



# ECONOMICS OF MODERN POWER SYSTEMS

## M6 – Behind-the-Meter (BTM) Energy Management Systems: PV + battery

# Learning Goals

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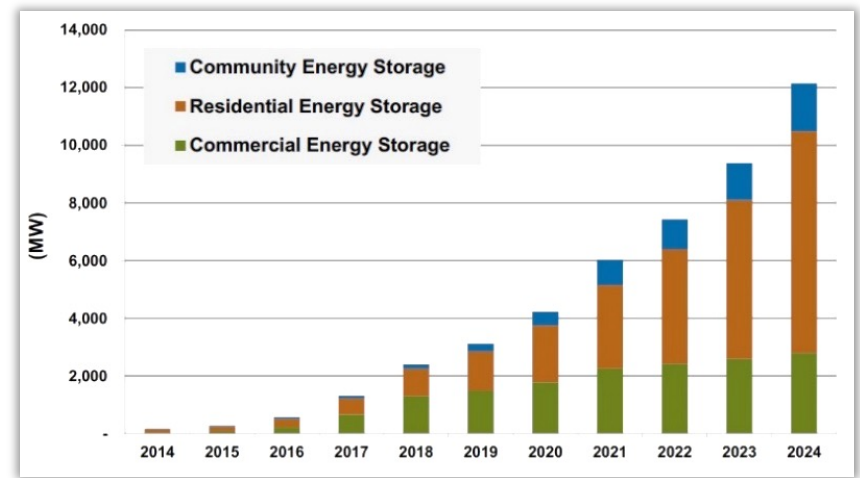
- Storage management
  - ▣ More on solving LPs in Python/R
  - ▣ case study for a customer with (PV + battery) system

# Energy Storage Management

Study case: Behind-the-meter Solar + Storage

# Approaches to Solar Storage

- Utility Scale Storage
- Customer Sited Storage
  - Commercial / Industrial
  - Residential



Projections of Energy storage growth in U.S.A in different levels (source: Navigant Research)

# Customer Sited Storage: Commercial/Industrial

- May be incentivized through existing or new tariffs
- If existing **demand charges** are  $\geq \$15/\text{kW}$ , customer storage may already be feasible with current technologies and pricing
- New tariffs with time-varying or dynamic rates to promote demand reduction are an option
  - ▣ High peak kW demand rate at certain hours reflecting seasonal patterns ( e.g. 4-6pm in May-Sep )
  - ▣ Low off-peak kW demand rate
  - ▣ Low energy rate for kWh ( 2-3 cents )
- According to 2017 NREL whitepaper on behind-the-meter battery energy storage, **demand rates in NC are as high as \$25.65 per kW, average \$15.61**

# Customer-sited storage: Residential

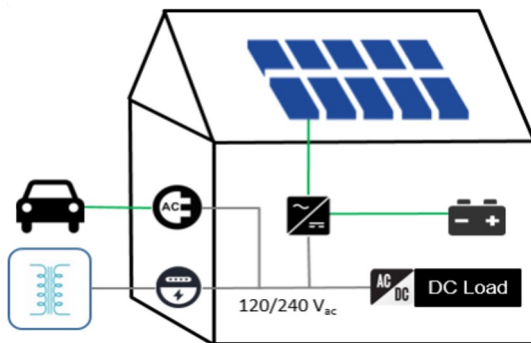
- DC and AC storage options
- Storage connects directly to DC service to avoid the loss incurred by DC/AC conversion
- Options for PV system and electric vehicle(s) and/or other storage

