# Inventory Control Models

Economic Order Quantity Model
Economic Product Quantity Model
Quantity Discount Model

# Learning objective

- After this class the students should be able to:
  - calculate the order quantity that minimize the total cost inventory, based on the EOQ model and
  - analyze the implication of this model.

# Inventory & inventory system

- Inventory is the set of items that an organization holds for later use by the organization.
- An inventory system is a set of policies that monitors and controls inventory.
- It determines
  - how much of each item should be kept,
  - when low items should be replenished, and
  - how many items should be ordered or made when replenishment is needed.

# 1/8 sheet: True or False

1. *The inventory decision may be summarized by two questions: to make or buy and how much.*
2. *Forecasting the demand for parts and products is a basic component of an inventory control system.*
3. *The aim of inventory models used in inventory management is to maximize profit.*

# Basic types of inventory

- independent demand,
- dependent demand, and
- supplies.

# Independent Demand

- ***Independent demand*** items are those items that we sell to customers.
- ***Dependent demand*** items are those items whose demand is determined by other items.
  - Demand for a car translates into demand for four tires, one engine, one transmission, and so on.
  - The items used in the production of that car (the independent demand item) are the dependent demand items.
- **Supplies** are items such as copier paper, cleaning materials, and pens that are not used directly in the production of independent demand items

# Four reasons

1. **To meet variations in demand;**
2. **To allow flexible production schedules;**
3. **As a safeguard against variations in delivery time; and**
4. **To get a lower price.**

# INVENTORY CONTROL MODELS

- **INVENTORY CONTROL** is the systematic storing of resources for current or future need.
- The purpose of inventory control is to minimize the total cost of inventory.

# The cost of Inventory

- *Holding costs*
- *Setup costs of production;*
- *Ordering costs*
  - cost of placing an order (for raw material stocks) *; and*
- *Shortage costs*
  - (difficult to measure and is often handled by establishing a service –level policy.

# Key Inventory Terms

- ***Lead time***: time interval between ordering and receiving the order
- ***Holding (carrying) costs***: cost to carry an item in inventory for a length of time, usually a year (heat, light, rent, security, deterioration, spoilage, breakage, depreciation, opportunity cost,…, etc.,)
- ***Ordering costs***: costs of ordering and receiving inventory (shipping cost, cost of preparing how much is needed, preparing invoices, cost of inspecting goods upon arrival for quality and quantity, moving the goods to temporary storage)
- ***Shortage costs***: costs when demand exceeds supply (the opportunity cost of not making a sale, loss of customer goodwill, late charges, the cost of lost of production or downtime)

# INVENTORY CONTROL

- *Assumptions*:
  1. There is an order quantity that will minimize cost.
  2. There is a safety stock quantity that will minimize costs.

# Economic Order Quantity Models

The question of how much to order is frequently determined by using an Economic Order Quantity (EOQ) model. EOQ models identify the optimal order quantity by minimizing the sum of certain annual costs that vary with order size. Three order size models are described:

- The basic economic order quantity model
- The economic production quantity model
- The quantity discount model

# The Economic Order Quantity Model

- Assumptions:
  - EOQ model assumes a constant demand.
  - EOQ calculation assumes that ordering costs and holding costs will remain constant.
- Limitations
  - Since no fluctuation in demand is considered in the EOQ calculation, business losses due to potential shortage of inventory are ignored.
  - EOQ model does not take into account the seasonal fluctuations in the cost of inventory. In seasonal industries, it would make sense to buy inventory in bulk when it is readily available at a lower price. Inventory may be harder to procure in off season and would usually cost more as well.
  - EOQ model does not take into account purchase discounts that could be obtained by buying inventory in bulk.

