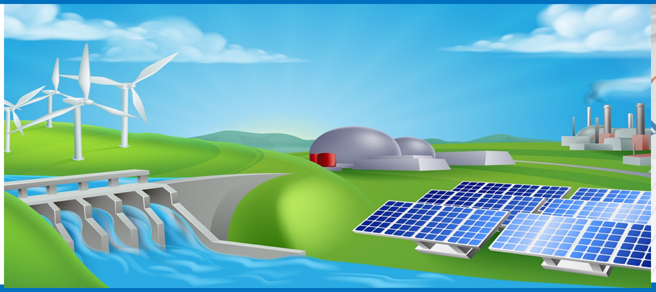


ECONOMICS OF MODERN POWER SYSTEMS

M9 – OPEX and Benchmark Analysis

Prof. Luana Medeiros Marangon Lima, Ph.D.

Recap M8

- Four modes of incentive based regulation
  - Revenue-cap, price-cap, yardstick and rate-of-return/cost of service
- Tariff design process
  - Step 1- Calculating utility's required revenue
  - Required revenue formula is mix of revenue-cap and rate-of-return

$$RR_t = (RB_t)R_t + OC_t + T_t$$

Learning Outcomes

- Explore another mode of incentive-based regulation
  - Yardstick or benchmark analysis
- Understand the Data Envelopment Analysis (DEA) model and its variations
- Study case

Benchmark Model for O&M

DEA Model

DEA Model – CCR or CRS

- Data Envelopment Analysis (DEA) is based on linear programming and seeks to measure firm efficiency
- Identifies best practice (Benchmarking)
- Provides a measure of the overall efficiency of each unit analyzed, called Decision Making Unit (DMU)
- Original formulation of the problem was given by Charnes, Cooper and Rhodes (1978) – CCR
- Assumes CRS – Constant Returns to Scale (production function)

William W Cooper
(1914 – 2012)

THE UNIVERSITY OF TEXAS AT AUSTIN

First DEA Paper

[PDF] Measuring the efficiency of decision making units

A **Charnes**, WW **Cooper**, E **Rhodes** - European journal of operational …, **1978** - farapaper.com

This paper is concerned with developing measures of'decision making efficiency'with special reference to possible use in evaluating public programs. As we shall use it, the term'program'will refer to a collection of decision making units (DMU's) with common inputs and outputs. These outputs and inpms will usually be multiple in character and may also assume a variety of forms which admit of only ordinal measurements. For example, in an educational program like'Follow Through'l, the efficiency of various schools, viewed as …

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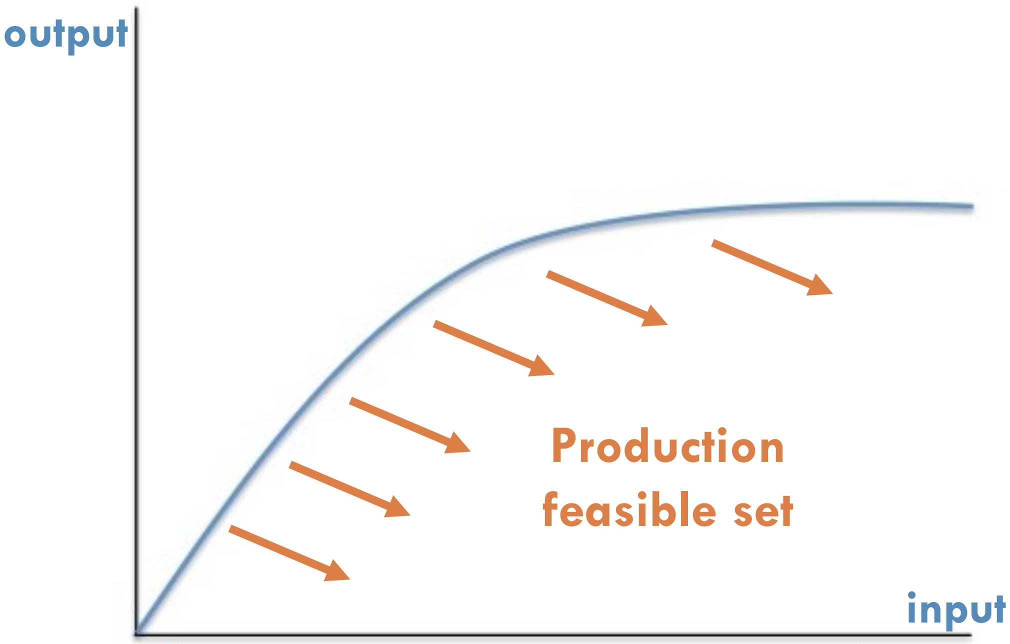
Production Function

- A production function can be an equation, table or graph presenting the **maximum amount of outputs** a firm can produce given a set of inputs during a period of time



Law of diminishing returns

- **Stage 1: average product increasing**
- **Stage 2: average product declining**

DEA Model – CCR or CRS (cont.)

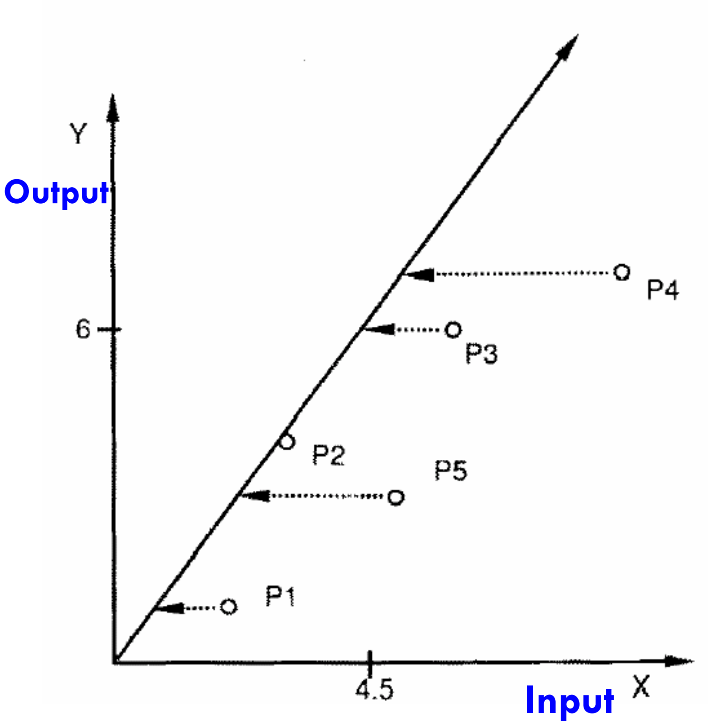
- Allows to work with multiple inputs and multiple products in different units of measurement
- Fundamental hypothesis - convexity of the production function
- Model assumption

