Multi-stage Stochastic Optimization for a PV-Storage Hybrid Unit in a Household

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Abstract — **In the face of increasing global energy supply challenges, renewable energy sources provide a cleaner and environmentally friendly energy alternative. To address the intermittency in PV power generation, battery storage can be used to store energy during lower demand periods. This requires the charging and discharging routine of the storage system to be controlled to achieve optimal economic benefits. In this paper, coordinated control between PV component and an accompanying storage unit is presented considering the stochastic nature of PV generation. The stochastic dual dynamic programming (SDDP) algorithm is employed to optimize the charge/discharge profiles with the goal to minimize the overall cost of satisfying the daily household load demand. The PV-storage hybrid unit can jointly contribute in reducing the consumer costs as shown through simulation analysis.**

Keywords: Control strategy, stochastic dual dynamic programming, energy storage, household load profile, solar power.

I. INTRODUCTION

To deal with the increasing electricity demand and reduce greenhouse gas emissions, renewable energy generation has received considerable attention as a source for clean energy. This motivated a world-wide integration of a vast amount of renewable generation such as wind and solar into the electric grids. For residential usage, solar PV is emerging as one of the most effective alternative for electricity generation. However, in many cases generated peak PV power cannot be used by the customer during the day time due to lower household demand at this time. In addition, peak load demand occurs during evenings, which cannot be mitigated by PV generation. The intermittency of photovoltaic (PV) power depends on both the day–night cycles and on the real-time weather conditions (such as cloud passing). Energy storage devices, such as batteries, can be used to store generated solar power. Energy storage option not only improves the utilization of solar PV power, but can also be used to reduce the household electricity consumption costs by an using optimal scheduling algorithm.

In a household unit, it is necessary to have an effective charge/discharge control strategy for the integrated PV-storage hybrid unit to minimize the electricity purchase cost from the utility grid. However, solar PV generation faces uncertainties, and forecasted PV generation cannot provide sufficient information to decision-making models that attempt to optimize the control strategy and reduce the overall system costs. Due to this intrinsic uncertainty, it is necessary to represent PV generation with different stochastic scenarios in order to better represent the problem at hand and derive a more robust analysis [1].

Energy storage devices can be used alongside renewable generation sources to address several challenges in electrical systems. For example, storage devices can be used to reduce variations in load demand output of a smart household system [2] and/or to address transient stability issues in smart grids [3]. Ramp rate control through energy storage has been considered as a solution in [4]. To exploit the price variations, energy storage can help shift the load demand to comparatively low-price periods [5]-[7]. Voltage violation may occur as a result of renewable energy integration in power grids, which can be regulated with the help of energy storage [8]. Smart grid residential users equipped with local solar PV power generation and an energy storage system are considered in other literature to minimize the cost of the system based on convex optimization approaches [9], [10]; however, the authors do not consider the representation of uncertainty in PV generation. Hourly charge/discharge schedule of battery with a short-term scheduling for unit commitment is analyzed in [11]. Energy management strategy has been applied to the energy storage of the solar PV plants to take part in the day and intra-day electricity markets [12].

In this paper, we develop a control strategy framework for a smart house that includes solar PV generation along with an energy storage device while considering the uncertainty in solar PV production. The control strategy is based on the stochastic dual dynamic programming (SDDP) algorithm applied in the context of stochastic PV generation and deterministic household electricity demand from single point forecasting. For deterministic cases, with a lower dimensional representation of the state and the decision vectors, the dynamic programming (DP) technique usually ensures global optimal solution and works efficiently. However, for stochastic cases, when the stochastic counterpart of DP (stochastic dynamic programming – SDP) is applied, the discretization of the state space and the different stochastic scenario representations contribute to the curse of dimensionality [13], [14] and to the intractability of real-sized models. On the other hand, SDDP constructs a piecewise linear approximation of the future cost function using Benders cuts that does not require the algorithm to discretize the state and decision spaces. As a result, it requires less computation time and memory [15]-[17].

The paper is organized as follows: Section II represents the household system model. Section III provides a mathematical formulation. SDDP algorithm is presented in Section IV.

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Section V presents the simulation results and Section VI gives a summary and the main conclusions of this paper.

