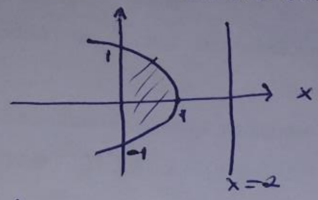
① $x=1-y^4$ eğrisi ve y-ekseni arasında kalan bölgenin $x=2$ doğrusu etrafında döndürülmesi ile oluşan cismin hacmini bulunuz

Çözüm



$x=1-y^4 \quad x=0 \Rightarrow 1-y^4=0$

$y^4=1 \Rightarrow (y^2-1)(y^2+1)=0$

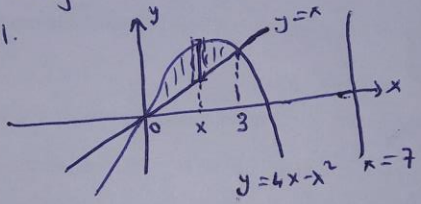
$y^2-1=0 \Rightarrow y=\mp 1$

$$V=\pi\int_{-1}^{1}\left[(0-2)^2-(1-y^4-2)^2\right]dy=\pi\int_{-1}^{1}\left(4-(-1-y^4)^2\right)dy$$

$$=\pi\int_{-1}^{1}\left(4-(y^4+1)^2\right)dy=2\pi\int_{0}^{1}\left(4-(y^4+1)^2\right)dy=\frac{224\pi}{45}$$

② $y=4x-x^2$ parabolü ve $y=x$ doğrusu arasında kalan bölgenin $x=7$ doğrusu etrafında döndürülmesi ile oluşan cismin hacmini bul.

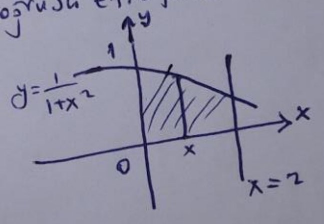
Çöz.



$$V=2\pi\int_{0}^{3}(7-x)(4x-x^2-x)dx$$

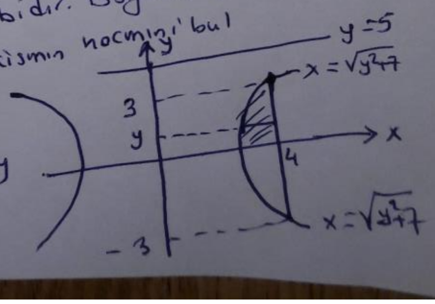
$$=2\pi\int_{0}^{3}(7-x)(-x^2+3x)dx$$

③ Şekildeki bölgenin $x=2$ doğrusu etrafında döndürülmesi ile oluşan cismin hacmini bul



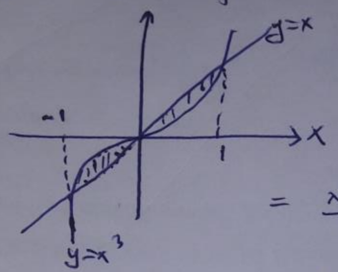
$$V=2\pi\int_{0}^{2}(2-x)\left(\frac{1}{1+x^2}\right)dx$$

④ $x^2-y^2=7$ hiperbolü şekildeki gibidir. Bölgenin $y=5$ doğrusu etrafında döndürülmesi ile oluşan cismin hacmini bul



$$V=2\pi\int_{-3}^{3}(5-y)\left(4-\sqrt{y^2+7}\right)dy$$

⑤ $y=x^3$ eğrisi ve $y=x$ doğrusu arasında kalan bölgenin alanını bul.



$x^3=x \Rightarrow x(x^2-1)=0\ ,\ x=0\ ,\ x=\mp 1$

Taralı alan $= \int_{-1}^{0}(x^3-x)+\int_{0}^{1}(x-x^3)dx$

$= \frac{x^4}{4}-\frac{x^2}{2}\Big]_{-1}^{0} + \frac{x^2}{2}-\frac{x^4}{4}\Big]_{0}^{1}$

$= -\left(\frac{1}{4}-\frac{1}{2}\right)+\frac{1}{2}-\frac{1}{4} = \frac{1}{2}-\frac{1}{4}+\frac{1}{2}-\frac{1}{4}$

