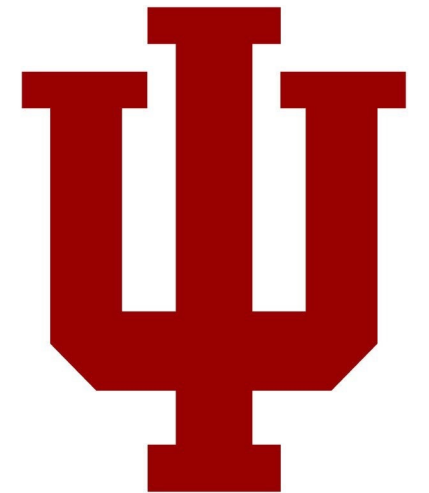Trey   Austin   Yifan   Clare

Grant   Chris   Max   Jack   Will   Michael

04 Cryptography II

@Andrew: assign groups!

Engr 399/599: Hardware Security
Andrew Lukefahr
Indiana University

Adapted from: Mark Tehranipoor of University of Florida

Course Website

engr599.github.io

Room

Passcode:

2-3-5

Write that down!

Last Time: Caesar Cypher Example

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z
D E F G H I J K L M N O P Q R S T U V W X Y Z A B C

HELLOWORLD ← plaintext

KHOOR ... ← cypher text

1-grams (Unigrams) for English

a	0.080	h	0.060	n	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c	0.030	j	0.005	p	0.020	v	0.010
d	0.040	k	0.005	q	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g	0.015					z	0.002

[cf. Barbara Endicott-Popovsky, U. Washington]

Polyalphabetic Substitution - Examples

- Example:

	A B C D E F G H I J K L M
Key1:	a d g j m p s v y b e h k
Key2:	n s x c h m r w b g l q v

	N O P Q R S T U V W X Y Z
Key1:	n q t w z c f i l o r u x
Key2:	a f k p u z e j o t y d i

- **Plaintext: TOUGH STUFF**
- **Ciphertext: ffirv zfjpm**

  use n (=2) keys in turn for consecutive P chars in P

- Note:
  - Different chars mapped into the same one: **T, O → f**
  - Same char mapped into different ones: **F → p, m**
  - '**f**' most frequent in C (0.30); in English: f(**f**) = 0.02 << f(**e**) = 0.13

Vigenere Tableaux (1)

Note: Row A – shift 0 (a->a)
Row B – shift 1 (a->b)
Row C – shift 2 (a->c)
…
Row Z – shift 25 (a->z)

[cf. J. Leiwo, VU, NL]

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	p
A	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	0
B	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	1
C	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	2
D	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	3
E	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	4
F	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	5
G	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	6
H	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	7
I	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	8
J	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	9
K	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	10
L	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	11
M	m	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	12
N	n	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	13
O	o	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	14
P	p	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	15
Q	q	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	16
R	r	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	17
S	s	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	18
T	t	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	19
U	u	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	20
V	v	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	21
W	w	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	22
X	x	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	23
Y	y	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	24
Z	z	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	25

Vigenère Tableaux (2)

- ## Example
  Key:
  EXODUS

  Plaintext P:
  YELLOW SUBMARINE FROM YELLOW RIVER

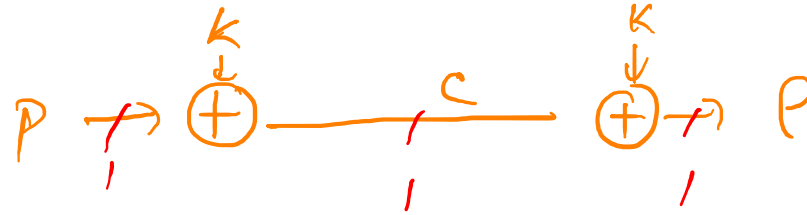 Extended keyword (re-applied to mimic words in P):
  YELLOW SUBMARINE FROM YELLOW RIVER

  EXODUS EXODUSEXO DUSE XODUSE XODUS

  Ciphertext:
  cbxoio wlppujmks ilgq vsofhb owyyj

Stream and Block Ciphers (1)

a. Stream ciphers

b. Problems with stream ciphers

c. Block ciphers

d. Pros / cons for stream and block ciphers

Stream Ciphers (1)

- ## Stream cipher: 1 char from P → 1 char for C
  - Example: polyalphabetic cipher
    - P and K (repeated 'EXODUS'):
      YELLOWSUBMARINEFROMYELLOWRIVER
      EXODUSEXODUSEXODUSEXODUSEXODUS

    - Encryption (char after char, using Vigenère Tableaux):
      (1) E(Y, E) → c   (2) E(E, X) → b   (3) E(L, O) → z . . .

