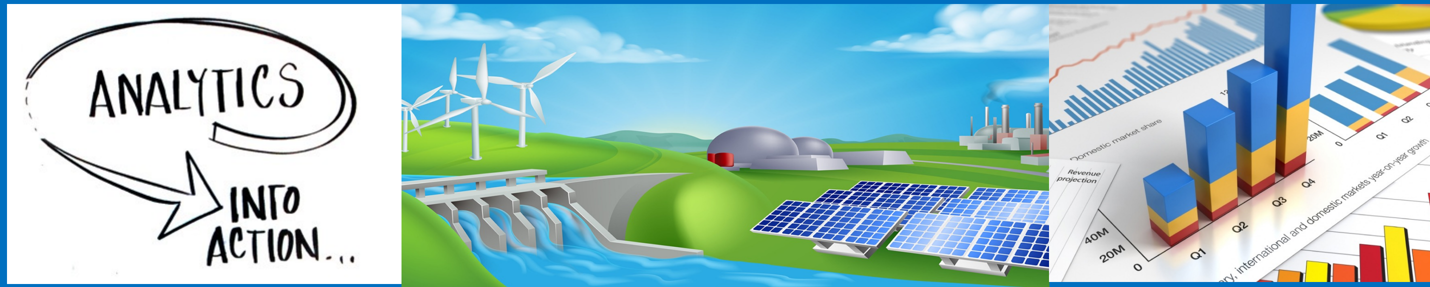


ECONOMICS OF MODERN POWER SYSTEMS

M10 – Network Pricing Step 2: Cost Allocation

Prof. Luana Medeiros Marangon Lima, Ph.D.

Recap: Tariff Design Process

The tariff design is a two-folded process

Allowed Revenue Definition

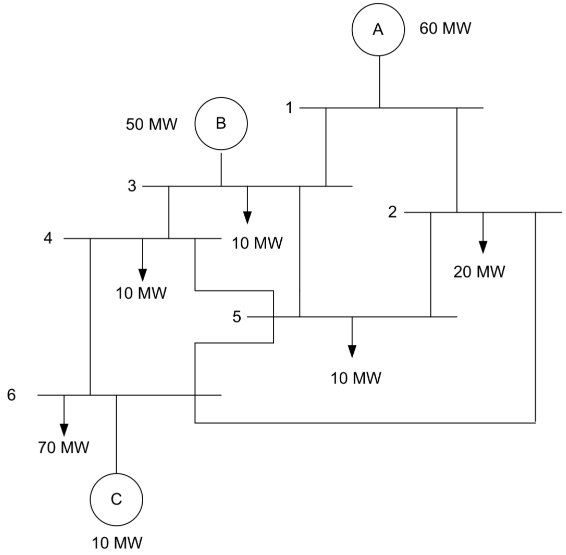
- ✓ O&M costs
- ✓ Base rate returns
- ✓ Assets depreciation
- ✓ Losses
- ✓ Ancillary Services

Tariff Structure

- ✓ Cost Drivers
- ✓ Cost Allocation Method
- ✓ Connection Charges
- ✓ DG Special Tariffs
- ✓ RTP and ToU
- ✓ Revenue Reconciliation

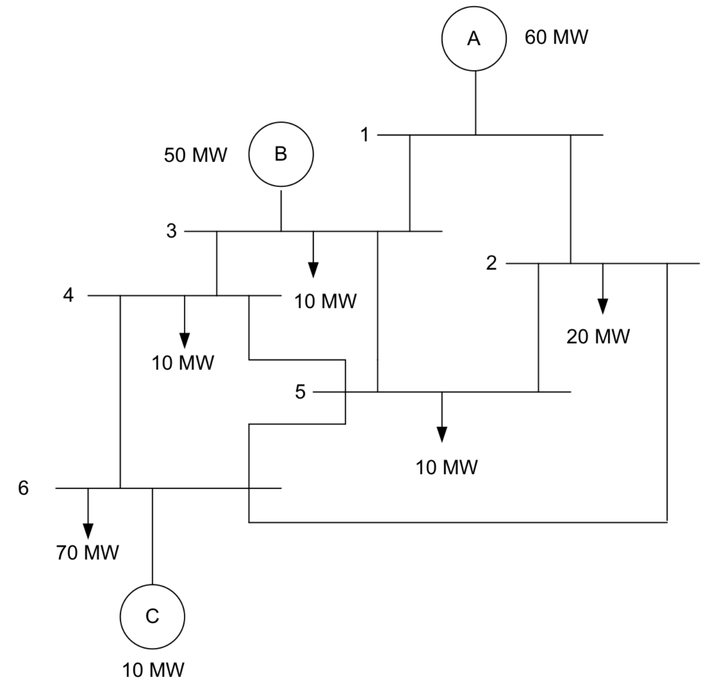
Learning Outcomes

- What is Cost Allocation?
- Review: power flow analysis
- Explore cost allocation methods
  - Postage Stamp
  - MW-mile
  - Modulus Method
  - Zero Counterflow
  - Incremental Cost
- Case Study



Learning Outcomes – Part2

- Explore cost allocation methods
  - Postage Stamp
  - MW-mile
  - Modulus Method
  - Zero Counterflow
  - Incremental Cost
- Case Study



Cost Allocation Principles

Basic Assumptions

- Economic Efficiency
  - Signaling for the rational use of the network by reducing costs in the expansion and operation (locational signaling)
  - No cross-subsidies
- Stability (continuity of the charge)
  - Security to the agents to conduct economic analyzes and feasibility of their new developments
- Remuneration of the grid
  - Generation of sufficient charges to cover annual required revenue (RR)
- Simplicity and ease of regulation
  - Ensuring transparency and reproducibility of results

Cost Allocation Methods

- Three paradigms:
  - Embedded cost
    - Easy to implement
    - Recover all required revenue
    - Lack of economic grounds
  - Incremental cost
    - Short term marginal cost (most popular)
    - Do not recover all transmission cost
  - Composite embedded/incremental cost
    - Solution to the problem
    - Exploit properties of both methods

Amount of capacity used (power flow)

Fixed costs

Cost per unit of capacity (marginal costs)

Embedded Cost

Main Embedded Cost Methods

- Cost are allocated proportionally to the “extent of use” of the network
- Easy to implement
- Some are based on power flow
- Examples
  - Postage Stamp
  - MW-mile
  - Modulus Method
  - Zero Counterflow Method

Postage Stamp

- Every system is affected uniformly for each network usage transaction
- Assessment is made on the basis of the amount of power
- Given a user k, the network charge is given by

$$T_k = RR \frac{D_k}{D_{total}}$$

Widely used for revenue reconciliation

  - T_k transmission cost to user k ($)
  - RR network required revenue ($)
  - D_k demand of user k (MW)
  - D_{total} total network demand (MW)

- Cross-subsidies - generation point close to the load versus point of injection and withdrawal distant

MW-mile

- Widely used for transmission network pricing
- Formulated based on the distance and active power flow in each line of the network
- Considered the first pricing strategy based on actual use of the system
- Costs are allocated in proportion to the ratio of flow over capacity

$$T_k = \sum_{i=1}^{N} C_i \frac{P_i^k}{\overline{P}_i}$$

  - N total number of network elements (line or transformer)
  - T_k network charge to user k ($)
  - C_i cost of asset i ($)
  - P_i^k power flow in line/transformer i by user k (MW)
  - $\overline{P}_i$ power flow capacity in /transformer i (MW)
- Since the total circuit power flows are usually smaller than the circuit capacities, this allocation rule **does not recover all embedded costs**

Modulus method or Usage method

- A simple way to **ensure recovery** of all embedded costs in the MW-mile is to replace the circuit capacities by the sum of absolute power flows

$$T_k = \sum_{i=1}^{N} C_i \frac{\left|P_i^k\right|}{\sum_{s \in S_i} \left|P_i^s\right|}$$

  - N total number of network elements (line or transformer)
  - S set of agents using i
  - T_k transmission cost to user k ($)
  - C_i cost of asset i ($)
  - P_i^k power flow in line/transformer i by user k (MW)
  - $\overline{P}_i$ power flow capacity in /transformer i (MW)
- Do not differ between user that go on an opposite direction than the net power flow

Zero Counterflow Method

- No charge for the agent whose power flow is in the opposite direction of the net flow
- Only the agents that use the circuit in the same direction of the net flow pay in proportion to their flow
- Given a user k, the network charge is given by

$$T_k = \begin{cases} \sum_{i=1}^{N} C_i \frac{P_i^k}{\sum_{j \in \Omega_{i+}} P_i^j} & if \quad P_i^k \geq 0 \\ 0 & if \quad P_i^k < 0 \end{cases}$$

  - N total number of network elements (line or transformer)
  - Ω_{i+}set of agents using asset I in same direction as net flow
  - T_k network charge to user k ($)
  - C_i cost of asset i ($)
  - P_i^k power flow in line/transformer i by user k (MW)

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  - Ω_{i+}set of agents using asset I in same direction as net flow
  - T_k network charge to user k ($)
  - C_i cost of asset i ($)
  - P_i^k power flow in line/transformer i by user k (MW)

Summary

Summary

Embedded Cost Methods

Postage Stamp

$$T_k = RR\frac{D_k}{D_{total}}$$

T_k transmission cost to user k ($)

RR network required revenue ($)

D_k demand of user k (MW)

D_{total} total network demand (MW)

MW-mile

$$T_k = \sum_{i=1}^{N} C_i \frac{P_i^k}{\overline{P}_i}$$

N # of network elements

T_k network charge to user k ($)

C_i cost of asset i ($)

P_i^k power flow in element i caused by user k (MW)

$\overline{P}_i$ power flow capacity in element i (MW)

Embedded Cost Methods

Modulus Method

$$T_k = \sum_{i=1}^{N} C_i \frac{|P_i^k|}{\sum_{s \in S_i} |P_i^s|}$$

N total number of network elements (line or transformer)

S set of agents using i

T_k transmission cost to user k ($)

C_i cost of asset i ($)

P_i^k power flow in line/transformer i by user k (MW)

$\overline{P}_i$ power flow capacity in line/transformer i (MW)

Zero Counterflow Method

$$T_k = \begin{cases} \sum_{i=1}^{N} C_i \frac{P_i^k}{\sum_{j \in \Omega_{i+}} P_i^j} & if \quad P_i^k \geq 0 \\ 0 & if \quad P_i^k < 0 \end{cases}$$

N total number of network elements (line or transformer)

Ω_{i+}set of agents using asset I in same direction as net flow

T_k network charge to user k ($)

C_i cost of asset i ($)

P_i^k power flow in line/transformer i by user k (MW)

Some methods require power flow analysis.