## Customer benefits

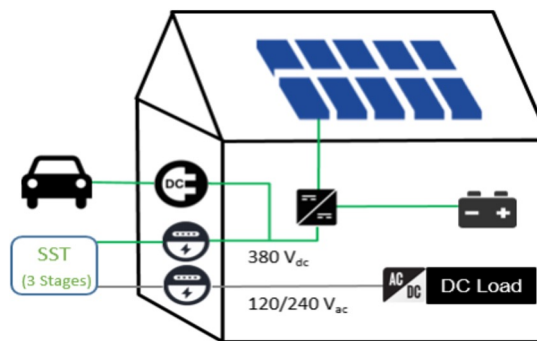
- ✓ Shift energy according to time-of-use rate
- ✓ Minimize PV curtailment
- ✓ Optimize EV charging
- ✓ Backup generation source

## Model, Data & Tools

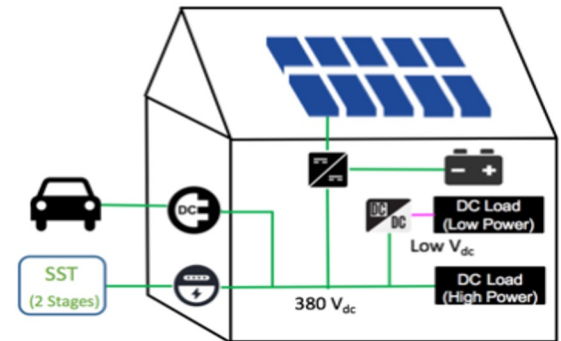
- ✓ Household load model
- ✓ Converter efficiency curve
- ✓ EV charging patterns
- ✓ Energy storage parameters