    - C: **cbzoiowlppujmksilgqvsofhbowyyj**

    - C as sent (in the right-to-left order):

      jyywobhfosvqgliskmjupplwoiozbc

Sender
S

Receiver
R

Stream Ciphers (2)

– Example: polyalphabetic cipher - cont.

- C as received (in the right-to-left order):

<table>
<tr><td>Sender
S</td><td>**jyywobhfosvqgliskmjupplwoiozbc**</td><td>Receiver
R</td></tr>
</table>

- C and K for decryption:

  cbzoiowlppujmksilgqvsofhbowyyj
  EXODUSEXODUSEXODUSEXODUSEXODUS

- Decryption:
  (1) D(**c**, **E**) → **Y** (2) D(**b**, **X**) → **E**   (3) D(**z**, **O**) → **L** ...

- Decrypted P:
  YEL...

  Q: Do you know how D uses Vigenère Tableaux?

A: Finds c under column e → Y

Problems with Stream Ciphers (1)

- Problems with stream ciphers
    - Dropping a char from key K results in wrong decryption
    - Example:
        - P and K (repeated '`EXODUS`') with a char in K missing:

          `YELLOWSUBMARINEFROMYELLOWRIVER`

          `EODUSEXODUSEXODUSEXODUSEXODUSE`

          missing X in K ! (no errors in repeated K later)

        - Encryption

          (using VT):

          1) E(`Y`,`E`) → `c`

          2) E(`E`,`O`) → `s`

          3) E(`L`,`D`) → `o`

          . . .

        - Ciphertext: `cso...`

          C in the order as sent (right-to-left):

          `...osc`

Problems with Stream Ciphers (2)

- C as received (in the right-to-left order):

  `...osc`

  →

- C and correct K ('`EXODUS`') for decryption:

  `cso...`
  `EXO...`

- Decryption (using VT, applying correct key):

  1) D(`c`, `E`) → `Y`

  2) D(`s`, `X`) → `V`

  3) D(`o`, `O`) → `A`

  `...`

- Decrypted P:

  `YVA...` - Wrong!

  – We know it's wrong, Receiver might not know it *yet*!

> What if message is corrupted in a noisy area?

Problems with Stream Ciphers (3)

- The problem might be recoverable
  - Example:

    If R had more characters decoded, R might be able to detect that S dropped a key char, and R could recover

    - E.g., suppose that R decoded:

      YELLOW SUBMAZGTR

      - R could guess, that the 2nd word should really be:

        SUBMARINE

      - => R would know that S dropped a char from K after sending "**SUBMA**"

      - => R could go back 4 chars, drop a char from K ("recalibrate K with C"), and get "resynchronized" with S

Block Ciphers (1)

- We can do better than using recovery for stream ciphers
  - Solution: use block ciphers

- Block cipher:
  1 *block* of chars from P → 1 *block* of chars for C
  - Example of block cipher: columnar transposition
  - Block size = "o(message length)"   (informally)

Block Ciphers (2)

- Why block size = "o(message length)" ?
  - Because R must wait for "almost" the entire C before R can decode some characters near beginning of P
  - E.g., for P = 'HELLO WORLD', block size is "o(10)"
  - Suppose that Key = 3 (3 columns):

        HEL
        LOW
        ORL
        DXX

  - C as sent (in the right-to-left order):

        xlwlxroedolh

    Sender S ───────────────────▶ Receiver R

Block Ciphers (3)

- C as received (in the right-to-left order): **xlwlxroedolh**

- R knows: K = 3, block size = 12   (=> 4 rows)

$$\begin{array}{|l|} \hline \texttt{123} \\ \texttt{456} \\ \texttt{789} \\ \texttt{abc} \\ \hline \end{array}$$

a=10
b=11
c=12

=> R knows that characters wil be sent in the order:
  1st-4th-7th-10th--2nd-5th-8th-11th--3rd-6th-9th-12th

- R must wait for at least:
  - 1 char of C to decode 1st char of P ('**h**')
  - 5 chars of C to decode 2nd char of P ('**he**')
  - 9 chars of C to decode 3rd, 4th, and 5th chars of P ('**hello**')
  - 10 chars of C to decode 6th, 7th, and 8th chars of P ('**hello wor**')
  - etc.