Therefore, before we proceed, we need to learn/review power flow…

DC Power Flow Approximation

Following Slides from Daniel Kirshen (University of Washington) - 2011

Power Flow Equations

$$P_k^I - \sum_{i=1}^{N} V_k V_i [G_{ki} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

$$Q_k^I - \sum_{i=1}^{N} V_k V_i [G_{ki} \sin(\theta_k - \theta_i) - B_{ki} \cos(\theta_k - \theta_i)] = 0$$

- Set of non-linear equations
- Need a simple linear relation for fast and intuitive analysis
- DC power flow provides such a relation but requires a number of approximations

First Approx.: Neglect Reactive Power

$$P_k^I - \sum_{i=1}^{N} V_k\, V_i [G_{ki}\cos(\theta_k - \theta_i) + B_{ki}\sin(\theta_k - \theta_i)] = 0$$

$$Q_k^I - \sum_{i=1}^{N} V_k\, V_i [G_{ki}\sin(\theta_k - \theta_i) - B_{ki}\cos(\theta_k - \theta_i)] = 0$$

$$\Downarrow$$

$$P_k^I - \sum_{i=1}^{N} V_k\, V_i [G_{ki}\cos(\theta_k - \theta_i) + B_{ki}\sin(\theta_k - \theta_i)] = 0$$

Impedance and Admittance

Impedance is
voltage/current

$$Z = R + jX$$

$$R = resistance = Re\,(Z)$$
$$X = reactance = Im\,(Z)$$

- Definition: how a group of components pushes against the current flowing
- Resistance is impeding the flow on resistors
- Reactance is impeding the flow in inductors or capacitors

Admittance is
current/ voltage

$$Y = \frac{1}{Z} = G + jB$$

$$G = conductance = Re\,(Y)$$
$$B = susceptance = Im\,(Y)$$

- Definition: inverse of impedance, i.e., ability to conduct power
- Conductance is the reciprocal of resistance
- Susceptance is the reciprocal of reactance

Second Approx.: Neglect Resistance of the Branches

$$P_k^I - \sum_{i=1}^{N} V_k V_i [\cancel{G_{ki}} \cos(\theta_k - \theta_i) + B_{ki} \sin(\theta_k - \theta_i)] = 0$$

$$\Downarrow$$

$$P_k^I - \sum_{i=1}^{N} V_k V_i B_{ki} \sin(\theta_k - \theta_i) = 0$$

Third Approx.: Assume All Voltage Magnitudes = 1.0 p.u.

$$P_k^I - \sum_{i=1}^{N} V_k V_i B_{ki} \sin(\theta_k - \theta_i) = 0$$

$$\Downarrow$$

$$P_k^I - \sum_{i=1}^{N} B_{ki} \sin(\theta_k - \theta_i) = 0$$

Fourth Approx.: Assume all angles are small

$$P_k^I - \sum_{i=1}^{N} B_{ki} \sin(\theta_k - \theta_i) = 0$$

If α is small: $\sin\alpha \approx \alpha$ (α in radians)

$$\Downarrow$$

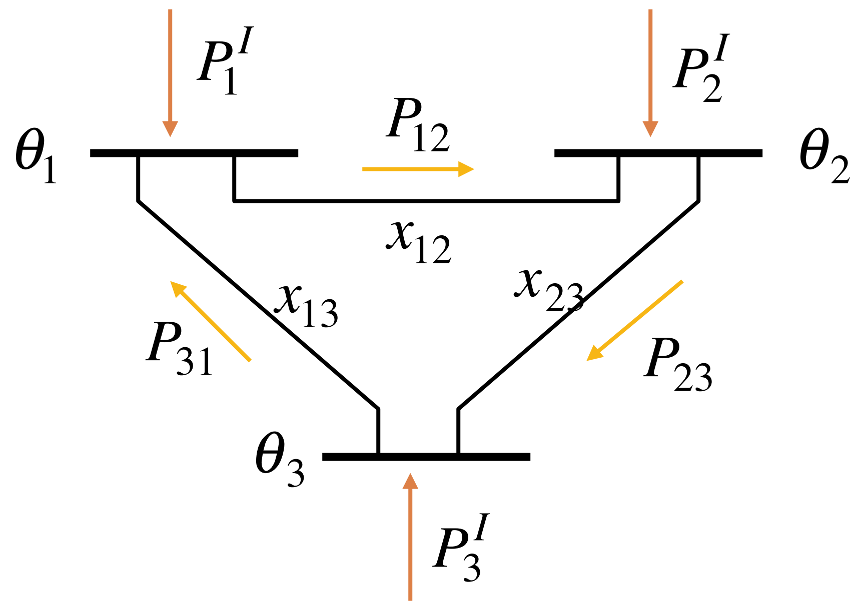
$$P_k^I - \sum_{i=1}^{N} B_{ki}(\theta_k - \theta_i) = 0 \quad \text{or} \quad P_k^I - \sum_{i=1}^{N} \frac{(\theta_k - \theta_i)}{x_{ki}} = 0$$

Interpretation of formulas

$$\boxed{P_k^I} - \boxed{\sum_{i=1}^{N} \frac{(\theta_k - \theta_i)}{x_{ki}} = 0}$$

Power In node k

Power Out node k



$$P_k^I - \sum_{i=1}^{N} P_{ki} = 0$$

$$P_{ki} = \frac{(\theta_k - \theta_i)}{x_{ki}}$$

$$P_{12} = \frac{(\theta_1 - \theta_2)}{x_{12}}; \quad P_{23} = \frac{(\theta_2 - \theta_3)}{x_{23}}; \quad P_{31} = \frac{(\theta_3 - \theta_1)}{x_{13}}$$

Why is it called dc power flow?

- Reactance plays the role of resistance in dc circuit
- Voltage angle plays the role of dc voltage
- Power plays the role of dc current

$$P_{ki} = \frac{(\theta_k - \theta_i)}{x_{ki}} \quad \Longleftrightarrow \quad I_{ki} = \frac{(V_k - V_i)}{R_{ki}}$$

Matrix representation

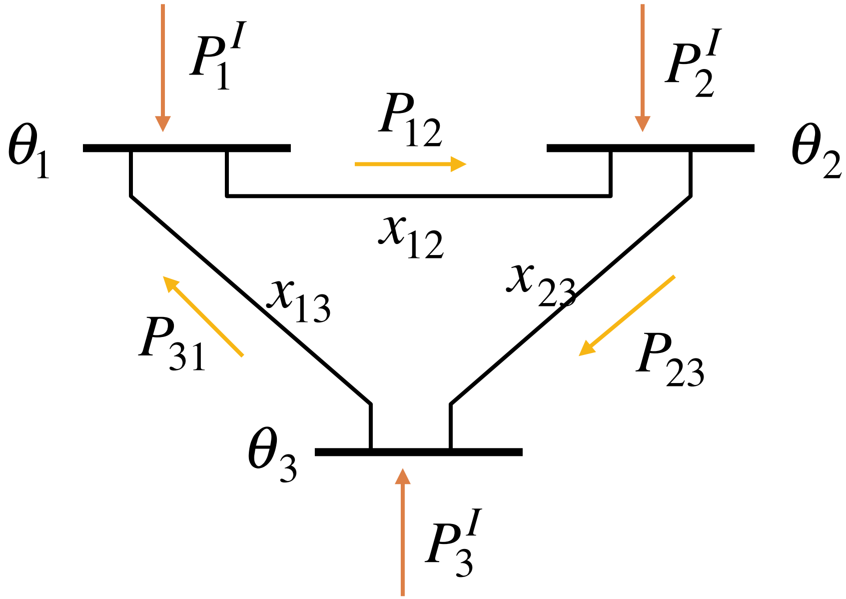
Node 1:

$$P_1^I - P_{12} + P_{31} = 0$$

$$P_1^I = P_{12} - P_{31} = \frac{\theta_1 - \theta_2}{x_{12}} - \frac{\theta_3 - \theta_1}{x_{31}}$$

$$P_1^I = \frac{\theta_1}{x_{12}} - \frac{\theta_2}{x_{12}} - \frac{\theta_3}{x_{31}} + \frac{\theta_1}{x_{31}}$$

$$P_1^I = \left[\frac{1}{x_{12}} + \frac{1}{x_{13}}\right]\theta_1 - \left[\frac{1}{x_{12}}\right]\theta_2 - \left[\frac{1}{x_{31}}\right]\theta_3$$



Node 2:

$$P_2^I = -\left[\frac{1}{x_{12}}\right]\theta_1 + \left[\frac{1}{x_{12}} + \frac{1}{x_{23}}\right]\theta_2 - \left[\frac{1}{x_{23}}\right]\theta_3$$

Node 3:

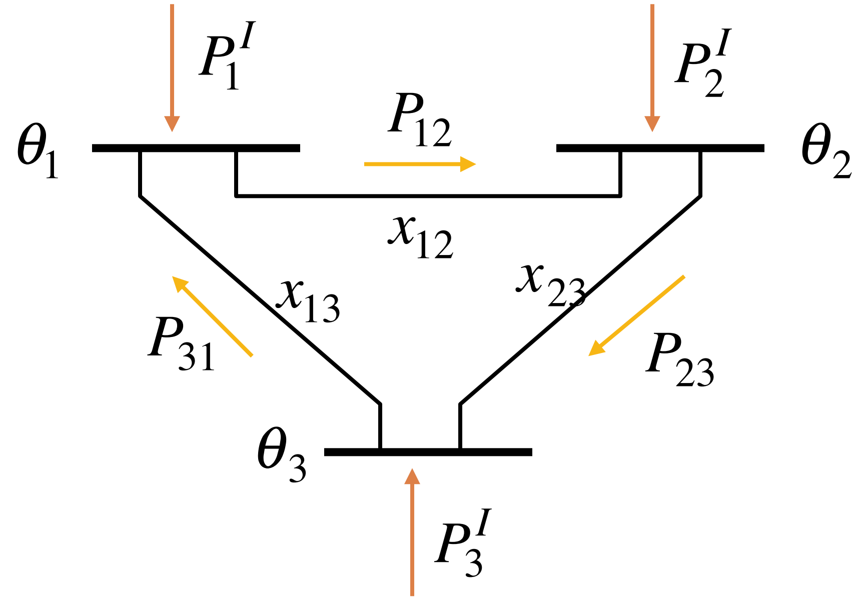
$$P_3^I = -\left[\frac{1}{x_{13}}\right]\theta_1 - \left[\frac{1}{x_{23}}\right]\theta_2 + \left[\frac{1}{x_{13}} + \frac{1}{x_{23}}\right]\theta_3$$

Matrix representation (cont'd)

In matrix format:

$$\begin{bmatrix} P_1^I \\ P_2^I \\ P_3^I \end{bmatrix} = \begin{bmatrix} \frac{1}{x_{12}} + \frac{1}{x_{13}} & -\frac{1}{x_{12}} & -\frac{1}{x_{13}} \\ -\frac{1}{x_{12}} & \frac{1}{x_{12}} + \frac{1}{x_{23}} & -\frac{1}{x_{23}} \\ -\frac{1}{x_{13}} & -\frac{1}{x_{23}} & \frac{1}{x_{13}} + \frac{1}{x_{23}} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

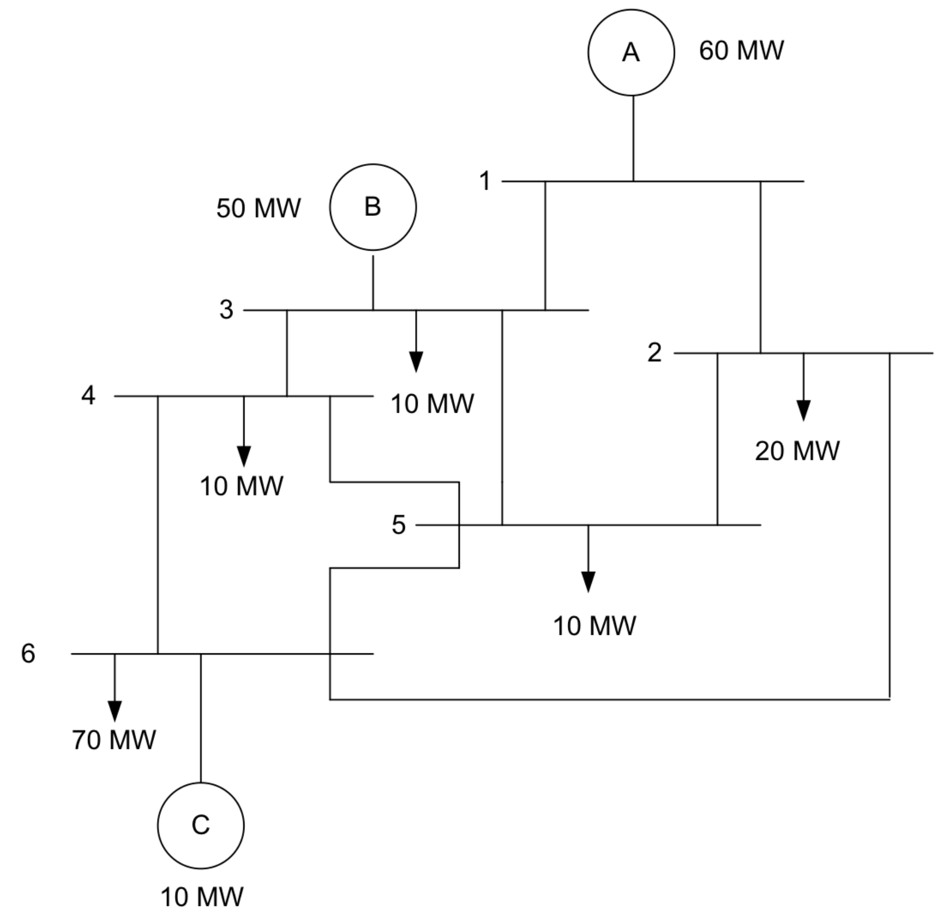
$$\boldsymbol{P} = \boldsymbol{B\theta}$$