$= 1-\frac{1}{2}=\frac{1}{2}$

⑥ $\int \frac{\arctan x}{x^2(1+x^2)}dx$, $u=\arctan x$, $du=\frac{1}{1+x^2}dx$

$dv=\frac{1}{x^2(1+x^2)}dx$

$v=\int\frac{1}{x^2(1+x^2)}dx=\int\left(\frac{1}{x^2}-\frac{1}{1+x^2}\right)dx$

$=-\frac{1}{x}-\arctan x$

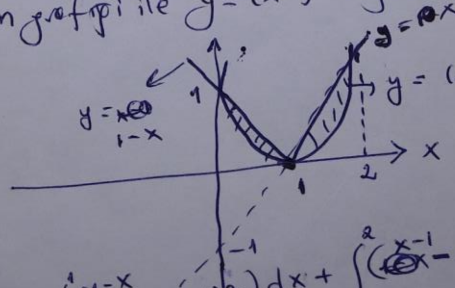
$=\arctan x\left(-\frac{1}{x}-\arctan x\right)-\int\left(-\frac{1}{x}-\arctan x\right)\frac{1}{1+x^2}dx$

$=\underbrace{-\arctan x\left(\frac{1}{x}+\arctan x\right)}+\int\frac{1}{x(1+x^2)}dx+\int\frac{\arctan x}{1+x^2}dx$

$=A+\int\left(\frac{1}{x}-\frac{x}{1+x^2}\right)dx+\frac{1}{2}(\arctan x)^2+C$

$=A+\ln|x|-\frac{1}{2}\ln(1+x^2)+\frac{1}{2}(\arctan x)^2+C$

⑦ $y=|x-1|$ in grafiği ile $y=(x-1)^2$ eğrisi arasında kalan alanı bulunuz



Taralı alan $=\int_{0}^{1}\left(1-x-(x-1)^2\right)dx+\int_{1}^{2}\left(x-1-(x-1)^2\right)dx$

⑧ $\displaystyle\int \frac{x^2dx}{\sqrt{4x-x^2}} = ?$ $\quad 4x-x^2 = -(x^2-4x) = -((x-2)^2-4) = 4-(x-2)^2$

$t = x-2 \Rightarrow dx = dt \quad \displaystyle\int \frac{x^2dx}{\sqrt{4x-x^2}} = \int \frac{(t+2)^2}{\sqrt{4-t^2}}dt \quad t = 2\sin\theta$

donusumu yapilir

$$= \int \frac{t^2+4t+4}{\sqrt{4-t^2}}dt = \int \frac{t^2}{\sqrt{4-t^2}}dt + 4\int \frac{t}{\sqrt{4-t^2}}dt + 4\int \frac{1}{\sqrt{4-t^2}}dt$$

$$= \int \frac{t^2}{\sqrt{4-t^2}}dt + \frac{-4}{2}\int \frac{-2t}{2\sqrt{4-t^2}}dt + 4\cdot\frac{1}{2}\arcsin t$$

$$= A - 2\sqrt{4-t^2} + 2\arcsin t \quad A = \int \frac{t^2}{\sqrt{4-t^2}}dt$$

③

⑨ $\displaystyle\int_0^1 x\sqrt{1-x^4}\,dx = \int_0^1 x\sqrt{1-(x^2)^2}\,dx \quad x^2 = t \Rightarrow 2x\,dx = dt$

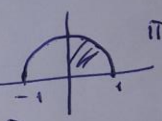
$x\,dx = \frac{dt}{2}$

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 1$

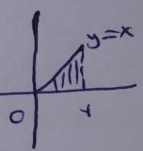
$$= \int_0^1 \frac{dt}{2}\sqrt{1-t^2}\,dt = \frac{1}{2}\int_0^1 \sqrt{1-t^2}\,dt$$

$$= \frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$$



$\pi\cdot 1^2 \to$ cember alan

⑩ $\displaystyle\int_0^1 (x+\sqrt{1-x^2})dx = \underbrace{\int_0^1 x\,dx}_{\frac{1}{2}} + \underbrace{\int_0^1 \sqrt{1-x^2}\,dx}_{\frac{\pi}{4}} = \frac{\pi}{4}+\frac{1}{2}$



⑪ $\displaystyle\int_{-1}^1 \frac{\sin x}{1+x^2}dx = ?$ (Cevap 0 cunki fonksiyon tek fonk)

⑫ $\displaystyle\int_0^1 (1-x)^9 dx = \int_1^0 t^9(-dt) = -\int_1^0 t^9 dt = \int_0^1 t^9 dt = \frac{t^{10}}{10}\Big]_0^1 = \frac{1}{10}$

