02_prob2

September 6, 2025

1 Problem 2: Y-Chromosome Threshold Analysis - Implementation

Objective: Analyze the relationship between maternal BMI and timing of Y-chromosome concentration reaching 4% threshold using interval-censored survival analysis.

Key Approach: Use Accelerated Failure Time (AFT) models with proper interval censoring to handle multiple tests per mother and determine optimal testing weeks.

1.1 Implementation Plan

This notebook implements the complete analysis following the detailed implementation guide:

1. **Section 1**: Event Interval Construction (Per Mother)
2. **Section 2**: Feature Matrix Preparation
3. **Section 3**: AFT Modeling & Inference (Core Analysis)
4. **Section 4**: Turnbull Non-parametric Validation
5. **Section 5**: BMI Grouping & Group-specific Optimal Weeks
6. **Section 6**: Monte Carlo Measurement Error Testing
7. **Section 7**: Baseline Two-step ML Comparison (Optional)
8. **Section 8**: Validation & Final Policy Table

[1]:
```
# Import libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
from pathlib import Path
import warnings
import sys
import re
from scipy import stats

# Core survival analysis
from lifelines import WeibullAFTFitter, LogLogisticAFTFitter, KaplanMeierFitter

# Machine learning
from sklearn.tree import DecisionTreeRegressor
from sklearn.model_selection import KFold
```

```python
from sklearn.linear_model import LogisticRegression
from sklearn.ensemble import RandomForestClassifier, RandomForestRegressor
from sklearn.metrics import roc_auc_score

# Add project root to Python path for imports
PROJECT_ROOT = Path("/home/richard/projects/cumcm")
if str(PROJECT_ROOT) not in sys.path:
    sys.path.insert(0, str(PROJECT_ROOT))

# Import our custom modules
from src.analysis.problem2 import (
    apply_qc_filters,
    construct_intervals,
    prepare_feature_matrix,
    perform_group_specific_analysis,
    create_group_optimal_weeks_summary,
    run_monte_carlo_robustness_test,
    create_monte_carlo_summary_table,
    run_ml_baseline_comparison,
    run_comprehensive_validation
)
from src.models.aft_models import (
    fit_aft_models,
    display_model_summary,
    predict_survival_curves,
    calculate_optimal_weeks,
    fit_turnbull_estimator,
    compare_aft_vs_turnbull,
    assess_aft_goodness_of_fit
)

# Configure environment
warnings.filterwarnings('ignore')
plt.style.use('seaborn-v0_8')
sns.set_palette("husl")
pd.set_option('display.max_columns', 50)
pd.set_option('display.width', 1000)

print("  Libraries imported successfully")
print("  Survival analysis tools ready")
print("  Machine learning tools ready")
```

```
 Libraries imported successfully
 Survival analysis tools ready
 Machine learning tools ready
```

[2]:
```
# Setup paths
DATA_PATH = PROJECT_ROOT / "src" / "data" / "data.xlsx"
OUTPUT_PATH = PROJECT_ROOT / "output"
OUTPUT_DATA_PATH = OUTPUT_PATH / "data"
OUTPUT_FIGURES_PATH = OUTPUT_PATH / "figures"
OUTPUT_RESULTS_PATH = OUTPUT_PATH / "results"

# Create output directories
OUTPUT_DATA_PATH.mkdir(parents=True, exist_ok=True)
OUTPUT_FIGURES_PATH.mkdir(parents=True, exist_ok=True)
OUTPUT_RESULTS_PATH.mkdir(parents=True, exist_ok=True)

print(f"  Paths configured - Data: {DATA_PATH}")
print(f"  Output paths ready")

# Load original male fetus data for interval construction
print("  Loading original male fetus data...")
male_data = pd.read_excel(DATA_PATH, sheet_name='    ')
print(f"  Loaded {len(male_data)} rows and {len(male_data.columns)} columns")

print("\n  Data structure preview:")
print(f"   Columns: {list(male_data.columns[:10])}...")
print(f"   Maternal IDs: {male_data['   '].nunique()} unique mothers")
print(f"   Total tests: {len(male_data)} test records")
```

```
 Paths configured - Data: /home/richard/projects/cumcm/src/data/data.xlsx
 Output paths ready
 Loading original male fetus data…
 Loaded 1082 rows and 31 columns

 Data structure preview:
  Columns: [' ', '   ', ' ', ' ', ' ', '   ', 'IVF ', '   ', '    ',
'   ']…
  Maternal IDs: 267 unique mothers
  Total tests: 1082 test records
```

1.2 Section 1: Event Interval Construction (Per Mother)

Goal: Convert multiple test records per mother into single interval-censored event records.

Event Definition: Y-chromosome concentration first reaches 4% threshold

Censoring Types: 1. **Left-censored**: First observation already 4% → Event occurred before first test 2. **Interval-censored**: Threshold crossed between visits → Event in interval (L, R] 3. **Right-censored**: Never reached threshold → Event beyond last observation

[3]:
```
### Step 1.1: Data Preparation and QC Filtering

# Apply same QC as preprocessing to get df with individual tests
```

```python
print("  Applying QC filters from preprocessing...")
df_tests, filter_stats = apply_qc_filters(male_data, verbose=True)

print(f"\n  Individual test dataset ready:")
df_tests.head(10)
```

```
 Applying QC filters from preprocessing…
 Parsing variables and applying QC filters…
 Variable parsing completed:
  Gestational weeks: 1082/1082 valid
  BMI: 1082/1082 valid
  Y concentration: 1082/1082 valid
  After gestational weeks filter (10-25): 1069 records
  After GC content filter (40-60%): 620 records
  After chromosome abnormality filter: 556 records
  After missing data filter: 556 records
 Applying IQR outlier detection…
  gestational_weeks:
    IQR outliers: 0 (0.00%) - bounds: [3.500, 29.214]
    After IQR filtering: 556 records (removed 0)
  bmi:
    IQR outliers: 13 (2.34%) - bounds: [24.982, 39.214]
    After IQR filtering: 543 records (removed 13)
  y_concentration:
    IQR outliers: 5 (0.92%) - bounds: [-0.017, 0.168]
    After IQR filtering: 538 records (removed 5)

 QC filtering completed: 538 records remaining (49.7% retention)
  Unique mothers: 238
  Tests per mother: 2.3 average

 Individual test dataset ready:
```

[3]:

	maternal_id	gestational_weeks	bmi	y_concentration
0	A002	13.857143	33.331832	0.059230
1	A003	13.000000	30.742188	0.065185
2	A003	20.285714	31.882812	0.052253
3	A004	11.000000	28.641243	0.049498
4	A004	15.857143	28.641243	0.066800
5	A004	23.571429	29.161993	0.082347
6	A005	15.285714	30.844444	0.081533
7	A006	12.142857	35.883634	0.069469
8	A007	12.285714	33.874064	0.021902
9	A007	16.000000	33.874064	0.038038

[4]:

```python
### Step 1.2: Interval Construction Logic

# Generate interval-censored observations using module function
```

```python
df_intervals = construct_intervals(df_tests, threshold=0.04, verbose=True)
```

```
Constructing interval-censored observations…
Interval construction completed: 238 mothers

Censoring type distribution:
 left: 203 (85.3%)
 interval: 22 (9.2%)
 right: 13 (5.5%)
```

[5]:
```python
### Step 1.3: Output Analysis

print(f"\n  Sample intervals:")
print(df_intervals.head(10)[['maternal_id', 'bmi', 'L', 'R', 'censor_type',␣
 ↪'n_tests', 'max_y_concentration']])
```

```

 Sample intervals:
  maternal_id        bmi          L          R censor_type  n_tests
max_y_concentration
0        A002  33.331832   0.000000  13.857143        left        1
0.059230
1        A003  30.742188   0.000000  13.000000        left        2
0.065185
2        A004  28.641243   0.000000  11.000000        left        3
0.082347
3        A005  30.844444   0.000000  15.285714        left        1
0.081533
4        A006  35.883634   0.000000  12.142857        left        1
0.069469
5        A007  33.874064  23.714286        inf       right        4
0.038038
6        A008  29.136316   0.000000  13.000000        left        2
0.060891
7        A009  33.333333   0.000000  13.857143        left        3
0.159267
8        A010  33.333333  20.857143        inf       right        2
0.031946
9        A011  36.250470  16.142857  20.571429    interval        4
0.068983
```

1.3 Section 2: Feature Matrix Preparation

Goal: Prepare covariate matrix for AFT modeling with proper standardization and quality checks.

[6]:
```python
### Step 2.1: Standardization and Feature Matrix Creation

# Create feature matrix using module function
```

```python
df_X = prepare_feature_matrix(df_intervals, verbose=True)
```

```
Preparing feature matrix for AFT modeling…
BMI standardization completed:
 Mean BMI: 31.88
 Std BMI: 2.59
 BMI range: 26.6 - 39.2
All intervals are valid (L < R)

Feature matrix summary:
 Observations: 238
 Features: bmi, bmi_z
 Interval bounds: L, R
 Censoring types: ['left' 'right' 'interval']

Dataset statistics:
```

	bmi	bmi_z	L	R
count	238.000000	2.380000e+02	238.000000	238.000000
mean	31.879303	-5.747037e-16	2.337335	inf
std	2.587130	1.000000e+00	5.824886	NaN
min	26.619343	-2.033125e+00	0.000000	11.000000
25%	29.917696	-7.582176e-01	0.000000	12.571429
50%	31.632725	-9.530933e-02	0.000000	13.428571
75%	33.453780	6.085806e-01	0.000000	16.142857
max	39.159843	2.814138e+00	24.285714	inf

```
Quality checks:
 Missing values: 0 total
 Interval validity check:
   Finite intervals: 225
   Right-censored: 13
   Left-censored: 203

Summary by censoring type:
```

	bmi			L	R
	count	mean	std	mean	<lambda>
censor_type					
interval	22	32.40	3.16	14.31	19.62
left	203	31.74	2.52	0.00	14.00
right	13	33.22	2.22	18.58	NaN

```
Feature matrix preparation completed - ready for AFT modeling
```

[7]:
```python
### Step 2.2: Quality Checks

# Quality checks are included in the prepare_feature_matrix function above
```

```python
print("  Feature matrix preparation completed with quality checks - ready for␣
↪AFT modeling")
```

```
 Feature matrix preparation completed with quality checks - ready for AFT
modeling
```

1.4 Section 3: AFT Modeling & Inference (Core Analysis)

Goal: Fit Accelerated Failure Time models to interval-censored data and derive BMI-specific survival functions.

Models: - Primary: Weibull AFT (log T = ’x + , ~ Gumbel) - Alternative: Log-logistic AFT (~ Logistic)

Key Output: Survival functions S(t|x) for calculating optimal testing weeks

[8]:
```python
### Step 3.1: Model Fitting

# Fit AFT models using module function
primary_model, primary_name, all_models = fit_aft_models(df_X, formula='~␣
↪bmi_z', verbose=True)
```

```
 Fitting AFT models to interval-censored data…

 Fitting Weibull AFT model…
 Weibull AFT model fitted successfully

 Fitting Log-logistic AFT model…
 Log-logistic AFT model fitted successfully

 Using Weibull AFT as primary model
```

[9]:
```python
### Step 3.2: Model Summary & Diagnostics

# Display model summary and diagnostics using module function
display_model_summary(primary_model, primary_name, verbose=True)
```

```
 Weibull AFT Model Summary:
==================================================
                       coef  exp(coef)  se(coef)  coef lower 95%  coef upper 95%
exp(coef) lower 95%  exp(coef) upper 95%  cmp to          z             p
-log2(p)
param   covariate
lambda_ Intercept  2.018579   7.527618  0.146708        1.731035        2.306122
5.646497             10.035431      0.0  13.759117  4.489752e-43  140.676272
        bmi_z      0.167024   1.181783  0.072897        0.024149        0.309899
1.024443             1.363288      0.0   2.291235  2.194982e-02    5.509647
rho_    Intercept  0.122470   1.130286  0.179968       -0.230260        0.475201
0.794327             1.608338      0.0   0.680512  4.961806e-01    1.011063
```

```
Model Parameters:
 ('lambda_', 'Intercept'): 2.0186
 ('lambda_', 'bmi_z'): 0.1670
 ('rho_', 'Intercept'): 0.1225

BMI Effect Analysis:
 BMI coefficient (standardized): 0.1670
 P-value: 0.0219
 Statistical significance: significant
 Interpretation: Higher BMI delays time to Y 4% threshold
 AFT interpretation: 1-unit BMI increase → 16.7% change in log-time scale

Model Fit Statistics:
 AIC: 253.54
 Log-likelihood: -123.77
```

[10]:

```python
### Step 3.3: Survival Function Prediction

# Generate survival curves using module function
time_grid = np.linspace(10, 25, 100)  # Gestational weeks 10-25
survival_curves = predict_survival_curves(
    primary_model,
    df_intervals,
    time_grid,
    quartiles=[0.25, 0.5, 0.75],
    verbose=True
)
```

```
Generating survival curves for different BMI levels…

BMI quartiles for analysis:
 Q25: 29.9
 Q50: 31.6
 Q75: 33.5

Computing survival functions…
   Q25: BMI 29.9 (z=-0.76)
   Q50: BMI 31.6 (z=-0.10)
   Q75: BMI 33.5 (z=0.61)

Survival Probabilities at Key Weeks:
Week  BMI_Q25     BMI_Q50     BMI_Q75
----------------------------------------
12    0.142       0.179       0.222
14    0.099       0.130       0.167
16    0.066       0.091       0.123
18    0.045       0.065       0.091
20    0.031       0.046       0.068
```

```
Survival function prediction completed for 3 BMI levels
```

[11]:
```python
### Step 3.4: Optimal Testing Week Calculation

# Calculate optimal testing weeks using module function
optimal_weeks_results = calculate_optimal_weeks(
    survival_curves,
    confidence_levels=[0.90, 0.95],
    verbose=True
)
```

```
Calculating optimal testing weeks for different confidence levels…

Optimal weeks for 90% confidence level:
 (When  90% of mothers have reached Y 4% threshold)
------------------------------------------------------------
  Q25: BMI 29.9 → Week 13.9
  Q50: BMI 31.6 → Week 15.6
  Q75: BMI 33.5 → Week 17.6

Optimal weeks for 95% confidence level:
 (When  95% of mothers have reached Y 4% threshold)
------------------------------------------------------------
  Q25: BMI 29.9 → Week 17.6
  Q50: BMI 31.6 → Week 19.7
  Q75: BMI 33.5 → Week 22.1

Optimal Testing Weeks Summary:
BMI Group     BMI Value  90% Conf   95% Conf
---------------------------------------------
Q25           29.9       13.9       17.6
Q50           31.6       15.6       19.7
Q75           33.5       17.6       22.1

Optimal testing week calculation completed
```

1.5 Section 4: Turnbull Non-parametric Validation

Goal: Validate AFT assumptions using non-parametric Turnbull estimator.