Power Flow Example

Example: 6 buses, 3 Gen, 5 Loads

- Total demand = 120 MW
- All network elements have capacity of 100 MW



DC Power Flow Computation

- $P = B\,\theta$, therefore $\theta = B^{-1}P$

		Reactance (X%)
1	2	12
1	3	8
2	5	5
2	6	12
3	4	15
3	5	16
4	5	12
4	6	9
5	6	3

Matrix B

$$\begin{array}{c|c} & \begin{matrix} \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} \end{matrix} \\ \begin{matrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \\ \mathbf{4} \\ \mathbf{5} \\ \mathbf{6} \end{matrix} & \begin{bmatrix} 20.83 & -8.33 & -12.5 & 0 & 0 & 0 \\ -8.33 & 36.67 & 0 & 0 & -20 & -8.33 \\ -12.5 & 0 & 25.42 & -6.67 & -6.25 & 0 \\ 0 & 0 & -6.67 & 26.11 & -8.33 & -11.11 \\ 0 & -20 & -6.25 & -8.33 & 67.92 & -33.33 \\ 0 & -8.33 & 0 & -11.11 & -33.33 & 52.78 \end{bmatrix} \end{array}$$

Consider reactance in p.u. for matrix B computation

- Vector P represents injection (generation) - withdrawal (load)

	Pg	Pl	**Vector P**
1	60	0	60
2	0	20	-20
3	50	10	40
4	0	10	-10
5	0	10	-10
6	10	70	-60

DC Power Flow Computation (cont'd)

- Choosing reference node as node 1, therefore $\theta_1 = 0$

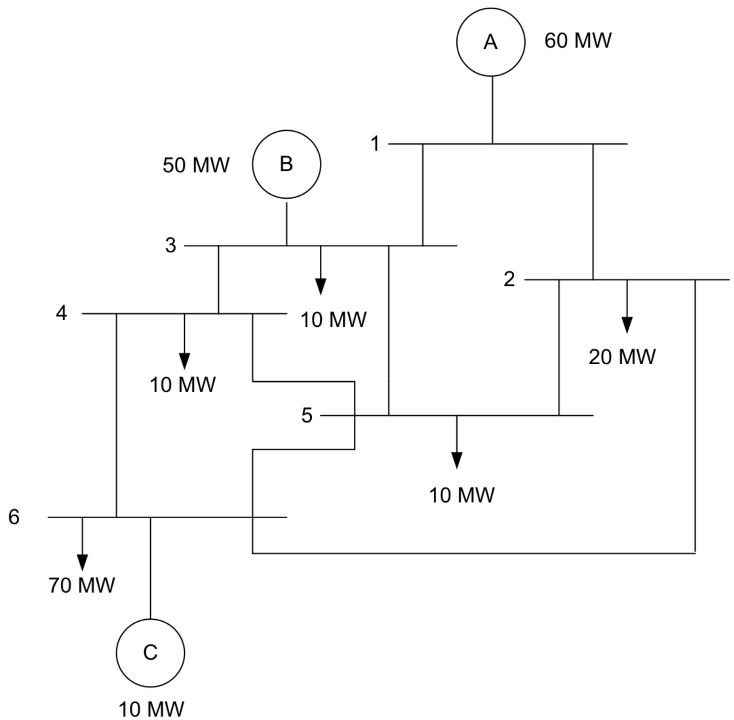
$$
\underbrace{\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}}_{\text{Vector } \theta}
=
\underbrace{\begin{bmatrix}
0.0756 & 0.0296 & 0.0542 & 0.0625 & 0.0628 \\
0.0296 & 0.0603 & 0.0439 & 0.0384 & 0.0381 \\
0.0542 & 0.0439 & 0.1028 & 0.0687 & 0.0736 \\
0.0625 & 0.0384 & 0.0687 & 0.0826 & 0.0765 \\
0.0628 & 0.0381 & 0.0736 & 0.0765 & 0.0927
\end{bmatrix}}_{\text{Inverse of B}}
\underbrace{\begin{bmatrix} -20 \\ 40 \\ -10 \\ -10 \\ -60 \end{bmatrix}}_{\text{Vector P}}
=
\underbrace{\begin{bmatrix} -5.263 \\ -1.289 \\ -5.459 \\ -5.817 \\ -6.795 \end{bmatrix}}_{\text{Vector } \theta}
$$

- Power flow in circuit ij:

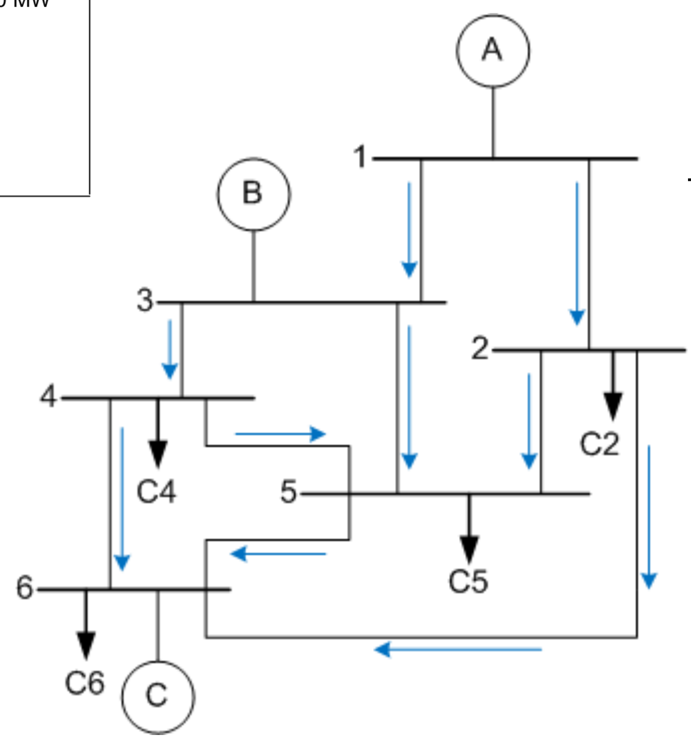
$$F_{ij} = \frac{(\theta_i - \theta_j)}{X_{ij}}$$

From	To	Reactance (X in p.u.)
1	2	0.12
1	3	0.08
2	5	0.05
2	6	0.12
3	4	0.15
3	5	0.16
4	5	0.12
4	6	0.09
5	6	0.03

Power Flow Results



From	To	Flow [MW]
1	2	43.87
1	3	16.13
2	5	11.13
2	6	12.75
3	4	27.81
3	5	28.32
4	5	3.00
4	6	14.81
5	6	32.44

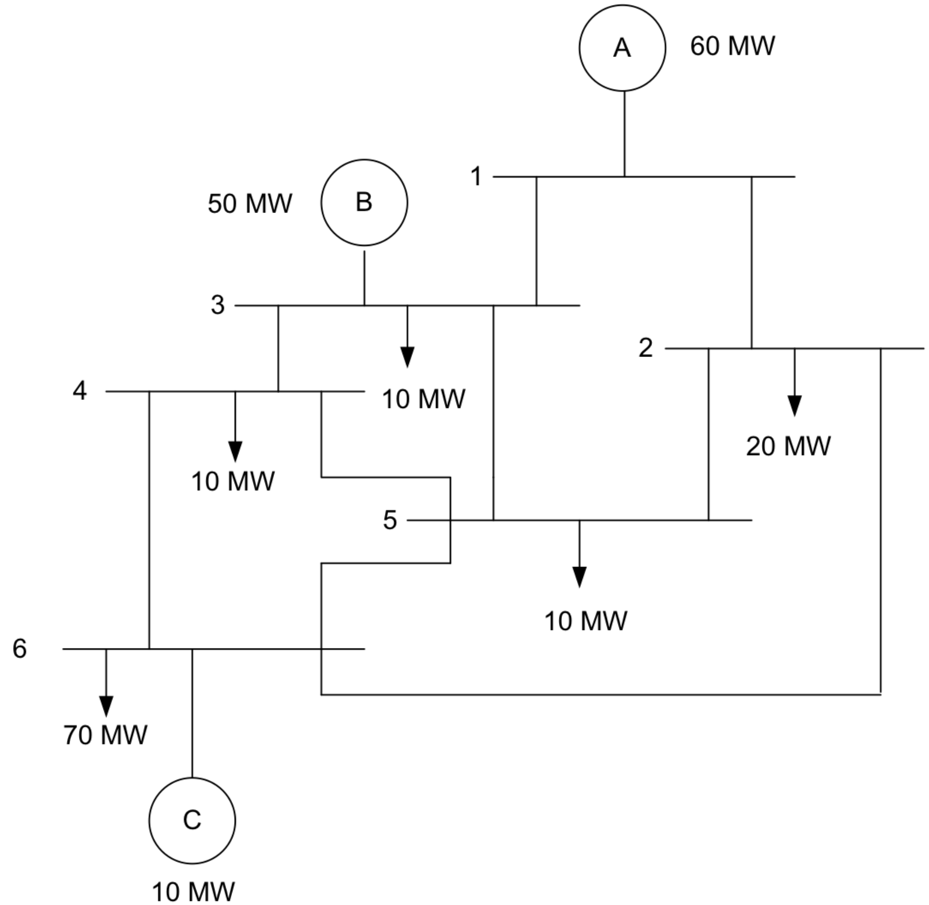


No losses!

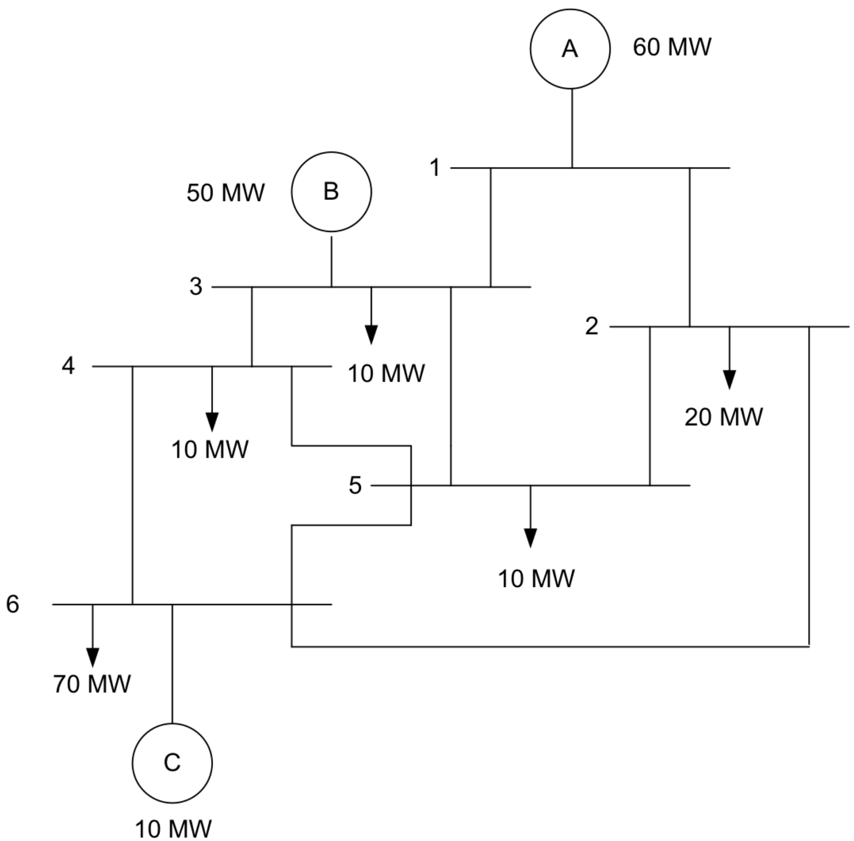
Case Study

Example: 6 buses, 3 Gen, 5 Loads

- Total demand = 120 MW
- Required revenue for the year is \$ 810.00
- Revenue is equally distributed within networks assets, i.e., all 9 circuits have an associated cost of \$90.00
- All network elements have capacity of 100 MW