Block Ciphers (4)

- *Informally*, we might call ciphers like the above example columnar transposition cipher "weak-block" ciphers

  - R can get some (even most) but not all chars of P before entire C is received

    - R can get one char of P immediately
      - » the 1st-after 1 of C (delay of 1 - 1 = 0)
    - R can get some chars of P with "small" delay
      - » e.g., 2nd-after 5 of C (delay of 5 - 2 = 3)
    - R can get some chars of P with "large" delay
      - » e.g., 3rd-after 9 of C (delay of 9 − 3 = 6)

- There are block ciphers when R cannot even start decoding C before receiving the entire C

  - *Informally*, we might call them "strong-block" ciphers

Pros / Cons for
Stream and Block Ciphers (1)

- Pros / cons for stream ciphers
  - + Low delay for decoding individual symbols
    - Can decode as soon as received
  - + Low error propagation
    - Error in $E(c_1)$ does not affect $E(c_2)$

  - - Low diffusion
    - Each char separately encoded => carries over its frequency info
  - - Susceptibility to malicious insertion / modification
    - Adversary can fabricate a new msg from pieces of broken msgs, even if he doesn't know E (just broke a few msgs)

- Pros / cons for block ciphers
  - + High diffusion
    - Frequency of a *char* from P diffused over (a few chars of) a *block* of C

  - + Immune to insertion
    - Impossible to insert a char into a block without easy detection (block size would change)
    - Impossible to modify a char in a block without easy detection (if checksums are used)

- Pros / cons for block ciphers — Part 2
  - - High delay for decoding individual chars
    - See example for '**hello worldxx**' above
      - For some E can't decode even the 1st char before whole k chars of a block are received

  - - High error propagation
    - It affects the block, not just a single char

Cryptanalysis (1)

- What cryptanalysts do when confronted with unknown?

  Four possible situations w.r.t. available info:
  1) C available
  2) Full P available
  3) Partial P available
  4) E available (or D available)

- (1) – (4) suggest 5 different approaches

Cryptanalysis (2)

- Cryptanalyst approaches

1) Ciphertext-only attack

    - We have shown examples for such attacks
      - E.g., for Caesar's cipher, columnar transposition cipher

2) Known plaintext attack

    - Analyst have C and P
      - Needs to deduce E such that C=E(P), then finds D

3) Probable plaintext attack

    - Partial decryption provides partial match to C
      - This provides more clues

Cryptanalysis (3)

- Cryptanalyst approaches – cont.

    4) Chosen plaintext attack

       - Analyst able to fabricate encrypted msgs
          - Then observe effects of msgs on adversary's actions
             » This provides further hints

    5) Chosen ciphertext attack

       - Analyst has both E and C
       - Run E for many candidate plaintexts to find P for which $E(P) = C$
          - Purpose: to find K_E

Symmetric and Asymmetric Cryptosystems (1)

- Symmetric encryption = secret key encryption
  - $K_E = K_D$ — called a secret key or a private key
  - Only sender S and receiver R know the key
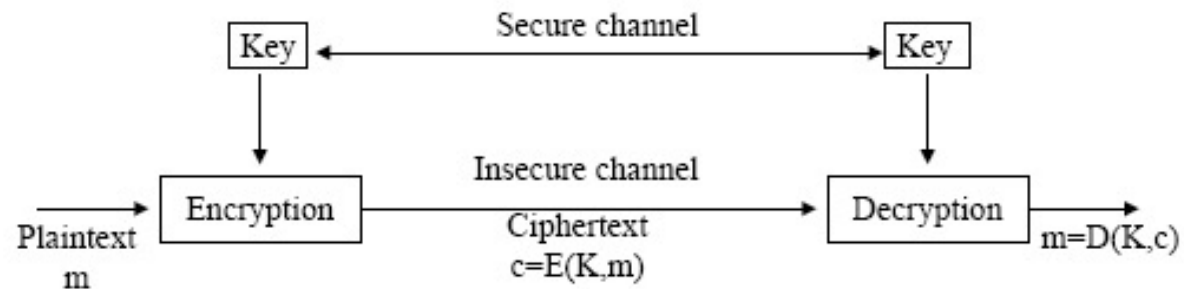
[cf. J. Leiwo]

  - As long as the key remains secret, it also provides authentication (= proof of sender's identity)

Symmetric and Asymmetric Cryptosystems (3)

- Asymmetric encryption = public key encryption (PKE)
  - $K_E \neq K_D$ — public and private keys

- PKE systems eliminate symmetric encryption problems
  - Need no secure key distribution channel
    - => easy key distribution

- One PKE approach:
  - R keeps her private key K_D

  - R can distribute the correspoding public key K_E to anybody who wants to send encrypted msgs to her
    - No need for secure channel to send K_E
    - Can even post the key on an open Web site — it is public!