DMU's are homogeneous

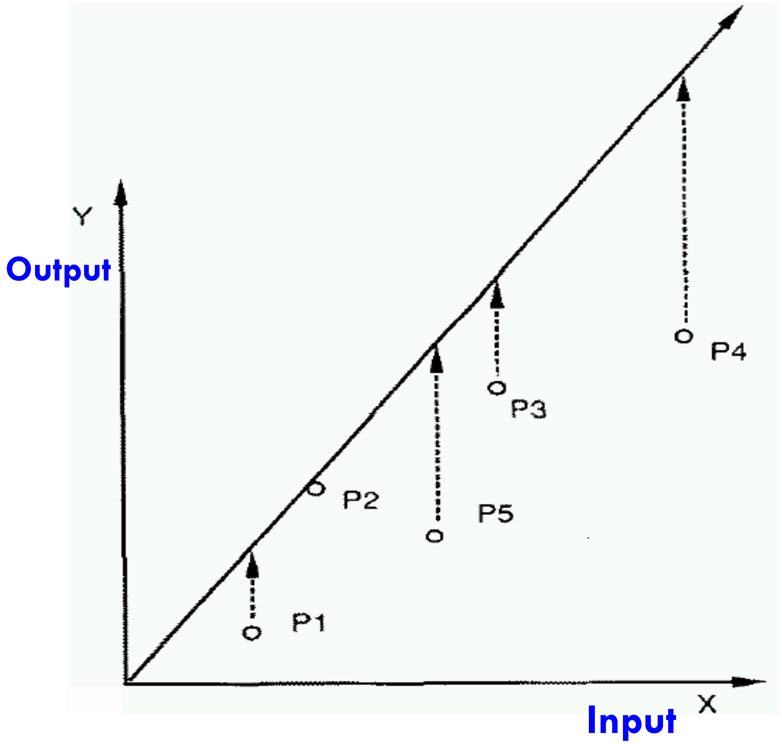
- Use the same inputs in the production of the same products
- Have same technology level

DEA Model Variations

- **Input-oriented model:** verifies whether DMU can reduce inputs used by keeping production at the same level



- **Output-oriented model:** verifies whether the DMU can increase production while maintaining the same resource utilization



DEA Model Variations

- The choice of one orientation or another depends on which variables are considered manageable by the firm
- For electricity transmission and distribution the manageable variables are the inputs (i.e. operational costs), since utility cannot choose how much to offer of its product
  - Ex. utility can not choose how many customers, how many lines, transformer, this is a result of concession area

DEA Model Formulation

DEA Model Variables

- <u>Problem data</u>

$j = 1, \ldots, n$ associated with the set of DMU's

$r = 1, \ldots, s$ associated to the set of products

$i = 1, \ldots, m$ associated to the set of inputs

X_{ij}: consumed quantity of input i by DMU j

Y_{rj}: amount of product r produced by DMU j

X_{i0}: consumed amount of input i by DMU 0, i.e. DMU under analysis

Y_{r0}: amount of product r produced by DMU 0, i.e. DMU under analysis

- <u>Decision Variables</u>

u_r: weight given to product r for DMU under analysis

v_i: weight given to input i for DMU under analysis

Input-oriented DEA-CRS

- The efficiency of a utility (DMU) is given by

$$Eff_0 = \frac{\sum_{r=1}^{s} u_r Y_{r0}}{\sum_{i=1}^{m} v_i X_{i0}}$$

- Mathematical formulation

$$\max \frac{\sum_{r=1}^{s} u_r Y_{r0}}{\sum_{i=1}^{m} v_i X_{i0}}$$

Objective: maximize efficiency of the DMU under analysis

$$\text{s.t.}$$

$$\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \leq 1, \qquad j = 1, \dots, n$$

Constraint 1: The efficiency off all DMUs should be less than equal to 1

$$u_r, v_i \geq 0, \qquad r = 1, \dots, s$$

$$i = 1, \dots, m$$

Constraint 2: nonnegativity, weights should be positive

Input-oriented DEA-CRS

- Transformation of the model into a linear programming problem

$$\max \frac{\sum_{r=1}^{s} u_r Y_{r0}}{\sum_{i=1}^{m} v_i X_{i0}}$$

s.t.

$$\frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \leq 1, \qquad j = 1, \dots, n$$

$$u_r, v_i \geq 0, \qquad r = 1, \dots, s$$
$$i = 1, \dots, m$$

$$\max \sum_{r=1}^{s} u_r Y_{r0}$$

s.t.