Results for Postage Stamp



$$T_{GA} = \frac{810}{2} * \frac{60}{120} = 202.50\ \$/year$$

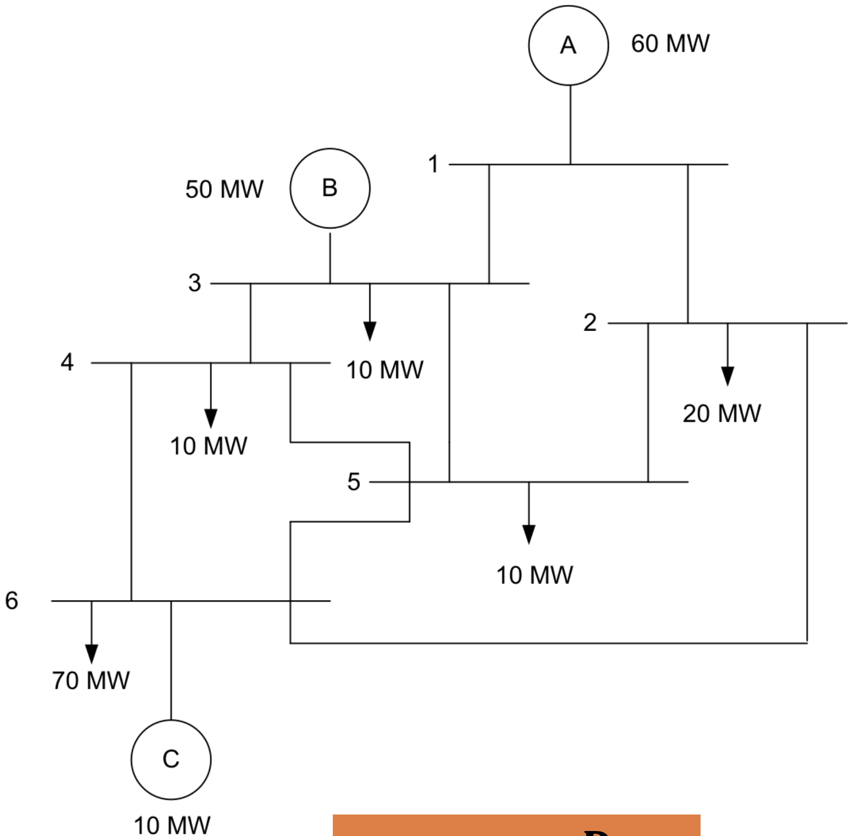
	G/D	Postage Stamp	
	MW	$/year	$/MW
GA	60	202.50	3.375
GB	50	?	?
GC	10	?	?
Total	120	?	?

L2	20	?	?
L3	10	?	?
L4	10	?	?
L5	10	?	?
L6	70	?	?
Total	120	?	?

$$T_k = RR \frac{D_k}{D_{total}}$$

Collected Revenue	?

Charge results for Postage Stamp



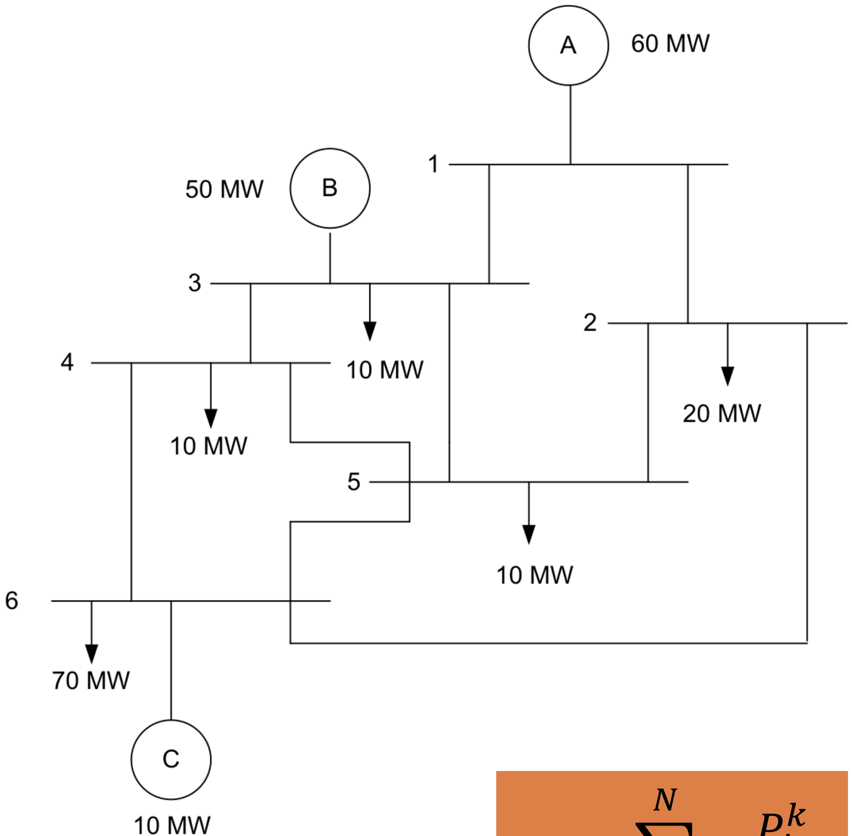
$$T_k = RR \frac{D_k}{D_{total}}$$

	G/D [MW]	Charge $/year Postage Stamp
GA	60	202.50
GB	50	168.75
GC	10	33.75
Total	120	405.00

L2	20	67.50
L3	10	33.75
L4	10	33.75
L5	10	33.75
L6	70	236.25
Total	120	405.00

Collected Revenue	810.00

Results for MW-mile



Missing P_i^k!!!

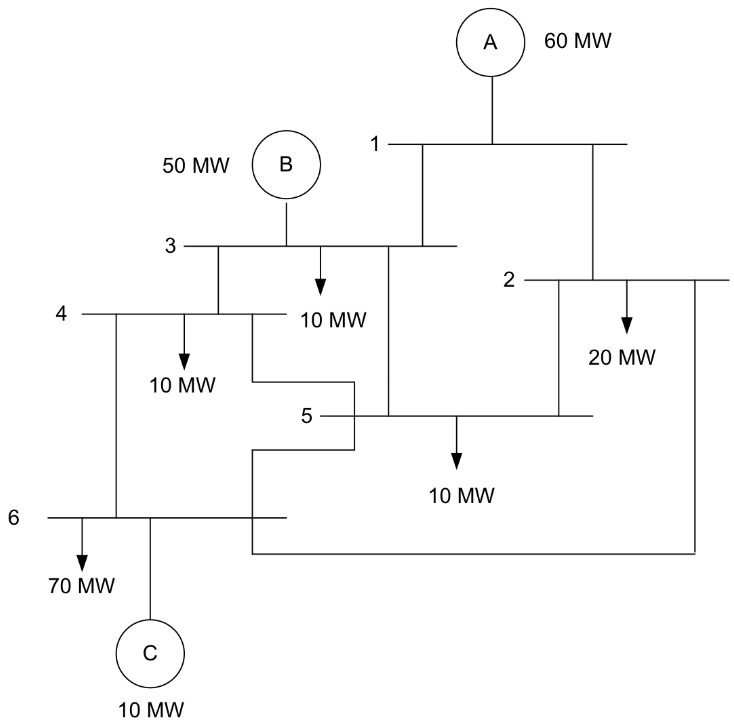
How do we get $P_i^{GA}, P_i^{GB}, P_i^{GC}$?

Is that enough to find all charges?

No, also need $P_i^{L2}, P_i^{L3}, P_i^{L4}, P_i^{L5}, P_i^{L6}$.

$$T_k = \sum_{i=1}^{N} C_i \frac{P_i^k}{\overline{P}_i}$$

How to find P_i^{GA}?



To find power flow on each element due to GA

1) remove GA by changing vector P

	Pg	Pl	Vector P
1	60	0	60
2	0	20	-20
3	50	10	40
4	0	10	-10
5	0	10	-10
6	10	70	-60

	Pg	Pl	New Vector P
1	0	0	0
2	0	20 – (20*60/120) = 10	-10
3	50	10 – (10*60/120) = 5	45
4	0	10 – (10*60/120) = 5	-5
5	0	10 – (10*60/120) = 5	-5
6	10	70 – (70*60/120) = 35	-25

2) Simulate power flow for the new vector P

3) Subtract power flow obtained in 2 from the base case

Power Flow without Generator A

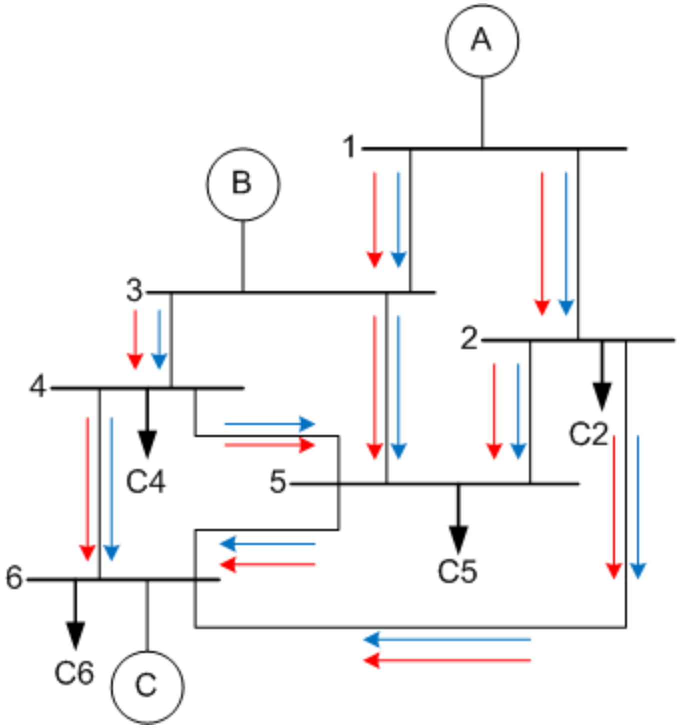
- Take generator A out (60MW) and same amount of load proportionally
- Find new angle vector θ

$$
\underbrace{\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}}_{\text{Vector } \theta} = \underbrace{\begin{bmatrix} 0.0756 & 0.0296 & 0.0542 & 0.0625 & 0.0628 \\ 0.0296 & 0.0603 & 0.0439 & 0.0384 & 0.0381 \\ 0.0542 & 0.0439 & 0.1028 & 0.0687 & 0.0736 \\ 0.0625 & 0.0384 & 0.0687 & 0.0826 & 0.0765 \\ 0.0628 & 0.0381 & 0.0736 & 0.0765 & 0.0927 \end{bmatrix}}_{\text{Inverse of B}} \boxed{\underbrace{\begin{bmatrix} -10 \\ 45 \\ -5 \\ -5 \\ -25 \end{bmatrix}}_{\text{Vector P}}} = \underbrace{\begin{bmatrix} -1.578 \\ 1.054 \\ -1.264 \\ -1.566 \\ -1.982 \end{bmatrix}}_{\text{Vector } \theta}
$$

- Find flow corresponding to generator A

De	Para	Reactance (X)	Angle Difference	**Flow without GA [MW]**
1	2	12	1.58	**13.16**
1	3	8	-1.05	**-13.16**
2	5	5	-0.01	**-0.19**
2	6	12	0.40	**3.35**
3	4	15	2.32	**15.46**
3	5	16	2.62	**16.39**
4	5	12	0.30	**2.52**
4	6	9	0.72	**7.94**
5	6	3	0.42	**13.72**

Results for Power Flow due to GA



From	To	Flow [MW]	Flow GA [MW]
1	2	43.87	30.72
1	3	16.13	29.28
2	5	11.13	11.32
2	6	12.75	9.40
3	4	27.81	12.35
3	5	28.32	11.93
4	5	3.00	0.47
4	6	14.81	6.88
5	6	32.44	18.73