  - Only private K_D can decode msgs encoded with public K_E!
    - Anybody (K_E is public) can encode
    - Only owner of K_D can decode

DES (Data Encryption Standard)

Background and History of DES (1)

- Early 1970's - NBS (Nat'l Bureau of Standards) recognized general public's need for a secure crypto system

    NBS – part of US gov't / Now: NIST – Nat'l Inst. of Stand's & Technology

    - "Encryption for the masses"   [A. Striegel]

    - Existing US gov't crypto systems were not meant to be made public
        - E.g. DoD, State Dept.

    - Problems with proliferation of commercial encryption devices
        - Incompatible
        - Not extensively tested by independent body

Background and History of DES (2)

- 1972 - NBS calls for proposals for a *public* crypto system
  - Criteria:
    - Highly secure / easy to understand / publishable / available to all / adaptable to diverse app's / economical / efficient to use / able to be validated / exportable
    - In truth: Not *too* strong (for NSA, etc.)

- 1974 – IBM proposed its Lucifer
  - DES *based* on it
  - Tested by NSA (Nat'l Security Agency) and the general public

- Nov. 1976 – DES adopted as US standard for *sensitive but unclassified* data / communication
  - Later adopted by ISO (Int'l Standards Organization)
  - Official name: DEA - Data Encryption Algorithm / DEA-1 abroad

Overview of DES

- DES - a block cipher
  - a product cipher
  - 16 rounds (iterations) on the input bits (of P)
    - substitutions (for confusion) and permutations (for diffusion)
  - Each round with a *round key*
    - Generated from the user-supplied key
- Easy to implement in S/W or H/W

- There are 72,000,000,000,000,000 (72 quadrillion) or more possible encryption keys that can be used.
- For each given message, the key can be chosen at random from among this enormous number of keys.

@ Andrew: Post DES verilog

Basic Structure

- Input: 64 bits (a block)
- Li/Ri– left/right half of the input block for iteration i (32 bits) – subject to substitution S and permutation P
- K - user-supplied key
- Ki - round key:
  - 56 bits used +8 unused

    (unused for E but often used for error checking)

- Output: 64 bits (a block)
- Note: Ri becomes L(i+1)
- All basic op's are simple logical ops
  - Left shift / XOR