1.5.1 Overview

- Fit Turnbull estimator to same interval-censored data
- Compare Turnbull vs AFT survival curves

- Assess goodness of fit for parametric assumptions

[12]:
```python
### Step 4.1: Turnbull Fitting

# Fit Turnbull non-parametric estimator using module function
turnbull_model = fit_turnbull_estimator(df_X, verbose=True)
```

```
 Fitting Turnbull non-parametric estimator…
Iteration 0
   delta log-likelihood: 113.6900846918
   log-like:             -153.399751
Iteration 1
   delta log-likelihood: -101.1526863254
   log-like:             -132.911080
Iteration 2
   delta log-likelihood: 2.9231947109
   log-like:             -129.987885
Iteration 3
   delta log-likelihood: -1.0849443151
   log-like:             -126.598301
Iteration 4
   delta log-likelihood: 1.0252113341
   log-like:             -125.573090
Iteration 5
   delta log-likelihood: -0.0043090313
   log-like:             -124.052893
Iteration 6
   delta log-likelihood: 0.6773956086
   log-like:             -122.989866
Iteration 7
   delta log-likelihood: -0.1124386356
   log-like:             -122.198954
Iteration 8
   delta log-likelihood: 0.4953306406
   log-like:             -121.587947
Iteration 9
   delta log-likelihood: -0.1470470728
   log-like:             -121.106272
Iteration 10
   delta log-likelihood: 0.3827816919
   log-like:             -120.719448
Iteration 11
   delta log-likelihood: -0.1542395080
   log-like:             -120.405148
Iteration 12
   delta log-likelihood: 0.1367757006
   log-like:             -120.268372
Iteration 13
   delta log-likelihood: 0.0208871686
   log-like:             -120.033195
```

```
Iteration 14
   delta log-likelihood: 0.1034543726
   log-like:             -119.837019
Iteration 15
   delta log-likelihood: 0.0060184068
   log-like:             -119.672051
Iteration 16
   delta log-likelihood: 0.0806968021
   log-like:             -119.532256
Iteration 17
   delta log-likelihood: -0.0024361723
   log-like:             -119.412988
Iteration 18
   delta log-likelihood: 0.0645611503
   log-like:             -119.310564
Iteration 19
   delta log-likelihood: -0.0071565792
   log-like:             -119.222084
Iteration 20
   delta log-likelihood: 0.0527439360
   log-like:             -119.145206
Iteration 21
   delta log-likelihood: -0.0096519134
   log-like:             -119.078056
Iteration 22
   delta log-likelihood: 0.0438424917
   log-like:             -119.019097
Iteration 23
   delta log-likelihood: -0.0108040344
   log-like:             -118.967082
Iteration 24
   delta log-likelihood: 0.0369699499
   log-like:             -118.920976
Iteration 25
   delta log-likelihood: -0.0111424797
   log-like:             -118.879927
Iteration 26
   delta log-likelihood: 0.0315475596
   log-like:             -118.843224
Iteration 27
   delta log-likelihood: -0.0109919317
   log-like:             -118.810273
Iteration 28
   delta log-likelihood: 0.0271871201
   log-like:             -118.780573
Iteration 29
   delta log-likelihood: -0.0105545888
   log-like:             -118.753704
```

```
Iteration 30
   delta log-likelihood: 0.0236217730
   log-like:             -118.729309
Iteration 31
   delta log-likelihood: -0.0099576962
   log-like:             -118.707084
Iteration 32
   delta log-likelihood: 0.0106061086
   log-like:             -118.696478
Iteration 33
   delta log-likelihood: 0.0006978986
   log-like:             -118.677050
Iteration 34
   delta log-likelihood: 0.0091457544
   log-like:             -118.659209
Iteration 35
   delta log-likelihood: 0.0003370091
   log-like:             -118.642779
Iteration 36
   delta log-likelihood: 0.0079509036
   log-like:             -118.627607
Iteration 37
   delta log-likelihood: 0.0000994209
   log-like:             -118.613559
Iteration 38
   delta log-likelihood: 0.0069607969
   log-like:             -118.600520
Iteration 39
   delta log-likelihood: -0.0000524145
   log-like:             -118.588390
Iteration 40
   delta log-likelihood: 0.0061306498
   log-like:             -118.577081
Iteration 41
   delta log-likelihood: -0.0001447958
   log-like:             -118.566515
Iteration 42
   delta log-likelihood: 0.0054272447
   log-like:             -118.556625
Iteration 43
   delta log-likelihood: -0.0001961852
   log-like:             -118.547351
Iteration 44
   delta log-likelihood: 0.0048256315
   log-like:             -118.538639
Iteration 45
   delta log-likelihood: -0.0002195820
   log-like:             -118.530442
```

```
Iteration 46
   delta log-likelihood: 0.0043068240
   log-like:             -118.522719
Iteration 47
   delta log-likelihood: -0.0002241542
   log-like:             -118.515431
Iteration 48
   delta log-likelihood: 0.0038561881
   log-like:             -118.508545
Iteration 49
   delta log-likelihood: -0.0002163672
   log-like:             -118.502032
Iteration 50
   delta log-likelihood: 0.0034623036
   log-like:             -118.495863
Iteration 51
   delta log-likelihood: -0.0002007698
   log-like:             -118.490014
Iteration 52
   delta log-likelihood: 0.0031161494
   log-like:             -118.484464
Iteration 53
   delta log-likelihood: -0.0001805473
   log-like:             -118.479191
Iteration 54
   delta log-likelihood: 0.0028105148
   log-like:             -118.474178
Iteration 55
   delta log-likelihood: -0.0001579131
   log-like:             -118.469407
Iteration 56
   delta log-likelihood: 0.0025395697
   log-like:             -118.464863
Iteration 57
   delta log-likelihood: -0.0001343867
   log-like:             -118.460533
Iteration 58
   delta log-likelihood: 0.0022985469
   log-like:             -118.456403
Iteration 59
   delta log-likelihood: -0.0001109930
   log-like:             -118.452461
Iteration 60
   delta log-likelihood: 0.0020835073
   log-like:             -118.448697
Iteration 61
   delta log-likelihood: -0.0000884053
   log-like:             -118.445099
```

```
Iteration 62
   delta log-likelihood: 0.0018911621
   log-like:             -118.441660
Iteration 63
   delta log-likelihood: -0.0000670477
   log-like:             -118.438369
Iteration 64
   delta log-likelihood: 0.0017187401
   log-like:             -118.435218
Iteration 65
   delta log-likelihood: -0.0000471686
   log-like:             -118.432201
Iteration 66
   delta log-likelihood: 0.0015638854
   log-like:             -118.429310
Iteration 67
   delta log-likelihood: -0.0000288944
   log-like:             -118.426539
Iteration 68
   delta log-likelihood: 0.0014245792
   log-like:             -118.423880
Iteration 69
   delta log-likelihood: -0.0000122670
   log-like:             -118.421329
Iteration 70
   delta log-likelihood: 0.0012990789
   log-like:             -118.418880
Iteration 71
   delta log-likelihood: 0.0000027290
   log-like:             -118.416528
Iteration 72
   delta log-likelihood: 0.0011858715
   log-like:             -118.414268
Iteration 73
   delta log-likelihood: 0.0000161462
   log-like:             -118.412095
Iteration 74
   delta log-likelihood: 0.0010836352
   log-like:             -118.410006
Iteration 75
   delta log-likelihood: 0.0000280613
   log-like:             -118.407996
Iteration 76
   delta log-likelihood: 0.0009912102
   log-like:             -118.406061
Iteration 77
   delta log-likelihood: 0.0000385649
   log-like:             -118.404199
```

```
Iteration 78
   delta log-likelihood: 0.0009075743
   log-like:             -118.402405
Iteration 79
   delta log-likelihood: 0.0000477548
   log-like:             -118.400676
Iteration 80
   delta log-likelihood: 0.0008318235
   log-like:             -118.399010
Iteration 81
   delta log-likelihood: 0.0000557313
   log-like:             -118.397404
Iteration 82
   delta log-likelihood: 0.0007631560
   log-like:             -118.395854
Iteration 83
   delta log-likelihood: 0.0000625942
   log-like:             -118.394359
Iteration 84
   delta log-likelihood: 0.0007008586
   log-like:             -118.392916
Iteration 85
   delta log-likelihood: 0.0000684401
   log-like:             -118.391523
Iteration 86
   delta log-likelihood: 0.0006442956
   log-like:             -118.390177
Iteration 87
   delta log-likelihood: 0.0000733618
   log-like:             -118.388876
Iteration 88
   delta log-likelihood: 0.0005928995
   log-like:             -118.387619
Iteration 89
   delta log-likelihood: 0.0000774467
   log-like:             -118.386404
Iteration 90
   delta log-likelihood: 0.0005461627
   log-like:             -118.385228
Iteration 91
   delta log-likelihood: 0.0000807770
   log-like:             -118.384091
Iteration 92
   delta log-likelihood: 0.0005036305
   log-like:             -118.382990
Iteration 93
   delta log-likelihood: 0.0000834292
   log-like:             -118.381925
```

```
Iteration 94
   delta log-likelihood: 0.0004648954
   log-like:             -118.380893
Iteration 95
   delta log-likelihood: 0.0000854739
   log-like:             -118.379894
Iteration 96
   delta log-likelihood: 0.0004295915
   log-like:             -118.378926
Iteration 97
   delta log-likelihood: 0.0000869767
   log-like:             -118.377987
Iteration 98
   delta log-likelihood: 0.0003973903
   log-like:             -118.377077
Iteration 99
   delta log-likelihood: 0.0000879973
   log-like:             -118.376195
Iteration 100
   delta log-likelihood: 0.0003679962
   log-like:             -118.375339
Iteration 101
   delta log-likelihood: 0.0000885906
   log-like:             -118.374509
Iteration 102
   delta log-likelihood: 0.0003411436
   log-like:             -118.373703
Iteration 103
   delta log-likelihood: 0.0000888068
   log-like:             -118.372920
Iteration 104
   delta log-likelihood: 0.0003165931
   log-like:             -118.372160
Iteration 105
   delta log-likelihood: 0.0000886915
   log-like:             -118.371422
Iteration 106
   delta log-likelihood: 0.0002941292
   log-like:             -118.370705
Iteration 107
   delta log-likelihood: 0.0000882861
   log-like:             -118.370009
Iteration 108
   delta log-likelihood: 0.0002735576
   log-like:             -118.369331
Iteration 109
   delta log-likelihood: 0.0000876282
   log-like:             -118.368673
```

```
Iteration 110
   delta log-likelihood: 0.0002547032
   log-like:             -118.368033
Iteration 111
   delta log-likelihood: 0.0000867516
   log-like:             -118.367410
Iteration 112
   delta log-likelihood: 0.0002374079
   log-like:             -118.366804
Iteration 113
   delta log-likelihood: 0.0000856871
   log-like:             -118.366214
Iteration 114
   delta log-likelihood: 0.0002215291
   log-like:             -118.365640
Iteration 115
   delta log-likelihood: 0.0000844621
   log-like:             -118.365081
Iteration 116
   delta log-likelihood: 0.0002069380
   log-like:             -118.364537
Iteration 117
   delta log-likelihood: 0.0000831015
   log-like:             -118.364007
Iteration 118
   delta log-likelihood: 0.0001935180
   log-like:             -118.363491
Iteration 119
   delta log-likelihood: 0.0000816274
   log-like:             -118.362988
Iteration 120
   delta log-likelihood: 0.0001811641
   log-like:             -118.362497
Iteration 121
   delta log-likelihood: 0.0000800597
   log-like:             -118.362020
Iteration 122
   delta log-likelihood: 0.0001697810
   log-like:             -118.361554
Iteration 123
   delta log-likelihood: 0.0000784162
   log-like:             -118.361099
Iteration 124
   delta log-likelihood: 0.0001592827
   log-like:             -118.360656
Iteration 125
   delta log-likelihood: 0.0000767126
   log-like:             -118.360224
```

```
Iteration 126
   delta log-likelihood: 0.0001495912
   log-like:             -118.359802
Iteration 127
   delta log-likelihood: 0.0000749629
   log-like:             -118.359391
Iteration 128
   delta log-likelihood: 0.0001406360
   log-like:             -118.358989
Iteration 129
   delta log-likelihood: 0.0000731795
   log-like:             -118.358597
Iteration 130
   delta log-likelihood: 0.0001323531
   log-like:             -118.358215
Iteration 131
   delta log-likelihood: 0.0000713734
   log-like:             -118.357841
Iteration 132
   delta log-likelihood: 0.0001246845
   log-like:             -118.357476
Iteration 133
   delta log-likelihood: 0.0000695543
   log-like:             -118.357120
Iteration 134
   delta log-likelihood: 0.0001175778
   log-like:             -118.356771
Iteration 135
   delta log-likelihood: 0.0000677305
   log-like:             -118.356431
Iteration 136
   delta log-likelihood: 0.0001109852
   log-like:             -118.356099
Iteration 137
   delta log-likelihood: 0.0000659094
   log-like:             -118.355774
Iteration 138
   delta log-likelihood: 0.0001048634
   log-like:             -118.355456
Iteration 139
   delta log-likelihood: 0.0000640975
   log-like:             -118.355146
Iteration 140
   delta log-likelihood: 0.0000991732
   log-like:             -118.354842
Iteration 141
   delta log-likelihood: 0.0000623003
   log-like:             -118.354545
```

```
Iteration 142
   delta log-likelihood: 0.0000938787
   log-like:             -118.354255
Iteration 143
   delta log-likelihood: 0.0000605226
   log-like:             -118.353971
Iteration 144
   delta log-likelihood: 0.0000889475
   log-like:             -118.353693
Iteration 145
   delta log-likelihood: 0.0000587685
   log-like:             -118.353421
Iteration 146
   delta log-likelihood: 0.0000843502
   log-like:             -118.353155
Iteration 147
   delta log-likelihood: 0.0000570413
   log-like:             -118.352895
Iteration 148
   delta log-likelihood: 0.0000800597
   log-like:             -118.352640
Iteration 149
   delta log-likelihood: 0.0000553441
   log-like:             -118.352391
Iteration 150
   delta log-likelihood: 0.0000760516
   log-like:             -118.352147
Iteration 151
   delta log-likelihood: 0.0000536792
   log-like:             -118.351908
Iteration 152
   delta log-likelihood: 0.0000723037
   log-like:             -118.351674
Iteration 153
   delta log-likelihood: 0.0000520485
   log-like:             -118.351445
Iteration 154
   delta log-likelihood: 0.0000687955
   log-like:             -118.351220
Iteration 155
   delta log-likelihood: 0.0000504537
   log-like:             -118.351001
Iteration 156
   delta log-likelihood: 0.0000655085
   log-like:             -118.350785
Iteration 157
   delta log-likelihood: 0.0000488959
   log-like:             -118.350574
```

```
Iteration 158
   delta log-likelihood: 0.0000624257
   log-like:             -118.350368
Iteration 159
   delta log-likelihood: 0.0000473761
   log-like:             -118.350165
Iteration 160
   delta log-likelihood: 0.0000595318
   log-like:             -118.349967
Iteration 161
   delta log-likelihood: 0.0000458949
   log-like:             -118.349772
Iteration 162
   delta log-likelihood: 0.0000568125
   log-like:             -118.349581
Iteration 163
   delta log-likelihood: 0.0000444527
   log-like:             -118.349394
Iteration 164
   delta log-likelihood: 0.0000542549
   log-like:             -118.349211
Iteration 165
   delta log-likelihood: 0.0000430498
   log-like:             -118.349032
Iteration 166
   delta log-likelihood: 0.0000518473
   log-like:             -118.348855
Iteration 167
   delta log-likelihood: 0.0000416861
   log-like:             -118.348683
Iteration 168
   delta log-likelihood: 0.0000495787
   log-like:             -118.348513
Iteration 169
   delta log-likelihood: 0.0000403616
   log-like:             -118.348347
Iteration 170
   delta log-likelihood: 0.0000474392
   log-like:             -118.348184
Iteration 171
   delta log-likelihood: 0.0000390758
   log-like:             -118.348024
Iteration 172
   delta log-likelihood: 0.0000454197
   log-like:             -118.347867
Iteration 173
   delta log-likelihood: 0.0000378286
   log-like:             -118.347714
```

```
Iteration 174
   delta log-likelihood: 0.0000435119
   log-like:             -118.347563
Iteration 175
   delta log-likelihood: 0.0000366194
   log-like:             -118.347415
Iteration 176
   delta log-likelihood: 0.0000417080
   log-like:             -118.347269
Iteration 177
   delta log-likelihood: 0.0000354475
   log-like:             -118.347127
Iteration 178
   delta log-likelihood: 0.0000400010
   log-like:             -118.346986
Iteration 179
   delta log-likelihood: 0.0000343125
   log-like:             -118.346849
Iteration 180
   delta log-likelihood: 0.0000383844
   log-like:             -118.346714
Iteration 181
   delta log-likelihood: 0.0000332135
   log-like:             -118.346582
Iteration 182
   delta log-likelihood: 0.0000368522
   log-like:             -118.346452
Iteration 183
   delta log-likelihood: 0.0000321499
   log-like:             -118.346324
Iteration 184
   delta log-likelihood: 0.0000353990
   log-like:             -118.346199
Iteration 185
   delta log-likelihood: 0.0000311208
   log-like:             -118.346076
Iteration 186
   delta log-likelihood: 0.0000340195
   log-like:             -118.345955
Iteration 187
   delta log-likelihood: 0.0000301254
   log-like:             -118.345836
Iteration 188
   delta log-likelihood: 0.0000327092
   log-like:             -118.345719
Iteration 189
   delta log-likelihood: 0.0000291629
   log-like:             -118.345605
```

```
Iteration 190
   delta log-likelihood: 0.0000314637
   log-like:             -118.345492
Iteration 191
   delta log-likelihood: 0.0000282325
   log-like:             -118.345382
Iteration 192
   delta log-likelihood: 0.0000302789
   log-like:             -118.345273
Iteration 193
   delta log-likelihood: 0.0000273333
   log-like:             -118.345167
Iteration 194
   delta log-likelihood: 0.0000291513
   log-like:             -118.345062
Iteration 195
   delta log-likelihood: 0.0000264644
   log-like:             -118.344959
Iteration 196
   delta log-likelihood: 0.0000280772
   log-like:             -118.344857
Iteration 197
   delta log-likelihood: 0.0000256249
   log-like:             -118.344758
Iteration 198
   delta log-likelihood: 0.0000270536
   log-like:             -118.344660
Iteration 199
   delta log-likelihood: 0.0000248140
   log-like:             -118.344564
Iteration 200
   delta log-likelihood: 0.0000260775
   log-like:             -118.344470
Iteration 201
   delta log-likelihood: 0.0000240307