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \leq 0, \qquad j = 1, \dots, n$$

$$\sum_{i=1}^{m} v_i X_{i0} = 1,$$

$$u_r, v_i \geq 0, \qquad r = 1, \dots, s$$
$$i = 1, \dots, m$$

Duality Theory

Side note: Duality Theory

- Every LP problem (P) has a dual problem (D) where the decision variables will be the shadow prices of P

$$(P)\quad \begin{aligned} \max z = cx & \\ Ax &\leq b \\ x &\geq 0 \end{aligned} \qquad\qquad (D)\quad \begin{aligned} \min w = yb & \\ yA &\geq c \\ y &\geq 0 \end{aligned}$$

- If model P has an optimal solution than D will also have an optimal solution and objective function values will be the same!!
- Rules for converting any primal into its dual
  - Transpose rows and columns
  - Transpose objective function coefficients and right hand side constants
  - Inequalities sign will depend on primal decision variables values
  - Change objective (max to min or min to max)

Side note: Duality Theory

- Primal – Dual conversion rules

	MINIMIZATION PROBLEM		MAXIMIZATION PROBLEM	
variables	≥ 0	$\Leftrightarrow$	$\leq$	constraints
	≤ 0	$\Leftrightarrow$	$\geq$	
	unrestricted	$\Leftrightarrow$	$=$	
constraints	$\geq$	$\Leftrightarrow$	≥ 0	variables
	$\leq$	$\Leftrightarrow$	≤ 0	
	$=$	$\Leftrightarrow$	unrestricted	

Side note: Duality Theory

- Small Example

$$\max 3x_1 + 5x_2$$

s.t.

$$x_1 \le 4$$

$$2x_2 \le 12$$

$$3x_1 + 2x_2 \le 18$$

$$x_1, x_2 \ge 0$$

$$\max 3x_1 + 5x_2$$

s.t.

$$x_1 + 0x_2 \le 4$$

$$0x_1 + 2x_2 \le 12$$

$$3x_1 + 2x_2 \le 18$$

$$x_1, x_2 \ge 0$$

$$\max \quad [3 \quad 5][x_1 \quad x_2]^T$$

s.t.

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \le \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix}$$

$$x_1, x_2 \ge 0$$

Side note: Duality Theory

$$\max \quad [3 \quad 5][x_1 \quad x_2]^T$$

s.t.

$$\begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 4 \\ 12 \\ 18 \end{bmatrix} \quad \begin{matrix} \boldsymbol{y_1} \\ \boldsymbol{y_2} \\ \boldsymbol{y_3} \end{matrix}$$

$$x_1, x_2 \geq 0$$

$$\min \quad [4 \quad 12 \quad 18][y_1 \quad y_2 \quad y_3]^T$$

s.t.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \geq \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$y_1, y_2, y_3 \geq 0$$

- Transpose rows and columns
- Transpose objective function coefficients and right hand side constants
- Inequalities sign will depend on primal decision variables values
- Change objective (max to min or min to max)
- Dual variables sign depend on primal constraints sign

Side note: Duality Theory

$$\min \quad \begin{bmatrix} 4 & 12 & 18 \end{bmatrix} \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}^T$$

$$\text{s.t.}$$

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \geq \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$y_1, y_2, y_3 \geq 0$$

$$\min 4y_1 + 12y_2 + 18y_3$$

$$\text{s.t.}$$

$$y_1 + 3y_3 \geq 3$$

$$2y_2 + 2y_3 \geq 5$$

$$y_1, y_2, y_3 \geq 0$$

Comparing Solutions

Primal	Dual
$\max 3x_1 + 5x_2$	$\min 4y_1 + 12y_2 + 18y_3$
s.t.	s.t.
$x_1 \leq 4$	$y_1 + 3y_3 \geq 3$
$2x_2 \leq 12$	$2y_2 + 2y_3 \geq 5$
$3x_1 + 2x_2 \leq 18$	$y_1, y_2, y_3 \geq 0$
$x_1, x_2 \geq 0$	$\boldsymbol{obj = 36}$
$\boldsymbol{obj = 36}$	$\boldsymbol{y_1 = 0, y_2 = 1.5, y_3 = 1}$
$\boldsymbol{x_1 = 2, x_2 = 6}$	$\boldsymbol{\lambda_1 = 2, \lambda_2 = 6}$
$\boldsymbol{\lambda_1 = 0, \lambda_2 = 1.5, \lambda_3 = 1}$	

Back to DEA…

Input-oriented DEA-CRS

- Input-oriented multiplier model

$$\max \sum_{r=1}^{s} u_r Y_{r0}$$

s.t.

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} \le 0, j = 1, \dots, n \quad : \lambda_j$$

$$\sum_{i=1}^{m} v_i X_{i0} = 1, \quad : \theta$$

$$u_r, v_i \ge 0, \qquad r = 1, \dots, s\,, i = 1, \dots, m$$

- Input-oriented envelopment model

$$\min \theta$$

s.t.

$$\theta X_{i0} - \sum_{j=1}^{n} \lambda_j X_{ij} \ge 0, \qquad i = 1, \dots, m$$

$$-Y_{r0} + \sum_{j=1}^{n} \lambda_j Y_{rj} \ge 0, \qquad r = 1, \dots, s$$

$$\lambda_j \ge 0, \qquad j = 1, \dots, n$$

From duality theory

Input-oriented DEA-CRS

- Input-oriented envelopment model

$$\min \theta$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} \lambda_j X_{ij} \leq \theta X_{i0}, \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{r0}, \qquad r = 1, \dots, s$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

- Input-oriented envelopment model

$$\min \theta$$

$$\text{s.t.}$$

$$\theta X_{i0} - \sum_{j=1}^{n} \lambda_j X_{ij} \geq 0, \qquad i = 1, \dots, m$$

$$-Y_{r0} + \sum_{j=1}^{n} \lambda_j Y_{rj} \geq 0, \qquad r = 1, \dots, s$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

Final formulation

Input-oriented DEA-CRS

- Input-oriented envelopment model

$$\min \theta$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} \lambda_j X_{ij} \leq \theta X_{i0}, \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{r0}, \qquad r = 1, \dots, s$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

- Note that the left side of the constraints is a linear combination of the DMUs under analysis, which can be interpreted as the ***virtual efficient utility***
- In this sense, the first constraint ensures that the efficient firm should use the same amount of inputs, or less, than the utility in analysis
- The second constraint ensures that the quantity of products generated by the efficient firm is greater, or equal, the quantity of products of the utility under analysis