II. System Representation

A household system consisting of a PV-storage hybrid unit is shown in Fig. 1. The solar PV panel provides energy to a storage system and also delivers power to satisfy the household load demand. The energy storage unit, however, is not connected to the grid, and can be used only to store energy from the PV panel or to satisfy the household demand. The household load is connected to the grid as well as to the solar PV panel along with the energy storage system.

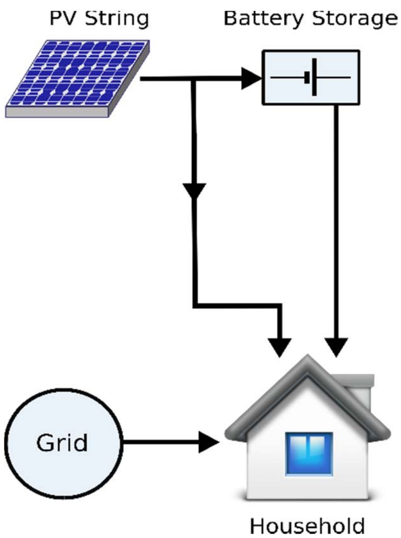


Fig. 1. PV-Storage hybrid unit in a household system.

The forecasted household load profile (single point forecast) gives the demand value that is used as the deterministic information in the decision-making model. For solar PV generation, a stochastic representation based on forecasted data for cloud covering and sudden weather changes is employed. To minimize the overall cost in a day, simulation of the proposed modeling framework is performed to determine the scheduling and usage of system components. The forecasted load demand information and the stochastic solar PV scenarios generated data for a 24-hour period are given as input to the mathematical model of the system and the SDDP algorithm, which computes an optimal state of charge (SOC) strategy.

III. Problem Formulation

Cost minimization of a household with optimal operation of the energy storage integrated PV system can significantly benefit a customer. In this study, a one-day cycle from 0hr to 24hr with a 15-minute resolution is considered. The total 24hr is divided into T time periods based on a resolution Δt. Let C_t be the time-of-use (TOU) electricity tariffs, $P_{load,t}$ is the day-ahead forecasted household load demand, $P_{PV,t}^{\omega_t}$ is the generated solar power, and $P_{grid,t}$ is the power demanded from grid at time t; the objective function J and constraints can then be written as -

$$\min J = \sum_{t=1}^{T} [C_t \cdot P_{grid,t}] \quad (1)$$

with the following structural constraints and operational limits.

A. Power balance constraint

$$P_{grid,t} - P_{b_{ch,t}} + P_{b_{disch,t}} - P_{slack,t} = P_{load,t} - P_{PV,t}^{\omega_t} \quad (2)$$

B. Charge balance constraint

$$SOC_{b,t} = SOC_{b,t-1} + \frac{P_{b_{ch,t}} \eta_{b,ch,t}}{Q_b \, \Delta t} - \frac{P_{b_{disch,t}}}{Q_b \, \eta_{b,disch,t} \Delta t}, \forall t \in T \quad (3)$$

C. Charge and discharge operational limits based on PV and load, respectively

$$P_{b\,ch,t} \leq P_{PV,t}^{\omega_t}, \forall t \in T \quad (4)$$

$$P_{b\,disch,t} \leq P_{load,t}, \forall t \in T \quad (5)$$

D. Non negativity requirement for purchases from the grid

$$P_{grid,t} \geq 0, \forall t \in T \quad (6)$$

E. Upper and lower bounds of the decision variables

$$SoC_{b,min} \leq SoC_{b,t} \leq SoC_{b,max}, \forall t \in T \quad (7)$$

$$P_{b_{ch}}^{min} \leq P_{b\,ch,t} \leq P_{b_{ch}}^{max}, \forall t \in T \quad (8)$$

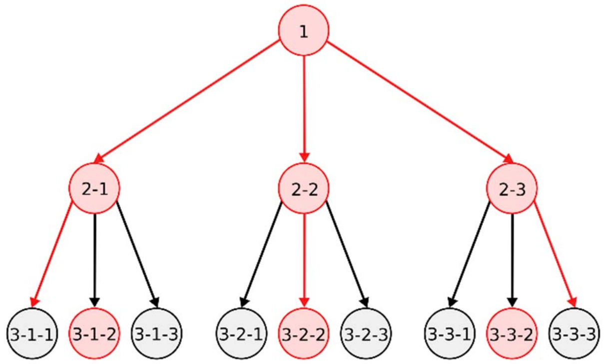
$$P_{b_{disch}}^{min} \leq P_{b\,disch,t} \leq P_{b_{disch}}^{max}, \forall t \in T \quad (9)$$