   log-like:             -118.344377
Iteration 202
   delta log-likelihood: 0.0000251461
   log-like:             -118.344286
Iteration 203
   delta log-likelihood: 0.0000232744
   log-like:             -118.344196
Iteration 204
   delta log-likelihood: 0.0000242569
   log-like:             -118.344108
Iteration 205
   delta log-likelihood: 0.0000225440
   log-like:             -118.344021
```

```
Iteration 206
   delta log-likelihood: 0.0000234076
   log-like:             -118.343936
Iteration 207
   delta log-likelihood: 0.0000218388
   log-like:             -118.343852
Iteration 208
   delta log-likelihood: 0.0000225958
   log-like:             -118.343769
Iteration 209
   delta log-likelihood: 0.0000211580
   log-like:             -118.343688
Iteration 210
   delta log-likelihood: 0.0000218196
   log-like:             -118.343608
Iteration 211
   delta log-likelihood: 0.0000205007
   log-like:             -118.343530
Iteration 212
   delta log-likelihood: 0.0000210769
   log-like:             -118.343453
Iteration 213
   delta log-likelihood: 0.0000198662
   log-like:             -118.343377
Iteration 214
   delta log-likelihood: 0.0000203661
   log-like:             -118.343302
Iteration 215
   delta log-likelihood: 0.0000192537
   log-like:             -118.343229
Iteration 216
   delta log-likelihood: 0.0000196855
   log-like:             -118.343156
Iteration 217
   delta log-likelihood: 0.0000186624
   log-like:             -118.343085
Iteration 218
   delta log-likelihood: 0.0000190333
   log-like:             -118.343015
Iteration 219
   delta log-likelihood: 0.0000180916
   log-like:             -118.342947
Iteration 220
   delta log-likelihood: 0.0000184083
   log-like:             -118.342879
Iteration 221
   delta log-likelihood: 0.0000175406
   log-like:             -118.342812
```

```
Iteration 222
   delta log-likelihood: 0.0000178090
   log-like:             -118.342746
Iteration 223
   delta log-likelihood: 0.0000170088
   log-like:             -118.342682
Iteration 224
   delta log-likelihood: 0.0000172342
   log-like:             -118.342618
Iteration 225
   delta log-likelihood: 0.0000164954
   log-like:             -118.342556
Iteration 226
   delta log-likelihood: 0.0000166825
   log-like:             -118.342494
Iteration 227
   delta log-likelihood: 0.0000159997
   log-like:             -118.342433
Iteration 228
   delta log-likelihood: 0.0000161529
   log-like:             -118.342374
Iteration 229
   delta log-likelihood: 0.0000155212
   log-like:             -118.342315
Iteration 230
   delta log-likelihood: 0.0000156443
   log-like:             -118.342257
Iteration 231
   delta log-likelihood: 0.0000150593
   log-like:             -118.342200
Iteration 232
   delta log-likelihood: 0.0000151557
   log-like:             -118.342144
Iteration 233
   delta log-likelihood: 0.0000146133
   log-like:             -118.342089
Iteration 234
   delta log-likelihood: 0.0000146861
   log-like:             -118.342034
Iteration 235
   delta log-likelihood: 0.0000141827
   log-like:             -118.341981
Iteration 236
   delta log-likelihood: 0.0000142347
   log-like:             -118.341928
Iteration 237
   delta log-likelihood: 0.0000137668
   log-like:             -118.341876
```

```
Iteration 238
   delta log-likelihood: 0.0000138005
   log-like:             -118.341825
Iteration 239
   delta log-likelihood: 0.0000133653
   log-like:             -118.341775
Iteration 240
   delta log-likelihood: 0.0000133829
   log-like:             -118.341725
Iteration 241
   delta log-likelihood: 0.0000129775
   log-like:             -118.341676
Iteration 242
   delta log-likelihood: 0.0000129810
   log-like:             -118.341628
Iteration 243
   delta log-likelihood: 0.0000126029
   log-like:             -118.341580
Iteration 244
   delta log-likelihood: 0.0000125941
   log-like:             -118.341533
Iteration 245
   delta log-likelihood: 0.0000122412
   log-like:             -118.341487
Iteration 246
   delta log-likelihood: 0.0000122216
   log-like:             -118.341442
Iteration 247
   delta log-likelihood: 0.0000118917
   log-like:             -118.341397
Iteration 248
   delta log-likelihood: 0.0000118629
   log-like:             -118.341353
Iteration 249
   delta log-likelihood: 0.0000115541
   log-like:             -118.341310
Iteration 250
   delta log-likelihood: 0.0000115172
   log-like:             -118.341267
Iteration 251
   delta log-likelihood: 0.0000112279
   log-like:             -118.341225
Iteration 252
   delta log-likelihood: 0.0000111841
   log-like:             -118.341183
Iteration 253
   delta log-likelihood: 0.0000109127
   log-like:             -118.341142
```

```
Iteration 254
   delta log-likelihood: 0.0000108630
   log-like:             -118.341101
Iteration 255
   delta log-likelihood: 0.0000106080
   log-like:             -118.341062
Iteration 256
   delta log-likelihood: 0.0000105534
   log-like:             -118.341022
Iteration 257
   delta log-likelihood: 0.0000103136
   log-like:             -118.340984
Iteration 258
   delta log-likelihood: 0.0000102548
   log-like:             -118.340945
Iteration 259
   delta log-likelihood: 0.0000100291
   log-like:             -118.340908
Iteration 260
   delta log-likelihood: 0.0000099668
   log-like:             -118.340871
Iteration 261
   delta log-likelihood: 0.0000097540
   log-like:             -118.340834
Iteration 262
   delta log-likelihood: 0.0000096888
   log-like:             -118.340798
Iteration 263
   delta log-likelihood: 0.0000094880
   log-like:             -118.340762
Iteration 264
   delta log-likelihood: 0.0000094205
   log-like:             -118.340727
Iteration 265
   delta log-likelihood: 0.0000092308
   log-like:             -118.340692
Iteration 266
   delta log-likelihood: 0.0000091615
   log-like:             -118.340658
Iteration 267
   delta log-likelihood: 0.0000089820
   log-like:             -118.340625
Iteration 268
   delta log-likelihood: 0.0000089114
   log-like:             -118.340591
Iteration 269
   delta log-likelihood: 0.0000087414
   log-like:             -118.340559
```

```
Iteration 270
   delta log-likelihood: 0.0000086699
   log-like:             -118.340526
Iteration 271
   delta log-likelihood: 0.0000085087
   log-like:             -118.340494
Iteration 272
   delta log-likelihood: 0.0000084365
   log-like:             -118.340463
Iteration 273
   delta log-likelihood: 0.0000082835
   log-like:             -118.340432
Iteration 274
   delta log-likelihood: 0.0000082110
   log-like:             -118.340401
Iteration 275
   delta log-likelihood: 0.0000080657
   log-like:             -118.340371
Iteration 276
   delta log-likelihood: 0.0000079930
   log-like:             -118.340341
Iteration 277
   delta log-likelihood: 0.0000078548
   log-like:             -118.340312
Iteration 278
   delta log-likelihood: 0.0000077823
   log-like:             -118.340283
Iteration 279
   delta log-likelihood: 0.0000076508
   log-like:             -118.340254
Iteration 280
   delta log-likelihood: 0.0000075786
   log-like:             -118.340226
Iteration 281
   delta log-likelihood: 0.0000074532
   log-like:             -118.340198
Iteration 282
   delta log-likelihood: 0.0000073815
   log-like:             -118.340170
Iteration 283
   delta log-likelihood: 0.0000072619
   log-like:             -118.340143
Iteration 284
   delta log-likelihood: 0.0000071909
   log-like:             -118.340116
Iteration 285
   delta log-likelihood: 0.0000070767
   log-like:             -118.340090
```

```
Iteration 286
   delta log-likelihood: 0.0000070065
   log-like:             -118.340063
Iteration 287
   delta log-likelihood: 0.0000068974
   log-like:             -118.340038
Iteration 288
   delta log-likelihood: 0.0000068280
   log-like:             -118.340012
Iteration 289
   delta log-likelihood: 0.0000067237
   log-like:             -118.339987
Iteration 290
   delta log-likelihood: 0.0000066553
   log-like:             -118.339962
Iteration 291
   delta log-likelihood: 0.0000065554
   log-like:             -118.339938
Iteration 292
   delta log-likelihood: 0.0000064880
   log-like:             -118.339913
Iteration 293
   delta log-likelihood: 0.0000063923
   log-like:             -118.339889
Iteration 294
   delta log-likelihood: 0.0000063260
   log-like:             -118.339866
Iteration 295
   delta log-likelihood: 0.0000062343
   log-like:             -118.339842
Iteration 296
   delta log-likelihood: 0.0000061692
   log-like:             -118.339819
Iteration 297
   delta log-likelihood: 0.0000060811
   log-like:             -118.339797
Iteration 298
   delta log-likelihood: 0.0000060172
   log-like:             -118.339774
Iteration 299
   delta log-likelihood: 0.0000059327
   log-like:             -118.339752
Iteration 300
   delta log-likelihood: 0.0000058700
   log-like:             -118.339730
Iteration 301
   delta log-likelihood: 0.0000057887
   log-like:             -118.339708
```

```
Iteration 302
   delta log-likelihood: 0.0000057273
   log-like:             -118.339687
Iteration 303
   delta log-likelihood: 0.0000056492
   log-like:             -118.339666
Iteration 304
   delta log-likelihood: 0.0000055890
   log-like:             -118.339645
Iteration 305
   delta log-likelihood: 0.0000055138
   log-like:             -118.339624
Iteration 306
   delta log-likelihood: 0.0000054549
   log-like:             -118.339604
Iteration 307
   delta log-likelihood: 0.0000053825
   log-like:             -118.339584
Iteration 308
   delta log-likelihood: 0.0000053249
   log-like:             -118.339564
Iteration 309
   delta log-likelihood: 0.0000052552
   log-like:             -118.339544
Iteration 310
   delta log-likelihood: 0.0000051988
   log-like:             -118.339525
Iteration 311
   delta log-likelihood: 0.0000051316
   log-like:             -118.339506
Iteration 312
   delta log-likelihood: 0.0000050765
   log-like:             -118.339487
Iteration 313
   delta log-likelihood: 0.0000050116
   log-like:             -118.339468
Iteration 314
   delta log-likelihood: 0.0000049578
   log-like:             -118.339449
Iteration 315
   delta log-likelihood: 0.0000048952
   log-like:             -118.339431
Iteration 316
   delta log-likelihood: 0.0000048426
   log-like:             -118.339413
Iteration 317
   delta log-likelihood: 0.0000047822
   log-like:             -118.339395
```

```
Iteration 318
   delta log-likelihood: 0.0000047309
   log-like:             -118.339377
Iteration 319
   delta log-likelihood: 0.0000046725
   log-like:             -118.339360
Iteration 320
   delta log-likelihood: 0.0000046224
   log-like:             -118.339343
Iteration 321
   delta log-likelihood: 0.0000045660
   log-like:             -118.339326
Iteration 322
   delta log-likelihood: 0.0000045170
   log-like:             -118.339309
Iteration 323
   delta log-likelihood: 0.0000044625
   log-like:             -118.339292
Iteration 324
   delta log-likelihood: 0.0000044147
   log-like:             -118.339275
Iteration 325
   delta log-likelihood: 0.0000043620
   log-like:             -118.339259
Iteration 326
   delta log-likelihood: 0.0000043154
   log-like:             -118.339243
Iteration 327
   delta log-likelihood: 0.0000042643
   log-like:             -118.339227
Iteration 328
   delta log-likelihood: 0.0000042189
   log-like:             -118.339211
Iteration 329
   delta log-likelihood: 0.0000041694
   log-like:             -118.339196
Iteration 330
   delta log-likelihood: 0.0000041251
   log-like:             -118.339180
Iteration 331
   delta log-likelihood: 0.0000040772
   log-like:             -118.339165
Iteration 332
   delta log-likelihood: 0.0000040339
   log-like:             -118.339150
Iteration 333
   delta log-likelihood: 0.0000039875
   log-like:             -118.339135
```

```
Iteration 334
   delta log-likelihood: 0.0000039454
   log-like:             -118.339120
Iteration 335
   delta log-likelihood: 0.0000039004
   log-like:             -118.339106
Iteration 336
   delta log-likelihood: 0.0000038593
   log-like:             -118.339091
Iteration 337
   delta log-likelihood: 0.0000038156
   log-like:             -118.339077
Iteration 338
   delta log-likelihood: 0.0000037755
   log-like:             -118.339063
Iteration 339
   delta log-likelihood: 0.0000037332
   log-like:             -118.339049
Iteration 340
   delta log-likelihood: 0.0000036941
   log-like:             -118.339035
Iteration 341
   delta log-likelihood: 0.0000036530
   log-like:             -118.339022
Iteration 342
   delta log-likelihood: 0.0000036149
   log-like:             -118.339008
Iteration 343
   delta log-likelihood: 0.0000035750
   log-like:             -118.338995
Iteration 344
   delta log-likelihood: 0.0000035379
   log-like:             -118.338981
Iteration 345
   delta log-likelihood: 0.0000034992
   log-like:             -118.338968
Iteration 346
   delta log-likelihood: 0.0000034629
   log-like:             -118.338955
Iteration 347
   delta log-likelihood: 0.0000034253
   log-like:             -118.338943
Iteration 348
   delta log-likelihood: 0.0000033900
   log-like:             -118.338930
Iteration 349
   delta log-likelihood: 0.0000033534
   log-like:             -118.338917
```

```
Iteration 350
   delta log-likelihood: 0.0000033190
   log-like:             -118.338905
Iteration 351
   delta log-likelihood: 0.0000032835
   log-like:             -118.338893
Iteration 352
   delta log-likelihood: 0.0000032499
   log-like:             -118.338881
Iteration 353
   delta log-likelihood: 0.0000032153
   log-like:             -118.338869
Iteration 354
   delta log-likelihood: 0.0000031826
   log-like:             -118.338857
Iteration 355
   delta log-likelihood: 0.0000031490
   log-like:             -118.338845
Iteration 356
   delta log-likelihood: 0.0000031171
   log-like:             -118.338833
Iteration 357
   delta log-likelihood: 0.0000030844
   log-like:             -118.338822
Iteration 358
   delta log-likelihood: 0.0000030532
   log-like:             -118.338810
Iteration 359
   delta log-likelihood: 0.0000030215
   log-like:             -118.338799
Iteration 360
   delta log-likelihood: 0.0000029911
   log-like:             -118.338788
Iteration 361
   delta log-likelihood: 0.0000029602
   log-like:             -118.338777
Iteration 362
   delta log-likelihood: 0.0000029305
   log-like:             -118.338766
Iteration 363
   delta log-likelihood: 0.0000029004
   log-like:             -118.338755
Iteration 364
   delta log-likelihood: 0.0000028715
   log-like:             -118.338744
Iteration 365
   delta log-likelihood: 0.0000028422
   log-like:             -118.338734
```

```
Iteration 366
   delta log-likelihood: 0.0000028140
   log-like:             -118.338723
Iteration 367
   delta log-likelihood: 0.0000027855
   log-like:             -118.338713
Iteration 368
   delta log-likelihood: 0.0000027579
   log-like:             -118.338703
Iteration 369
   delta log-likelihood: 0.0000027301
   log-like:             -118.338692
Iteration 370
   delta log-likelihood: 0.0000027033
   log-like:             -118.338682
Iteration 371
   delta log-likelihood: 0.0000026762
   log-like:             -118.338672
Iteration 372
   delta log-likelihood: 0.0000026500
   log-like:             -118.338662
Iteration 373
   delta log-likelihood: 0.0000026236
   log-like:             -118.338653
Iteration 374
   delta log-likelihood: 0.0000025980
   log-like:             -118.338643
Iteration 375
   delta log-likelihood: 0.0000025723
   log-like:             -118.338633
Iteration 376
   delta log-likelihood: 0.0000025473
   log-like:             -118.338624
Iteration 377
   delta log-likelihood: 0.0000025223
   log-like:             -118.338614
Iteration 378
   delta log-likelihood: 0.0000024979
   log-like:             -118.338605
Iteration 379
   delta log-likelihood: 0.0000024735
   log-like:             -118.338596
Iteration 380
   delta log-likelihood: 0.0000024497
   log-like:             -118.338587
Iteration 381
   delta log-likelihood: 0.0000024258
   log-like:             -118.338577
```

```
Iteration 382
   delta log-likelihood: 0.0000024026
   log-like:             -118.338568
Iteration 383
   delta log-likelihood: 0.0000023794
   log-like:             -118.338560
Iteration 384
   delta log-likelihood: 0.0000023567
   log-like:             -118.338551
Iteration 385
   delta log-likelihood: 0.0000023340
   log-like:             -118.338542
Iteration 386
   delta log-likelihood: 0.0000023118
   log-like:             -118.338533
Iteration 387
   delta log-likelihood: 0.0000022897
   log-like:             -118.338525
Iteration 388
   delta log-likelihood: 0.0000022681
   log-like:             -118.338516
Iteration 389
   delta log-likelihood: 0.0000022465
   log-like:             -118.338508
Iteration 390
   delta log-likelihood: 0.0000022254
   log-like:             -118.338500
Iteration 391
   delta log-likelihood: 0.0000022043
   log-like:             -118.338491
Iteration 392
   delta log-likelihood: 0.0000021836
   log-like:             -118.338483
Iteration 393
   delta log-likelihood: 0.0000021631
   log-like:             -118.338475
Iteration 394
   delta log-likelihood: 0.0000021429
   log-like:             -118.338467
Iteration 395
   delta log-likelihood: 0.0000021228
   log-like:             -118.338459
Iteration 396
   delta log-likelihood: 0.0000021031
   log-like:             -118.338451
Iteration 397
   delta log-likelihood: 0.0000020835
   log-like:             -118.338444
```

```
Iteration 398
   delta log-likelihood: 0.0000020642
   log-like:             -118.338436
Iteration 399
   delta log-likelihood: 0.0000020450
   log-like:             -118.338428
Iteration 400
   delta log-likelihood: 0.0000020262
   log-like:             -118.338421
Iteration 401
   delta log-likelihood: 0.0000020075
   log-like:             -118.338413
Iteration 402
   delta log-likelihood: 0.0000019891
   log-like:             -118.338406
Iteration 403
   delta log-likelihood: 0.0000019708
   log-like:             -118.338398
Iteration 404
   delta log-likelihood: 0.0000019528
   log-like:             -118.338391
Iteration 405
   delta log-likelihood: 0.0000019349
   log-like:             -118.338384
Iteration 406
   delta log-likelihood: 0.0000019174
   log-like:             -118.338377
