1) $\int \frac{\arctan x}{x^2(1+x^2)}\,dx$ $u=\arctan x$ $dv=\frac{1}{x^2}dx$
$du=\frac{1}{1+x^2}dx$ $v=-\frac{1}{x}$

$= -\frac{1}{x}\arctan x - \int -\frac{1}{x}\cdot\frac{1}{1+x^2}\,dx = -\frac{1}{x}\arctan x + \int \frac{1}{x(1+x^2)}\,dx$

$= -\frac{1}{x}\arctan x + \int\left(\frac{1}{x} - \frac{x}{1+x^2}\right)dx = -\frac{1}{x}\arctan x + \ln|x| - \frac{1}{2}\ln(1+x^2) + C$

2) $\int \frac{x\arctan x}{\sqrt{1+x^2}}\,dx$, $x=\tan t$ $dx=\sec^2 t\,dt$
$t=\arctan x$ $\sqrt{1+x^2}=\sec t$

$= \int \frac{\tan t\,.\,t}{\sec t}\sec^2 t\,dt = \int t\tan t\sec t\,dt = \int \frac{\sin t}{\cancel{\cos t}}\cdot\frac{\cancel{1}}{\cos t}\,t\,dt$

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$\Rightarrow \int \underbrace{x}_{u}\underbrace{\sec x\tan x\,dx}_{dv}$, $u=x$ $dv=\sec x\tan x$
$du=dx$ $v=\sec x$

$= x\sec x - \int \sec x\,dx = x\sec x - \ln|\sec x + \tan x|$

$= t\sec t - \ln|\sec t + \tan t| = \arctan x\sqrt{1+x^2} - \ln\left|\sqrt{1+x^2}+x\right|$

$=$

3) $f(x,y) = y\,e^{\frac{x}{y}} + x\sin\left(\frac{y}{x}\right)$ ise $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f(x,y)$

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Coz. $\frac{\partial f}{\partial x} = -\frac{1}{y}\cdot y\,e^{\frac{x}{y}} + \sin\left(\frac{y}{x}\right) + x\left(-\frac{y}{x^2}\right)\cos\left(\frac{y}{x}\right)$

$= -e^{\frac{x}{y}} + \sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)$

$\frac{\partial f}{\partial y} = e^{\frac{x}{y}} + y\left(-\frac{x}{y^2}e^{\frac{x}{y}}\right) + x\cdot\frac{1}{x}\cos\left(\frac{y}{x}\right)$

$= e^{\frac{x}{y}} - \frac{x}{y}e^{\frac{x}{y}} + \cos\left(\frac{y}{x}\right)$

$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = x\left(-e^{\frac{x}{y}} + \sin\left(\frac{y}{x}\right) - \frac{y}{x}\cos\left(\frac{y}{x}\right)\right)$
$+ y\left(e^{\frac{x}{y}} - \frac{x}{y}e^{\frac{x}{y}} + \cos\left(\frac{y}{x}\right)\right)$

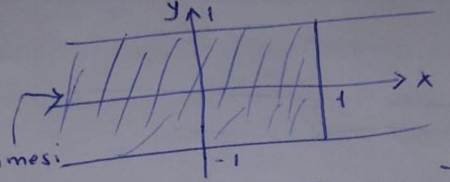
$= -\cancel{x e^{\frac{x}{y}}} + x\sin\left(\frac{y}{x}\right) - \cancel{y\cos\left(\frac{y}{x}\right)}$
$+ y e^{\frac{x}{y}} - \cancel{x e^{\frac{x}{y}}} + \cancel{y\cos\left(\frac{y}{x}\right)}$
$= x\sin\left(\frac{y}{x}\right) + y e^{\frac{x}{y}} = f(x,y)$ olur.

④ $f(x,y) = y^2\sqrt{1-x} + x^2\sqrt{1-y^2}$

ile f nın tanım kümesini bulunuz $\frac{\partial f}{\partial x} = ?$ $\frac{\partial f}{\partial y} = ?$

Çözüm: $1-x \geq 0 \Rightarrow x \leq 1$ $\quad 1-y^2 \geq 0 \Rightarrow y^2 \leq 1 \Rightarrow |y| \leq 1$

$-1 \leq y \leq 1$



f nın tanım kümesi

$$\frac{\partial f}{\partial x} = y^2\left(-\frac{1}{2\sqrt{1-x}}\right) + 2x\sqrt{1-y^2} = \frac{-y^2}{2\sqrt{1-x}} + 2x\sqrt{1-y^2}$$

$$\frac{\partial f}{\partial y} = 2y\sqrt{1-x} + x^2\left(\frac{-2y}{2\sqrt{1-y^2}}\right) = 2y\sqrt{1-x} + \frac{-yx^2}{\sqrt{1-y^2}}$$

⑤ Parametrik denklemleri: $x = e^t\cos t$ $\; y = e^t\sin t$ ile verilen $t=0$ $\; t=\ln\pi$ için elde edilen noktalar arasındaki eğrinin yay uzunluğunu bulunuz. $\quad x = x(t) = e^t\cos t$ $\; y = y(t) = e^t\sin t$

②

$$\text{yay uzunluğu} = \int_0^{\ln\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{\ln\pi} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt \qquad \begin{aligned} x'(t) &= e^t\cos t - e^t\sin t \\ y'(t) &= e^t\sin t + e^t\cos t \end{aligned}$$

$$(x'(t))^2 + (y'(t))^2 = (e^t(\cos t - \sin t))^2 + (e^t(\sin t + \cos t))^2$$

$$= e^{2t}\left((\cos t - \sin t)^2 + (\sin t + \cos t)^2\right) = e^{2t}(1+1) = 2e^{2t}$$

$$\text{yay uzunluğu} = \int_0^{\ln\pi} \sqrt{2e^{2t}}\, dt = \sqrt{2}\int_0^{\ln\pi} e^t\, dt = \sqrt{2}\left(e^t\Big]_0^{\ln\pi}\right)$$

$$= \sqrt{2}\left(e^{\ln\pi} - e^0\right) = \sqrt{2}(\pi - 1)$$

⑥ $y^2 = \frac{4}{9}(1+x^2)^3$ ile verilen eğrinin $0 \leq x \leq 3$ aralığındaki yayının uzunluğunu bulunuz. $\quad 2yy' = \frac{4}{9}(1+x^2)^2 \cdot 3 \cdot 2x$

Çöz: $\text{yay uzunluğu} = \int_0^3 \sqrt{1+(y')^2}\, dx$

$$y' = \frac{\frac{12}{9}(1+x^2)^2 \cdot x}{y} = \frac{\frac{4}{3}(1+x^2)^2 \cdot x}{y} \Rightarrow (y')^2 = \frac{\left(\frac{4}{3}\right)^2(1+x^2)^4 x^2}{y^2} = \frac{\frac{16}{9}(1+x^2)^4 x^2}{\frac{4}{9}(1+x^2)^3}$$

$$= \frac{16}{9}\cdot\frac{9}{4}(1+x^2)\cdot x^2 = 4x^2(1+x^2)$$

$$1+(y')^2 = 1 + 4x^4 + 4x^2 = (2x^2+1)^2$$

$$\int_0^3 \sqrt{1+(y')^2}\, dx = \int_0^3 \sqrt{(2x^2+1)^2}\, dx = \int_0^3 (2x^2+1)\, dx = \frac{2}{3}x^3 + x\Big]_0^1 = \frac{2}{3} + 1$$