In the system under consideration, the PV unit does not provide power to the grid. Therefore, if there is more PV generation than the electricity demand at a specific time and the storage unit if fully charged, then $P_{slack,t}$ will take care of the excess generation as deferred PV energy. $P_{b_{ch,t}}$ and $P_{b_{disch,t}}$ are the instantaneous charging and discharging power of the storage device. $SOC_{b,t}$ is the state of charge of the energy storage at time t. The lower and upper bounds of decision variables are provided in Table I. The parameters Q_b is the total capacity of the storage unit and $\eta_{b,ch,t}$ and $\eta_{b,disch,t}$ are the efficiency of the charger for charge and discharge.

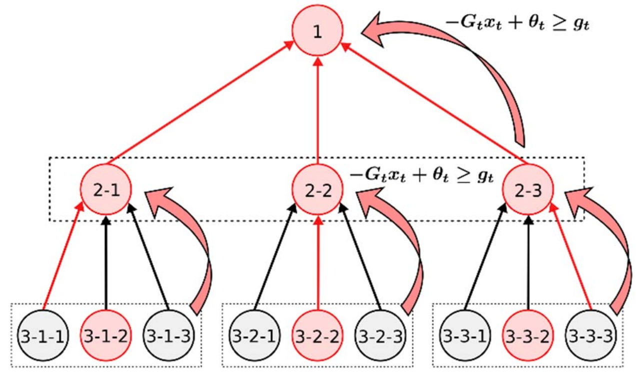
If the household demand needs to be satisfied by both PV generation and battery discharged power, then according to (2) and (3) it is less efficient to charge the battery with PV power. In that case, PV generation will be used directly to satisfy the demand with the option to use energy that is already stored in the battery as well as purchase power from the grid. On the other hand, if PV generation is higher than the demand, the surplus will be stored in the battery (if there is storage capacity available) and there will be no discharge. Thus, charging and discharging the storage device simultaneously is not possible. The different scenario representations for solar PV based on the forecasted solar generation $P_{PV,t}$ is obtained using (10) [18].

$$P_{PV,t}^{\omega_t} = P_{PV,t} \pm \rho_t^{\omega_t} P_{PV,t}; \; \omega_t \epsilon \forall \Omega_t, \forall t \in T \quad (10)$$

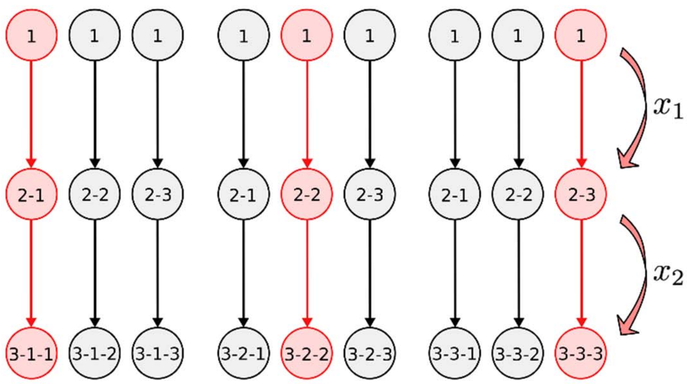
where ω_t is a scenario within Ω_t that is the set of all scenarios that represent the PV power generation, and $\rho_t^{\omega_t}$ is a normally distributed random parameter.



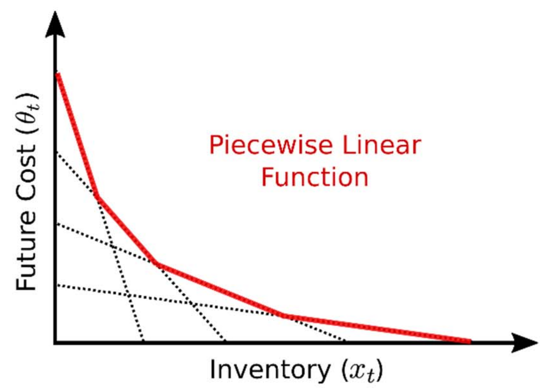
(a) Scenario Tree.