# Notation

- $D$ = Annual Demand rate
- $C_o$ = Ordering Cost per order
- $C_h$ = Holding Cost per unit per year
- $L$ = Lead time
- $W$ = Working days per year

# Steps in EOQ (Q*)

1. **Determine the EOQ (Q*)**

$$Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

2. **Determine the total inventory cost.**
   a. Compute the number of orders (**O**):

$$O = \frac{D}{Q^*}$$

   b. Compute the total ordering cost (**$C_o$**):

$$Total\,C_o = OC_o$$

   c. Compute the average inventory (***I***)

$$I = \frac{Beginning\,Inventory + Ending\,Inventory}{2}$$

# Steps in EOQ (Q*)

d. Compute the total holding cost ($C_h$):

$$TotalC_h = I * C_h$$

e. Compute the total annual inventory Cost

(***Total*** $C_I$):

$$TotalC_I = TotalC_O + TotalC_h$$

# Economic Order Quantity

## Two Methods for Carrying Cost

Carry cost ($C_h$) can be expressed either:

1. As a fixed cost, such as

   $C_h$ = \$0.50 per unit per year

2. As a percentage of the item's purchase cost (P)

   $C_h = I \times P$

   I = a percentage of the purchase cost

# Economic Production Quantity (EPQ)

- Inventory continuously flows or builds up over a period of time after an order has been placed or when units are produced and sold simultaneously.
- Inventory arrives gradually
- Daily demand rate must be taken into account
- Assumes inventory is being produced at a rate of p units per day
- There is a setup cost each time production begins

# Steps in EOQ (Q*)

3. **Determine the Reorder Point:**

   a. Compute the daily demand (***d***):

$$d = \frac{D}{W}$$

   b. Compute the Reorder Point(***ROP***):

$$ROP = dL$$

# Let's try this!!!

- Manuel, the Purchasing Manager of JVC Flat Screen TV, wants to determine the most optimum inventory practice for the company. He was able to determine the demand and inventory costs needed in his analysis. The annual demand is 1,000 units. The ordering cost is $10,000 per order and **the holding cost is $500 per unit per year. It** will take 20 days for an order to arrive and there are 250 working days per year.
- What is the economic order quantity? How much is the total annual inventory cost? What is the reorder point?

# It's your turn!

- Identify the given and show solutions of your answers to the question that follow.
    1. Demand for the Child Cycle at Best Buy is 500 units per month. Best Buy incurs a fixed order placement, transportation, and receiving cost of Php 4,000 each time an order is placed. Each cycle costs Php 500 and the retailer has a holding cost of 20 percent. Evaluate the number of computers that the store manager should order in each replenishment lot.

# Economic Production Quantity (EPQ)

- Inventory continuously flows or builds up over a period of time after an order has been placed or when units are produced and sold simultaneously.
- Inventory arrives gradually
- Daily demand rate must be taken into account
- Assumes inventory is being produced at a rate of $p$ units per day
- There is a setup cost each time production begins

# Economic Production Quantity (EPQ)

- Production done in batches or lots
- Capacity to produce a part exceeds the part's usage or demand rate
- Assumptions of EPQ are similar to EOQ except orders are received incrementally during production

# Economic Production Quantity (EPQ)

- Annual Carrying Cost for Production Run Model
  - $Q^*$ = number of pieces per order or production run
  - $D$ = annual demand
  - $C_s$ = setup cost
  - $C_h$ = holding or carrying cost per unit per year
  - $p$ = daily production rate
  - $d$ = daily demand rate
  - $t$ = length of production run in days

# Economic Production Quantity (EPQ)

- We will need the average inventory level for finding carrying cost
- Average inventory level is ½ the maximum
  - Max inventory = $Q\left(1-\frac{d}{p}\right)$
  - Ave inventory = $\frac{Q}{2}\left(1-\frac{d}{p}\right)$
  - Annual Holding Cost = $\frac{Q}{2}\left(1-\frac{d}{p}\right)C_h$

# Economic Production Quantity (EPQ)

- Annual Setup Cost
  - Setup cost replaces ordering cost

$$Annual\_Setup\_Cost = \frac{D}{Q} C_s$$

$$Annual\_Ordering\_Cost = \frac{D}{Q} C_s$$

# Economic Production Quantity (EPQ)

- Determining the Optimal Production Quantity
  - Costs are minimized when the setup costs equals the holding cost

$$Q^* = \sqrt{\frac{2DC_s}{C_h\left(1-\frac{d}{p}\right)}}$$

# Example

- Brown Manufacturing produces commercial refrigeration units in batches. The firm's estimated demand for the year is 10,000 units. It costs about $100 to set the manufacturing process, and the carrying cost is about 50 cents per unit per year. When the production process has been set up, 80 refrigeration units can be manufactured daily. The demand during the production period has traditionally been 60 units per day. Brown operates its refrigeration unit production area 167 days per year. How many refrigeration units should Brown Manufacturing produce each batch? How long should the production part of the cycle last? How many production runs there will be in a year?

