1) $\int_0^2 \frac{dx}{(x-2)^{\frac{2}{3}}}$ Genelleştirilmiş integraller

in yakınsak old. göstererek değerini bulunuz.

Çözüm 2 noktasında fonk. tanımlı değil

$$\lim_{c\to 2^-} \int_0^c \frac{dx}{(x-2)^{\frac{3}{2}}} = \lim_{c\to 2^-} \left(3(x-2)^{\frac{1}{3}}\Big]_0^c\right)$$

$$= \lim_{c\to 2^-} \left(3(c-2)^{\frac{1}{3}} - 3(0-2)^{\frac{1}{3}}\right) = 0 - 3(-2)^{\frac{1}{3}} = 3\cdot 2^{\frac{1}{3}} = 3\sqrt[3]{2}$$

yakınsak ve değeri $= 3\sqrt[3]{2}$ dır

2) $\int_2^4 \frac{dx}{(x-2)^{\frac{2}{3}}}$ in yak. old. göster.

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Çözüm $\lim_{c\to 2^-} \int_c^4 \frac{dx}{(x-2)^{\frac{2}{3}}} = \lim_{c\to 2^-} \left(3(x-2)^{\frac{1}{3}}\right]_c^4$

$$= \lim_{c\to 2^-} \left(3(4-2)^{\frac{1}{3}} - 3(c-2)^{\frac{1}{3}}\right) = 3\cdot 2^{\frac{1}{3}} - 0 = 3\sqrt[3]{2} \text{ dur.}$$

3) $\int_0^4 \frac{dx}{(x-2)^{\frac{3}{2}}}$ nin yak old. göster değerini bul.

Çözüm 2 de problem var $\int_0^2 \frac{dx}{(x-2)^{\frac{3}{2}}}$ ve $\int_2^4 \frac{dx}{(x-2)^{\frac{1}{3}}}$ yakınsak

olduğundan $\int_0^4 \frac{dx}{(x-2)^{\frac{2}{3}}}$ yakınsak ve değeri $= 3\sqrt[3]{2} + 3\sqrt[3]{2}$
$= 6\sqrt[3]{2}$ dur.

4) $\int_0^1 x\ln x\,dx$ in yak. old. göster

Çözüm: 0 da fonk tanımlı değil

$$\lim_{c\to 0^+} \int_c^1 x\ln x\,dx = \lim_{c\to 0^+} \left(\frac{x^2}{2}\ln x - \frac{x^2}{2}\Big]_c^1\right)$$

$$= \lim_{c\to 0^+} \left(\frac{1}{2}\overset{0}{\ln 1} - \frac{1}{2} - \left(\frac{c^2}{2}\ln c - \frac{c^2}{2}\right)\right)$$

$$= \lim_{c\to 0^+} \left(-\frac{1}{2} - \frac{c^2}{2}\ln c + \frac{c^2}{2}\right) = -\frac{1}{2} - \lim_{c\to 0^+}\frac{c^2}{2}\ln c + \lim_{c\to 0^+}\frac{c^2}{2}$$

$$= -\frac{1}{2} + 0 + 0$$

$$\lim_{c\to 0^+}\frac{c^2}{2}\ln c = \underbrace{\lim_{c\to 0^+}\frac{c}{2}}_{0} \cdot \underbrace{\lim_{c\to 0^+} c\ln c}_{0} = 0$$

$= -\frac{1}{2}$ olur.

⑤ $\int_1^{+\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$ in yak. old. göster ve değerini bul.

Çözüm: $\int \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx$ i hesaplayalım $t=-\sqrt{x}$ $dt = -\frac{1}{2\sqrt{x}}dx$

$\Rightarrow \frac{1}{\sqrt{x}}dx = -2dt$ $\quad \int \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx = \int e^t(-2dt)$

$= -2\int e^t dt = -2e^t = -2e^{-\sqrt{x}}$

$\lim\limits_{b\to+\infty}\int_1^b \frac{e^{-\sqrt{x}}}{\sqrt{x}}dx = \lim\limits_{b\to+\infty}\left(-2e^{-\sqrt{x}}\Big]_1^b\right) = \lim\limits_{b\to+\infty}\left(-2e^{-\sqrt{b}}+2e^{-1}\right)$

$= \lim\limits_{b\to+\infty} -2e^{-\sqrt{b}}+2 = -2.0+2e^{-1} = \frac{2}{e}$ $\left(\lim\limits_{b\to+\infty} e^{-\sqrt{b}} = \lim\limits_{b\to+\infty}\frac{1}{e^{\sqrt{b}}} = 0\right)$

Genelleştirilmiş integralin değeri $\frac{2}{e}$ dir.

⑥ $\int_2^{+\infty}\frac{dx}{x(\ln x)^2} = \lim\limits_{b\to+\infty}\int_2^b \frac{dx}{x(\ln x)^2} =$

$= \lim\limits_{b\to+\infty}\left(-\frac{1}{\ln x}\Big]_2^b\right) = \lim\limits_{b\to+\infty}\left(-\frac{1}{\ln b}+\frac{1}{\ln 2}\right) = \frac{1}{\ln 2}$ olur.

⑦ $\int_3^{+\infty}\frac{dx}{x^2-1} = \lim\limits_{b\to+\infty}\int_3^b \frac{dx}{x^2-1} = \lim\limits_{b\to+\infty}\left(\frac12\ln\left|\frac{x-1}{x+1}\right|\Big]_3^b\right)$

$= \lim\limits_{b\to+\infty}\left(\ln\left|\frac{b-1}{b+1}\right| - \frac12\ln\left|\frac{3-1}{3+1}\right|\right) = -\frac12\ln\frac24 = -\frac12\ln\frac12$

$\int\frac{1}{x^2-1}dx = \int\frac12\left(\frac{1}{x-1}-\frac{1}{x+1}\right)dx = \frac12\left(\ln|x-1|-\ln|x+1|\right)$

$= \frac12\left(\ln\left|\frac{x-1}{x+1}\right|\right)$ dir

problemler

1) $\int_{-1}^8 \frac{1}{\sqrt[3]{x}}dx = \frac92$ old. göst $\quad$ 2) $\int_{-\infty}^{+\infty}\frac{1}{1+4x^2}dx = \frac{\pi}{2}$ old. göst

3) $\int_{-\infty}^6 \frac{dx}{(4-x)^2}$ in yak. olmadığını göster $\quad$ 4) $\int_{-2}^2\frac{dx}{\sqrt{4-x^2}} = \pi$ old. göst

5) $\int_1^2 \frac{x\,dx}{\sqrt{x-1}}$ in yak olmadığını göster $\quad$ 6) $\int_0^1 \frac{e^{\frac1x}}{x^3}dx$ in yak. olmadığını göster.

7) $\int_1^{+\infty}\frac{\ln x}{x^2}dx = 1$ old. göster $\quad$ 8) $\int_0^{+\infty}\frac{\ln(1+x^2)}{x^2} = ?$