(c) Backward Recursion.



(b) Forward Paths Selection.



(d) Future cost function represented by Benders' cut.

Fig. 2. Optimization solution process via SDDP.

IV. Multi-stage Stochastic Optimization for Controlling a Hybrid PV-Battery System

A general T-stage stochastic linear program for the problem at hand can be formulated as follows:

$$\min_{x_1} \; c_1 \, x_1 + \mathbb{E}_{b_2|b_1} h_2(x_1, b_2) \tag{11}$$

$$\text{s.t.} \quad A_1 x_1 = B_1 x_0 + b_1: \qquad \pi_1 \tag{12}$$

$$x_1 \geq 0 \tag{13}$$

where for $t = 2, \ldots, T$,

$$h_t(x_{t-1}, b_t) = \min_{x_t} \; c_t x_t \; + \mathbb{E}_{b_{t+1}|b_t} h_{t+1}(x_t, b_{t+1}) \tag{14}$$

$$\text{s.t.} \; A_t x_t = B_t x_{t-1} + b_t: \qquad \pi_t \tag{15}$$

$$x_t \geq 0 \tag{16}$$

The decision variables of a particular stage t are considered as a vector x_t, which includes electricity purchases from the grid, charge and discharge power of the battery and *SoC* of the battery for the problem. The parameter b_t represents stochastic PV supply at stage t. Equations (11) and (14) represent the objective functions to minimize the total cost that includes first (present) and t^{th} (expected future) stage costs, respectively. Equations (12) and (15) are the model's structural constraints which include power balance and charge balance equations (2) and (3) for our case. Dual variables (denoted by π_t) derived from the structural constraints (or from the dual optimization model) are used later to construct a piece-wise linear approximation of the future cost function following Benders' decomposition scheme. Equations (13) and (16) are simple bounds on the decision variables such as (4)-(9). In the objective function defined in (10), $\mathbb{E}_{b_2|b_1} h_2(x_1, b_2)$ represents the expected cost function of stage 2 based on decisions x_1 taken in stage 1. The realization of the random parameter b_2 affects the system condition at stage 2. Similarly, for equation (14), $\mathbb{E}_{b_{t+1}|b_t} h_{t+1}(x_t, b_{t+1})$ calculates the expected cost function of stage $t+1$ given the decisions x_t in stage t. The realization of the random parameter is b_{t+1}.

To solve our version of the general model (11)-(16) we implement and employ a version of SDDP originated from the work of Pereira and Pinto [16], [17]. The SDDP is, to date, the state-of-the-art method for solving multi-stage stochastic linear programs. The SDDP algorithm avoids the well-known curse of dimensionality of DP by constructing an approximation of the future cost function with piecewise linear functions represented through Benders' cuts that are added iteratively. This process stops when a stopping criterion is reached.

A visualization of how SDDP works to solve this problem is depicted in Fig. 2 which shows the process for a simple three stage problem [15]; however, it is important to mention that the tree sizes of interest are quite large. For example, in our system a tree with 10 scenarios per stage with 96 stages is considered. Once a sampled scenario tree like Fig. 2 (a) is available for the SDDP, the process is started by sampling the forward paths in this tree as highlighted in Fig. 2 (b). These paths are considered for the problem to proceed for the forward pass. During the forward pass, a sequence of models like (14)-(16) is solved at each time stage using the simplex method. During the solution process, Benders' cuts, which are accumulated from previous iterations for the certain stage, are used as additional constraints to create a better approximation of the future costs and improve the decision-making process. Then the sample mean of the costs associated with all the sampled forward paths provides an estimate for the expected future cost. At the final stage of the forward pass, the total expected cost is estimated which is considered as the upper bound of the problem. The lower bound for the sampled problem is calculated from solving the first stage problem. After a finite number of iterations, the upper and lower bounds tend to converge and the algorithm can be stopped. A stopping criteria based on the desired level of precision in the convergence process is used. If upper and lower bound costs do not reach the desired convergence level, then another SDDP iteration is needed. At each iteration, new forward paths are sampled independently.