# Quantity Discount Model

- Quantity discounts are price reductions for large orders offered to customers to induce them to buy in large quantities.
  - In this case the price per unit decreases as order quantity increases.
- If the quantity discounts are offered, the buyer must weigh the potential benefits of reduced purchase price and fewer orders that will result from buying in large quantities against the increase in carrying cost caused by higher average inventories.
- The buyer's goal with quantity discounts is to select the order quantity that will minimize the total cost, where the total cost is the sum of carrying cost, ordering cost, and purchasing (i.e., product) cost.

# Quantity Discount Model

- Purpose
  - Minimize total inventory costs, which now include actual material cost
  - A quantity discount is a reduced unit price based on purchasing a large quantity

**TABLE 12.2**

Quantity Discount Schedule

| DISCOUNT NUMBER | DISCOUNT QUANTITY | DISCOUNT | DISCOUNT COST |
|---|---|---|---|
| 1 | 0 to 999 | 0% | $5.00 |
| 2 | 1,000 to 1,999 | 4% | $4.80 |
| 3 | 2,000 and over | 5% | $4.75 |

# Quantity Discount Model

- The purchase cost or material cost becomes a relevant cost, as it changes based on the order quantity

$$Total\_Cost = MaterialCost + OrderingCost + CarryingCost$$

$$Total\_Cost = DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

- Where:
  - ***D*** = annual demand in units
  - $C_o$ = ordering cost of each order
  - **C**= cost per unit
  - $C_h$ = holding or carrying cost per unit per year
  - $C_h$ = *IC where* ***I*** *= holding cost as a percentage of the unit cost* (*C*)