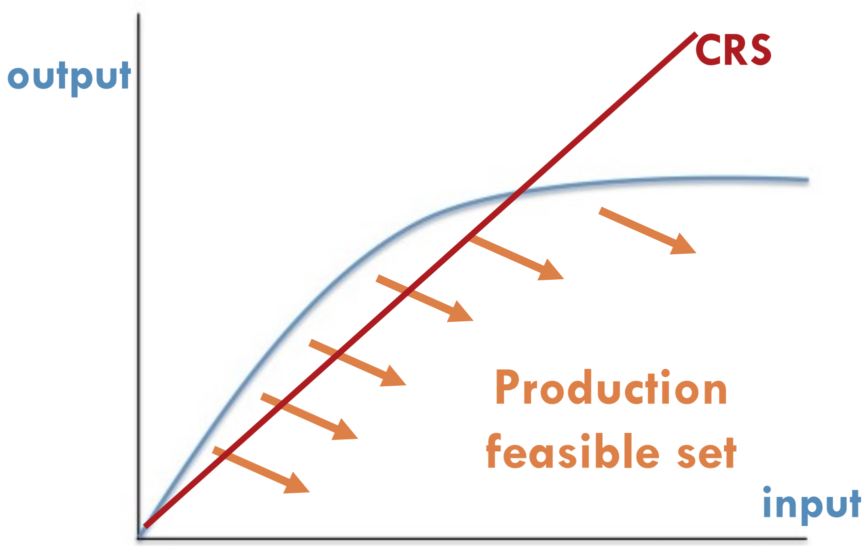
If θ * = 1, then the current level of inputs do not need to be reduced by indicating that the DMU 0 is in the efficient frontier

DEA – BCC or VRS

- Banker (1980) and Banker, Charnes and Cooper (1984) (BCC) extended the model of constant returns to scale to variable returns of scale (VRS)

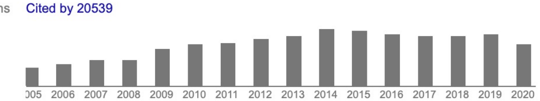
Rajiv Banker

President of the International DEA Society



Some models for estimating technical and scale inefficiencies in data envelopment analysis

Authors	Rajiv D Banker, Abraham Charnes, William Wager Cooper
Publication date	1984/9
Journal	Management science
Volume	30
Issue	9
Pages	1078-1092
Publisher	INFORMS
Description	In management contexts, mathematical programming is usually used to evaluate a collection of possible alternative courses of action *en route* to selecting one which is best. In this capacity, mathematical programming serves as a planning aid to management. Data Envelopment Analysis reverses this role and employs mathematical programming to obtain *ex post facto* evaluations of the relative efficiency of management accomplishments, however they may have been planned or executed. Mathematical programming is thereby extended for use as a tool for *control* and evaluation of past accomplishments as well as a tool to aid in *planning* future activities. The CCR ratio form introduced by Charnes, Cooper and Rhodes, as part of their Data Envelopment Analysis approach, comprehends both technical and scale inefficiencies via the optimal value of the ratio form, as obtained directly from the data without requiring *a* …
Total citations	Cited by 20539

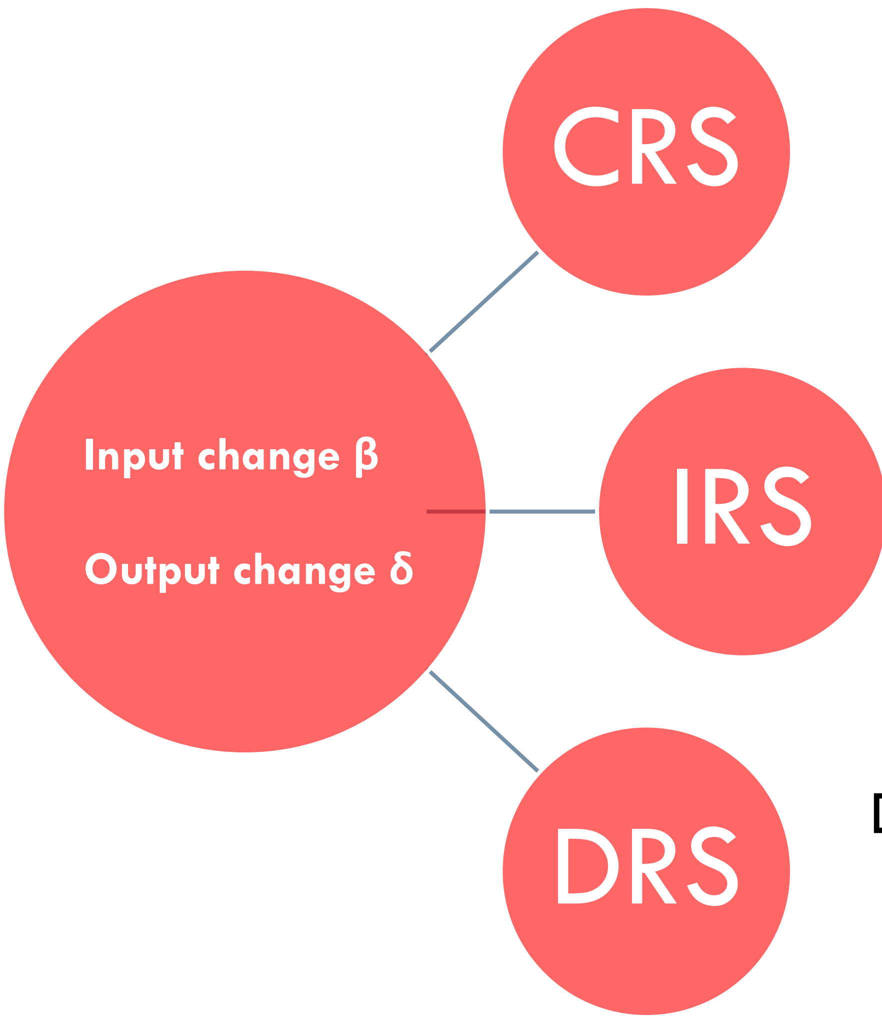


Scholar articles: Some models for estimating technical and scale inefficiencies in data envelopment analysis
RD Banker, A Charnes, WW Cooper - Management science, 1984
Cited by 20539 Related articles All 21 versions