For reaching the desired convergence level, the algorithm proceeds to the backward pass shown in Fig. 2(c). In the backward pass, the algorithm computes new Benders cuts for certain stage to better approximate the expected future cost function. As there is no future cost after the final stage T, cuts are not used in that stage. In Fig. 2(c), the highlighted nodes are selected in this iteration's backward pass. Fig. 2(d) depicts the sets of cuts corresponding to all the nodes on each stage for each iteration. If the future cost obtained from forward pass is θ_t for a sampled path ω_t, and from the solved linear problem for stage $(t+1)$ the dual variables are represented by $\pi_{t+1}^{\omega_{t+1}}$ and optimal cost by $h_{t+1}^{\omega_{t+1}}$, then the Benders' cut for the stage t is calculated using (17)-(19).

$$\theta_t - G_t x_t \geq g_t \tag{17}$$

$$G_t = \sum_{\omega_{t+1} \in \Delta(\omega_t)} \pi_{t+1}^{\omega_{t+1}} B_{t+1} \tag{18}$$

$$g_t = \sum_{\omega_{t+1} \in \Delta(\omega_t)} h_{t+1}^{\omega_{t+1}} - G_t [x_t^{\omega_t}]^k \tag{19}$$

All the sampled forward paths solved in the forward pass do not have to be solved in the backward pass. The SDDP can select a subset of sampled forward paths in order to compute the Benders' cuts. As the backward pass calculation takes more time than the forward pass calculation, the number of selected paths can be reduced to enhance the speed of convergence. After the completion of the backward pass for all the stages, a different set of forward paths are sampled from the scenario tree to imply the accumulated cuts, which are obtained from the backward pass of each iteration for each stage of the system.

V. Simulation Results And Analysis

The parameters used to represent the system characteristics for simulation analysis are listed in Table I. Forecasted solar power profile for a summer day and a typical household summer load profile are obtained from [19] and [20], and shown in Fig. 3. The summer TOU rate for residential customer varies over the day based on off-peak, partial peak and peak hours which is represented in Fig. 4. The solar energy production is considered to be a stochastic parameter since solar PV generation can often be considered as a random resource depending on climate and weather characteristics. Thus, the normally distributed random noise parameter $\rho_t^{\omega_t}$ in Equation (10) is sampled from the set $\mathcal{N}(0,1)$[kW] to generate different solar PV energy profiles. In a heuristic control strategy, the storage device gets charged when there is excess of solar generation and discharged with an increase in demand than the generated solar energy [22], [23]. The storage device will not be charged when there is no PV generation since it is not connected to the grid. The heuristic control strategy does not address the maximization of benefits to the homeowner. Our objective is to develop an improved storage energy control strategy to minimize the overall cost of the customer for a particular day based on SDDP. SDDP solves the multi-stage stochastic program designed for the problem and provides an optimal policy to charge and discharge of the storage.

TABLE I. System Parameters

Parameter	**Value**
PV panel installed power	3 kW
Battery capacity, (Q_b)	4 kWh
Efficiency, η_b^c	0.92
Initial *SOC*	20%
SOC_{min}	20%
SOC_{max}	80%
$P_{b_{ch}}^{min}$	0 kW
$P_{b_{ch}}^{max}$	3 kW
$P_{b_{disch}}^{min}$	0 kW
$P_{b_{disch}}^{max}$	3 kW
Battery type	Li-ion
Time period, t	15 min

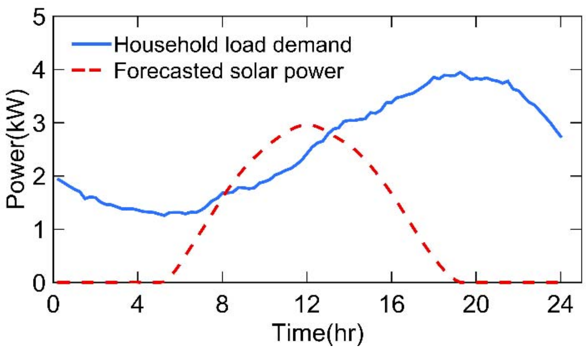


Fig. 3. Household and solar generation profile.