AC House

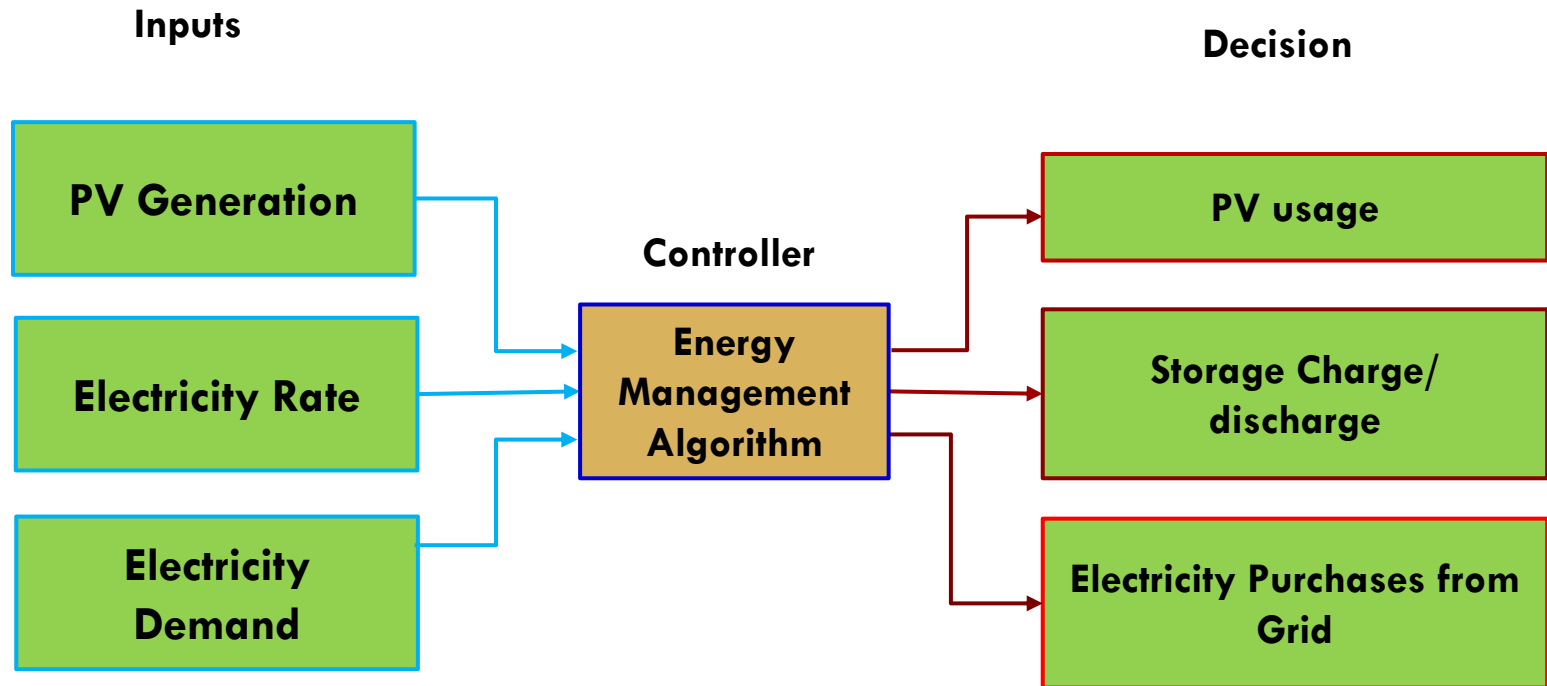


Hybrid House (both AC & DC)



Pure DC house

# PV-based Storage Control



# Study Case Description

- Customer Sited PVs with Storage
- We will use data from Assignment #1
  - ▣ Same Residential Customer
  - ▣ Same PV system
  - ▣ But now he will also have battery and he needs help with storage management to minimize cost of electricity
- Assumptions
  - ▣ He will not send power to the grid (e.g. suppose there is no net metering, so that is no incentive for him to feed the grid)
  - ▣ Inverter DC to AC ratio is 1 (matches PV installed capacity)



# Try to think about this problem

- Planning horizon – 1 day
- Time step hours
- Write down your decision variables
  
- Write down your constraints
  - ▣ If you can't come up with a mathematical expression, just describe with words what they would be

# Mathematical Model Formulation



- Our goal is

**minimize cost**

- Cost here is not related to investment, but daily expenses related to electricity supply
- Cost function depends on how much electricity I am using from utility and electricity rate I am paying, assuming there is no cost to generate and/store electricity with the PV + battery system
- And we want to minimize cost for all hours of the day

# Mathematical Model Formulation




$$\min \quad z = \sum_{t=1}^T [P_{grid,t} \cdot C_t]$$

Diagram illustrating the components of the objective function:

- $P_{grid,t}$  is labeled **Electricity from grid** (red box).
- $C_t$  is labeled **ToU Rate** (blue box).

# Mathematical Model Formulation


$$\min z = \sum_{t=1}^T [P_{grid,t} \cdot C_t]$$



- Now let's think about constraints...

## Power Balance at each time $t$

Power in  $\geq$  Power out

$$\text{Power}_{\text{grid}} + P_{\text{PV}} + P_{\text{disch}} \geq P_{\text{charge}} + P_{\text{load}}$$

input 

input 

$$\text{Power}_{\text{grid}} + P_{\text{disch}} - P_{\text{charge}} \geq + P_{\text{load}} - P_{\text{PV}}$$

$$\text{Power}_{\text{grid}} + P_{\text{disch}} - P_{\text{charge}} - P_{\text{slack}} = + P_{\text{load}} - P_{\text{PV}}$$

# Mathematical Model Formulation

$$\min \quad z = \sum_{t=1}^T [P_{grid,t} \cdot C_t]$$

Electricity  
from grid

ToU  
Rate

subjected to:

**Equality constraints:**

(i) Power Balance: **Input and output power should be equivalent**

$$\square \quad P_{grid,t} - P_{b_{ch},t} + P_{b_{disch},t} - P_{slack,t} = P_{load,t} - P_{PV,t}^{\omega_t} \quad (\forall t \in T)$$


Charge

Discharge

Actual  
Demand

PV

# Mathematical Model Formulation


$$\min \quad z = \sum_{t=1}^T [P_{grid,t} \cdot C_t]$$

Electricity  
from grid

ToU  
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Charge      Discharge      Actual Demand      PV

□ What's next?