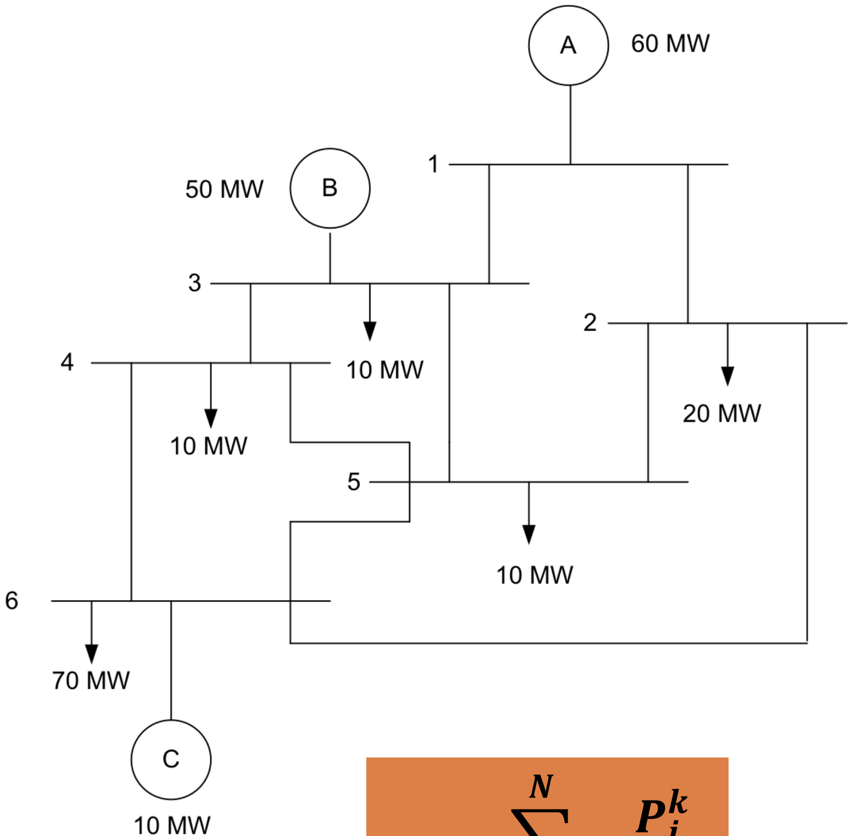
Repeat the process for other generators and loads!

Results summary: analysis extended to all loads

From	To	Flow [MW]	Flow G1	Flow G2	Flow G3
1	2	43,86	30,71	13,26	-0,11
1	3	16,11	29,28	-13,29	0,12
2	5	11,08	11,31	0,63	-0,85
2	6	12,77	9,40	4,29	-0,93
3	4	27,80	12,35	15,76	-0,31
3	5	28,30	11,93	16,78	-0,41
4	5	2,98	0,47	2,68	-0,16
4	6	14,84	6,87	8,95	-0,98
5	6	32,60	18,75	16,13	-2,28

From	To	Flow [MW]	Flow C2	Flow C3	Flow C4	Flow C5	Flow C6
1	2	43,86	9,67	1,00	3,05	3,74	26,39
1	3	16,11	0,32	4,00	1,95	1,26	8,58
2	5	11,08	-7,16	0,80	1,94	3,06	12,45
2	6	12,77	-3,14	0,21	1,11	0,66	13,92
3	4	27,80	3,80	-0,83	4,19	2,28	18,38
3	5	28,30	4,85	-1,00	1,92	3,13	19,39
4	5	2,98	1,73	-0,29	-2,67	1,33	2,89
4	6	14,84	2,09	-0,55	-3,15	0,96	15,50
5	6	32,60	-0,62	-0,51	1,23	-2,44	34,94

Charge results for MW Mile



$$T_k = \sum_{i=1}^{N} C_i \frac{P_i^k}{\overline{P_i}}$$

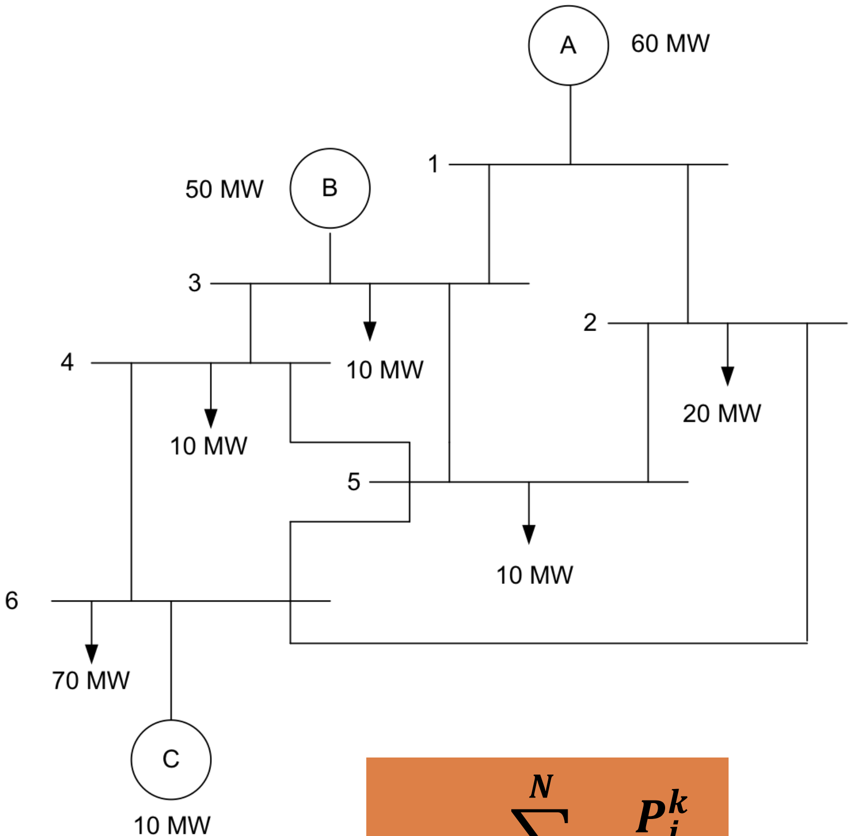
	G/D MW	MW Mile $/year	MW Mile $/MW
GA	60	58.98	0.983
GB	50	?	?
GC	10	?	?
Total	120	?	?

L2	20	?	?
L3	10	?	?
L4	10	?	?
L5	10	?	?
L6	70	?	?
Total	120	?	

Collected Revenue	?

Does this method recover all required revenue?

Charge results for MW Mile



$$T_k = \sum_{i=1}^{N} C_i \frac{P_i^k}{\overline{P_i}}$$

	G/D [MW]	Charge $/year MW Mile
G1	60	58,98
G2	50	29,29
G3	10	-2,66
Total	120	85,61
C2	20	5,19
C3	10	1,28
C4	10	4,30
C5	10	6,30
C6	70	68,55
Total	120	85,61
Collected Revenue		171,23

Do not recover all required revenue cost!

Network Usage x Reserve Capacity

- Required Revenue = $ 810.00
- Collected Revenue = $ 171.23, why?

From	To	Cost	Capacity [MW]	Flow [MW]	Cost*Flow/Capacity
1	2	90	100	43,87	39,49
1	3	90	100	16,13	14,51
2	5	90	100	11,13	10,01
2	6	90	100	12,75	11,47
3	4	90	100	27,81	25,03
3	5	90	100	28,32	25,49
4	5	90	100	3,00	2,70
4	6	90	100	14,81	13,33
5	6	90	100	32,44	29,20
				Total [$]	**171,23**

- Reserve Capacity Cost

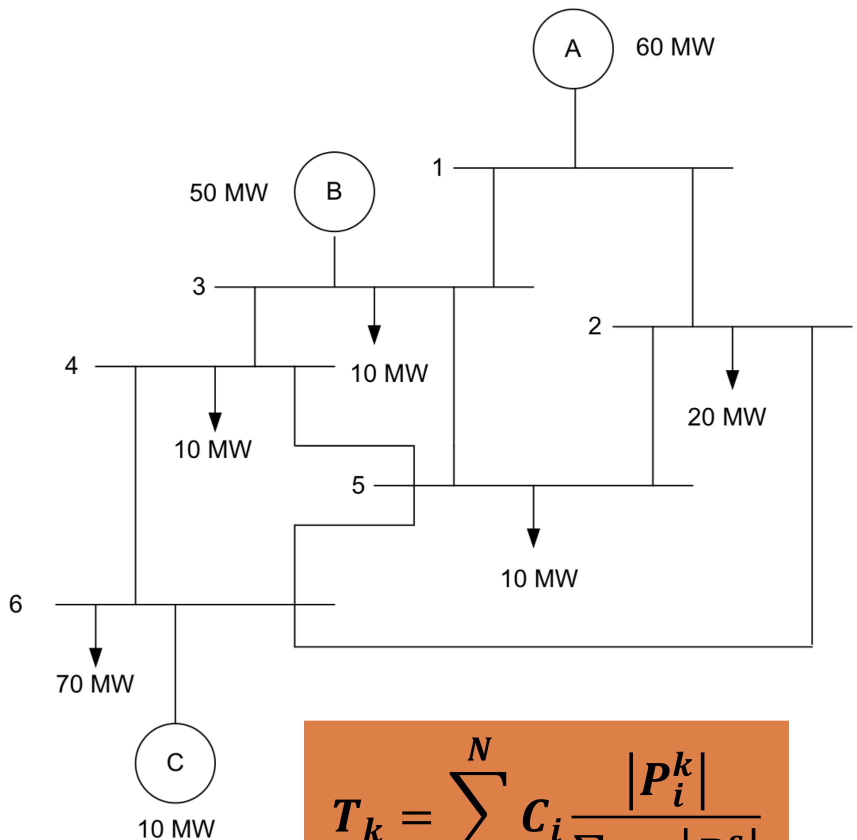
  = $ 810 – $ 173.23 = $ 638.77

Modulus Method – Flow summary table

From	To	Flow [MW]	Flow GA	Flow GB	Flow GC	Flow |GA|+|GB|+|GC|
1	2	43,86	30,71	13,26	-0,11	44,09
1	3	16,11	29,28	-13,29	0,12	42,69
2	5	11,08	11,31	0,63	-0,85	12,79
2	6	12,77	9,40	4,29	-0,93	14,62
3	4	27,80	12,35	15,76	-0,31	28,42
3	5	28,30	11,93	16,78	-0,41	29,12
4	5	2,98	0,47	2,68	-0,16	3,31
4	6	14,84	6,87	8,95	-0,98	16,80
5	6	32,60	18,75	16,13	-2,28	37,15

From	To	Flow [MW]	Flow C2	Flow C3	Flow C4	Flow C5	Flow C6	Flow |C2|+|C3|+|C4|+|C5|+|C6|
1	2	43,86	9,67	1,00	3,05	3,74	26,39	43,86
1	3	16,11	0,32	4,00	1,95	1,26	8,58	16,11
2	5	11,08	-7,16	0,80	1,94	3,06	12,45	25,41
2	6	12,77	-3,14	0,21	1,11	0,66	13,92	19,04
3	4	27,80	3,80	-0,83	4,19	2,28	18,38	29,47
3	5	28,30	4,85	-1,00	1,92	3,13	19,39	30,30
4	5	2,98	1,73	-0,29	-2,67	1,33	2,89	8,90
4	6	14,84	2,09	-0,55	-3,15	0,96	15,50	22,26
5	6	32,60	-0,62	-0,51	1,23	-2,44	34,94	39,73

Results for Modulus Method



$$T_k = \sum_{i=1}^{N} C_i \frac{\left|P_i^k\right|}{\sum_{s \in S_i} \left|P_i^s\right|}$$

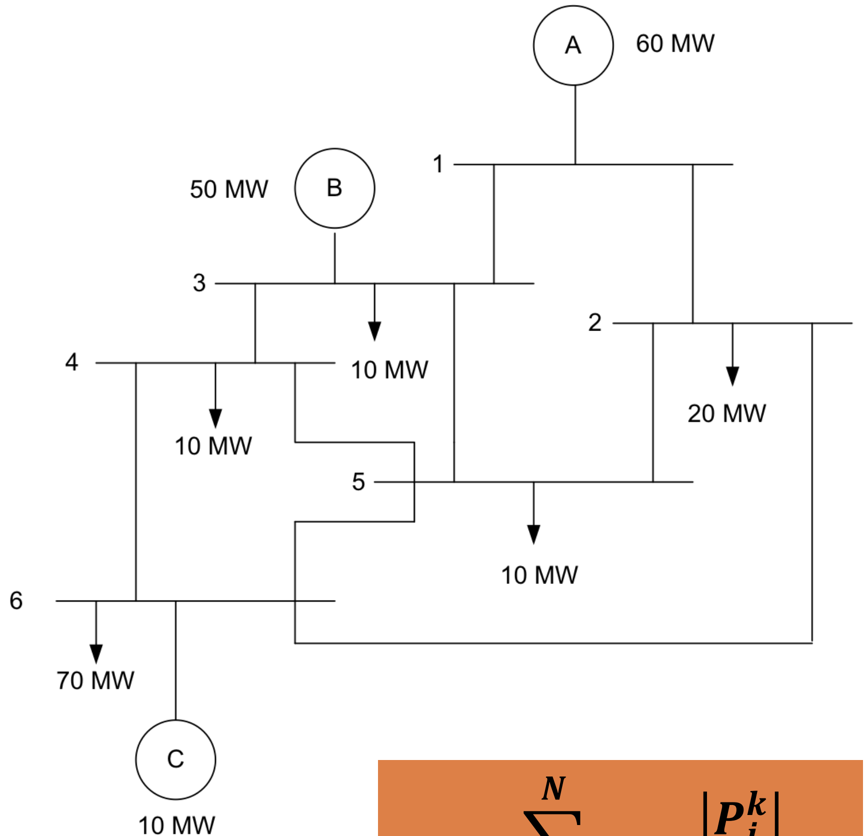
	G/D MW	Modulus $/year	Modulus $/MW
GA	60	216.51	3.6085
GB	50	?	?
GC	10	?	?
Total	120	?	?

