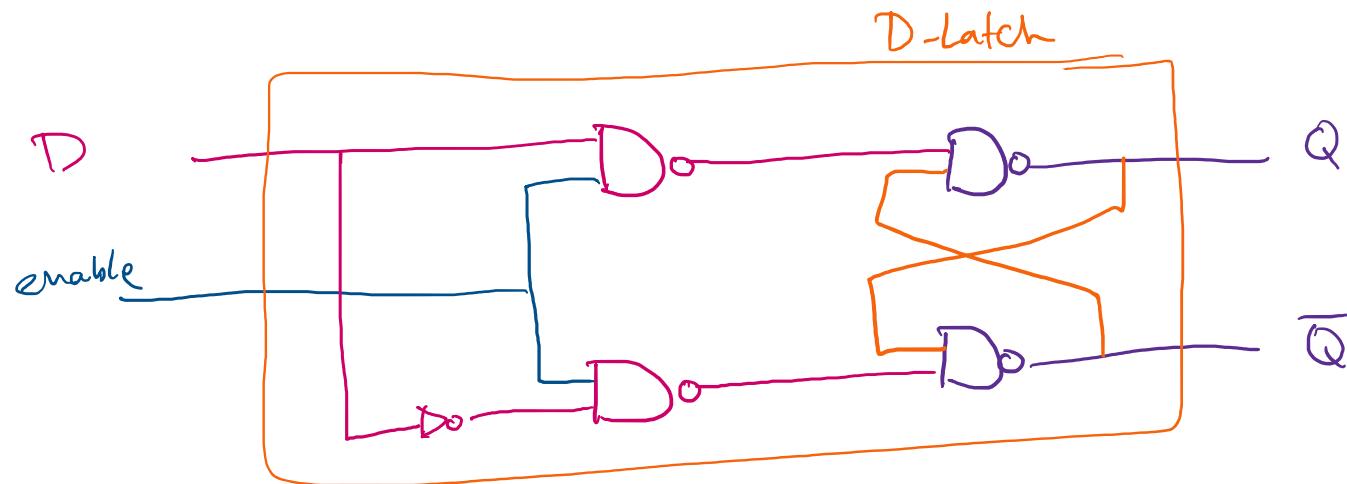


D	<u>en</u>	Q	\bar{Q}
-----	-----------	-----	-----------

D-Latch

 = 1

 = 0

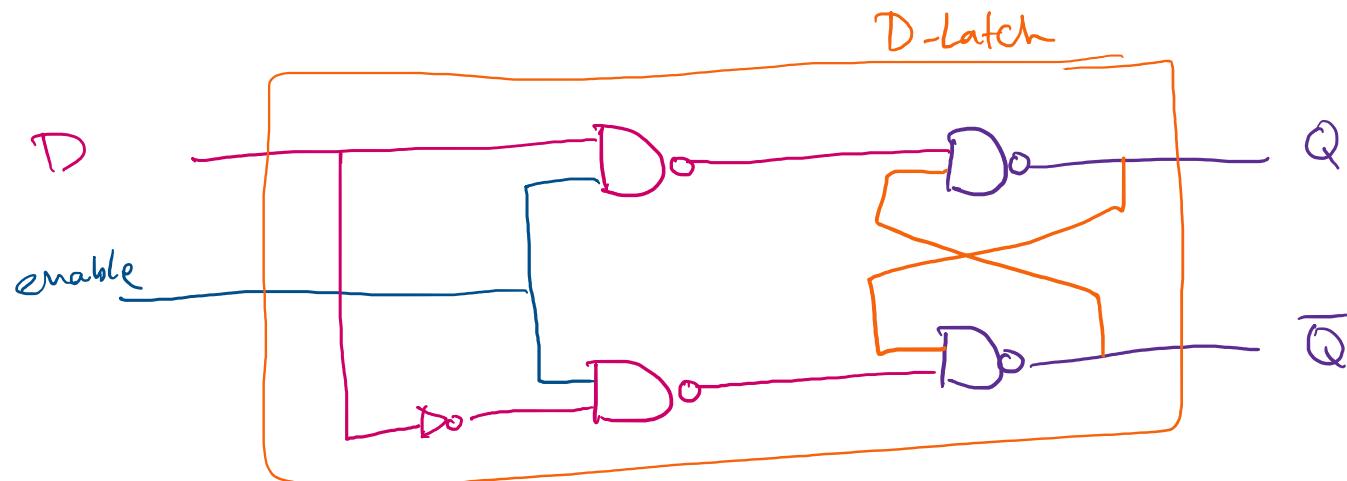


D	<u>en</u>	\underline{Q}	$\bar{\underline{Q}}$
0	0	Q	\bar{Q}
1	0	Q	\bar{Q}

D-Latch

 = 1

 = 0



D	<u>en</u>	\underline{Q}	$\bar{\underline{Q}}$
0	0	Q	\bar{Q}
1	0	\bar{Q}	Q
0	1	0	1

Next Time

- D Flip-Flops
- Latches / Flops in Verilog
- `always_comb` and `always_ff` blocks