**Storage balance constraint**

$$\text{Storage\_level\_t} = \text{Storage\_level\_t-1} + P_{\text{charge\_t}} - P_{\text{discharge\_t}}$$

# Mathematical Model Formulation



$$\min \quad z = \sum_{t=1}^T [P_{grid,t} \cdot C_t]$$

Electricity  
from grid

ToU  
Rate

subjected to:

**Equality constraints:**

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$$P_{grid,t} - P_{bch,t} + P_{bdisch,t} - P_{slack,t} = P_{load,t} - P_{PV,t}^{\omega_t} \quad (\forall t \in T)$$

Charge      Discharge      Actual Demand      PV

(ii) Charge Balance: **State of charge will change based on charging/ discharging power**

$$SoC_t = SoC_{t-1} + \frac{P_{bch,t} \cdot \eta_b^{ch}}{Q_b \cdot \Delta t} - \frac{P_{bdisch,t}}{Q_b \cdot \Delta t \cdot \eta_b^{disch}} \quad (\forall t \in T)$$

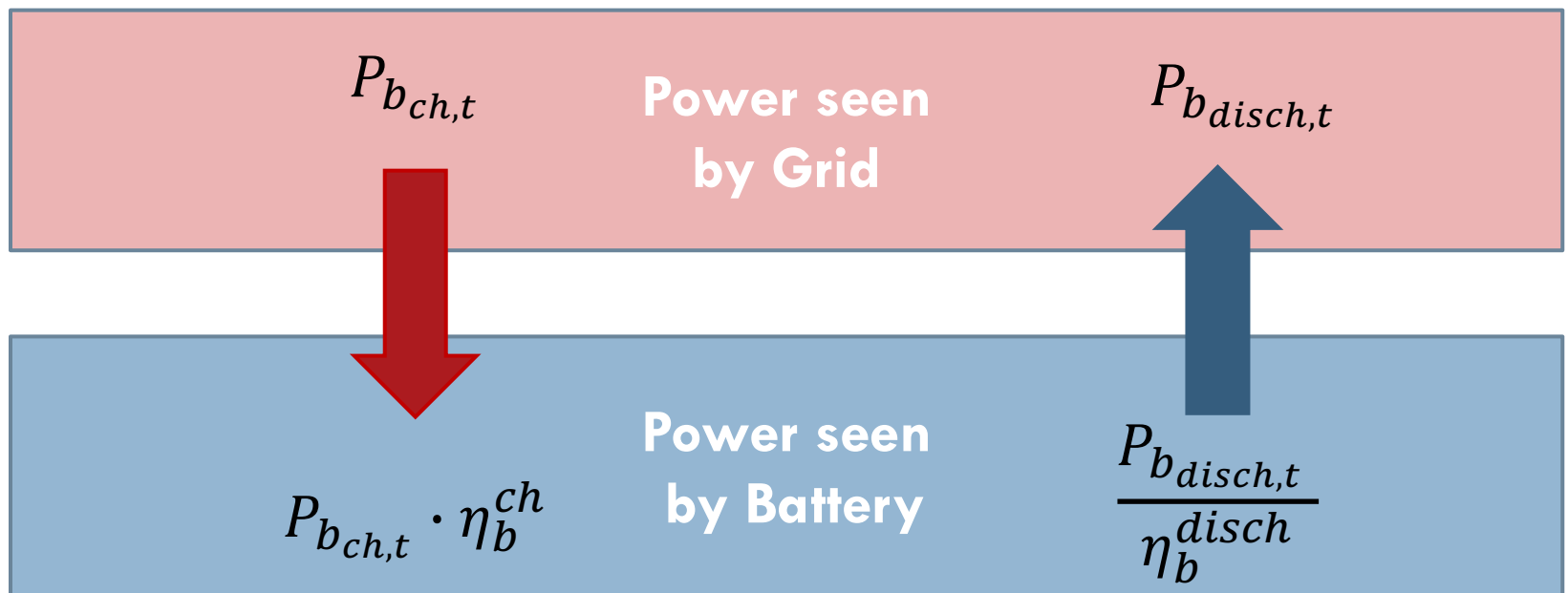
State of charge for storage  
device

Think of it as  $\frac{\text{power}_{in} \cdot \text{eff}}{\text{bat\_capacity}}$   
so the SoC is in %