# Four Steps to Analyze Quantity Discount Model

1. For each discount price (**C**), compute *EOQ*

$$EOQ = Q^* = \sqrt{\frac{2DC_o}{IC}}$$

2. If *EOQ* < *minimum for discount*, adjust the quantity to Q = minimum for discount

3. For each EOQ or adjusted Q, compute Total Cost

$$Total\_Cost = DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

4. Select the Q* with the lowest total cost

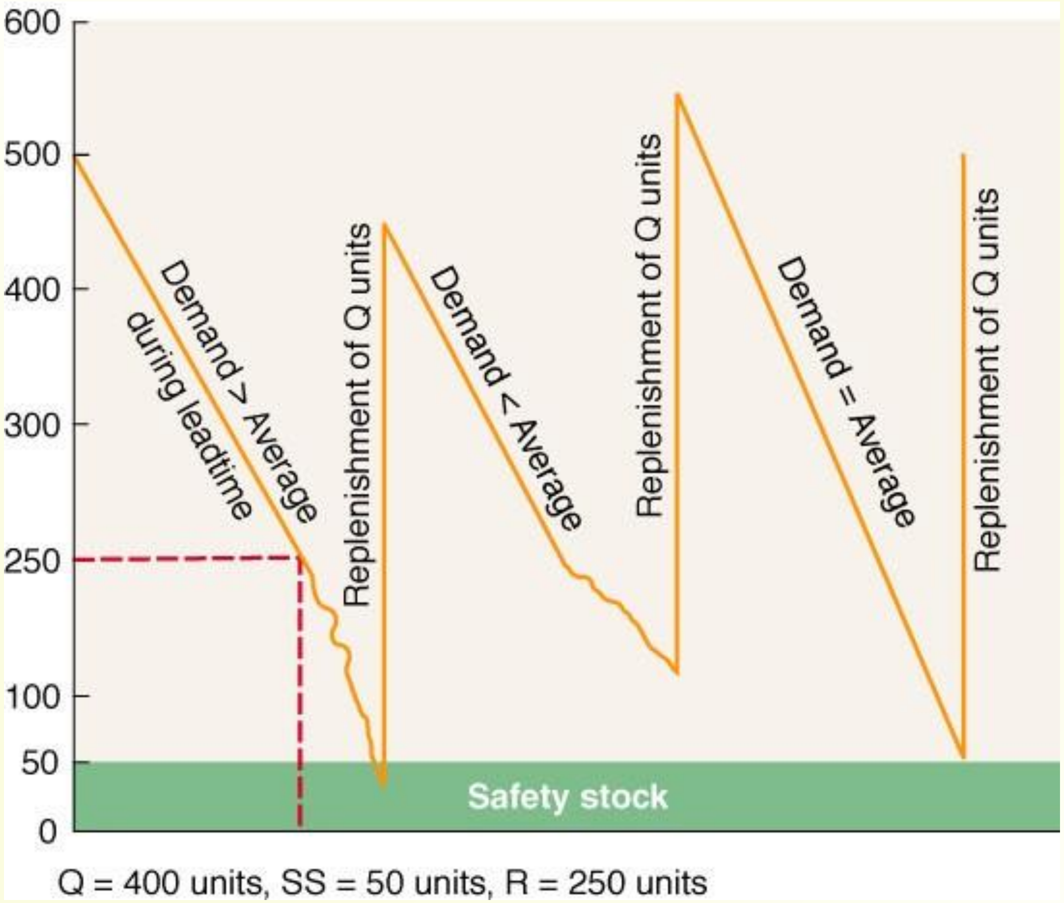
# Example

- Brass Department Store stocks toy race cars. Recently, the store was given a quantity discount schedule for the cars; this quantity discount schedule is shown below. The normal cost for the toy race cars is $5.

| Discount Number | Unit Price (C) | Order Quantity (Q) |
|---|---|---|
| 1 | $5 | 700 |
| 2 | 4.80 | 1,000 |
| 3 | 4.75 | 2,000 |

- The ordering cost is $49 per order, the annual demand is 5,000 race cars, and the inventory carrying charge as percentage of cost, I, is 20%.
- What order quantity will minimize the total inventory cost?

# What if Demand is Uncertain?



# When to Reorder with EOQ Ordering

- ***Reorder Point*** - When the quantity on hand of an item drops to this amount, the item is reordered
- ***Safety Stock*** - Stock that is held in excess of expected demand due to variable demand rate and/or lead time.
  - When variability is present in demand or lead time, it creates the possibility that actual demand will exceed expected demand.
  - Consequently, it becomes necessary to carry additional inventory, called "safety stock", to reduce the risk of running out of stock during lead time.
  - The reorder point then increases by the amount of the safety stock.

# Safety Stock and Service Level

- ***Service Level*** is the probability that demand during lead time won't exceed on-hand inventory.
- Risk of a stockout = 1 – (service level)
- More safety stock means greater service level and smaller risk of stockout