	G/D MW	Modulus $/year	Modulus $/MW
L2	20	?	?
L3	10	?	?
L4	10	?	?
L5	10	?	?
L6	70	?	?
Total	120	?	?

Collected Revenue	810

This method will recover all required revenue!

Results for Modulus Method



$$T_k = \sum_{i=1}^{N} C_i \frac{|P_i^k|}{\sum_{s \in S_i} |P_i^s|}$$

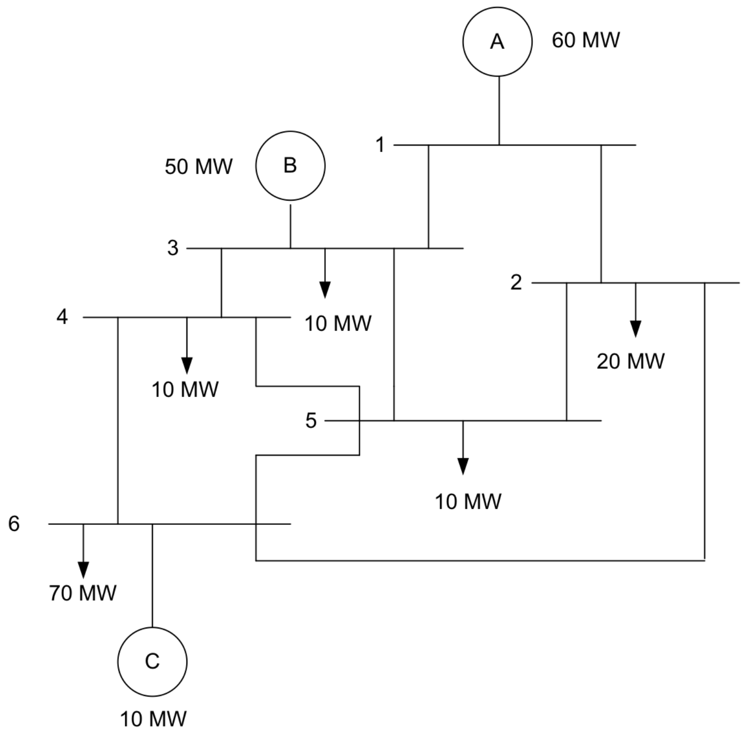
	G/D [MW]	Charge $/year
		Modulus
G1	60	216,51
G2	50	173,67
G3	10	14,82
Total	120	405,00
C2	20	57,54
C3	10	19,96
C4	10	45,09
C5	10	33,95
C6	70	248,45
Total	120	405,00
Collected Revenue		810,00

Zero Counterflow Method – Flow Summary table

From	To	Flow [MW]	Flow GA	Flow GB	Flow GC	Positive Flow Ratio GA	Positive Flow Ratio GB	Positive Flow Ratio GC
1	2	43,86	30,71	13,26	-0,11	0,70	0,30	0,00
1	3	16,11	29,28	-13,29	0,12	1,00	0,00	0,004
2	5	11,08	11,31	0,63	-0,85	0,95	0,05	0,00
2	6	12,77	9,40	4,29	-0,93	0,69	0,31	0,00
3	4	27,80	12,35	15,76	-0,31	0,44	0,56	0,00
3	5	28,30	11,93	16,78	-0,41	0,42	0,58	0,00
4	5	2,98	0,47	2,68	-0,16	0,15	0,85	0,00
4	6	14,84	6,87	8,95	-0,98	0,43	0,57	0,00
5	6	32,60	18,75	16,13	-2,28	0,54	0,46	0,00

From	To	Flow [MW]	Flow C2	Flow C3	Flow C4	Flow C5	Flow C6	Positive Flow Ratio C2	Positive Flow Ratio C3	Positive Flow Ratio C4	Positive Flow Ratio C5	Positive Flow Ratio C6
1	2	43,86	9,67	1,00	3,05	3,74	26,39	0,22	0,02	0,07	0,09	0,60
1	3	16,11	0,32	4,00	1,95	1,26	8,58	0,02	0,25	0,12	0,08	0,53
2	5	11,08	-7,16	0,80	1,94	3,06	12,45	0,00	0,04	0,11	0,17	0,68
2	6	12,77	-3,14	0,21	1,11	0,66	13,92	0,00	0,01	0,07	0,04	0,88
3	4	27,80	3,80	-0,83	4,19	2,28	18,38	0,13	0,00	0,15	0,08	0,64
3	5	28,30	4,85	-1,00	1,92	3,13	19,39	0,17	0,00	0,07	0,11	0,66
4	5	2,98	1,73	-0,29	-2,67	1,33	2,89	0,29	0,00	0,00	0,22	0,49
4	6	14,84	2,09	-0,55	-3,15	0,96	15,50	0,11	0,00	0,00	0,05	0,84
5	6	32,60	-0,62	-0,51	1,23	-2,44	34,94	0,00	0,00	0,03	0,00	0,97

Results for Zero Counterflow Method



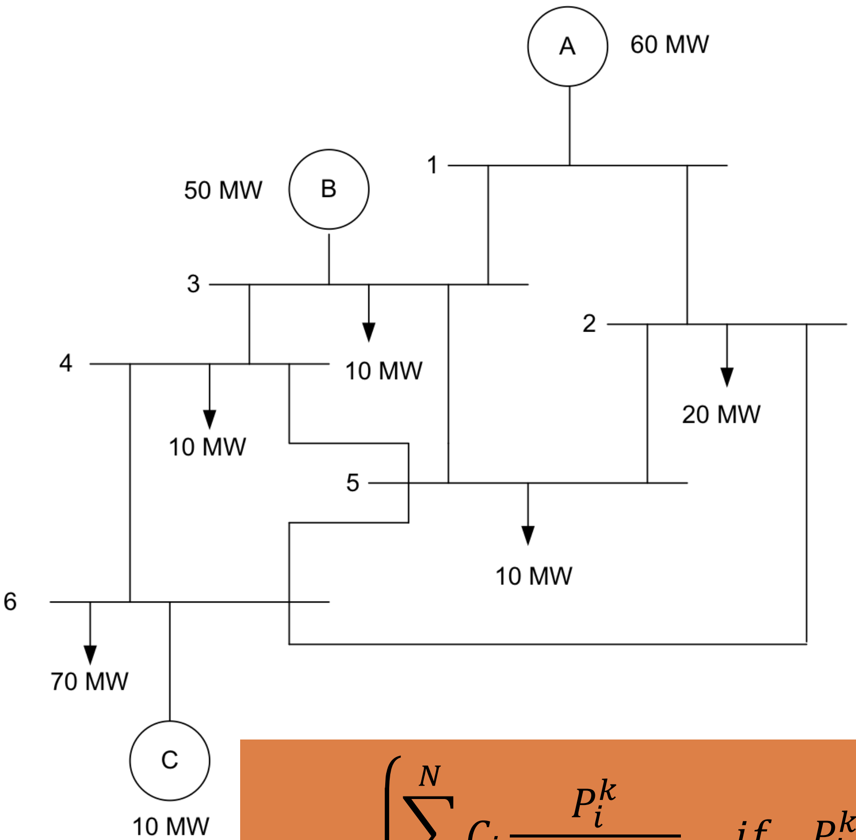
$$T_k = \sum_{\substack{i=1 \\ if\ P_i^k \geq 0}}^{N} C_i \frac{P_i^k}{\sum_{j \in \Omega_{i+}} P_i^j}$$

	G/D MW	Zero Counterflow $/year	$/MW
GA	60	238.77	3.9795
GB	50	?	
GC	10	?	
Total	120	?	

L2	20	?	
L3	10	?	
L4	10	?	
L5	10	?	
L6	70	?	
GA	120	?	