Input

Input Permutation

L0   R0

S ← K

P

⊕

L1   R1 ← K1

L16   R16 ← K16

Final Permutation

Output

31
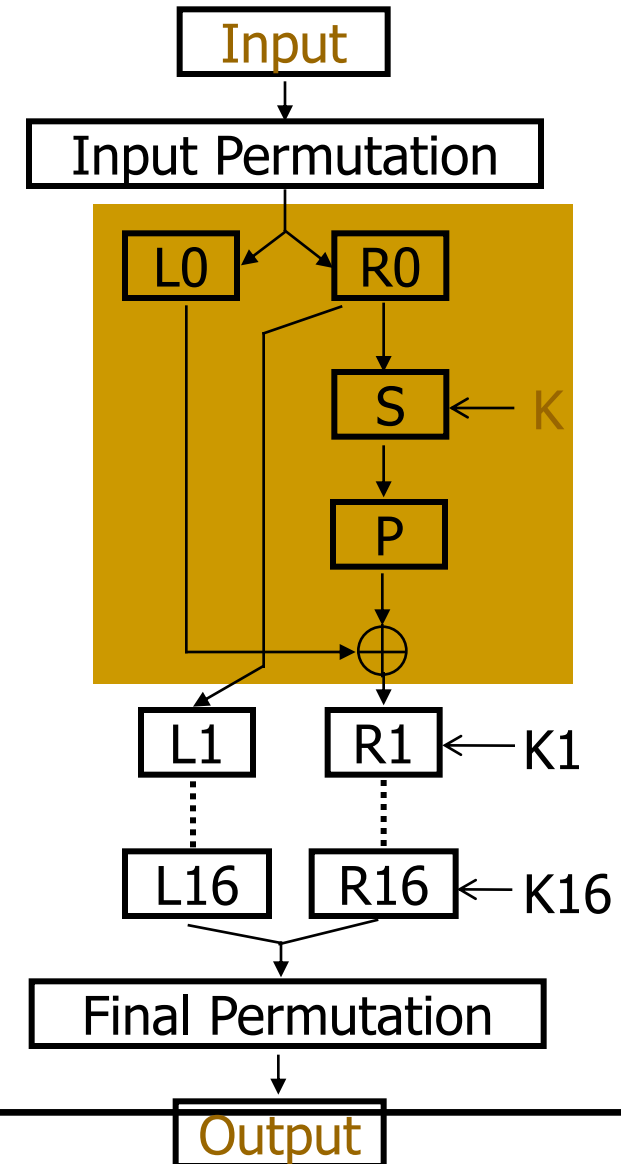
Generation of Round Keys
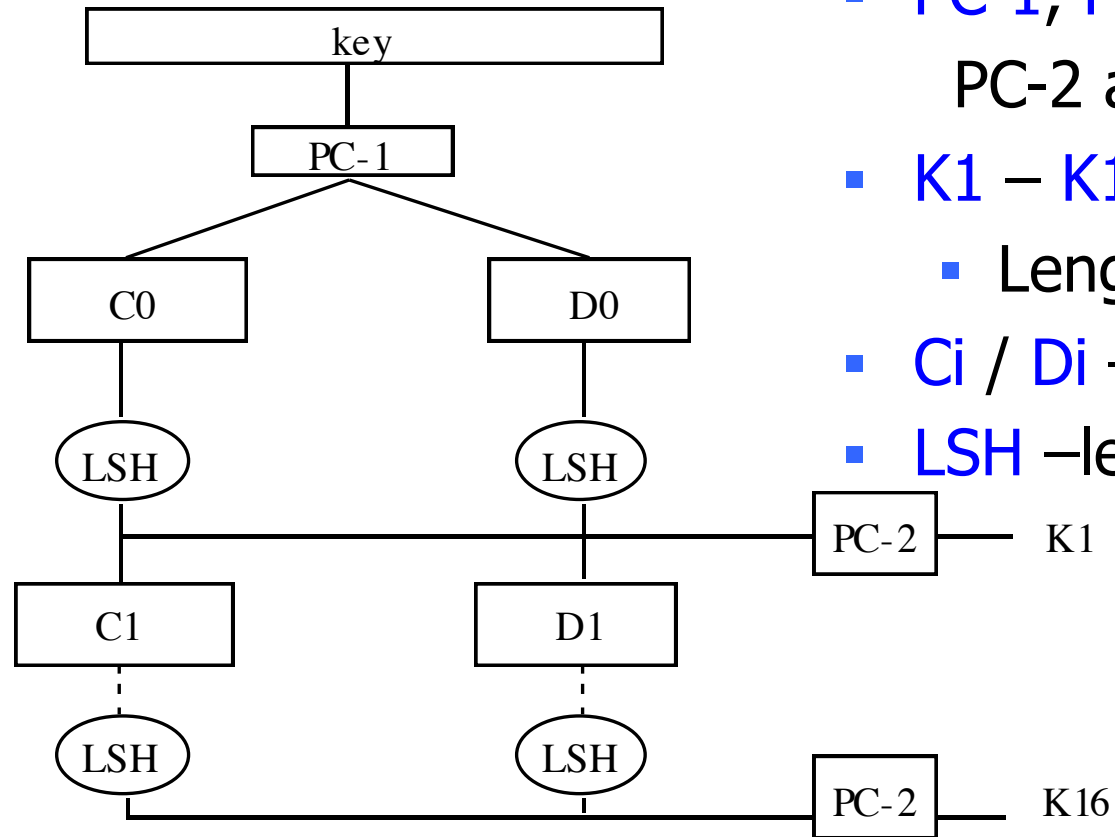
- key – user-supplied key (input)
- PC-1, PC-2 – permutation tables
    PC-2 also extracts 48 of 56 bits
- K1 – K16 – round keys (outputs)
  - Length(Ki) = 48
- Ci / Di – confusion / diffusion (?)
- LSH –left shift (rotation) tables
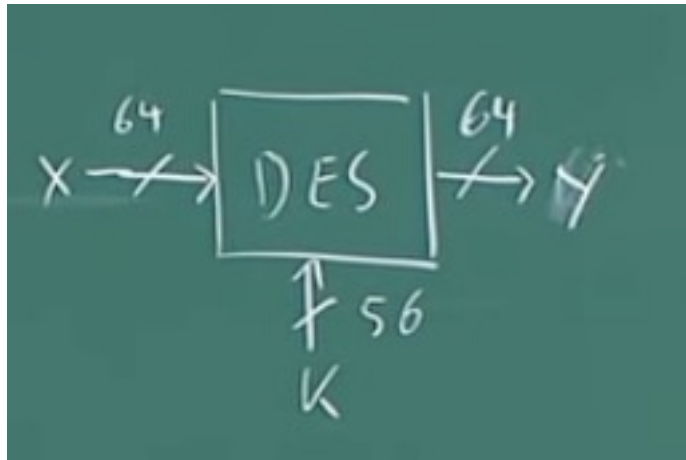
[Fig: cf. Barbara Endicott-Popovsky, U. Washington]