Side note: Return to scale

- Returns to scale is related to **production function**
- It measures the levels of **change in output with respect to changes in input** levels
- Can be variable, either increasing or decreasing, or they can be constant
- In a **competitive environment**, firms either have **increasing or decreasing** returns to scale

Variable Return to Scale



Constant returns to scale

$$\beta = \delta$$

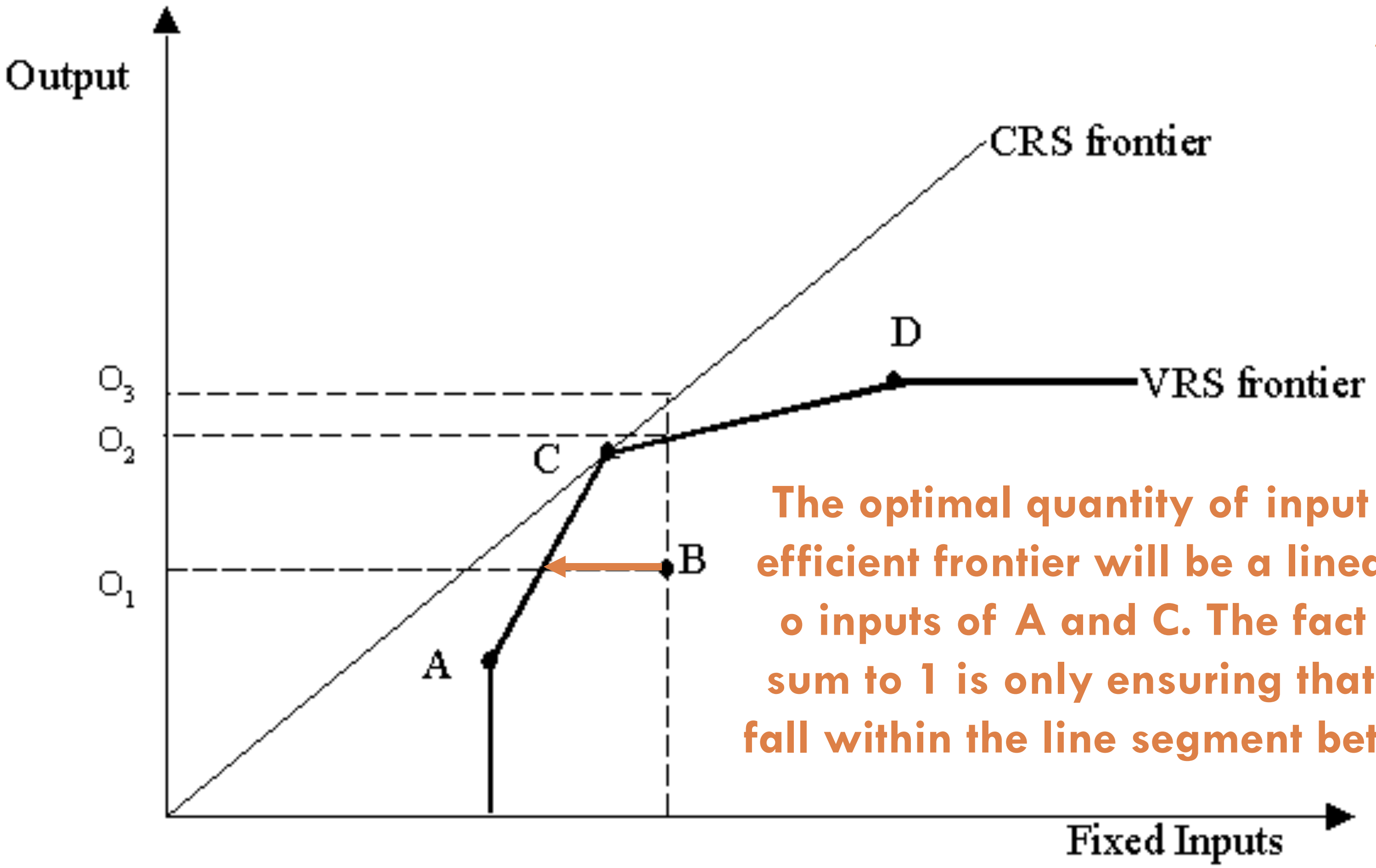
Increasing returns to scale

- β , δ increase $\longrightarrow \delta > \beta$
- β , δ decrease $\longrightarrow \delta < \beta$

Decreasing return to scale

- β , δ increase $\longrightarrow \delta < \beta$
- β , δ decrease $\longrightarrow \delta > \beta$

Variable Returns to Scale on DEA



Add one more constraint

$$\sum_{j=1}^{n} \lambda_j = 1$$

The optimal quantity of input for B to reach efficient frontier will be a linear combination o inputs of A and C. The fact that lambdas sum to 1 is only ensuring that optimal input fall within the line segment between A and C.

Input-oriented VRS

- Input-oriented multiplier model

$$\max \sum_{r=1}^{s} u_r Y_{r0} + u_0$$

$$\text{s.t.}$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} + u_0 \le 0, j = 1, \dots, n$$

$$\sum_{i=1}^{m} v_i X_{i0} = 1,$$

$$u_r, v_i \ge 0, \qquad r = 1, \dots, s\ , i = 1, \dots, m$$

$$u_0 \in \mathbb{R}$$

- Input-oriented envelopment model

$$\min \theta$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} \lambda_j X_{ij} \le \theta X_{i0}, \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \ge Y_{r0}, \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \ge 0, \qquad j = 1, \dots, n$$

Input-oriented NDRS

- Input-oriented multiplier model

$$\max \sum_{r=1}^{s} u_r Y_{r0} + u_0$$

$$\text{s.t.}$$

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} + u_0 \leq 0, j = 1, \dots, n$$

$$\sum_{i=1}^{m} v_i X_{i0} = 1,$$

$$u_r, v_i \geq 0, \quad r = 1, \dots, s, i = 1, \dots, m$$

$$u_0 \geq 0$$

- Input-oriented envelopment model

$$\min \theta$$

$$\text{s.t.}$$

$$\sum_{j=1}^{n} \lambda_j X_{ij} \leq \theta X_{i0}, \quad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{r0}, \quad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j \geq 1$$

$$\lambda_j \geq 0, \quad j = 1, \dots, n$$

If $u_0^* > 0$ then we are dealing with increasing returns to scale, or if $u_0^* \geq 0$ non-decreasing

Input-oriented NIRS

- Input-oriented multiplier model

$$\max \sum_{r=1}^{s} u_r Y_{r0} + u_0$$

s.t.