$1-x = t$

$dx = -dt$

⑬ $\displaystyle\int_1^{+\infty} \frac{1}{(3x+1)^2}dx = \lim_{b\to+\infty} \int_1^b \frac{1}{(3x+1)^2}dx$ $\quad t = 3x+1 \quad \frac{dt}{3} = dx$

$\displaystyle\int \frac{1}{(3x+1)^2}dx = \int \frac{dt}{3t^2} = -\frac{1}{3t}$

$$= \lim_{b\to+\infty}\left(-\frac{1}{3(3x+1)}\Big]_1^b\right)$$

$$= \lim_{b\to+\infty}\left(-\frac{1}{3(3b+1)} + \frac{1}{12}\right) = \frac{1}{12}$$

14) $\displaystyle\int_1^{+\infty} \frac{\ln x}{x^3}\,dx = ?$ $\quad u = \ln x \quad du = \frac{1}{x}dx \quad \frac{1}{x^3}dx = dv$

$v = \int x^{-3}dx = \frac{x^{-3+1}}{-3+1} = -\frac{1}{2x^2}$

$= \ln x\left(-\frac{1}{2x^2}\right) - \int -\frac{1}{2x^2}\cdot\frac{1}{x}\,dx$

$= \ln x\left(-\frac{1}{2x^2}\right) + \frac{1}{2}\int \frac{1}{x^3}\,dx = -\frac{1}{2x^2}\ln x + \frac{1}{2}\left(-\frac{1}{2x^2}\right)$

$\displaystyle\int_1^{+\infty} \frac{\ln x}{x^3} = \lim_{b\to+\infty} \int_1^b \frac{\ln x}{x^3}\,dx = \lim_{b\to+\infty}\left(-\frac{1}{2x^2}\ln x - \frac{1}{4x^2}\Big]_1^b\right)$

$\displaystyle= \lim_{b\to+\infty}\left(-\frac{1}{2b^2}\ln b - \frac{1}{4b^2} + \frac{1}{4}\right) = \frac{1}{4} \qquad \lim_{b\to+\infty}\frac{\ln b}{b^2} = 0$

15) $\displaystyle\int_0^{+\infty} \frac{1}{(x^2+3x+2)}\,dx = \int_0^{+\infty} \frac{1}{\left(x+\frac{3}{2}\right)^2 + 2 - \frac{9}{4}}\,dx = \int_0^{+\infty} \frac{1}{\left(x+\frac{3}{2}\right)^2 + \frac{1}{4}}$

$\displaystyle\lim_{b\to+\infty}\left(2\arctan 2\left(x+\frac{3}{2}\right)\right]_0^b = \lim_{b\to+\infty}\left(2\arctan(2b+3) - 2\arctan 3\right)$

$= 2\frac{\pi}{2} - 2\arctan 3$

4

16) $\displaystyle\int_0^2 (2^x - x^2)\,dx = \frac{2^x}{\ln 2} - \frac{x^3}{3}\Big]_0^2 = \frac{2^2}{\ln 2} - \frac{8}{3} - \left(\frac{2^0}{\ln 2} - \frac{0}{3}\right)$

$= \frac{4}{\ln 2} - \frac{8}{3} - \frac{1}{\ln 2} = \frac{3}{\ln 2} - \frac{8}{3}$

17) $\displaystyle\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx = -\arctan(\cos x)\Big]_0^{\pi}$ $\qquad \displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx \quad t = \cos x \quad dt = -\sin x\,dx$

$\displaystyle= \int \frac{-dt}{1+t^2} = -\arctan t = -\arctan(\cos x)$

$= \cancel{\arccos x}$

$= -\arctan(\cos\pi) + \arctan(\cos 0)$

$= -\arctan(-1) + \arctan 1$

$= -\left(-\frac{\pi}{4}\right) + \frac{\pi}{4} = \frac{\pi}{2}$

18) $\displaystyle\int_0^1 \frac{x+1}{1+x^2}\,dx = \int_0^1 \frac{x}{1+x^2}\,dx + \int_0^1 \frac{1}{1+x^2}\,dx = \frac{1}{2}\ln(1+x^2)\Big]_0^1 + \arctan x\Big]_0^1$

$= \frac{1}{2}\ln 2 - 0 + \arctan 1 - \underbrace{\arctan 0}_{0}$

$= \frac{\ln 2}{2} + \frac{\pi}{4}$