many times to build a
strong block cipher

"product cipher"
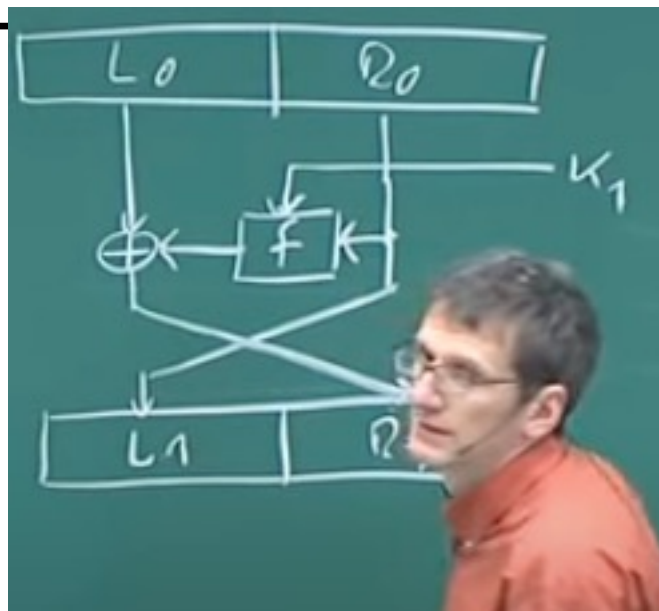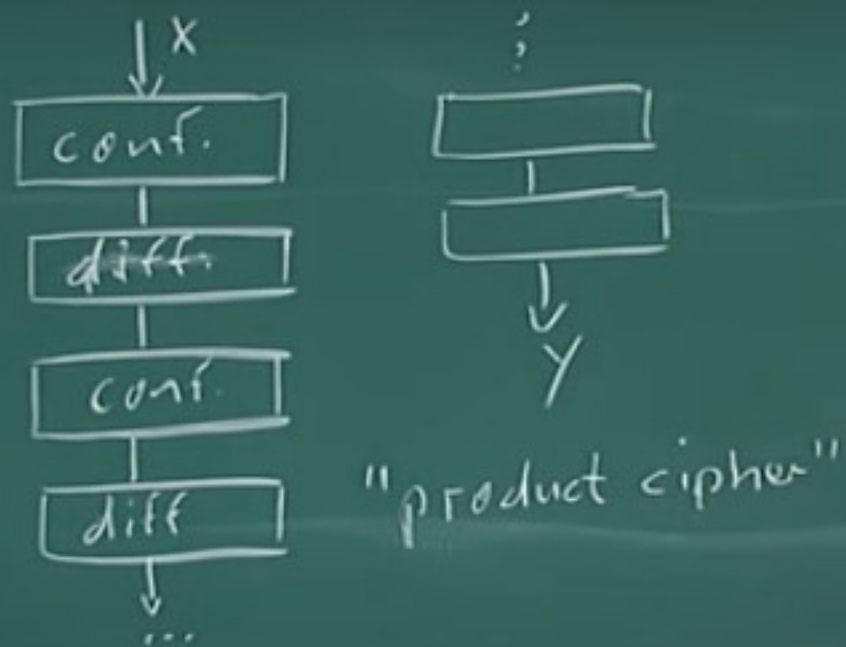
Problems with DES

- Diffie, Hellman 1977 prediction: "In a few years, technology would allow DES to be broken in days."