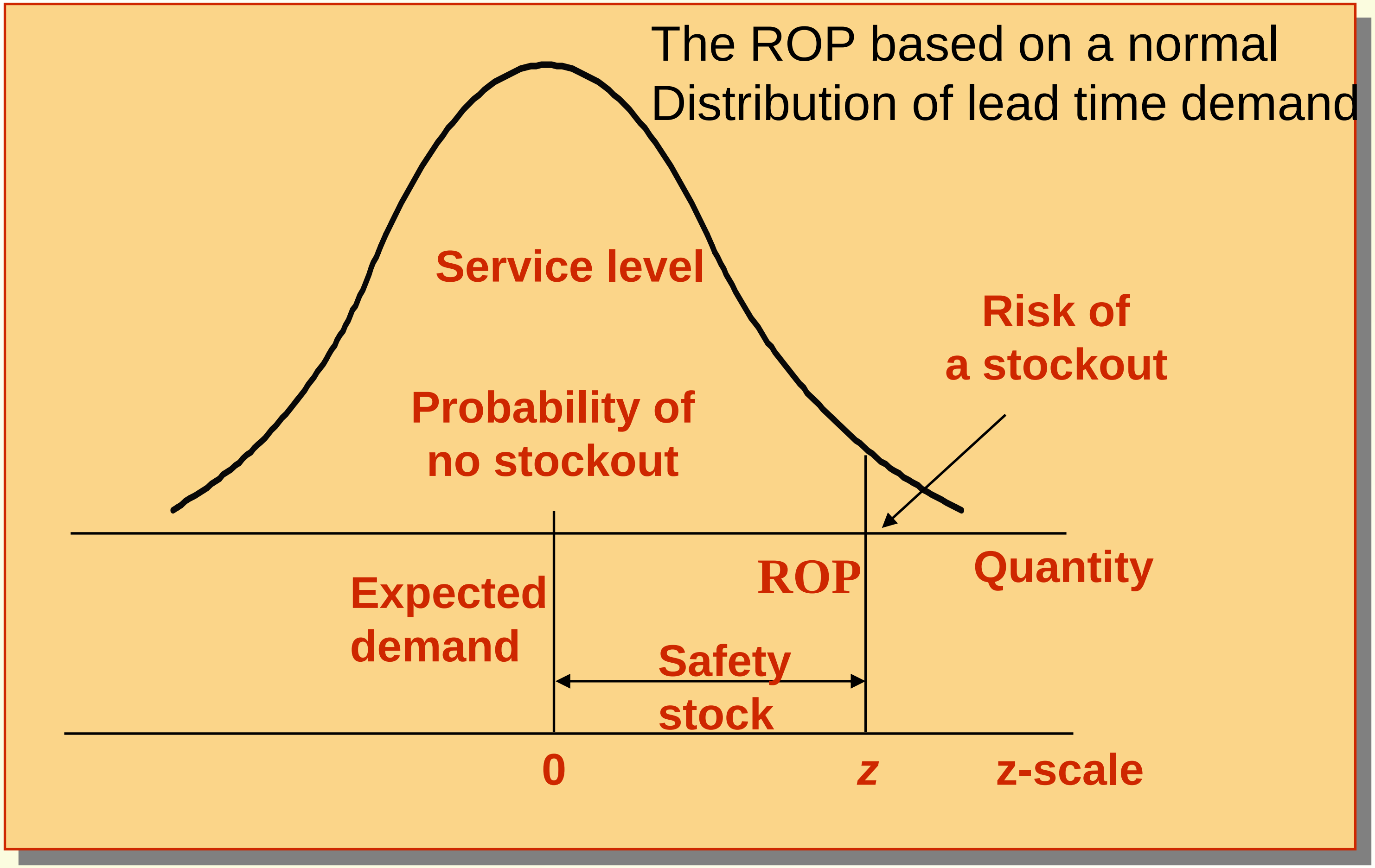
# Safety stock

- Because it costs money to hold safety stock, a manager must carefully weigh the cost of carrying safety stock against the reduction in stockout risk it provides.
- The customer service level increases as the risk of stockout decreases.
- The order cycle "service level" can be defined as the probability that demand will not exceed supply during lead time. This means a service level 95% implies a probability of 95% that demand will not exceed supply during lead time.
- The "risk of stockout" is the complement of "service level"
- **The amount of safety stock depends on:**

1. The average demand rate and average lead time
2. Demand and lead time variability
3. The desired service level

# Reorder Point

Figure 11.13



# Safety Stock and Reorder Point

- Without safety stock:

$$ROP = dL$$

where $ROP$ = reorder point in units

$d$ = daily demand in units

$L$ = lead time in days

- With safety stock:

$$R = dL + SS$$

where $SS$ = safety stock in units

# When to Reorder? Reorder Point Determination

1. **The expected demand during lead time and its standard deviation are available. In this case the formula is:**