Iteration 407
   delta log-likelihood: 0.0000018999
   log-like:             -118.338370
Iteration 408
   delta log-likelihood: 0.0000018827
   log-like:             -118.338363
Iteration 409
   delta log-likelihood: 0.0000018656
   log-like:             -118.338356
Iteration 410
   delta log-likelihood: 0.0000018488
   log-like:             -118.338349
Iteration 411
   delta log-likelihood: 0.0000018321
   log-like:             -118.338342
Iteration 412
   delta log-likelihood: 0.0000018157
   log-like:             -118.338335
Iteration 413
   delta log-likelihood: 0.0000017994
   log-like:             -118.338328
```

```
Iteration 414
   delta log-likelihood: 0.0000017833
   log-like:             -118.338322
Iteration 415
   delta log-likelihood: 0.0000017673
   log-like:             -118.338315
Iteration 416
   delta log-likelihood: 0.0000017516
   log-like:             -118.338309
Iteration 417
   delta log-likelihood: 0.0000017360
   log-like:             -118.338302
Iteration 418
   delta log-likelihood: 0.0000017206
   log-like:             -118.338296
Iteration 419
   delta log-likelihood: 0.0000017053
   log-like:             -118.338289
Iteration 420
   delta log-likelihood: 0.0000016903
   log-like:             -118.338283
Iteration 421
   delta log-likelihood: 0.0000016753
   log-like:             -118.338277
Iteration 422
   delta log-likelihood: 0.0000016606
   log-like:             -118.338271
Iteration 423
   delta log-likelihood: 0.0000016460
   log-like:             -118.338264
Iteration 424
   delta log-likelihood: 0.0000016316
   log-like:             -118.338258
Iteration 425
   delta log-likelihood: 0.0000016173
   log-like:             -118.338252
Iteration 426
   delta log-likelihood: 0.0000016031
   log-like:             -118.338246
Iteration 427
   delta log-likelihood: 0.0000015891
   log-like:             -118.338240
Iteration 428
   delta log-likelihood: 0.0000015753
   log-like:             -118.338234
Iteration 429
   delta log-likelihood: 0.0000015616
   log-like:             -118.338229
```

```
Iteration 430
   delta log-likelihood: 0.0000015481
   log-like:             -118.338223
Iteration 431
   delta log-likelihood: 0.0000015347
   log-like:             -118.338217
Iteration 432
   delta log-likelihood: 0.0000015215
   log-like:             -118.338211
Iteration 433
   delta log-likelihood: 0.0000015083
   log-like:             -118.338206
Iteration 434
   delta log-likelihood: 0.0000014954
   log-like:             -118.338200
Iteration 435
   delta log-likelihood: 0.0000014825
   log-like:             -118.338195
Iteration 436
   delta log-likelihood: 0.0000014698
   log-like:             -118.338189
Iteration 437
   delta log-likelihood: 0.0000014572
   log-like:             -118.338184
Iteration 438
   delta log-likelihood: 0.0000014448
   log-like:             -118.338178
Iteration 439
   delta log-likelihood: 0.0000014325
   log-like:             -118.338173
Iteration 440
   delta log-likelihood: 0.0000014203
   log-like:             -118.338168
Iteration 441
   delta log-likelihood: 0.0000014082
   log-like:             -118.338162
Iteration 442
   delta log-likelihood: 0.0000013963
   log-like:             -118.338157
Iteration 443
   delta log-likelihood: 0.0000013844
   log-like:             -118.338152
Iteration 444
   delta log-likelihood: 0.0000013727
   log-like:             -118.338147
Iteration 445
   delta log-likelihood: 0.0000013612
   log-like:             -118.338142
```

```
Iteration 446
   delta log-likelihood: 0.0000013497
   log-like:             -118.338137
Iteration 447
   delta log-likelihood: 0.0000013384
   log-like:             -118.338132
Iteration 448
   delta log-likelihood: 0.0000013271
   log-like:             -118.338127
Iteration 449
   delta log-likelihood: 0.0000013160
   log-like:             -118.338122
Iteration 450
   delta log-likelihood: 0.0000013050
   log-like:             -118.338117
Iteration 451
   delta log-likelihood: 0.0000012941
   log-like:             -118.338112
Iteration 452
   delta log-likelihood: 0.0000012833
   log-like:             -118.338107
Iteration 453
   delta log-likelihood: 0.0000012726
   log-like:             -118.338103
Iteration 454
   delta log-likelihood: 0.0000012621
   log-like:             -118.338098
Iteration 455
   delta log-likelihood: 0.0000012516
   log-like:             -118.338093
Iteration 456
   delta log-likelihood: 0.0000012412
   log-like:             -118.338089
Iteration 457
   delta log-likelihood: 0.0000012310
   log-like:             -118.338084
Iteration 458
   delta log-likelihood: 0.0000012208
   log-like:             -118.338080
Iteration 459
   delta log-likelihood: 0.0000012108
   log-like:             -118.338075
Iteration 460
   delta log-likelihood: 0.0000012008
   log-like:             -118.338071
Iteration 461
   delta log-likelihood: 0.0000011909
   log-like:             -118.338066
```

```
Iteration 462
   delta log-likelihood: 0.0000011812
   log-like:             -118.338062
Iteration 463
   delta log-likelihood: 0.0000011715
   log-like:             -118.338057
Iteration 464
   delta log-likelihood: 0.0000011619
   log-like:             -118.338053
Iteration 465
   delta log-likelihood: 0.0000011524
   log-like:             -118.338049
Iteration 466
   delta log-likelihood: 0.0000011431
   log-like:             -118.338044
Iteration 467
   delta log-likelihood: 0.0000011337
   log-like:             -118.338040
Iteration 468
   delta log-likelihood: 0.0000011245
   log-like:             -118.338036
Iteration 469
   delta log-likelihood: 0.0000011154
   log-like:             -118.338032
Iteration 470
   delta log-likelihood: 0.0000011064
   log-like:             -118.338028
Iteration 471
   delta log-likelihood: 0.0000010974
   log-like:             -118.338024
Iteration 472
   delta log-likelihood: 0.0000010886
   log-like:             -118.338020
Iteration 473
   delta log-likelihood: 0.0000010798
   log-like:             -118.338015
Iteration 474
   delta log-likelihood: 0.0000010711
   log-like:             -118.338011
Iteration 475
   delta log-likelihood: 0.0000010625
   log-like:             -118.338008
Iteration 476
   delta log-likelihood: 0.0000010539
   log-like:             -118.338004
Iteration 477
   delta log-likelihood: 0.0000010455
   log-like:             -118.338000
```

```
Iteration 478
   delta log-likelihood: 0.0000010371
   log-like:             -118.337996
Iteration 479
   delta log-likelihood: 0.0000010288
   log-like:             -118.337992
Iteration 480
   delta log-likelihood: 0.0000010206
   log-like:             -118.337988
Iteration 481
   delta log-likelihood: 0.0000010125
   log-like:             -118.337984
Iteration 482
   delta log-likelihood: 0.0000010044
   log-like:             -118.337981
Iteration 483
   delta log-likelihood: 0.0000009964
   log-like:             -118.337977
Iteration 484
   delta log-likelihood: 0.0000009885
   log-like:             -118.337973
Iteration 485
   delta log-likelihood: 0.0000009807
   log-like:             -118.337970
Iteration 486
   delta log-likelihood: 0.0000009729
   log-like:             -118.337966
Iteration 487
   delta log-likelihood: 0.0000009652
   log-like:             -118.337962
Iteration 488
   delta log-likelihood: 0.0000009576
   log-like:             -118.337959
Iteration 489
   delta log-likelihood: 0.0000009500
   log-like:             -118.337955
Iteration 490
   delta log-likelihood: 0.0000009426
   log-like:             -118.337952
Iteration 491
   delta log-likelihood: 0.0000009351
   log-like:             -118.337948
Iteration 492
   delta log-likelihood: 0.0000009278
   log-like:             -118.337945
Iteration 493
   delta log-likelihood: 0.0000009205
   log-like:             -118.337941
```

```
Iteration 494
   delta log-likelihood: 0.0000009133
   log-like:             -118.337938
Iteration 495
   delta log-likelihood: 0.0000009061
   log-like:             -118.337934
Iteration 496
   delta log-likelihood: 0.0000008991
   log-like:             -118.337931
Iteration 497
   delta log-likelihood: 0.0000008920
   log-like:             -118.337928
Iteration 498
   delta log-likelihood: 0.0000008851
   log-like:             -118.337924
Iteration 499
   delta log-likelihood: 0.0000008782
   log-like:             -118.337921
Iteration 500
   delta log-likelihood: 0.0000008714
   log-like:             -118.337918
Iteration 501
   delta log-likelihood: 0.0000008646
   log-like:             -118.337915
Iteration 502
   delta log-likelihood: 0.0000008579
   log-like:             -118.337912
Iteration 503
   delta log-likelihood: 0.0000008512
   log-like:             -118.337908
Iteration 504
   delta log-likelihood: 0.0000008447
   log-like:             -118.337905
Iteration 505
   delta log-likelihood: 0.0000008381
   log-like:             -118.337902
Iteration 506
   delta log-likelihood: 0.0000008317
   log-like:             -118.337899
Iteration 507
   delta log-likelihood: 0.0000008252
   log-like:             -118.337896
Iteration 508
   delta log-likelihood: 0.0000008189
   log-like:             -118.337893
Iteration 509
   delta log-likelihood: 0.0000008126
   log-like:             -118.337890
```

```
Iteration 510
   delta log-likelihood: 0.0000008063
   log-like:             -118.337887
Iteration 511
   delta log-likelihood: 0.0000008001
   log-like:             -118.337884
Iteration 512
   delta log-likelihood: 0.0000007940
   log-like:             -118.337881
Iteration 513
   delta log-likelihood: 0.0000007879
   log-like:             -118.337878
Iteration 514
   delta log-likelihood: 0.0000007819
   log-like:             -118.337875
Iteration 515
   delta log-likelihood: 0.0000007759
   log-like:             -118.337872
Iteration 516
   delta log-likelihood: 0.0000007700
   log-like:             -118.337869
Iteration 517
   delta log-likelihood: 0.0000007641
   log-like:             -118.337866
Iteration 518
   delta log-likelihood: 0.0000007583
   log-like:             -118.337864
Iteration 519
   delta log-likelihood: 0.0000007525
   log-like:             -118.337861
Iteration 520
   delta log-likelihood: 0.0000007468
   log-like:             -118.337858
Iteration 521
   delta log-likelihood: 0.0000007412
   log-like:             -118.337855
Iteration 522
   delta log-likelihood: 0.0000007355
   log-like:             -118.337852
Iteration 523
   delta log-likelihood: 0.0000007300
   log-like:             -118.337850
Iteration 524
   delta log-likelihood: 0.0000007244
   log-like:             -118.337847
Iteration 525
   delta log-likelihood: 0.0000007190
   log-like:             -118.337844
```

```
Iteration 526
   delta log-likelihood: 0.0000007135
   log-like:             -118.337842
Iteration 527
   delta log-likelihood: 0.0000007082
   log-like:             -118.337839
Iteration 528
   delta log-likelihood: 0.0000007028
   log-like:             -118.337836
Iteration 529
   delta log-likelihood: 0.0000006975
   log-like:             -118.337834
Iteration 530
   delta log-likelihood: 0.0000006923
   log-like:             -118.337831
Iteration 531
   delta log-likelihood: 0.0000006871
   log-like:             -118.337829
Iteration 532
   delta log-likelihood: 0.0000006819
   log-like:             -118.337826
Iteration 533
   delta log-likelihood: 0.0000006768
   log-like:             -118.337824
Iteration 534
   delta log-likelihood: 0.0000006718
   log-like:             -118.337821
Iteration 535
   delta log-likelihood: 0.0000006667
   log-like:             -118.337819
Iteration 536
   delta log-likelihood: 0.0000006617
   log-like:             -118.337816
Iteration 537
   delta log-likelihood: 0.0000006568
   log-like:             -118.337814
Iteration 538
   delta log-likelihood: 0.0000006519
   log-like:             -118.337811
Iteration 539
   delta log-likelihood: 0.0000006471
   log-like:             -118.337809
Iteration 540
   delta log-likelihood: 0.0000006422
   log-like:             -118.337806
Iteration 541
   delta log-likelihood: 0.0000006375
   log-like:             -118.337804
```

```
Iteration 542
   delta log-likelihood: 0.0000006327
   log-like:             -118.337802
Iteration 543
   delta log-likelihood: 0.0000006280
   log-like:             -118.337799
Iteration 544
   delta log-likelihood: 0.0000006234
   log-like:             -118.337797
Iteration 545
   delta log-likelihood: 0.0000006188
   log-like:             -118.337795
Iteration 546
   delta log-likelihood: 0.0000006142
   log-like:             -118.337792
Iteration 547
   delta log-likelihood: 0.0000006096
   log-like:             -118.337790
Iteration 548
   delta log-likelihood: 0.0000006051
   log-like:             -118.337788
Iteration 549
   delta log-likelihood: 0.0000006007
   log-like:             -118.337786
Iteration 550
   delta log-likelihood: 0.0000005962
   log-like:             -118.337783
Iteration 551
   delta log-likelihood: 0.0000005918
   log-like:             -118.337781
Iteration 552
   delta log-likelihood: 0.0000005875
   log-like:             -118.337779
Iteration 553
   delta log-likelihood: 0.0000005832
   log-like:             -118.337777
Iteration 554
   delta log-likelihood: 0.0000005789
   log-like:             -118.337775
Iteration 555
   delta log-likelihood: 0.0000005746
   log-like:             -118.337772
Iteration 556
   delta log-likelihood: 0.0000005704
   log-like:             -118.337770
Iteration 557
   delta log-likelihood: 0.0000005662
   log-like:             -118.337768
```

```
Iteration 558
   delta log-likelihood: 0.0000005621
   log-like:             -118.337766
Iteration 559
   delta log-likelihood: 0.0000005580
   log-like:             -118.337764
Iteration 560
   delta log-likelihood: 0.0000005539
   log-like:             -118.337762
Iteration 561
   delta log-likelihood: 0.0000005499
   log-like:             -118.337760
Iteration 562
   delta log-likelihood: 0.0000005459
   log-like:             -118.337758
Iteration 563
   delta log-likelihood: 0.0000005419
   log-like:             -118.337756
Iteration 564
   delta log-likelihood: 0.0000005379
   log-like:             -118.337754
Iteration 565
   delta log-likelihood: 0.0000005340
   log-like:             -118.337752
Iteration 566
   delta log-likelihood: 0.0000005301
   log-like:             -118.337750
Iteration 567
   delta log-likelihood: 0.0000005263
   log-like:             -118.337748
Iteration 568
   delta log-likelihood: 0.0000005225
   log-like:             -118.337746
Iteration 569
   delta log-likelihood: 0.0000005187
   log-like:             -118.337744
Iteration 570
   delta log-likelihood: 0.0000005149
   log-like:             -118.337742
Iteration 571
   delta log-likelihood: 0.0000005112
   log-like:             -118.337740
Iteration 572
   delta log-likelihood: 0.0000005075
   log-like:             -118.337738
Iteration 573
   delta log-likelihood: 0.0000005038
   log-like:             -118.337736
```

```
Iteration 574
   delta log-likelihood: 0.0000005002
   log-like:             -118.337735
Iteration 575
   delta log-likelihood: 0.0000004966
   log-like:             -118.337733
Iteration 576
   delta log-likelihood: 0.0000004930
   log-like:             -118.337731
Iteration 577
   delta log-likelihood: 0.0000004895
   log-like:             -118.337729
Iteration 578
   delta log-likelihood: 0.0000004859
   log-like:             -118.337727
Iteration 579
   delta log-likelihood: 0.0000004824
   log-like:             -118.337725
Iteration 580
   delta log-likelihood: 0.0000004790
   log-like:             -118.337724
Iteration 581
   delta log-likelihood: 0.0000004755
   log-like:             -118.337722
Iteration 582
   delta log-likelihood: 0.0000004721
   log-like:             -118.337720
Iteration 583
   delta log-likelihood: 0.0000004687
   log-like:             -118.337718
Iteration 584
   delta log-likelihood: 0.0000004654
   log-like:             -118.337717
Iteration 585
   delta log-likelihood: 0.0000004621
   log-like:             -118.337715
Iteration 586
   delta log-likelihood: 0.0000004588
   log-like:             -118.337713
Iteration 587
   delta log-likelihood: 0.0000004555
   log-like:             -118.337711
Iteration 588
   delta log-likelihood: 0.0000004522
   log-like:             -118.337710
Iteration 589
   delta log-likelihood: 0.0000004490
   log-like:             -118.337708
```

```
Iteration 590
   delta log-likelihood: 0.0000004458
   log-like:             -118.337706
Iteration 591
   delta log-likelihood: 0.0000004426
   log-like:             -118.337705
Iteration 592
   delta log-likelihood: 0.0000004395
   log-like:             -118.337703
Iteration 593
   delta log-likelihood: 0.0000004364
   log-like:             -118.337702
Iteration 594
   delta log-likelihood: 0.0000004333
   log-like:             -118.337700
Iteration 595
   delta log-likelihood: 0.0000004302
   log-like:             -118.337698
Iteration 596
   delta log-likelihood: 0.0000004271
   log-like:             -118.337697
Iteration 597
   delta log-likelihood: 0.0000004241
   log-like:             -118.337695
Iteration 598
   delta log-likelihood: 0.0000004211
   log-like:             -118.337694
Iteration 599
   delta log-likelihood: 0.0000004181
   log-like:             -118.337692
Iteration 600
   delta log-likelihood: 0.0000004152
   log-like:             -118.337690
Iteration 601
   delta log-likelihood: 0.0000004122
   log-like:             -118.337689
Iteration 602
   delta log-likelihood: 0.0000004093
   log-like:             -118.337687
Iteration 603
   delta log-likelihood: 0.0000004064
   log-like:             -118.337686
Iteration 604
   delta log-likelihood: 0.0000004035
   log-like:             -118.337684
Iteration 605
   delta log-likelihood: 0.0000004007
   log-like:             -118.337683
```

```
Iteration 606
   delta log-likelihood: 0.0000003979
   log-like:             -118.337681
Iteration 607
   delta log-likelihood: 0.0000003951
   log-like:             -118.337680
Iteration 608
   delta log-likelihood: 0.0000003923
   log-like:             -118.337678
Iteration 609
   delta log-likelihood: 0.0000003895
   log-like:             -118.337677
Iteration 610
   delta log-likelihood: 0.0000003868
   log-like:             -118.337676
Iteration 611
   delta log-likelihood: 0.0000003841
   log-like:             -118.337674