Collected Revenue	810.00

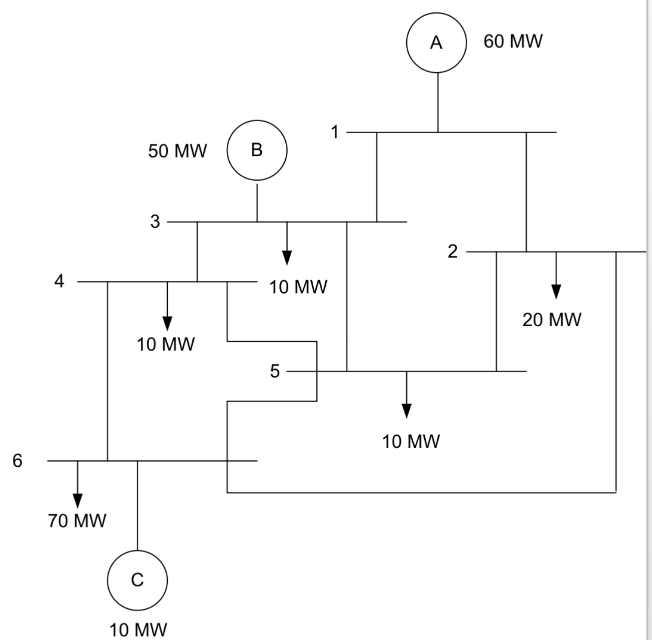
Results for Zero Counterflow Method

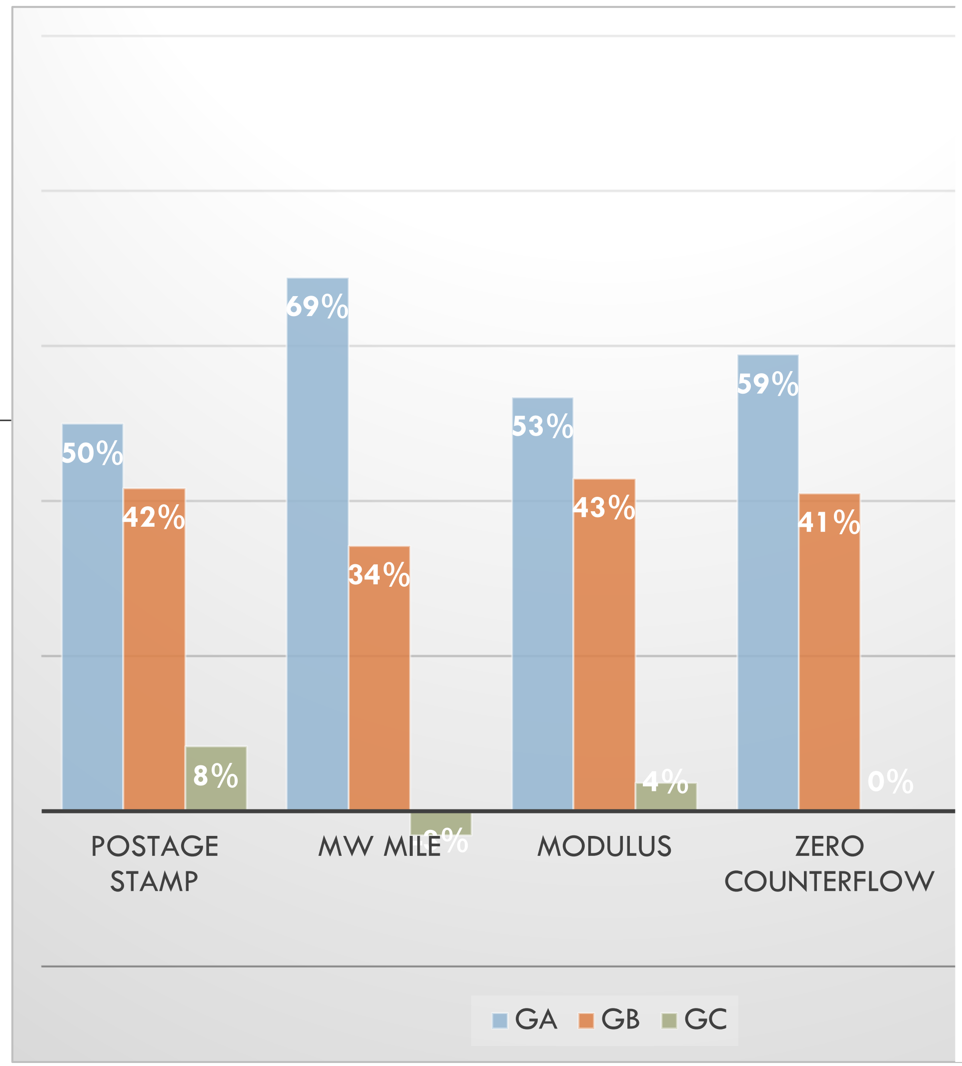


$$T_k = \begin{cases} \sum_{i=1}^{N} C_i \frac{P_i^k}{\sum_{j \in \Omega_{i+}} P_i^j} & if \quad P_i^k \geq 0 \\ 0 & if \quad P_i^k < 0 \end{cases}$$

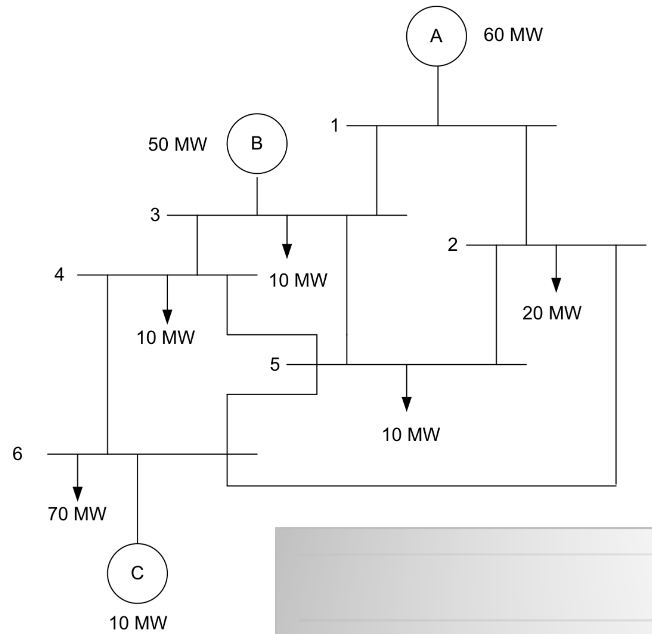
	G/D [MW]	Charge $/year
		Zero Counterflow
G1	60	238,77
G2	50	166,05
G3	10	0,18
Total	120	405,00
C2	20	42,31
C3	10	14,72
C4	10	27,52
C5	10	37,58
C6	70	282,86
Total	120	405,00
Collected Revenue		810,00

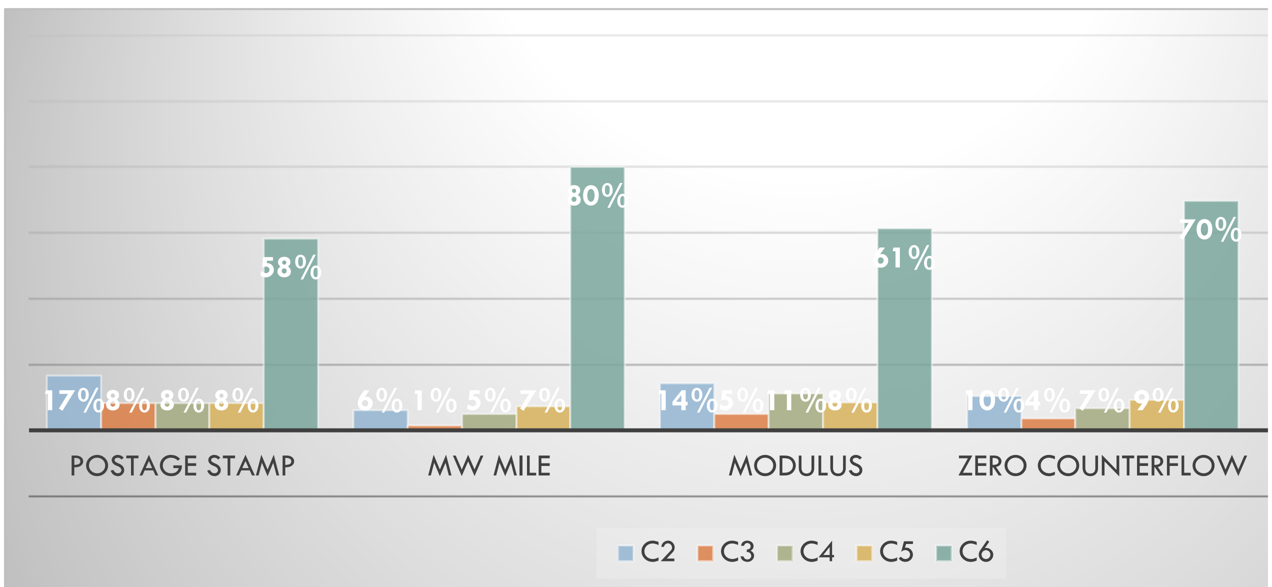
Comparing Generation Charges





Comparing Load Charges





Incremental Costs

Incremental Costs

- Variation of total cost due to the entry of a particular agent
- Long or short term
- Incremental cost = difference between the total cost before and after including a new agent/transport transaction
- It can be computed simulating the two cases (before and after) or using the Lagrange multipliers

ICRP method

- Proxy of the long term incremental cost - considers an specific system condition and configuration
- Provide generation and consumption tariff at each node of the network
- Don't actually solve optimization model all the time, instead the network charges at each bus are given by

$$\pi_i^g = \sum_{j=1}^{N} C_j \boxed{\beta_{ji}} \quad \text{and} \qquad \pi_i^c = -\pi_i^g$$

Sensitivity matrix

  - N # of network elements
  - π_i^g network charge for generators at node i ($/kW)
  - π_i^c network charge for consumers at node i ($/kW)
  - C_j cost of asset j ($)

What is β_{ji}?

- $\boldsymbol{\beta}$ is a sensitivity matrix
- Element β_{ji} power flow variation on element j when we increment 1 pu power in node i;
- How de we compute $\boldsymbol{\beta}$?

1. Matrix Form

$$\beta = D * C * X$$

D = Susceptibility Diagonal Matrix (nl x nl)

C = Connectivity Matrix (nl x nb)

X = Inverse of B with zeros in row and column of the reference node (nb x nb).

2. Power Flow computation with and without the 1 pu at node i

Back to our case study

Now let's compute charges with incremental cost

Beta Matrix Computation

Circuit	From	To	Resistance (R)	Reactance (X%)
1	1	2	0	12
2	1	3	0	8
3	2	5	0	5
4	2	6	0	12
5	3	4	0	15
6	3	5	0	16
7	4	5	0	12
8	4	6	0	9
9	5	6	0	3

Matrix D (nl x nl)

	l1	l2	l3	l4	l5	l6	l7	l8	l9
l1	8,33	0	0	0	0	0	0	0	0
l2	0	12,50	0	0	0	0	0	0	0
l3	0	0	20,00	0	0	0	0	0	0
l4	0	0	0	8,33	0	0	0	0	0
l5	0	0	0	0	6,67	0	0	0	0
l6	0	0	0	0	0	6,25	0	0	0
l7	0	0	0	0	0	0	8,33	0	0
l8	0	0	0	0	0	0	0	11,11	0
l9	0	0	0	0	0	0	0	0	33,33

Matrix C (nl x nb)

	b1	b2	b3	b4	b5	b6
l1	1	-1	0	0	0	0
l2	1	0	-1	0	0	0
l3	0	1	0	0	-1	0
l4	0	1	0	0	0	-1
l5	0	0	1	-1	0	0
l6	0	0	1	0	-1	0
l7	0	0	0	1	-1	0
l8	0	0	0	1	0	-1
l9	0	0	0	0	1	-1

Matrix X (nb x nb)

	b1	b2	b3	b4	b5	b6
b1	0	0	0	0	0	0
b2	0	0,0756	0,0296	0,0542	0,0625	0,0628
b3	0	0,0296	0,0603	0,0439	0,0384	0,0381
b4	0	0,0542	0,0439	0,1028	0,0687	0,0736
b5	0	0,0625	0,0384	0,0687	0,0826	0,0765
b6	0	0,0628	0,0381	0,0736	0,0765	0,0927

Beta matrix computation (cont'd)

Matrix Beta = D C X

	b1	b2	b3	b4	b5	b6
l1	0	-0,6297	-0,2466	-0,4515	-0,5206	-0,5231
l2	0	-0,3700	-0,7538	-0,5488	-0,4800	-0,4763
l3	0	0,2620	-0,1760	-0,2900	-0,4020	-0,2740
l4	0	0,1066	-0,0708	-0,1616	-0,1166	-0,2491
l5	0	-0,1641	0,1094	-0,3929	-0,2021	-0,2368
l6	0	-0,2056	0,1369	-0,1550	-0,2763	-0,2400
l7	0	-0,0691	0,0458	0,2841	-0,1158	-0,0242
l8	0	-0,0955	0,0644	0,3244	-0,0867	-0,2122
l9	0	-0,0100	0,0100	-0,1633	0,2033	-0,5399

- Beta matrix can also be used for power flow calculation F = β P

Vector F = **Matrix β** **Vector P** = **Vector F**

$$\begin{bmatrix} F_{12} \\ F_{13} \\ F_{25} \\ F_{26} \\ F_{34} \\ F_{35} \\ F_{45} \\ F_{46} \\ F_{56} \end{bmatrix} = \begin{bmatrix} 0 & -0{,}6297 & -0{,}2466 & -0{,}4515 & -0{,}5206 & -0{,}5231 \\ 0 & -0{,}3700 & -0{,}7538 & -0{,}5488 & -0{,}4800 & -0{,}4763 \\ 0 & 0{,}2620 & -0{,}1760 & -0{,}2900 & -0{,}4020 & -0{,}2740 \\ 0 & 0{,}1066 & -0{,}0708 & -0{,}1616 & -0{,}1166 & -0{,}2491 \\ 0 & -0{,}1641 & 0{,}1094 & -0{,}3929 & -0{,}2021 & -0{,}2368 \\ 0 & -0{,}2056 & 0{,}1369 & -0{,}1550 & -0{,}2763 & -0{,}2400 \\ 0 & -0{,}0691 & 0{,}0458 & 0{,}2841 & -0{,}1158 & -0{,}0242 \\ 0 & -0{,}0955 & 0{,}0644 & 0{,}3244 & -0{,}0867 & -0{,}2122 \\ 0 & -0{,}0100 & 0{,}0100 & -0{,}1633 & 0{,}2033 & -0{,}5399 \end{bmatrix} \begin{bmatrix} 60 \\ -20 \\ 40 \\ -10 \\ -10 \\ -60 \end{bmatrix} = \boxed{\begin{bmatrix} 43{,}84 \\ 16{,}11 \\ 11{,}08 \\ 12{,}76 \\ 27{,}82 \\ 28{,}30 \\ 2{,}98 \\ 14{,}84 \end{bmatrix}}$$

Exactly what we had before!