- Key length is fixed (= 56)
  - 2^{56} keys ~ 10^{15} keys
  - "Becoming" too short for faster computers
    - 1997: 3,500 machines – 4 months
    - 1998: special "DES cracker" h/w – 4 days

- Design decisions not public
  - Suspected of having backdoors
    - Speculation: To facilitate government access?

Double and Triple DES

- Double DES:
    - Use double DES encryption

        C = E(k2, E(k1, P))

    - Expected to multiply difficulty of breaking the encryption
        - Not true!
            - In general, 2 encryptions are not better than one

                [Merkle, Hellman, 1981]

        - Only doubles the attacker's work

Double and Triple DES (2)

- Triple DES:
  - Is it C = E(k3, E(k2, E(k1, P))) ?

  - Not soooo simple!

Double and Triple DES (3)

- Triple DES: *Is it C=E(k3, E(k2, E(k1, P))?*
  - Tricks used:
    D not E in the 2nd step, k1 used twice (in steps 1 & 3)
  - It is:

    C = E(k1, D(k2, E(k1, P)))

    and

    P = D(k1, E(k2, D(k1, C))

  - Doubles the effective key length
    - 112-bit key is quite strong
      - Even for today's computers
      - For all feasible known attacks

Security of DES

- So, is DES insecure?

- No, not yet
  - 1997 attack required a lot of cooperation
  - The 1998 special-purpose machine is still very expensive
  - Triple DES still beyond the reach of these 2 attacks

- But …
  - In 1995, NIST (formerly NBS) began search for new strong encryption standard

The AES Contest (1)

- 1997 – NIST calls for proposals NIST (Nat'l Institute of Standards and Technology)

  - Criteria:

    - Unclassifed code

    - Publicly disclosed

    - Royalty-free worldwide

    - Symmetric block cipher for 128-bit blocks

    - Usable with keys of 128, 192, and 256 bits

- 1998 – 15 algorithms selected

The AES Contest (2)

- 1999 – 5 finalists   [cf. J. Leiwo]
  - MARS by IBM
  - RC6 by RSA Laboratories
  - Rijndael (RINE-dahl) by Joan Daemen and Vincent Rijmen
  - Serpent by Ross Anderson, Eli Biham and Lars Knudsen
  - Twofish by Bruce Schneier, John Kelsey, Doug Whiting, Dawid Wagner, Chris Hall and Niels Ferguson

- Evaluation of finalists
  - Public and private scrutiny
  - Key evaluation areas:

    security / cost or efficiency of operation /

    ease of software implementation

The AES Contest (3)

- 2001- … and the winner is …

  Rijndael   (RINE-dahl)

  Authors: Vincent Rijmen + Joan Daemen (Dutchmen)

- Adopted by US gov't as

  Federal Info Processing Standard 197 (FIPS 197)

Overview of Rijndael/AES

- Similar to DES – cyclic type of approach
  - 128-bit blocks of P
  - # of iterations based on key length
    - 128-bit key => 9 "rounds" (called rounds, not cycles)
    - 192-bit key => 11 rounds
    - 256-bit key => 13 rounds

- Basic ops for a round:
  - Substitution – byte level (confusion)
  - Shift row (transposition) – depends on key length (diff.)
  - Mix columns – LSH and XOR (confusion +diffusion)
  - Add subkey – XOR used (confusion)

Strengths of AES

- Extensive cryptanalysis by US gov't and independent experts
- Dutch inventors have no ties to NSA or other US gov't bodies   (less suspicion of trapdoor)
- Solid math basis
  - Despite seemingly simple steps within rounds

Comparison of DES & AES (1)

	DES	AES
Date	1976	1999
Block size [bits]	64	128
Key length [bits]	56 (effect.)	128, 192, 256, or more
Encryption Primitives	substitution, permutation	substitution, shift, bit mixing
Cryptographic Primitives	confusion, diffusion	confusion, diffusion
Design	open	open
Design Rationale	closed	open
Selection process	secret	secret, but accepted public comments
Source	IBM, enhanced by NSA	independent Dutch cryptographers

Comparison of DES & AES (2)

- Weaknesses in AES?
  - 20+ yrs of experience with DES eliminated fears of its weakness (intentional or not)
    - Might be naïve…
  - Experts pored over AES for 2-year review period

Comparison of DES & AES (3)