Think of it as  $\frac{\text{power}_{out}/\text{eff}}{\text{bat\_capacity}}$   
so the SoC is in %

# Understanding SOC equation

$$SoC_t = SoC_{t-1} + \frac{P_{b_{ch,t}} \cdot \eta_b^{ch}}{Q_b \cdot \Delta t} - \frac{P_{b_{disch,t}}}{Q_b \cdot \Delta t \cdot \eta_b^{disch}}$$





# Defining Boundary Conditions



- What's next?

**Boundary conditions**

# Defining Boundary Conditions

## Inequality Constraints:

- Storage device will be charged only from PV-generated power

$$P_{b_{ch},t} \leq P_{PV,t}^{\omega_t}, \forall t \in T$$

- Storage device will deliver power only to the household

$$P_{b_{disch},t} \leq P_{load,t}, \forall t \in T$$

- There will be no back-feeding of power to the grid

$$P_{grid,t} \geq 0, \forall t \in T$$

## Upper and lower bounds:

- $SoC_{b,min} \leq SoC_{b,t} \leq SoC_{b,max}, \forall t \in T$

- $P_{b_{ch}}^{min} \leq P_{b_{ch},t} \leq P_{b_{ch}}^{max}, \forall t \in T$

- $P_{b_{disch}}^{min} \leq P_{b_{disch},t} \leq P_{b_{disch}}^{max}, \forall t \in T$

*The less your battery is discharged before being recharged again, the longer it will last*

*The default SoC for Li-ion batteries is 95%*

## Uncertainty generation:

$P_{PV,t}^{\omega_t}$  where  $\omega_t$  is a scenario within  $\Omega_t$  that is the set of all scenarios,  $\forall t \in T$



# Uncertainty: A Challenge

Linear  
programming



But PV generation  
is not fixed



How to deal **uncertainty**

with PV generation?



Stochastic Dynamic programming

ensures global minimum

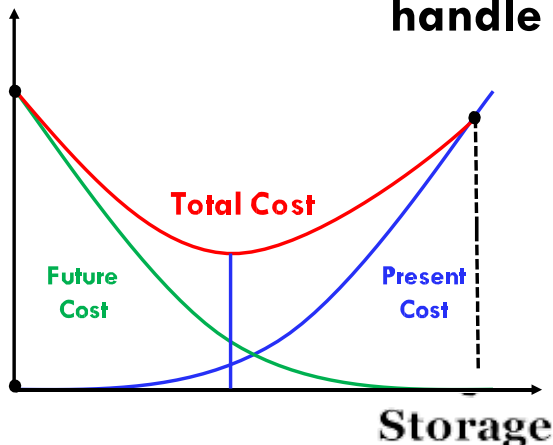
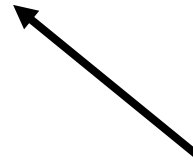


Face 'Curse of dimensionality'

Requires a lot of time to calculate

Require Huge memory

**Stochastic Dual  
Dynamic  
Programming can  
handle the issue**



We could also incorporate load uncertainty!

# Defining Boundary Conditions



## Inequality Constraints:

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- $P_{b\,ch}^{min} \leq P_{b\,ch,t} \leq P_{b\,ch}^{max}, \forall t \in T$

- $P_{b\,disch}^{min} \leq P_{b\,disch,t} \leq P_{b\,disch}^{max}, \forall t \in T$

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# Uncertainty: A Challenge

Linear programming

or Dynamic

Programming could

solve the problem

But PV generation  
is not fixed

How to deal **uncertainty**

with PV generation?