Iteration 612
   delta log-likelihood: 0.0000003814
   log-like:             -118.337673
Iteration 613
   delta log-likelihood: 0.0000003787
   log-like:             -118.337671
Iteration 614
   delta log-likelihood: 0.0000003761
   log-like:             -118.337670
Iteration 615
   delta log-likelihood: 0.0000003734
   log-like:             -118.337668
Iteration 616
   delta log-likelihood: 0.0000003708
   log-like:             -118.337667
Iteration 617
   delta log-likelihood: 0.0000003682
   log-like:             -118.337666
Iteration 618
   delta log-likelihood: 0.0000003657
   log-like:             -118.337664
Iteration 619
   delta log-likelihood: 0.0000003631
   log-like:             -118.337663
Iteration 620
   delta log-likelihood: 0.0000003606
   log-like:             -118.337662
Iteration 621
   delta log-likelihood: 0.0000003581
   log-like:             -118.337660
```

```
Iteration 622
   delta log-likelihood: 0.0000003556
   log-like:             -118.337659
Iteration 623
   delta log-likelihood: 0.0000003531
   log-like:             -118.337658
Iteration 624
   delta log-likelihood: 0.0000003507
   log-like:             -118.337656
Iteration 625
   delta log-likelihood: 0.0000003482
   log-like:             -118.337655
Iteration 626
   delta log-likelihood: 0.0000003458
   log-like:             -118.337654
Iteration 627
   delta log-likelihood: 0.0000003434
   log-like:             -118.337652
Iteration 628
   delta log-likelihood: 0.0000003410
   log-like:             -118.337651
Iteration 629
   delta log-likelihood: 0.0000003387
   log-like:             -118.337650
Iteration 630
   delta log-likelihood: 0.0000003363
   log-like:             -118.337649
Iteration 631
   delta log-likelihood: 0.0000003340
   log-like:             -118.337647
Iteration 632
   delta log-likelihood: 0.0000003317
   log-like:             -118.337646
Iteration 633
   delta log-likelihood: 0.0000003294
   log-like:             -118.337645
Iteration 634
   delta log-likelihood: 0.0000003271
   log-like:             -118.337644
Iteration 635
   delta log-likelihood: 0.0000003248
   log-like:             -118.337643
Iteration 636
   delta log-likelihood: 0.0000003226
   log-like:             -118.337641
Iteration 637
   delta log-likelihood: 0.0000003204
   log-like:             -118.337640
```

```
Iteration 638
   delta log-likelihood: 0.0000003182
   log-like:             -118.337639
Iteration 639
   delta log-likelihood: 0.0000003160
   log-like:             -118.337638
Iteration 640
   delta log-likelihood: 0.0000003138
   log-like:             -118.337637
Iteration 641
   delta log-likelihood: 0.0000003117
   log-like:             -118.337635
Iteration 642
   delta log-likelihood: 0.0000003095
   log-like:             -118.337634
Iteration 643
   delta log-likelihood: 0.0000003074
   log-like:             -118.337633
Iteration 644
   delta log-likelihood: 0.0000003053
   log-like:             -118.337632
Iteration 645
   delta log-likelihood: 0.0000003032
   log-like:             -118.337631
Iteration 646
   delta log-likelihood: 0.0000003011
   log-like:             -118.337630
Iteration 647
   delta log-likelihood: 0.0000002990
   log-like:             -118.337629
Iteration 648
   delta log-likelihood: 0.0000002970
   log-like:             -118.337628
Iteration 649
   delta log-likelihood: 0.0000002950
   log-like:             -118.337626
Iteration 650
   delta log-likelihood: 0.0000002929
   log-like:             -118.337625
Iteration 651
   delta log-likelihood: 0.0000002909
   log-like:             -118.337624
Iteration 652
   delta log-likelihood: 0.0000002890
   log-like:             -118.337623
Iteration 653
   delta log-likelihood: 0.0000002870
   log-like:             -118.337622
```

```
Iteration 654
   delta log-likelihood: 0.0000002850
   log-like:             -118.337621
Iteration 655
   delta log-likelihood: 0.0000002831
   log-like:             -118.337620
Iteration 656
   delta log-likelihood: 0.0000002812
   log-like:             -118.337619
Iteration 657
   delta log-likelihood: 0.0000002793
   log-like:             -118.337618
Iteration 658
   delta log-likelihood: 0.0000002774
   log-like:             -118.337617
Iteration 659
   delta log-likelihood: 0.0000002755
   log-like:             -118.337616
Iteration 660
   delta log-likelihood: 0.0000002736
   log-like:             -118.337615
Iteration 661
   delta log-likelihood: 0.0000002717
   log-like:             -118.337614
Iteration 662
   delta log-likelihood: 0.0000002699
   log-like:             -118.337613
Iteration 663
   delta log-likelihood: 0.0000002681
   log-like:             -118.337612
Iteration 664
   delta log-likelihood: 0.0000002662
   log-like:             -118.337611
Iteration 665
   delta log-likelihood: 0.0000002644
   log-like:             -118.337610
Iteration 666
   delta log-likelihood: 0.0000002627
   log-like:             -118.337609
Iteration 667
   delta log-likelihood: 0.0000002609
   log-like:             -118.337608
Iteration 668
   delta log-likelihood: 0.0000002591
   log-like:             -118.337607
Iteration 669
   delta log-likelihood: 0.0000002574
   log-like:             -118.337606
```

```
Iteration 670
   delta log-likelihood: 0.0000002556
   log-like:             -118.337605
Iteration 671
   delta log-likelihood: 0.0000002539
   log-like:             -118.337604
Iteration 672
   delta log-likelihood: 0.0000002522
   log-like:             -118.337603
Iteration 673
   delta log-likelihood: 0.0000002505
   log-like:             -118.337602
Iteration 674
   delta log-likelihood: 0.0000002488
   log-like:             -118.337601
Iteration 675
   delta log-likelihood: 0.0000002471
   log-like:             -118.337600
Iteration 676
   delta log-likelihood: 0.0000002455
   log-like:             -118.337599
Iteration 677
   delta log-likelihood: 0.0000002438
   log-like:             -118.337598
Iteration 678
   delta log-likelihood: 0.0000002422
   log-like:             -118.337598
Iteration 679
   delta log-likelihood: 0.0000002406
   log-like:             -118.337597
Iteration 680
   delta log-likelihood: 0.0000002389
   log-like:             -118.337596
Iteration 681
   delta log-likelihood: 0.0000002373
   log-like:             -118.337595
Iteration 682
   delta log-likelihood: 0.0000002358
   log-like:             -118.337594
Iteration 683
   delta log-likelihood: 0.0000002342
   log-like:             -118.337593
Iteration 684
   delta log-likelihood: 0.0000002326
   log-like:             -118.337592
Iteration 685
   delta log-likelihood: 0.0000002311
   log-like:             -118.337591
```

```
Iteration 686
   delta log-likelihood: 0.0000002295
   log-like:             -118.337591
Iteration 687
   delta log-likelihood: 0.0000002280
   log-like:             -118.337590
Iteration 688
   delta log-likelihood: 0.0000002265
   log-like:             -118.337589
Iteration 689
   delta log-likelihood: 0.0000002249
   log-like:             -118.337588
Iteration 690
   delta log-likelihood: 0.0000002234
   log-like:             -118.337587
Iteration 691
   delta log-likelihood: 0.0000002220
   log-like:             -118.337586
Iteration 692
   delta log-likelihood: 0.0000002205
   log-like:             -118.337586
Iteration 693
   delta log-likelihood: 0.0000002190
   log-like:             -118.337585
Iteration 694
   delta log-likelihood: 0.0000002176
   log-like:             -118.337584
Iteration 695
   delta log-likelihood: 0.0000002161
   log-like:             -118.337583
Iteration 696
   delta log-likelihood: 0.0000002147
   log-like:             -118.337582
Iteration 697
   delta log-likelihood: 0.0000002132
   log-like:             -118.337581
Iteration 698
   delta log-likelihood: 0.0000002118
   log-like:             -118.337581
Iteration 699
   delta log-likelihood: 0.0000002104
   log-like:             -118.337580
Iteration 700
   delta log-likelihood: 0.0000002090
   log-like:             -118.337579
Iteration 701
   delta log-likelihood: 0.0000002076
   log-like:             -118.337578
```

```
Iteration 702
   delta log-likelihood: 0.0000002063
   log-like:             -118.337578
Iteration 703
   delta log-likelihood: 0.0000002049
   log-like:             -118.337577
Iteration 704
   delta log-likelihood: 0.0000002036
   log-like:             -118.337576
Iteration 705
   delta log-likelihood: 0.0000002022
   log-like:             -118.337575
Iteration 706
   delta log-likelihood: 0.0000002009
   log-like:             -118.337575
Iteration 707
   delta log-likelihood: 0.0000001995
   log-like:             -118.337574
Iteration 708
   delta log-likelihood: 0.0000001982
   log-like:             -118.337573
Iteration 709
   delta log-likelihood: 0.0000001969
   log-like:             -118.337572
Iteration 710
   delta log-likelihood: 0.0000001956
   log-like:             -118.337572
Iteration 711
   delta log-likelihood: 0.0000001943
   log-like:             -118.337571
Iteration 712
   delta log-likelihood: 0.0000001931
   log-like:             -118.337570
Iteration 713
   delta log-likelihood: 0.0000001918
   log-like:             -118.337569
Iteration 714
   delta log-likelihood: 0.0000001905
   log-like:             -118.337569
Iteration 715
   delta log-likelihood: 0.0000001893
   log-like:             -118.337568
Iteration 716
   delta log-likelihood: 0.0000001880
   log-like:             -118.337567
Iteration 717
   delta log-likelihood: 0.0000001868
   log-like:             -118.337567
```

```
Iteration 718
   delta log-likelihood: 0.0000001856
   log-like:             -118.337566
Iteration 719
   delta log-likelihood: 0.0000001844
   log-like:             -118.337565
Iteration 720
   delta log-likelihood: 0.0000001832
   log-like:             -118.337565
Iteration 721
   delta log-likelihood: 0.0000001820
   log-like:             -118.337564
Iteration 722
   delta log-likelihood: 0.0000001808
   log-like:             -118.337563
Iteration 723
   delta log-likelihood: 0.0000001796
   log-like:             -118.337563
Iteration 724
   delta log-likelihood: 0.0000001784
   log-like:             -118.337562
Iteration 725
   delta log-likelihood: 0.0000001772
   log-like:             -118.337561
Iteration 726
   delta log-likelihood: 0.0000001761
   log-like:             -118.337561
Iteration 727
   delta log-likelihood: 0.0000001749
   log-like:             -118.337560
Iteration 728
   delta log-likelihood: 0.0000001738
   log-like:             -118.337559
Iteration 729
   delta log-likelihood: 0.0000001727
   log-like:             -118.337559
Iteration 730
   delta log-likelihood: 0.0000001715
   log-like:             -118.337558
Iteration 731
   delta log-likelihood: 0.0000001704
   log-like:             -118.337557
Iteration 732
   delta log-likelihood: 0.0000001693
   log-like:             -118.337557
Iteration 733
   delta log-likelihood: 0.0000001682
   log-like:             -118.337556
```

```
Iteration 734
   delta log-likelihood: 0.0000001671
   log-like:             -118.337555
Iteration 735
   delta log-likelihood: 0.0000001660
   log-like:             -118.337555
Iteration 736
   delta log-likelihood: 0.0000001650
   log-like:             -118.337554
Iteration 737
   delta log-likelihood: 0.0000001639
   log-like:             -118.337554
Iteration 738
   delta log-likelihood: 0.0000001628
   log-like:             -118.337553
Iteration 739
   delta log-likelihood: 0.0000001618
   log-like:             -118.337552
Iteration 740
   delta log-likelihood: 0.0000001607
   log-like:             -118.337552
Iteration 741
   delta log-likelihood: 0.0000001597
   log-like:             -118.337551
Iteration 742
   delta log-likelihood: 0.0000001587
   log-like:             -118.337551
Iteration 743
   delta log-likelihood: 0.0000001576
   log-like:             -118.337550
Iteration 744
   delta log-likelihood: 0.0000001566
   log-like:             -118.337549
Iteration 745
   delta log-likelihood: 0.0000001556
   log-like:             -118.337549
Iteration 746
   delta log-likelihood: 0.0000001546
   log-like:             -118.337548
Iteration 747
   delta log-likelihood: 0.0000001536
   log-like:             -118.337548
Iteration 748
   delta log-likelihood: 0.0000001526
   log-like:             -118.337547
Iteration 749
   delta log-likelihood: 0.0000001516
   log-like:             -118.337547
```

```
Iteration 750
   delta log-likelihood: 0.0000001507
   log-like:             -118.337546
Iteration 751
   delta log-likelihood: 0.0000001497
   log-like:             -118.337545
Iteration 752
   delta log-likelihood: 0.0000001487
   log-like:             -118.337545
Iteration 753
   delta log-likelihood: 0.0000001478
   log-like:             -118.337544
Iteration 754
   delta log-likelihood: 0.0000001468
   log-like:             -118.337544
Iteration 755
   delta log-likelihood: 0.0000001459
   log-like:             -118.337543
Iteration 756
   delta log-likelihood: 0.0000001449
   log-like:             -118.337543
Iteration 757
   delta log-likelihood: 0.0000001440
   log-like:             -118.337542
Iteration 758
   delta log-likelihood: 0.0000001431
   log-like:             -118.337542
Iteration 759
   delta log-likelihood: 0.0000001422
   log-like:             -118.337541
Iteration 760
   delta log-likelihood: 0.0000001413
   log-like:             -118.337541
Iteration 761
   delta log-likelihood: 0.0000001404
   log-like:             -118.337540
Iteration 762
   delta log-likelihood: 0.0000001395
   log-like:             -118.337540
Iteration 763
   delta log-likelihood: 0.0000001386
   log-like:             -118.337539
Iteration 764
   delta log-likelihood: 0.0000001377
   log-like:             -118.337539
Iteration 765
   delta log-likelihood: 0.0000001368
   log-like:             -118.337538
```

```
Iteration 766
   delta log-likelihood: 0.0000001359
   log-like:             -118.337538
Iteration 767
   delta log-likelihood: 0.0000001351
   log-like:             -118.337537
Iteration 768
   delta log-likelihood: 0.0000001342
   log-like:             -118.337536
Iteration 769
   delta log-likelihood: 0.0000001334
   log-like:             -118.337536
Iteration 770
   delta log-likelihood: 0.0000001325
   log-like:             -118.337536
Iteration 771
   delta log-likelihood: 0.0000001317
   log-like:             -118.337535
Iteration 772
   delta log-likelihood: 0.0000001308
   log-like:             -118.337535
Iteration 773
   delta log-likelihood: 0.0000001300
   log-like:             -118.337534
Iteration 774
   delta log-likelihood: 0.0000001292
   log-like:             -118.337534
Iteration 775
   delta log-likelihood: 0.0000001284
   log-like:             -118.337533
Iteration 776
   delta log-likelihood: 0.0000001276
   log-like:             -118.337533
Iteration 777
   delta log-likelihood: 0.0000001267
   log-like:             -118.337532
Iteration 778
   delta log-likelihood: 0.0000001259
   log-like:             -118.337532
Iteration 779
   delta log-likelihood: 0.0000001251
   log-like:             -118.337531
Iteration 780
   delta log-likelihood: 0.0000001244
   log-like:             -118.337531
Iteration 781
   delta log-likelihood: 0.0000001236
   log-like:             -118.337530
```

```
Iteration 782
   delta log-likelihood: 0.0000001228
   log-like:             -118.337530
Iteration 783
   delta log-likelihood: 0.0000001220
   log-like:             -118.337529
Iteration 784
   delta log-likelihood: 0.0000001212
   log-like:             -118.337529
Iteration 785
   delta log-likelihood: 0.0000001205
   log-like:             -118.337528
Iteration 786
   delta log-likelihood: 0.0000001197
   log-like:             -118.337528
Iteration 787
   delta log-likelihood: 0.0000001190
   log-like:             -118.337528
Iteration 788
   delta log-likelihood: 0.0000001182
   log-like:             -118.337527
Iteration 789
   delta log-likelihood: 0.0000001175
   log-like:             -118.337527
Iteration 790
   delta log-likelihood: 0.0000001167
   log-like:             -118.337526
Iteration 791
   delta log-likelihood: 0.0000001160
   log-like:             -118.337526
Iteration 792
   delta log-likelihood: 0.0000001153
   log-like:             -118.337525
Iteration 793
   delta log-likelihood: 0.0000001146
   log-like:             -118.337525
Iteration 794
   delta log-likelihood: 0.0000001138
   log-like:             -118.337525
Iteration 795
   delta log-likelihood: 0.0000001131
   log-like:             -118.337524
Iteration 796
   delta log-likelihood: 0.0000001124
   log-like:             -118.337524
Iteration 797
   delta log-likelihood: 0.0000001117
   log-like:             -118.337523
```

```
Iteration 798
   delta log-likelihood: 0.0000001110
   log-like:             -118.337523
Iteration 799
   delta log-likelihood: 0.0000001103
   log-like:             -118.337522
Iteration 800
   delta log-likelihood: 0.0000001096
   log-like:             -118.337522
Iteration 801
   delta log-likelihood: 0.0000001090
   log-like:             -118.337522
Iteration 802
   delta log-likelihood: 0.0000001083
   log-like:             -118.337521
Iteration 803
   delta log-likelihood: 0.0000001076
   log-like:             -118.337521
Iteration 804
   delta log-likelihood: 0.0000001069
   log-like:             -118.337520
Iteration 805
   delta log-likelihood: 0.0000001063
   log-like:             -118.337520
Iteration 806
   delta log-likelihood: 0.0000001056
   log-like:             -118.337520
Iteration 807
   delta log-likelihood: 0.0000001049
   log-like:             -118.337519
Iteration 808
   delta log-likelihood: 0.0000001043
   log-like:             -118.337519
Iteration 809
   delta log-likelihood: 0.0000001036
   log-like:             -118.337518
Iteration 810
   delta log-likelihood: 0.0000001030
   log-like:             -118.337518
Iteration 811
   delta log-likelihood: 0.0000001024
   log-like:             -118.337518
Iteration 812
   delta log-likelihood: 0.0000001017
   log-like:             -118.337517
Iteration 813
   delta log-likelihood: 0.0000001011
   log-like:             -118.337517
```

```
Iteration 814
   delta log-likelihood: 0.0000001005
   log-like:             -118.337517
Iteration 815
   delta log-likelihood: 0.0000000999
   log-like:             -118.337516
 Turnbull estimator fitted successfully
  Timeline range: 0.0 - 24.4
  Number of intervals: 27
```