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} + u_0 \leq 0, j = 1, \dots, n$$

$$\sum_{i=1}^{m} v_i X_{i0} = 1,$$

$$u_r, v_i \geq 0, \qquad r = 1, \dots, s, i = 1, \dots, m$$

$$u_0 \leq 0$$

- Input-oriented envelopment model

$$\min \theta$$

s.t.

$$\sum_{j=1}^{n} \lambda_j X_{ij} \leq \theta X_{i0}, \qquad i = 1, \dots, m$$

$$\sum_{j=1}^{n} \lambda_j Y_{rj} \geq Y_{r0}, \qquad r = 1, \dots, s$$

$$\sum_{j=1}^{n} \lambda_j \leq 1$$

$$\lambda_j \geq 0, \qquad j = 1, \dots, n$$

If $u_0^* < 0$ then we are dealing with decreasing returns to scale, or if $u_0^* \leq 0$ non-increasing

VRS model: u_0 sign explained

- Input-oriented multiplier model

$$\max \sum_{r=1}^{s} u_r Y_{r0} + u_0$$

s.t.

$$\sum_{r=1}^{s} u_r Y_{rj} - \sum_{i=1}^{m} v_i X_{ij} + u_0 \le 0, j = 1, \dots, n$$

$$\sum_{i=1}^{m} v_i X_{i0} = 1,$$

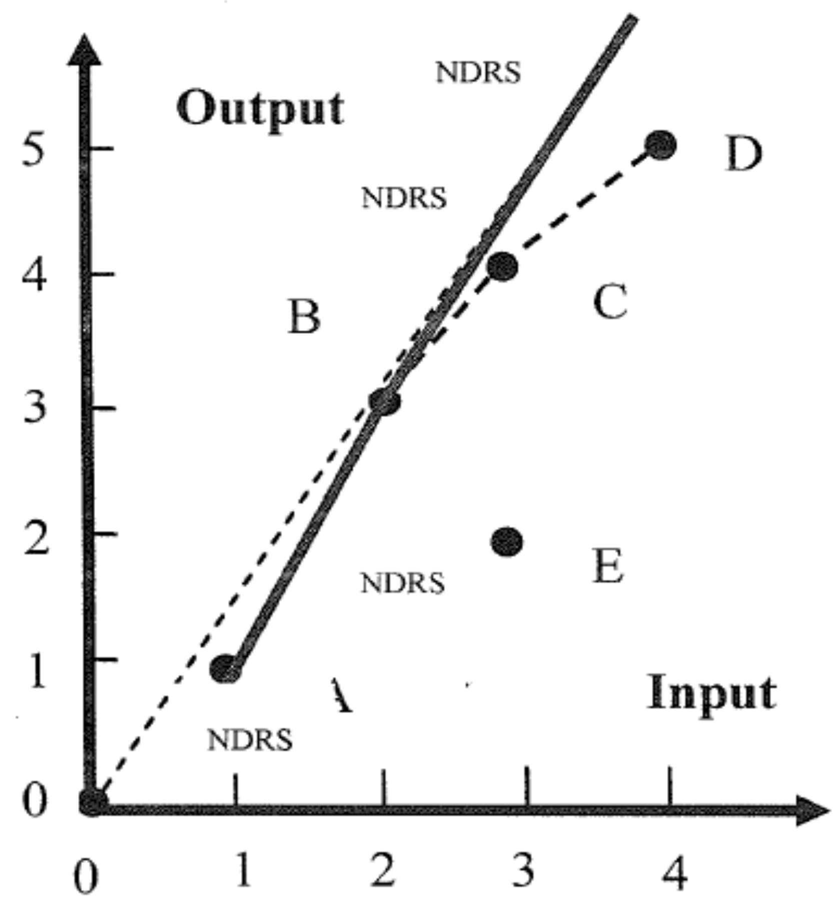
$$u_r, v_i \ge 0, \qquad r = 1, \dots, s\ , i = 1, \dots, m$$

$$u_0 \in \mathbb{R}$$

- When $u_0 = 0$, then the CRS efficiency is equal VRS efficiency
- If $u_0 \ge 0$ non-decreasing (NDRS) leads to an improve in the efficiency score
- From duality theory: $u_0 \ge 0$ means in the Dual the restriction will be $\sum_{j=1}^{n} \lambda_j \ge 1$
- If $u_0 \le 0$ non-increasing (NIRS) leads to an improve in the efficiency score
- From duality theory: $u_0 \le 0$ means in the Dual the restriction will be $\sum_{j=1}^{n} \lambda_j \le 1$

Increasing or Non-Decreasing (NDRS)