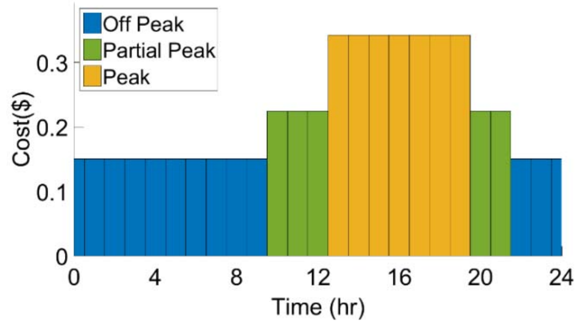


Fig. 4. TOU rate

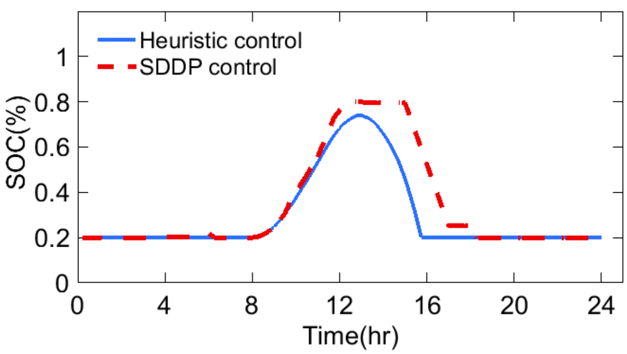


Fig. 5. SOC profiles for heuristic and SDDP based control of a scenario.

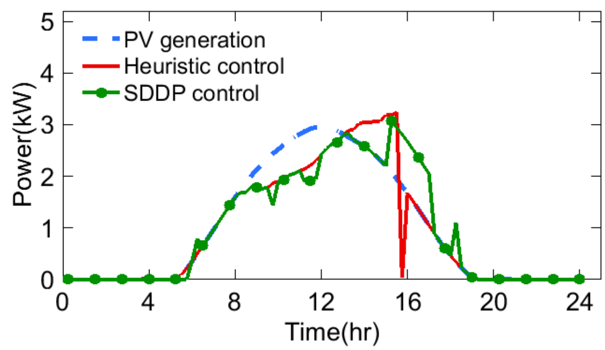


Fig. 6. PV usage profiles for heuristic and SDDP based control policies of a scenario.

The SOC profiles of the energy storage and PV energy usages by the load for a day with a solar generation scenario similar to the forecasted one for both strategies, SDDP based control and heuristic control, are provided in Fig. 5 and Fig. 6, respecively. For the SDDP based control strategy, it can be seen that when the TOU rate is lower and there is enough solar energy production only then the storage system starts charging. But all the available PV generation during the off-peak hours are not used to charge the storage though the TOU rate is the lowest rate during that time. The storage needs to have enough capacity to store excess PV generation, which occurs on partial peak period to avoid the waste of solar energy. Thus, SOC keeps increasing during off-peak and a little in the partial peak period. Due to the use of PV energy to charge the storage device, its usage for directly supplying the load sometimes becomes lower for SDDP based control when compared to the heuristic control strategy during off-peak and partial peak periods. In the SDDP based control, during peak hours the available generated PV and energy stored in the battery are used mostly to satisfy the household demand and reduce the overall costs for the customer. The SOC level decreases and PV energy usage (the combination of the available PV energy and the stored PV energy) by the load during peak period becomes higher than the heuristic control strategy for the same time interval. Therefore, TOU rate and PV generation will influence the SOC level of the storage device and PV usage by the load for SDDP based control strategy. The overall electricity purchases from the grid for both control strategies for this solar generation profile are given in Fig. 7.

Heuristic control strategy and deterministic dual dynamic programming control (DDDP) [16] are compared with SDDP based control in a policy evaluation procedure. The goal of such procedure is to assess how the different control policies perform using out-of-sample test cases representing the problem uncertainty; for further details the reader should refer to [23]. For DDDP, only a deterministic solar generation scenario is used to create an instance of model (1)-(9), instead of stochastic scenario tree (Fig. 2(a)) which is adopted when using the SDDP algorithm.

In order to perform policy evaluation analysis, 100 solar profiles are generated using normal distributed random values sampled from a $\mathcal{N}(0,1)$[kW] for $\rho_t^{\omega t}$ at each time stage and used as data in equation (10). These randomly generated PV profiles are used to find out the average usages of PV energy, peak hour savings and electricity purchase costs for different control policies. Fig. 8 shows that the average operational costs of 100 different solar profiles for a summer day is lower for SDDP than other control policies. Table II gives the idea of improvement of average PV usage by load, average peak hour savings and reduction of average electricity purchase cost using the SDDP control policy. If the system is controlled by DDDP, the average cost saving is 3.6% per day compared to the average cost of heuristic control. The control strategy with SDDP algorithm reduces the total cost by 9.2% per day compared to 5.7% per day with the DDDP algorithm. Therefore, using the SDDP control strategy for the PV-storage hybrid unit, an average of $25.8 per month can be saved. The peak hour saving is not only beneficial to the homeowner but also helps utility companies to reduce large load variations during peak periods.