Stochastic Dynamic programming

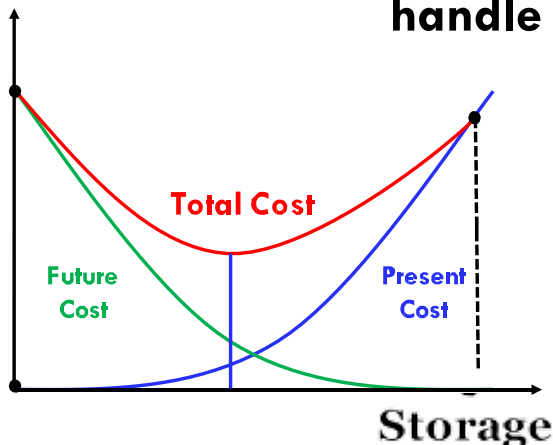
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Programming can  
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We could also incorporate load uncertainty!

# Really???

... Thanks god I took Time Series !!

...I knew I should have taken Time series !!




□ But we will not handle uncertainty today...

□ Today we will go over a **DETERMINISTIC** approach!

# Our final formulation will be...

$$\min \quad z = \sum_{t=1}^T [P_{grid,t} \cdot C_t]$$

Because we are doing deterministic approach,  
no need to write  $\forall \omega_t \in \Omega_t$



$$\text{s.t.} \quad P_{grid,t} - P_{bch,t} + P_{bdisch,t} - P_{slack,t} = P_{load,t} - P_{PV,t}^{\omega_t} \quad \forall t \in T \quad (\text{Power Balance})$$

$$SoC_t = SoC_{t-1} + \frac{P_{bch,t} \cdot \eta_b^{ch}}{Q_b \cdot \Delta t} - \frac{P_{bdisch,t}}{Q_b \cdot \Delta t \cdot \eta_b^{disch}} \quad \forall t \in T \quad (\text{Charge Balance})$$

$$P_{bch,t} \leq P_{PV,t}^{\omega_t} \quad \forall t \in T \quad (\text{Storage device only charged from PV})$$

$$P_{bdisch,t} \leq P_{load,t} \quad \forall t \in T \quad (\text{Storage deliver power only to household})$$

$$P_{grid,t} \geq 0 \quad \forall t \in T \quad (\text{No back-feeding of power to the grid})$$

$$SoC_{b,min} \leq SoC_{b,t} \leq SoC_{b,max} \quad \forall t \in T$$

$$P_{bch}^{min} \leq P_{bch,t} \leq P_{bch}^{max} \quad \forall t \in T \quad (\text{Upper and lower bounds})$$

$$P_{bdisch}^{min} \leq P_{bdisch,t} \leq P_{bdisch}^{max} \quad \forall t \in T$$

# Study Case Parameters

## Known parameters

PV installed capacity	7,92	kW
Battery capacity	4	kWh
Battery Efficiency for charging and discharging	0,92	
Initial State of Charge (SOC_0)	20%	
Minimum SOC	20%	
Maximum SOC	80%	
Pb_ch_min	0	kW
Pb_ch_max	3	kW
Pb_disch_min	0	kW
Pb_disch_max	3	kW
Battery type	Li-ion	
Time steps for this analysis: t	1	h
Time Horizon: T	24	h