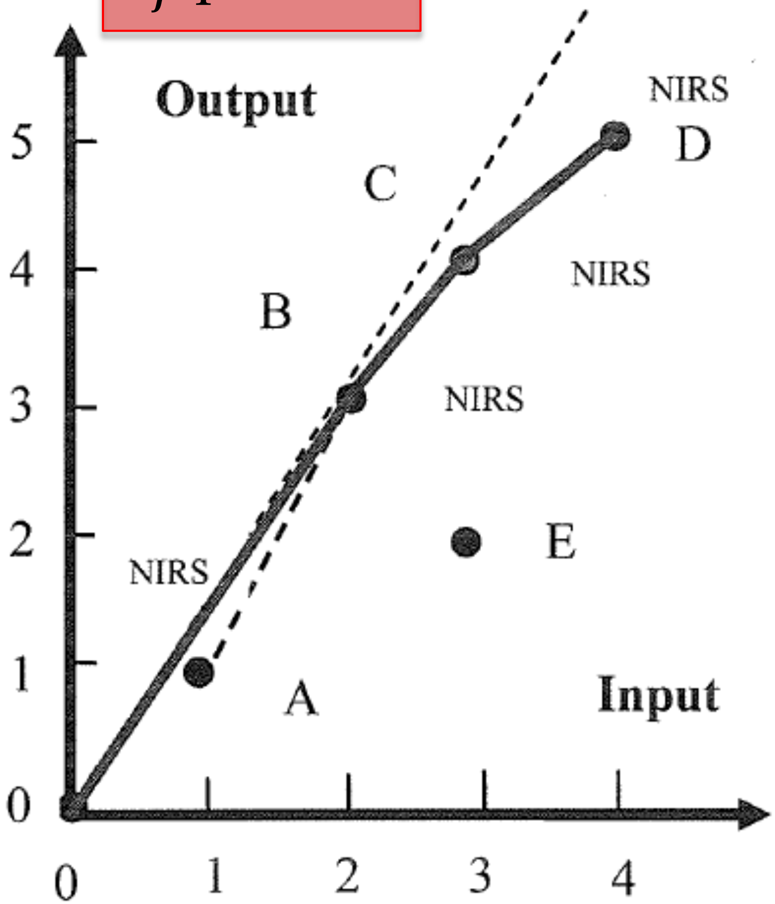
- Dashed line is the CRS frontier
- Bold line is NDRS frontier
- Slope of NDRS frontier is larger than CRS
- NDRS favor smaller firms, note that for larger firms, NDRS fronter is equal to CRS frontier
- More usual for DEA applications to electric utilities