$$SS = z\sigma_{dL}$$

i.e.,

$$R = dL + z\sigma_{dL}$$

$R$ = reorder point with safety stock

$d$ = average daily demand

$L$ = lead time in days

$z$ = number of standard deviations associated with desired service level

$\sigma$ = standard deviation of demand during lead time

# Z-values corresponding to service level

| Service Level | Service Factor | | Service Level | Service Factor |
|---|---|---|---|---|
| 50.00% | 0 | | 90.00% | 1.28 |
| 55.00% | 0.13 | | 91.00% | 1.34 |
| 60.00% | 0.25 | | 92.00% | 1.41 |
| 65.00% | 0.39 | | 93.00% | 1.48 |
| 70.00% | 0.52 | | 94.00% | 1.55 |
| 75.00% | 0.67 | | 95.00% | 1.64 |
| 80.00% | 0.84 | | 96.00% | 1.75 |
| 81.00% | 0.88 | | 97.00% | 1.88 |
| 82.00% | 0.92 | | 98.00% | 2.05 |
| 83.00% | 0.95 | | 99.00% | 2.33 |
| 84.00% | 0.99 | | 99.50% | 2.58 |
| 85.00% | 1.04 | | 99.60% | 2.65 |
| 86.00% | 1.08 | | 99.70% | 2.75 |
| 87.00% | 1.13 | | 99.80% | 2.88 |
| 88.00% | 1.17 | | 99.90% | 3.09 |
| 89.00% | 1.23 | | 99.99% | 3.72 |

# Example

- Suppose the manager of a construction supply house determined from historical records that demand for sand during lead time averages 50 tons. In addition, suppose the manager determined that demand during lead time could be described by a normal distribution that has a mean of 50 tons and standard deviation of 5 tons. Answer the questions, assuming that the manager is willing to accept a stock out risk of no more than 3 percent.

a. What value of Z is appropriate?

b. How much safety stock should be held?

c. What reorder point should be used?

# solution

The expected lead time demand = 50 tons

$\sigma_{dL}$ = 5 tons

Risk = 3 percent

Service level = 1- risk = 0.97

a. From Appendix B, table B, using a service level 0.97, you obtain a value of Z = + 1.88
b. Safety stock = Z $\sigma_{dL}$ = 1.88(5) = 9.4 tons = 10
c. *ROP* = expected demand during lead time + safety stock

   = 50 + 10 = 60 tons

# ROP Models

**2. If only demand is variable**, then

$$\sigma_{dL} = \sigma_d \sqrt{L}$$

and the reorder point with safety stock is :

$$R = \bar{d} \times L + z\sigma_d \sqrt{L}$$

Where:

$R$ = reorder point with safety stock

$\bar{d}$ =average daily or weekly demand

$\sigma_d$ = standard deviation of demand per day or week

$L$ = lead time in days or weeks

# ROP Models

**3. If only lead time is variable**, then

$$\sigma_{dL} = D\sigma_L$$

and then the reorder point with safety stock is:

$$R = d \times \overline{L} + zd\sigma_L$$

Where:

$R$ = reorder point with safety stock

$d$ = daily or weekly demand

$\overline{L}$ = average lead time in days or weeks

$\sigma_L$ = standard deviation of lead time in days or weeks

# ROP models

4. If both demand and lead time are variable, then

$$\sigma_{dL} = \sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2}$$

And the reorder point with safety stock is:

$$R = \bar{d} \times \bar{L} + z\sqrt{\bar{L}\sigma_d^2 + \bar{d}^2\sigma_L^2}$$

Note: each of these models assume that demand and lead time are independent

# Example

- A restaurant uses an average of 50 jars of a special sauce each week. Weekly usage of sauce has a standard deviation of 3 jars. The manager is willing to accept no more than a 10 percent risk of stockout during lead time, which is two weeks. Assume the distribution of usage is normal.

a. Which of the above formulas is appropriate for this situation? Why?

b. Determine the value of z?

c. Determine the ROP?

# Solution

$\bar{d}$ = 50 jars/week, $L$ = 2 weeks, $\sigma_d$ = 3 jars/week

Acceptable risk = 10 percent, so service level is 0.90

a. Because only demand is variable ( i.e., has a standard deviation) the second model is appropriate.

b. From appendix B, table B, using a service level of 0.90, you obtain z = **1.28**.

c. $R =$ $\bar{d} \times L + z\sigma_d \sqrt{L}$

= 50(2) + 1.28√2(3) = 100 + 5.43 = 105.43 = **106**

Because the inventory is discrete units (jars) we round this amount to 106 (generally, round up.)

# Example

- A restaurant uses an average of 50 jars of a special sauce each week. Weekly usage of sauce has a standard deviation of 3 jars. The manager is willing to accept no more than a 10 percent risk of stockout during lead time, which is two weeks. Assume the distribution of usage is normal.

a. How many are you going to order?

b. Determine the ROP?