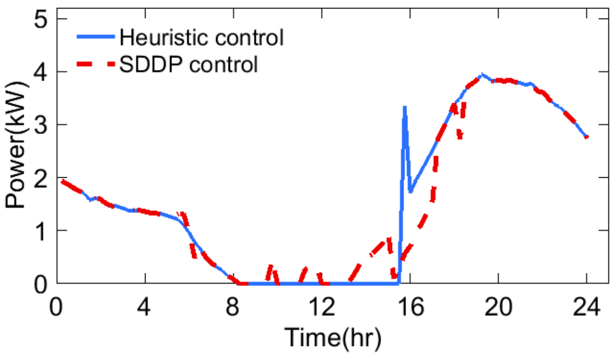


Fig 7. Electricity demands from grid for heuristic and SDDP based control policy for a scenario.

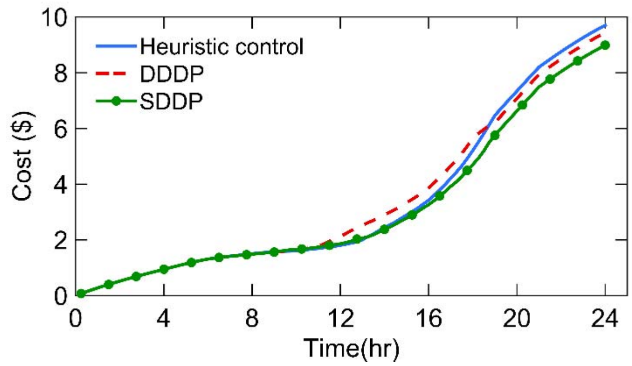


Fig. 8. Policy evaluation of overall cost for heuristic control, DDDP and SDDP.

TABLE II. COMPARISON OF DIFFERENT CONTROL STRATEGIES

Control Strategy	**PV Usage per day (%)**	**Peak hour Saving per day (%)**	**Electricity purchase cost per day($)**
Heuristic Control	87.4	29.6	9.8
DDDP	88.1	48.1	9.4
SDDP	97.3	48.7	8.9

VI. CONCLUSION

The research presented in this paper shows the importance of considering proper models and algorithms that account for uncertainty when considering an integrated solar PV and energy storage hybrid unit in a household. SDDP helps the decision maker to control the charge and discharge profiles of the energy storage component and minimize the overall cost for the customer. As expected, the strategy obtained with the SDDP also provides more reliable results than the deterministic control strategy provided by the DDDP. The results from the SDDP based control strategy show that by controlling the energy storage and solar PV power flow in the system, it is possible to optimally purchase electricity from the grid and minimize the customer cost and simultaneously satisfy the household demand. The methodology discussed in this study can be further expanded to coordinate control among many different storage units and renewable sources with uncertainties.

VI. ACKNOWLEDGEMENT

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REFERENCES

[1] E. Ela, V. Diakov, E. Ibanez, and M. Heaney, “Impacts of Variability and Uncertainty in Solar Photovoltaic Generation at Multiple Timescales,” *National Renewable Energy Laboratory.*

[2] F. Hafiz, P. Fajri and I. Husain, “Load Regulation of a Smart Household with PV-Storage and Electric Vehicle by Dynamic Programming Successive Algorithm Technique,” *in Proc. Power and Energy System General Meeting (PESGM)*, 2016.

[3] A. K. Srivastava, A. A. Kumar and N. N. Schulz, “Impact of Distributed Generations with Energy Storage Devices on the Electric Grid,” *IEEE Trans. Syst*., vol. 6, no. 1, pp. 110–117, Mar. 2012.

[4] J. Marcos, O. Storkel, L. Marroyo, M. Garcia and E. Lorenzo, “Storage requirements for PV power ramp-rate control,” *Elsevier, Solar Energy.*

[5] P. M. van de Ven, N. Hegde, L. Massoulié and Theodoros Salonidis, “Optimal Control of End-User Energy Storage,” *IEEE Trans. On Smart Grid*, vol. 4, no. 2, pp. 789–797, June 2013.