- Longevity of AES?
  - DES is nearly 40 yrs old (1976)
    - DES-encrypted message can be cracked in days

  - Longevity of AES more difficult to answer
    - Can extend key length to > 256 bits   (DES: 56)
      - 2 * key length => 4 * number of keys
    - Can extend number of rounds   (DES: 16)

  - Extensible AES seems to be significantly better than DES, but..
    - Human ingenuity is unpredicatble!
    - => Need to incessantly search for better and better encryption algorithms

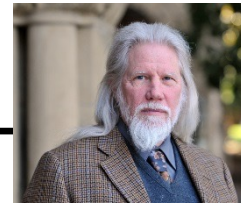# Motivation for PKE (1)

- So far - cryptosystems with secret keys

- Problems:
  - A lot of keys
    - $o(n^2)$ keys for n users   (n * (n-1) /2 keys)
      - — if each must be able to communicate with each
  - Distributing so many keys securely
  - Secure storage for the keys
    - User with n keys can't just memorize them

- Can have a system with significantly fewer keys?
    Yes!

Motivation for PKE (2)

- 1976 — Diffie and Hellman — new kind of cryptosystem:

public key cryptosystem = asymmetric cryptosystem

- – Key pairs: $< k_{PRIVATE}, k_{PUBLIC} >$
- – Each user owns one private key
- – Each user shares the corresponding public key with n-1 remaining users   => n users share each public key
- – Only 2n keys for n users $2n = n * (1 + n * 1/n)$
  - » Since public key is shared by n people: 1 "owner" + (n-1) others = n
  - » 1/n since each part "owns" 1/n of the public key

- Even if each communicates with each
- Reduction from $o(n^2)$ to $o(n)$!
- n key pairs are:

$$< k_{PRIV-1}, k_{PUB-1} >, < k_{PRIV-2}, k_{PUB-2} >, ..., < k_{PRIV-n}, k_{PUB-n} >$$

Characteristics of PKE (1)

- PKE requirements
  1. It must be computationally easy to encipher or decipher a message given the appropriate key

  2. It must be computationally infeasible to derive k_{PRIV} from k_{PUB}

  3. It must be computationally infeasible to determine k_{PRIV} from a chosen plaintext attack

Characteristics of PKE (2)

- Key pair characteristics

  - One key is inverse of the other key of the pair
    - i.e., it can undo encryption provided by the other:
      - $D(k_{PRIV}, E(k_{PUB}, P)) = P$
      - $D(k_{PUB}, E(k_{PRIV}, P)) = P$

  - One of the keys can be public since each key does only half of E "+" D
    - As shown above – need both E and D to get P back

Characteristics of PKE (3)

- Two E/D possibilities for key pair $<k_{PRIV}, k_{PUB}>$
  - $P = D(k_{PRIV}, E(k_{PUB}, P))$
    - User encrypts msg with k_{PUB} (k_{PUB} "locks")
    - Recipient decrypts msg with k_{PRIV} (k_{PRIV} "unlocks")

  OR
  - $P = D(k_{PUB}, E(k_{PRIV}, P))$ (e.g., in RSA)
    - User encrypts msg with k_{PRIV} (k_{PRIV} "locks")
    - Recipient decrypts msg with key k_{PUB} (k_{PUB} "unlocks")

- Do we still need symmetric encryption (SE) systems?
  - Yes, PKEs are 10,000+ times (!) slower than SEs
    - PKEs use exponentiation – involves multiplication and division
    - SEs use bit operations (add,XOR< substitute, shift)–much faster

RSA Encryption (1)

- RSA = Rivest, Shamir, and Adelman (MIT), 1978
- **RSA** is one of the first practical public-key cryptosystems and is widely used for secure data transmission.

- Underlying hard problem:
  - Number theory – determining prime factors of a given (large) number (ex. factoring of small #: 5 → 5, 6 → 2 *3)
  - Arithmetic modulo n

- How secure is RSA?
  - So far remains secure (after all these years...)
  - Will quantum computing break it?   TBD

RSA Encryption (2)

- In RSA:

  P = E (D(P)) = D(E(P))   (order of D/E does not matter)

  - More precisely: $P = E(k_E, D(k_D, P)) = D(k_D, E(k_E, P))$

- Encryption: $C = P^e \bmod n$ $K_E = e$

  - Given C, it is very difficult to find P without knowing K_D

- Decryption: $P = C^d \bmod n$ $K_D = d$