[13]:
```python
### Step 4.2: Model Comparison

# Compare AFT vs Turnbull using median BMI as representative
median_bmi = df_intervals['bmi'].median()
comparison_results = compare_aft_vs_turnbull(
    primary_model,
    turnbull_model,
    time_grid,
    median_bmi,
    df_intervals,
    verbose=True
)
```

```
 Comparing AFT model vs Turnbull non-parametric estimator…
  Restricting comparison to clinically meaningful range: 12.0 - 24.7 weeks
  Using 84 time points (avoids unreliable early extrapolation)
 Model comparison completed

 Population-Level Survival Curve Comparison:
  Turnbull: Non-parametric estimate using all 238 observations
  AFT: Parametric prediction averaged across all observations

 Comparison Metrics:
  Mean Absolute Error: 0.0141
  Root Mean Square Error: 0.0186
  Maximum Absolute Error: 0.0559
  KS Statistic: 0.0559
    Excellent agreement (MAE < 0.05)

 Survival Probabilities at Key Weeks:
Week   Turnbull     AFT          Difference
-------------------------------------------------
12     0.195        0.184        0.011
14     0.108        0.139        0.031
16     0.108        0.100        0.008
18     0.066        0.074        0.007
20     0.049        0.054        0.005
```

[14]:
```
### Step 4.3: Goodness of Fit Assessment

# Assess overall AFT model adequacy using module function
fit_assessment = assess_aft_goodness_of_fit(comparison_results, verbose=True)
```

```
 AFT Model Goodness of Fit Assessment:
==================================================

 Fit Quality Metrics:
  MAE: 0.0141 (threshold: < 0.1)
  RMSE: 0.0186 (threshold: < 0.15)
  KS: 0.0559 (threshold: < 0.2)

 Criteria Assessment:
  MAE:   PASS
  RMSE:   PASS
  KS:   PASS

 Overall Assessment:
  Criteria met: 3/3
  Overall fit quality: Good
    Recommendation: AFT model provides adequate fit to data
     The parametric AFT assumptions appear reasonable.

 Validation Summary:
  • Turnbull provides the non-parametric gold standard
  • AFT model offers interpretable covariate effects
  • Model agreement suggests good parametric fit
```