[6] S. Hambridge, A. Q. Huang and R. Yu, “Solid State Transformer (SST) as an Energy Router: Economic Dispatch Based Energy Routing Strategy,” in *Proc. IEEE Energy Conv. Congr. Expo. (ECCE)*, pp. 2356–2360, Sept. 2015.

[7] S. Hambridge, A. Q. Huang and N. Lu, “Proposing a Frequency Based Real-Time Energy Market and Economic Dispatch Strategy,” *in Proc. Power and Energy System General Meeting (PESGM)*, 2016.

[8] S. R. Deeba, R. Sharma, T. K. Saha and A. Thomas, “Investigation of Voltage Performance of an LV Distribution Network for Improving Rooftop Photovoltaic Uptake in Australia,” *Proc. Power and Energy System General Meeting (PESGM)*, 2016.

[9] Y. Wang, X. Lin and M. Pedram, “A Near-Optimal Model-Based Control Algorithm for Households Equipped With Residential Photovoltaic Power Generation and Energy Storage Systems,” *IEEE Trans. On Sustainable Energy*, vol. 7, no. 1, pp. 77–86, Jan. 2016.

[10] Y. Wang, Xue Lin and M. Pedram, “Adaptive Control for Energy Storage Systems in Households with Photovoltaic Modules,” *IEEE Trans. On Sustainable Energy*, vol. 5, no. 2, pp. 992–1001, Mar. 2014.

[11] L. Bo and M. Shahidehpour, “Short-term scheduling of battery in a grid connected PV/battery system,” *IEEE Trans. Power Syst*., vol. 20, no. 2, pp. 1053–1061, May 2005.

[12] H. Beltran, E. Pérez, N. Aparicio and P. Rodríguez, “Daily Solar Energy Estimation for Minimizing Energy Storage Requirements in PV Power Plants,” *IEEE Trans. On Sustainable Energy*, vol. 4, no. 2, pp., April 2013.

[13] Dimitri P. Bertsekas. *Dynamic Programming and Optimal Control*, 3rd edition. Athena Scientific, 2005.

[14] F. Hafiz, P. Fajri and I. Husain, “Effect of Brake Power Distribution on Dynamic Programming Technique in Plug-in Series Hybrid Electric Vehicle Control Strategy,” *Proc. IEEE Energy Conversion Congress & Exposition (ECCE)*, pp. 100–105, Sept. 2015.

[15] A. R. de Queiroz and D. P. Morton, “Sharing cuts under aggregated forecasts when decomposing multi-stage stochastic programs,” *Elsevier, Operation Research Letters,* 2013.

[16] M.V.F. Pereira and L.M.V.G. Pinto, “Multi-stage stochastic optimization applied to energy planning,” *Math Programming* 52, pp. 359-375, 1991

[17] A. R. de Queiroz, "Stochastic hydro-thermal scheduling optimization: An overview," *Elsevier, Renewable and Sustainable Energy Reviews, 2016*.

[18] Averill M. Law, "Simulation Modeling and Analysis", *5th Edition,* McGraw-Hill Series in Industrial Engineering and Management, 2014

[19] Solar Resource & Meteorological Assessment Project (SOLRMAP) [Online] http://www.nrel.gov/midc/lmu/.

[20] Historical Residential Load Data [Online]. https://www.pseg.com/business/energy_choice/third_party/historical.jsp#anchor16

[21] N. B. Amor, O. Kanoun and N. Derbel1, "Logically controlled energy management circuit," *Proc. IEEE International Muti Conference on Systems, Signals and Devices*, March 2012.

[22] Jayalakshmi N.S.I, D.N. Gaonkar, Adarsh S. and Sunil S., "A Control Strategy for Power Management in a PV-Battery Hybrid System with MPPT," *Proc. IEEE International Conference on Power Electronics. Intelligent Control and Energy Systems (ICPEICES-2016)*, July 2016.

[23] A. R. de Queiroz, "A Sampling-based Decomposition Algorithm with Application to Hydrothermal Scheduling: Cut Formation and Solution Quality," PhD Thesis, University of Texas Austin,Texas, 2011.

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