(1) $\int_0^{\frac{\pi}{4}} \frac{dx}{1+2\sin^2 x} = \int_0^{\frac{\pi}{4}} \frac{dx}{\sin^2 x+\cos^2 x+2\sin^2 x} dx = \int_0^{\pi/4} \frac{dx}{3\sin^2 x+\cos^2 x}$

$= \int_0^{\pi/4} \frac{\frac{1}{\cos^2 x}}{3\frac{\sin^2 x}{\cos^2 x}+1} dx = \int_0^{\pi/4} \frac{\sec^2 x}{1+3\tan^2 x}$ $\quad t=\tan x \quad dt=\sec^2 x dx$ (1)

$x=0 \Rightarrow t=0$

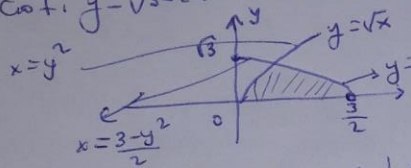
$x=\frac{\pi}{4} \Rightarrow t=1$

$= \int_0^1 \frac{dt}{1+3t^2} = \int_0^1 \frac{1}{3(\frac{1}{3}+t^2)} dt = \frac{1}{3}\int_0^1 \frac{dt}{(\frac{1}{\sqrt{3}})^2+t^2} = \frac{1}{3}\cdot\frac{\sqrt{3}}{1} \arctan\sqrt{3}t \Big]_0^1$

$= \frac{\sqrt{3}}{3}(\arctan\sqrt{3} - \arctan 0) = \frac{\sqrt{3}}{3}\arctan\sqrt{3} = \frac{\sqrt{3}}{3}\cdot\frac{\pi}{3}$ olur

(2) $y=\sqrt{x}$ ve $y=\sqrt{3-2x}$ eğrileri arasında kalan bölgenin alanını bulunuz.

Çözüm: $y=\sqrt{3-2x}$ in tanımlılması $3-2x\geq 0 \Rightarrow 2x\leq 3 \Rightarrow x\leq\frac{3}{2}$ olacak



$y=\sqrt{3-2x}$ eğrisi $x=0$ için $y=\sqrt{3}$

$x=\frac{3}{2}$ için $y=0$ olur

Kesim noktası

$y^2=3-2x$

$2x=3-y^2$

$x=\frac{3-y^2}{2}$

$\sqrt{x}=\sqrt{3-2x}$

$x=3-2x$

$3x=3$

$x=1$

$y=\sqrt{3-2x}$

$y=\sqrt{3-2}=1$

alanı y-eksenine pare düşünecegiz

Alan $= \int_0^1$ (sağdaki eğri − soldaki eğri) dx

$= \int_0^1 (\frac{3-y^2}{2} - y^2) dy$

$= \frac{1}{2}\int_0^1 (3-3y^2) dy = \frac{1}{2}\left(3y-y^3\Big]_0^1\right) = \frac{1}{2}(3-1-0) = 1$

(3) $\int \sqrt{x}\cos\sqrt{x}\,dx = ?$

Çözüm: $t=\sqrt{x}$ $\quad t^2=x$ $\quad dx=2t\,dt$

$= \int t\cdot\cos t\cdot 2t\,dt = 2\int t^2\cos t\,dt$ $\quad \int x^2\cos x\,dx$ hesaplanır sonucaçıdılır

(4) $\int x^2 \arcsin x\,dx = ?$ $\quad$ Çözüm: $u=\arcsin x$ $\quad du=\frac{1}{\sqrt{1-x^2}}dx$ $\quad dv=x^2dx$ $\quad v=\frac{x^3}{3}$

$= \frac{x^3}{3}\arcsin x - \int \frac{x^3}{3}\cdot\frac{1}{\sqrt{1-x^2}} dx = \frac{x^3}{3}\arcsin x - \frac{1}{3}\int \frac{x^3}{\sqrt{1-x^2}} dx$

$\int \frac{x^3}{\sqrt{1-x^2}} dx = \int \frac{x^2 x\,dx}{\sqrt{1-x^2}}$ $\quad t=\sqrt{1-x^2} \Rightarrow t^2=1-x^2 \Rightarrow x^2=1-t^2$