1.6 Section 5: BMI Grouping & Group-specific Optimal Weeks

Goal: Create BMI groups and calculate optimal testing weeks for each group.

1.6.1 Overview

- Calculate predicted median times for CART-based grouping
- Apply multiple BMI grouping strategies (clinical, tertile, CART)
- Evaluate grouping strategies using risk-based scoring
- Calculate group-specific optimal weeks for different confidence levels
- Generate final policy recommendations by BMI group

[15]:
```
### Step 5.1: Comprehensive Group-specific Analysis

# Perform group-specific analysis including:
# - Predicted median time calculation
# - Multiple BMI grouping strategies
# - Grouping strategy evaluation
# - Group-specific optimal weeks calculation
```

```python
group_analysis_results = perform_group_specific_analysis(
    df_intervals,
    primary_model,
    confidence_levels=[0.90, 0.95],
    time_grid=time_grid,
    grouping_methods=['clinical', 'tertile', 'cart'],  # Include CART if data␣
 ↪allows
    verbose=True
)
```

```
Performing group-specific optimal weeks analysis…
Calculating individual predicted median survival times…
Predicted median times calculated: 238/238 valid

Predicted Median Time Statistics:
 Mean: 5.52 weeks
 Std: 0.98 weeks
 Range: 3.88 - 8.71 weeks

Evaluating 4 grouping strategies…
Evaluating grouping strategy: bmi_group_cart
 Number of groups: 6
 Risk score: 11.522
 Weighted median: 5.522 weeks
 Complexity penalty: 6.000
 Between-group variance: 1.187
 Within-group variance: 0.083

 Group sizes:
   CART_G2: 58 (24.4%)
   CART_G5: 46 (19.3%)
   CART_G1: 38 (16.0%)
   CART_G3: 35 (14.7%)
   CART_G4: 31 (13.0%)
   CART_G6: 30 (12.6%)
Evaluating grouping strategy: bmi_group_clinical
 Number of groups: 3
 Risk score: 8.522
 Weighted median: 5.522 weeks
 Complexity penalty: 3.000
 Between-group variance: 2.240
 Within-group variance: 0.227

 Group sizes:
   Obese I (30-35): 147 (61.8%)
   Overweight (25-30): 61 (25.6%)
   Obese II+ ( 35): 30 (12.6%)
Evaluating grouping strategy: bmi_group_tertile
```

```
Number of groups: 3
Risk score: 8.522
Weighted median: 5.522 weeks
Complexity penalty: 3.000
Between-group variance: 1.068
Within-group variance: 0.240

Group sizes:
  Low BMI (T1): 80 (33.6%)
  High BMI (T3): 79 (33.2%)
  Medium BMI (T2): 79 (33.2%)

Best grouping strategy: clinical
Risk score: 8.522

Calculating optimal weeks for 3 BMI groups…

  Analyzing group: Obese I (30-35)
   90% confidence: Week 15.9
   95% confidence: Week 20.2

  Analyzing group: Overweight (25-30)
   90% confidence: Week 13.3
   95% confidence: Week 16.7

  Analyzing group: Obese II+ ( 35)
   90% confidence: Week 21.1
   95% confidence: Week Never

Group-specific analysis completed for 3 groups
```

[16]:
```python
### Step 5.2: Generate Group-specific Optimal Weeks Summary

# Create comprehensive summary table for group-specific recommendations
group_summary_table = create_group_optimal_weeks_summary(group_analysis_results, verbose=True)
```

```
Group-Specific Optimal Testing Weeks Summary:
======================================================================
         BMI_Group  n_mothers  representative_BMI BMI_range optimal_week_90
optimal_week_95
   Obese I (30-35)        147           32.017138 30.0-34.9            15.9
20.2
Overweight (25-30)         61           29.136316 26.6-30.0            13.3
16.7
   Obese II+ ( 35)        30           36.326687 35.1-39.2            21.1
Never
```

```
Summary Statistics:
 Total mothers: 238
 Number of BMI groups: 3
 Grouping method: clinical
```

1.7 Section 6: Monte Carlo Measurement Error Testing

Goal: Assess robustness to Y-chromosome concentration measurement errors.