## “Unknown” parameters – Deterministic Approach

	P_PV_t	P_load_t	C_t
Sep 18, 12:00 am	0	2.05	0.09996372
Sep 18, 1:00 am	0	0.32	0.09996372
Sep 18, 2:00 am	0	1.72	0.09996372
Sep 18, 3:00 am	0	0.34	0.09996372
Sep 18, 4:00 am	0	1.58	0.09996372
Sep 18, 5:00 am	0	0.34	0.09996372
Sep 18, 6:00 am	0.230828	0.83	0.09996372
Sep 18, 7:00 am	1.53247	1.55	0.09996372
Sep 18, 8:00 am	3.19997	0.51	0.09996372
Sep 18, 9:00 am	4.53936	1.98	0.09996372
Sep 18, 10:00 am	4.32765	0.37	0.09996372
Sep 18, 11:00 am	2.07893	2.5	0.09996372
Sep 18, 12:00 pm	3.82706	2.08	0.09996372
Sep 18, 1:00 pm	5.54551	1.86	0.09996372
Sep 18, 2:00 pm	4.79316	3.42	0.09996372
Sep 18, 3:00 pm	3.04991	1.55	0.09996372
Sep 18, 4:00 pm	1.38626	2.88	0.09996372
Sep 18, 5:00 pm	0.353036	2.34	0.09996372
Sep 18, 6:00 pm	0	3.1	0.09996372
Sep 18, 7:00 pm	0	2.22	0.09996372
Sep 18, 8:00 pm	0	2.79	0.09996372
Sep 18, 9:00 pm	0	1.05	0.09996372
Sep 18, 10:00 pm	0	1.1	0.09996372
Sep 18, 11:00 pm	0	1.47	0.09996372

From SAM ↑      From Customer ↑      From Duke Energy ↑



# Questions

- How many decision variables do we have?
  - ▣  $5 \times 24 = 120$  decision variables
  
- How many constraints do we have?
  - ▣  $4 \times 24 = 96$  technical constraints
  - ▣  $6 \times 24 = 144$  simple bounds
  - ▣  $2 \times 24 = 48$  nonnegativity
  
- Can we use Excel Solver to find optimal solution?
  - ▣ We might, but would require some rewriting
  - ▣ As it is, the problem is too big for the Solver



# Model Implementation in R

# Study Case Model Implementation

- Download data file from Sakai
- Recall: number of columns is number of dec. variables
  - ▣  $5 \times 24 = 120$  columns
  - ▣ Keep track of the order of the variables in the LP
  - ▣ Our example
    - 1 to 24       $P_{grid,t}$
    - 25 to 48       $P_{b_{ch},t}$
    - 49 to 72       $P_{b_{disch},t}$
    - 73 to 96       $P_{slack,t}$
    - 97 to 120       $SoC_t$

# Power Balance Constraint ( $t = 1$ )

$$P_{grid,t} - P_{b_{ch},t} + P_{b_{disch},t} - P_{slack,t} = P_{load,t} - P_{PV,t}^{\omega_t}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
$P_{grid,t}$	1																							
	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
$P_{b_{ch},t}$	-1																							
	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
$P_{b_{disch},t}$	1																							
	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96
$P_{slack,t}$	-1																							
	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
$SoC_t$																								

All other cells should be zero !

# Power Balance Constraint ( $t = 2$ )

$$P_{grid,t} - P_{b_{ch},t} + P_{b_{disch},t} - P_{slack,t} = P_{load,t} - P_{PV,t}^{\omega_t}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
$P_{grid,t}$		1																						
	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
$P_{b_{ch},t}$		-1																						
	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
$P_{b_{disch},t}$		1																						
	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96
$P_{slack,t}$		-1																						
	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
$SoC_t$																								

All other cells should be zero !

# Power Balance Constraint ( $t = 24$ )

$$P_{grid,t} - P_{b_{ch},t} + P_{b_{disch},t} - P_{slack,t} = P_{load,t} - P_{PV,t}^{\omega_t}$$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
$P_{grid,t}$																								1
	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
$P_{b_{ch},t}$																								-1
	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72
$P_{b_{disch},t}$																								1
	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96
$P_{slack,t}$																								-1
	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
$SoC_t$																								

All other cells should be zero !

# R code

- Running Simple Example ?
  - ▣ IpSolveAPI running ?
- Importing Data using read.table ?
- Understanding how R store decision variables vector
  - ▣ Jump by 24
- Understanding how constraints are entered
- Understanding how to access optimal variables values
- Plotting graphs in R



# Model Implementation in Python



# Study Case Model Implementation



- Please refer to the ipynb file on Sakai



# THANK YOU !

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