$2t\,dt=-2x\,dx$

$x\,dx=-t\,dt$

$= \int \frac{(1-t^2)(-t\,dt)}{t}$

$= \int (t^2-1)\,dt = \frac{t^3}{3}-t = \frac{(\sqrt{1-x^2})^3}{3} - \sqrt{1-x^2}$

yerine yazılacak

(5) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}\,dx$ $\quad t=\sin x \quad x=0 \Rightarrow t=0$ $\quad dt=\cos x\,dx \quad x=\frac{\pi}{2} \Rightarrow t=1$

$$= \int_0^1 \frac{dt}{1+t^2} = \arctan t\Big]_0^1 = \arctan 1 - \arctan 0 = \frac{\pi}{4} - 0 = \frac{\pi}{4}$$

(6) $\int_0^{\sqrt{3}} \sqrt{4-x^2}\,dx = ?$ $\quad x=2\sin t \quad \sqrt{4-x^2}=2\cos t$ $\quad dx=2\cos t\,dt$

$t=\arcsin\left(\frac{x}{2}\right)$ $\quad x=0 \Rightarrow t=0$ $\quad x=\sqrt{3} \Rightarrow t=\arcsin\left(\frac{\sqrt{3}}{2}\right)=\frac{\pi}{3}$

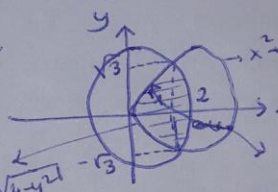
$$= \int_0^{\frac{\pi}{3}} 2\cos t \cdot 2\cos t\,dt = 4\int_0^{\frac{\pi}{3}} \cos^2 t\,dt = 4\int_0^{\frac{\pi}{3}} \frac{(1+\cos 2t)}{2}\,dt$$

$$= 2\int_0^{\frac{\pi}{3}} (1+\cos 2t)\,dt = 2\left(t+\frac{1}{2}\sin 2t\right]_0^{\frac{\pi}{3}}\right)$$

$$= 2\left(\frac{\pi}{3} + \underbrace{\sin\frac{2\pi}{3} - 0 - \frac{1}{2}\sin 0}_{0}\right) = 2\left(\frac{\pi}{3}+\frac{1}{2}\sin\frac{\pi}{3}\right) = 2\left(\frac{\pi}{3}+\frac{\sqrt{3}}{4}\right)$$

$$= \frac{2\pi}{3}+\frac{\sqrt{3}}{2}$$

(7) $x^2+y^2=4$ çemberi ile $x^2+y^2-4x=0$ çemberi arasında kalan bölgenin alanını bulunuz.



Çözüm kesim noktaları

$$\left.\begin{aligned} x^2+y^2&=4\\ x^2+y^2-4x&=0\end{aligned}\right\} \quad 4=4x \Rightarrow x=1 \Rightarrow y^2=4-x^2=4-1=3,\ y=\pm\sqrt{3}$$

$$\left.\begin{aligned} x^2+y^2&=4\\ x^2+y^2&=4x\end{aligned}\right\}$$

Taralı alan $= \int_{-\sqrt{3}}^{\sqrt{3}} \left(2+\sqrt{4-y^2}\right)\left(\sqrt{4-y^2}\right) - \left(2-\sqrt{4-y^2}\right)dy$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} \left(2\sqrt{4-y^2}-2\right)dy = 2\int_{-\sqrt{3}}^{\sqrt{3}} \left(\sqrt{4-y^2}-1\right)dy$$

$$= 4\int_0^{\sqrt{3}} \left(\sqrt{4-y^2}-1\right)dy = 4\left(\int_0^{\sqrt{3}} \sqrt{4-y^2}\,dy - \int_0^{\sqrt{3}} 1\,dy\right)$$

$$= 4\left(\frac{2\pi}{3}+\frac{\sqrt{3}}{2}-\sqrt{3}\right) = 4\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right) = \frac{8\pi}{3}-2\sqrt{3}$$