1.7.1 Overview

- Model measurement error as Gaussian noise: y_observed = y_true + , ~ N(0, ²)
- Run Monte Carlo simulations with noisy measurements
- Refit AFT models and recalculate group-specific optimal weeks
- Assess stability and provide uncertainty quantification
- Generate robustness summary with confidence intervals

[17]:
```python
### Step 6.1: Monte Carlo Robustness Testing

# Run Monte Carlo simulation to assess robustness to measurement errors
# Use smaller sample size for demonstration (increase for production)
print("  Starting Monte Carlo robustness testing...")
print("   Note: Using reduced simulation count for demonstration")

mc_results = run_monte_carlo_robustness_test(
    df_tests,  # Original test data
    construct_intervals,  # Function to construct intervals
    fit_aft_models,  # Function to fit AFT models
    perform_group_specific_analysis,  # Function for group analysis
    n_simulations=300,  # Production setting for smooth CI and stable assessment
    sigma_error=0.002,  # 0.2% absolute concentration error
    confidence_levels=[0.90, 0.95],
    random_seed=42,
    n_workers=1,  # Single-threaded for stability
    verbose=True
)
```

```
Starting Monte Carlo robustness testing…
  Note: Using reduced simulation count for demonstration
Running Monte Carlo robustness test…
 Simulations: 300
 Measurement error  : 0.0020 (0.20%)
 Confidence levels: ['90%', '95%']
Running simulations (single-threaded)…
 Progress: 30/300 (10.0%)
 Progress: 60/300 (20.0%)
 Progress: 90/300 (30.0%)
 Progress: 120/300 (40.0%)
 Progress: 150/300 (50.0%)
```

```
 Progress: 180/300 (60.0%)
 Progress: 210/300 (70.0%)
 Progress: 240/300 (80.0%)
 Progress: 270/300 (90.0%)
 Progress: 300/300 (100.0%)
Analyzing Monte Carlo results…
Monte Carlo Analysis:
 Successful iterations: 300/300 (100.0%)

   Group: Obese I (30-35)
    90% confidence:
      Mean: 15.90 ± 0.38 weeks
      Median: 15.91 weeks
      95% CI: [15.15, 16.52]
      CV: 0.024
    95% confidence:
      Mean: 20.03 ± 0.56 weeks
      Median: 20.00 weeks
      95% CI: [19.09, 21.21]
      CV: 0.028

   Group: Overweight (25-30)
    90% confidence:
      Mean: 13.55 ± 0.67 weeks
      Median: 13.48 weeks
      95% CI: [12.12, 14.85]
      CV: 0.049
    95% confidence:
      Mean: 17.05 ± 0.78 weeks
      Median: 16.97 weeks
      95% CI: [15.45, 18.64]
      CV: 0.046

   Group: Obese II+ ( 35)
    90% confidence:
      Mean: 20.27 ± 1.08 weeks
      Median: 20.15 weeks
      95% CI: [18.33, 22.42]
      CV: 0.053
    95% confidence:
      Mean: 24.06 ± 0.80 weeks
      Median: 24.24 weeks
      95% CI: [22.18, 25.00]
      CV: 0.033

   Group: High BMI (T3)
    90% confidence:
      Mean: 18.24 ± 0.56 weeks
```

```
    Median: 18.33 weeks
    95% CI: [16.89, 19.24]
    CV: 0.031
  95% confidence:
    Mean: 23.03 ± 0.92 weeks
    Median: 23.18 weeks
    95% CI: [20.98, 24.48]
    CV: 0.040

 Group: Medium BMI (T2)
  90% confidence:
    Mean: 15.62 ± 0.39 weeks
    Median: 15.61 weeks
    95% CI: [14.92, 16.36]
    CV: 0.025
  95% confidence:
    Mean: 19.75 ± 0.59 weeks
    Median: 19.70 weeks
    95% CI: [18.71, 20.91]
    CV: 0.030

 Group: Low BMI (T1)
  90% confidence:
    Mean: 13.78 ± 0.60 weeks
    Median: 13.79 weeks
    95% CI: [12.58, 14.70]
    CV: 0.044
  95% confidence:
    Mean: 17.40 ± 0.73 weeks
    Median: 17.42 weeks
    95% CI: [15.91, 18.56]
    CV: 0.042

Robustness Assessment:
 Obese I (30-35) (90%): Good stability
  CV: 0.024 (Excellent)
  95% CI width: 1.37 weeks (Good)
 Obese I (30-35) (95%): Good stability
  CV: 0.028 (Excellent)
  95% CI width: 2.12 weeks (Moderate)
 Overweight (25-30) (90%): Good stability
  CV: 0.049 (Excellent)
  95% CI width: 2.73 weeks (Moderate)
 Overweight (25-30) (95%): Good stability
  CV: 0.046 (Excellent)
  95% CI width: 3.18 weeks (Moderate)
 Obese II+ ( 35) (90%): Poor stability
  CV: 0.053 (Good)
```

```
    95% CI width: 4.09 weeks (Poor)
  Obese II+ ( 35) (95%): Good stability
    CV: 0.033 (Excellent)
    95% CI width: 2.82 weeks (Moderate)
  High BMI (T3) (90%): Good stability
    CV: 0.031 (Excellent)
    95% CI width: 2.35 weeks (Moderate)
  High BMI (T3) (95%): Good stability
    CV: 0.040 (Excellent)
    95% CI width: 3.50 weeks (Moderate)
  Medium BMI (T2) (90%): Good stability
    CV: 0.025 (Excellent)
    95% CI width: 1.44 weeks (Good)
  Medium BMI (T2) (95%): Good stability
    CV: 0.030 (Excellent)
    95% CI width: 2.20 weeks (Moderate)
  Low BMI (T1) (90%): Good stability
    CV: 0.044 (Excellent)
    95% CI width: 2.12 weeks (Moderate)
  Low BMI (T1) (95%): Good stability
    CV: 0.042 (Excellent)
    95% CI width: 2.66 weeks (Moderate)

  Overall Stability: Good
    Good: 11
    Moderate: 0
    Poor: 1
```

[18]:
```python
### Step 6.2: Robustness Summary & Uncertainty Quantification

# Generate comprehensive robustness summary table
mc_summary_table = create_monte_carlo_summary_table(mc_results, verbose=True)
```

```
 Monte Carlo Robustness Summary:
================================================================================
         BMI_Group optional_week_90_mean optimal_week_90_std optimal_week_90_ci
stability_90 optimal_week_95_mean optimal_week_95_std optimal_week_95_ci
stability_95
   Obese I (30-35)                15.90                ±0.38     [15.15, 16.52]
Good                 20.03                ±0.56     [19.09, 21.21]         Good
Overweight (25-30)                13.55                ±0.67     [12.12, 14.85]
Good                 17.05                ±0.78     [15.45, 18.64]         Good
   Obese II+ ( 35)               20.27                ±1.08     [18.33, 22.42]
Poor                 24.06                ±0.80     [22.18, 25.00]         Good
     High BMI (T3)                18.24                ±0.56     [16.89, 19.24]
Good                 23.03                ±0.92     [20.98, 24.48]         Good
   Medium BMI (T2)                15.62                ±0.39     [14.92, 16.36]
Good                 19.75                ±0.59     [18.71, 20.91]         Good
```

```
     Low BMI (T1)                13.78               ±0.60     [12.58, 14.70]
Good                17.40               ±0.73     [15.91, 18.56]         Good

 Monte Carlo Test Summary:
  Measurement error  : 0.0020 (0.20%)
  Simulations: 300
  Success rate: 100.0%
  Overall stability: Good
```

1.8 Section 7: Baseline Two-step ML Comparison (Optional)

Goal: Compare AFT survival approach against traditional machine learning methods.

1.8.1 Overview

- **Classification Component**: Binary outcome (ever reached 4% threshold)
- **Regression Component**: Time to threshold (interval midpoint approximation)

- **Feature Engineering**: BMI + derived features (squared, log-transformed)
- **Model Comparison**: Logistic Regression, Random Forest for both components
- **Group Mapping**: Map ML predictions to group-level recommendations
- **Benchmarking**: Compare AFT vs ML optimal weeks side-by-side

[19]:
```python
### Step 7.1: Complete ML Baseline Comparison Pipeline

# Run comprehensive ML baseline comparison including:
# - Binary classification (ever reached threshold)
# - Time-to-event regression (interval midpoint approximation)
# - Group-level recommendation mapping
# - Side-by-side comparison with AFT results

print("  Running ML baseline comparison against AFT survival analysis...")

ml_comparison_results = run_ml_baseline_comparison(
    df_X,  # Use feature matrix with standardized BMI (bmi_z)
    group_analysis_results,  # AFT group-specific results from Section 5
    confidence_levels=[0.90, 0.95],
    cv_folds=5,
    random_state=42,
    verbose=True
)
```

```
 Running ML baseline comparison against AFT survival analysis…
 Running ML baseline comparison pipeline…
 Preparing dataset for ML baseline comparison…
 ML dataset prepared:
  Samples: 238
  Features: 4 (bmi, bmi_z, bmi_squared, bmi_log)
  Ever reached threshold: 0.945
```

```
 Mean time approximation: 8.66 weeks
Training classification models for binary outcome…

  Training Logistic Regression…
   AUC (CV): 0.657 ± 0.207
   AUC (full): 0.677
   Brier score: 0.051

  Training Random Forest…
   AUC (CV): 0.565 ± 0.229
   AUC (full): 0.998
   Brier score: 0.015

Best classification model: Logistic Regression
 Cross-validation AUC: 0.657
Training regression models for time to threshold…

  Training Random Forest…
   MAE (CV): 3.499 ± 0.280 weeks
   RMSE (CV): 5.323 ± 0.470 weeks
   MAE (full): 1.575 weeks
   R²: 0.677

Best regression model: Random Forest
 Cross-validation MAE: 3.499 weeks
Mapping ML predictions to group-level recommendations…

  Group: Obese I (30-35)
   n = 147, BMI = 32.1
   Threshold prob: 0.945
   Predicted time: 8.74 ± 2.80
   ML optimal week (90%): 6.40
   ML optimal week (95%): 6.22

  Group: Overweight (25-30)
   n = 61, BMI = 29.0
   Threshold prob: 0.973
   Predicted time: 7.53 ± 1.93
   ML optimal week (90%): 6.18
   ML optimal week (95%): 6.00

  Group: Obese II+ ( 35)
   n = 30, BMI = 36.7
   Threshold prob: 0.889
   Predicted time: 10.46 ± 3.27
   ML optimal week (90%): 7.05
   ML optimal week (95%): 6.82
Comparing AFT vs ML baseline recommendations…
```

```
 Comparing 3 common BMI groups…
 AFT vs ML Comparison:
================================================================================
         BMI_Group  n_mothers_AFT  n_mothers_ML  bmi_mean_AFT  bmi_mean_ML
AFT_week_90 ML_week_90 diff_90 AFT_week_95 ML_week_95 diff_95
Overweight (25-30)            61            61     29.136316    28.991183
13.33        6.18   +7.15       16.67       6.00  +10.66
   Obese I (30-35)           147           147     32.017138    32.087644
15.91        6.40   +9.51       20.15       6.22  +13.93
   Obese II+ ( 35)            30            30     36.326687    36.730942
21.06        7.05  +14.01       Never       6.82     N/A

 Comparison Summary:
  90% confidence level:
    Mean difference (AFT - ML): +10.22 ± 2.85 weeks
    Agreement: Poor
  95% confidence level:
    Mean difference (AFT - ML): +12.29 ± 1.63 weeks
    Agreement: Poor

 ML baseline comparison completed
```

1.9 Section 8: Validation & Final Policy Table

Goal: Generate final recommendations with comprehensive validation and uncertainty quantification.

1.9.1 Overview

- **Cross-validation**: K-fold validation for model robustness assessment
- **Sensitivity Analysis**: Test different confidence levels and model assumptions
- **Final Policy Table**: Comprehensive recommendations with uncertainty bounds
- **Results Export**: Save final policy table for clinical implementation
- **Quality Assurance**: Complete validation summary with confidence metrics

[20]:
```python
### Step 8.1: Comprehensive Validation & Final Policy Generation

# Run complete validation pipeline including:
# - K-fold cross-validation for model robustness
# - Sensitivity analysis across confidence levels
# - Final policy table with uncertainty quantification
# - Automatic export to CSV for clinical use

print("  Running comprehensive validation and final policy generation...")

final_validation_results = run_comprehensive_validation(
    df_intervals,  # Interval-censored dataset
    group_analysis_results,  # AFT group-specific results
```

```
    construct_intervals,  # Function to construct intervals
    fit_aft_models,  # Function to fit AFT models
    perform_group_specific_analysis,  # Function for group analysis
    mc_results=mc_results,  # Monte Carlo robustness results
    k_folds=5,  # Cross-validation folds
    confidence_levels=[0.90, 0.95],
    export_results=True,  # Export to CSV
    output_path=None,  # Use default path
    verbose=True
)
```

```
Running comprehensive validation and final policy generation…
Running comprehensive validation and final policy generation…
Performing 5-fold cross-validation of AFT pipeline…

    Fold 1/5
     Train: 190 intervals from 190 mothers
     Test: 48 intervals from 48 mothers
       Fold completed: 48 test predictions

    Fold 2/5
     Train: 190 intervals from 190 mothers
     Test: 48 intervals from 48 mothers
       Fold completed: 48 test predictions

    Fold 3/5
     Train: 190 intervals from 190 mothers
     Test: 48 intervals from 48 mothers
       Fold completed: 48 test predictions

    Fold 4/5
     Train: 191 intervals from 191 mothers
     Test: 47 intervals from 47 mothers
       Fold completed: 47 test predictions

    Fold 5/5
     Train: 191 intervals from 191 mothers
     Test: 47 intervals from 47 mothers
       Fold completed: 47 test predictions

Cross-validation Summary:
 Successful folds: 5/5
Performing sensitivity analysis…

    Testing different confidence levels…
     Variant 1: ['85%', '90%', '95%']
         Success
     Variant 2: ['90%', '95%']
```

```
      Success
    Variant 3: ['80%', '90%', '95%']
      Success

   Testing AFT model robustness…
    Weibull: AIC = 253.54
    LogLogistic: AIC = 254.20

  Sensitivity analysis completed
  Creating final policy recommendation table…
  Final Policy Recommendation Table:
================================================================================
====================
         BMI_Range  n_mothers  representative_BMI bmi_range optimal_week_90
threshold_prob_at_optimal_90 optimal_week_95 threshold_prob_at_optimal_95
mc_ci_90     mc_ci_95 cv_success_rate
   Obese I (30-35)       147          32.017138 30.0-34.9            15.9
0.900           20.2                         0.951 [15.1, 16.5] [19.1, 21.2]
1.00
Overweight (25-30)        61          29.136316 26.6-30.0            13.3
0.903           16.7                         0.950 [12.1, 14.8] [15.5, 18.6]
1.00
   Obese II+ ( 35)        30          36.326687 35.1-39.2            21.1
0.901          Never                           N/A [18.3, 22.4] [22.2, 25.0]
1.00

  Policy Summary:
   Analysis method: clinical
   Total mothers: 238
   BMI groups: 3
   Confidence levels: ['90%', '95%']
   Monte Carlo stability: Good
   Cross-validation success rate: 1.00
  Final policy table exported to:
/home/richard/projects/cumcm/output/results/prob2_policy_recommendations.csv
   File size: 0.5 KB

  Comprehensive validation completed
   Cross-validation success: 1.00
   Final policy groups: 3
   Results exported to:
/home/richard/projects/cumcm/output/results/prob2_policy_recommendations.csv
```