Variable Returns to Scale

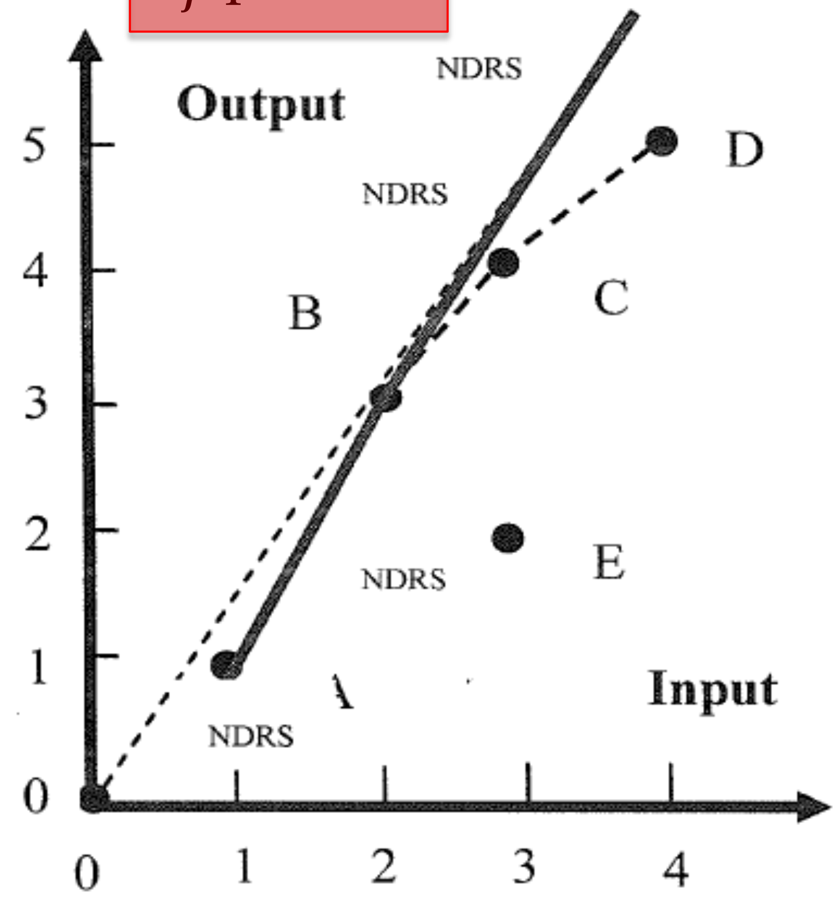
$$\sum_{j=1}^{n} \lambda_j \leq 1$$

Non-Increasing

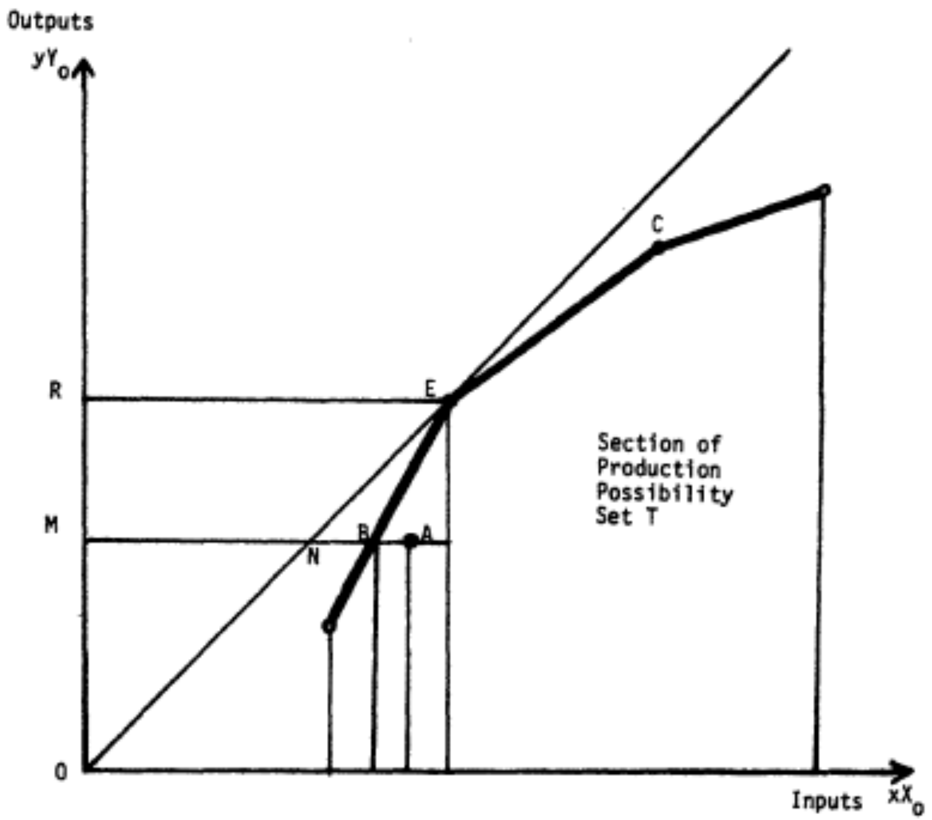


$$\sum_{j=1}^{n} \lambda_j \geq 1$$

Non-Decreasing



Efficiency Decomposition



Banker (1984)

$$\text{Technical efficiency} = Ef^B = \theta^*_{VRS}$$

$$\text{Scale efficiency} = \frac{Ef^C}{Ef^B} = \frac{\theta^*_{CRS}}{\theta^*_{VRS}}$$

$$\text{Global efficiency} = Ef^B \times \frac{Ef^C}{Ef^B} = \theta^*_{CRS}$$

Technical efficiency $= \frac{MB}{MA}$

Scale efficiency $= \frac{MN}{MB}$

Global efficiency $= \frac{MN}{MA}$

DEA for OPEX

Inputs and Outputs

Inputs	Outputs
OPEX Or TOTEX - CAPEX + OPEX	Network extension Number of costumers Energy consumption (MWh) Quality index

How efficient score becomes efficient OPEX?

- Recall

$$RR_t = (RB_t)R_t + OC_t + T_t$$

- The efficient operational cost is

Used as input to the DEA model

$$OC_t = \theta * OPEX_{actual} + D_t$$

From DEA results

Depreciation (can appear inside OC_t or directly in RR_t

- But usually commissions will give utilities a time frame to reach that efficient cost if θ is too low

More on operational expenses

- Operating expenses include labor, power purchases, outside consultants, purchased maintenance services, fuel, insurance and others
- Some operating expenses are sporadic: storm damage, rate cases
- We also have depreciation expense that increases the revenue requirement
  - Accumulated depreciation is a deduction to the utility's rate base, reducing the revenue requirement
  - Depreciation expense is the return of capital, utilities are allowed to collect that to pay for eventual replacement costs

More on Capital Expenditures (CAPEX)

- The utilities industry has capital expenditures and depreciation that are very different
- Growth capex include buying land, fixing machinery, building a new plant, upgrading the power system, or many other items. Some of these items are to reduce expenses, increase production, or improve the production process

Case Study

Benchmarking 20 utilities

Assignment #4

Problem Data

20 utilities

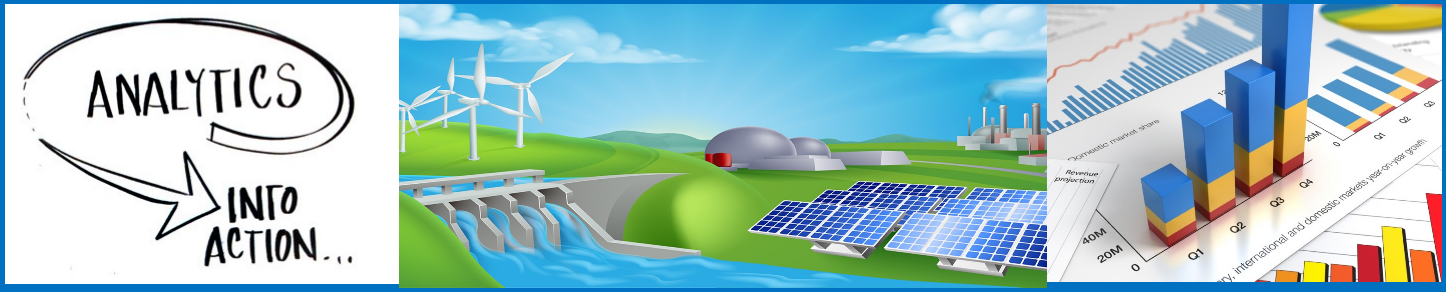
Utility	OPEX	Network extension	Number of customers	Energy Consumption
	$ 10,000.00	km (10000)	10000	10000 kWh
U1	25	7	124	366
U2	28	3	160	495
U3	58	7	153	391
U4	79	15	251	822
U5	63	21	251	553
U6	31	12	213	280
U7	37	14	117	308
U8	187	50	746	1561
U9	28	8	106	155
U10	67	25	519	715
U11	38	13	308	476
U12	120	23	405	1139
U13	16	5	121	211
U14	25	2	150	510
U15	39	11	231	625
U16	143	5	646	2032
U17	26	9	88	215
U18	25	6	133	332
U19	67	6	358	1052
U20	21	8	134	380
	Input	Outputs		

How can you compare this utilities?

Note: We will start with constant returns to scale

Guideline for A#4

- Implement primal model for U1
  - Solve and look at sensitivity analysis
- Implement dual model for U1
  - Solve and look at sensitivity analysis
- Check dual variable and objective function to see duality theory!!
- Then pick primal or dual and fill the tables on handout
  - Hint: Do CRS first, then VRS, analyze the results
  - Then implement NDRS and NIRS and check results again



THANK YOU !

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