Computing Network Charge π

Bus	Generation	Load	πg	πc
1	60	0	0,000	0,000
2	0	20	-1,058	1,058
3	50	10	-0,793	0,793
4	0	10	-1,399	1,399
5	0	10	-1,797	1,797
6	10	70	-2,498	2,498

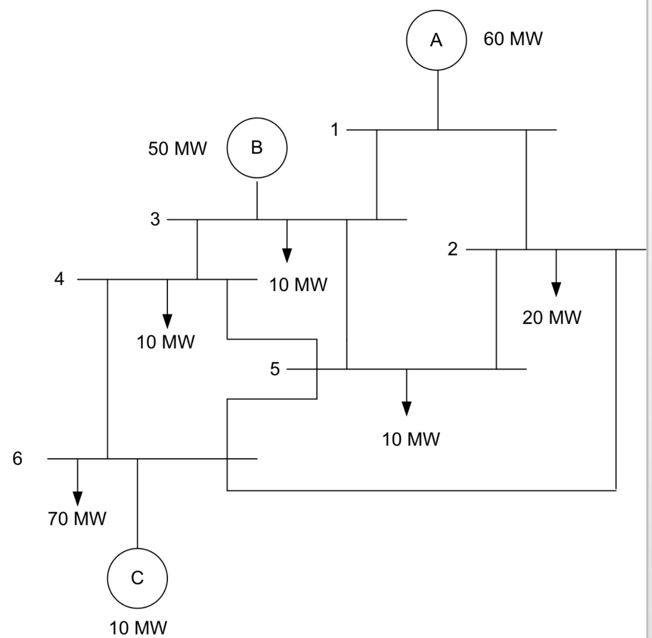
- Network charge for generator Eg = πg * G
- Network charge for load Ec = πc * D

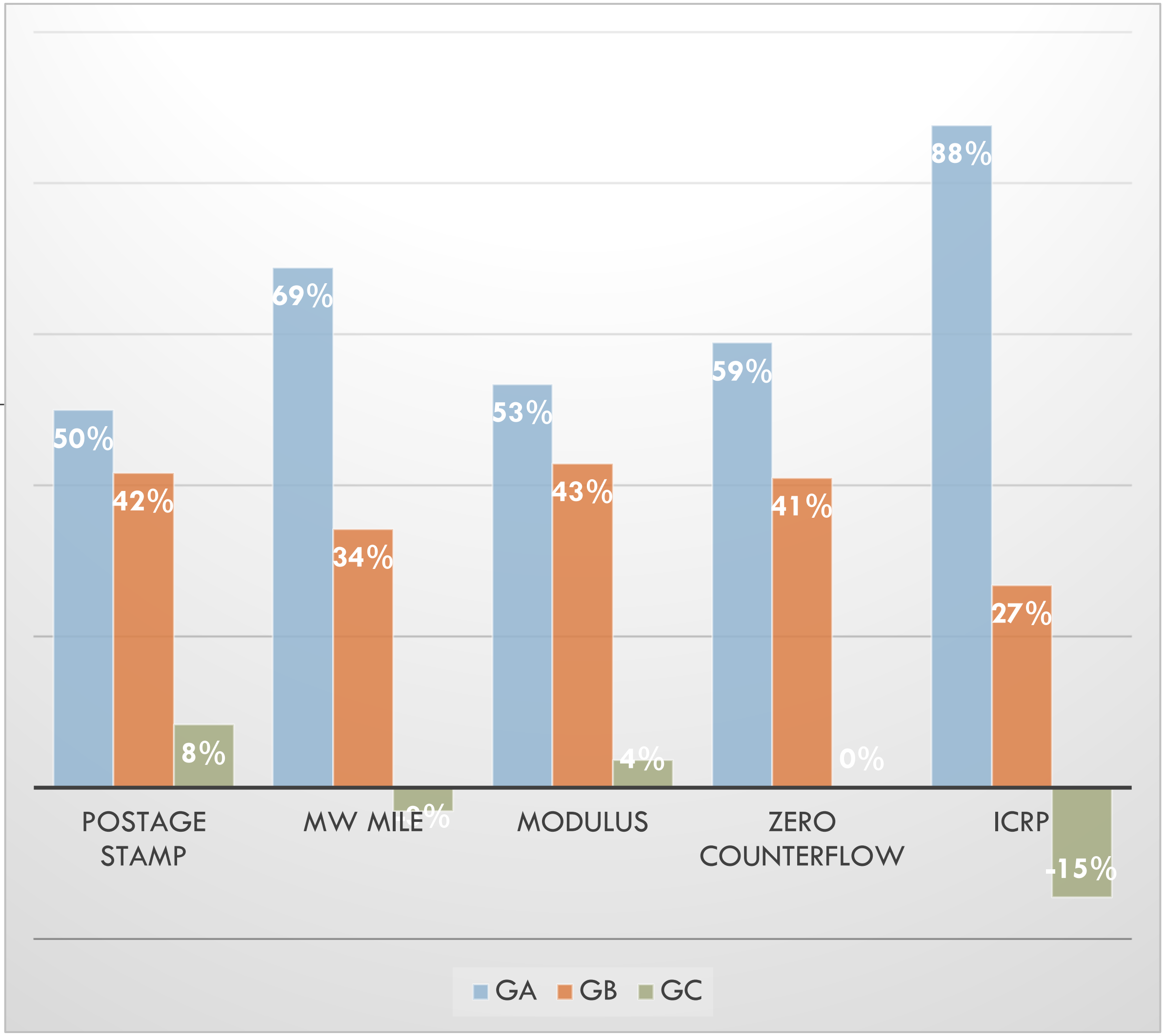
Bus	Generation	Load	πg	Eg	πc	Ec
1	60	0	0,000	0,00	0,000	0,00
2	0	20	-1,058	0,00	1,058	21,16
3	50	10	-0,793	-39,63	0,793	7,93
4	0	10	-1,399	0,00	1,399	13,99
5	0	10	-1,797	0,00	1,797	17,97
6	10	70	-2,498	-24,98	2,498	174,86
				-64,61		**235,91**

Collected Revenue [$] 171,30

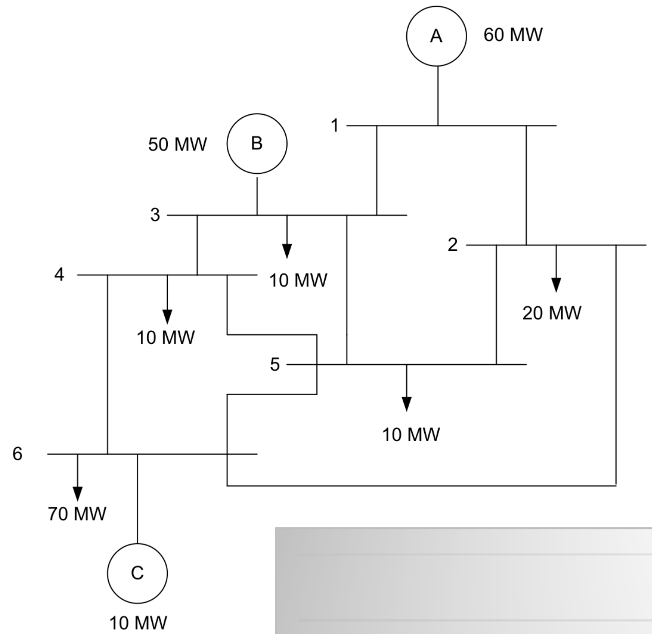
Just as in MW mile it recovers only the cost related to the used portion of the network

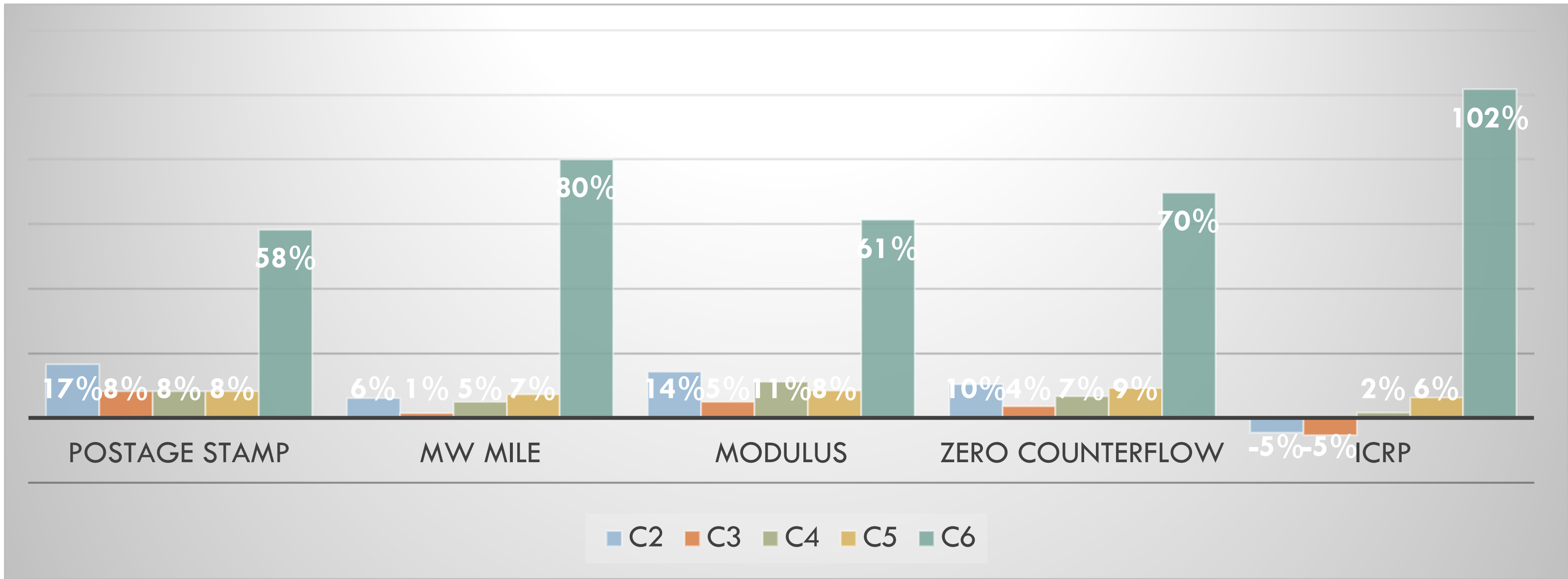
Comparing Gen Charges

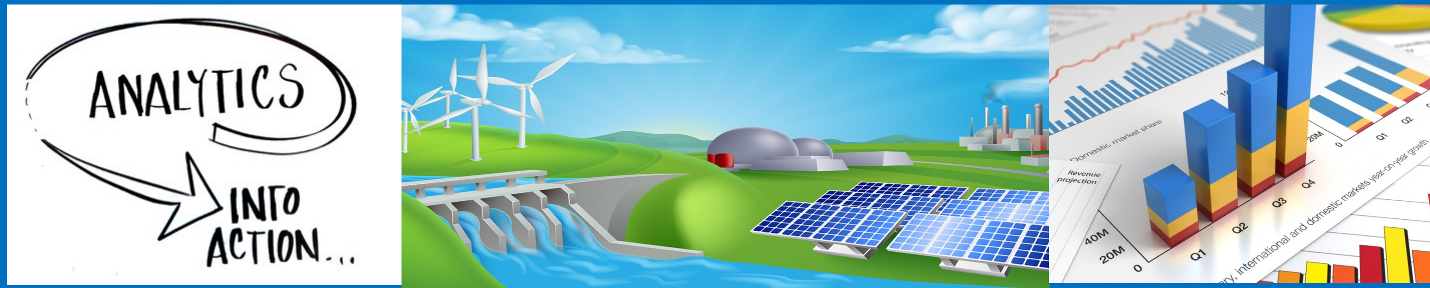




Comparing Load Charges







THANK YOU !

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