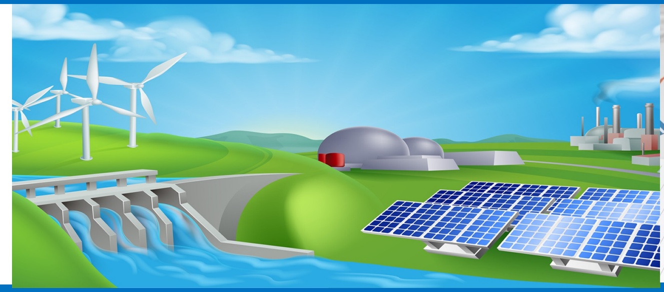



ECONOMICS OF MODERN POWER SYSTEMS

M7 – Economic Dispatch with Renewables

Prof. Luana Medeiros Marangon Lima, Ph.D.

Learning Goals

- Understand the economic dispatch problem definition
- Understand the challenged posed by renewables and storage
  - Present decisions impact future cost
  - Add another level of uncertainty – generation
- Discuss ways to handle uncertainty in decision problems
- Understand the hydro-thermal scheduling problem

Economic Dispatch Problem Definition

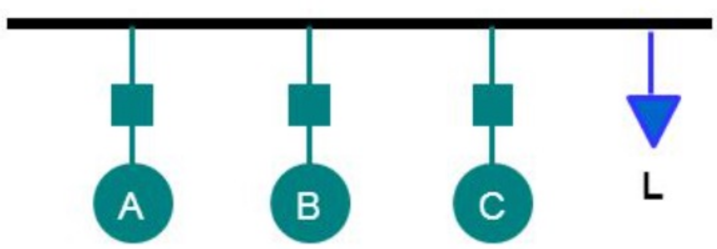
Context

- In power systems
  - Power plants are not located at the same distance from the center loads
  - Their fuel costs are different
  - Generation capacity is more than total load demand and losses

Thus, there are many options for scheduling generation!

Economic Dispatch Definition

Economic dispatch is the problem of finding the optimal electricity generation targets for all power plants in order to meet the system load at the lowest possible cost subject to generation, transmission and operational constraints



What share of the load should each generating unit produce?

Economic Dispatch Definition

- Economic Dispatch Components
  - Planning for tomorrow's dispatch (Day-ahead)
  - Dispatching the power system today (Real-time)

- Variations of the problem
  1. Thermal units only
  2. Hydro and thermal units
  3. Hydro-thermal with wind and solar

Planning for tomorrow's dispatch

Scheduling generating units for each hour of the next day's based on

- Forecasted load for the next day
- Generating unit's operational limits
  - Maximum and minimum generation levels
  - Ramp rate (how quickly generator's output can be changed)
- Generating unit's characteristics
  - Cost of generating (heat rate, fuel or non-fuel)
  - Start-up costs
  - Environmental compliance

Dispatching the power system today

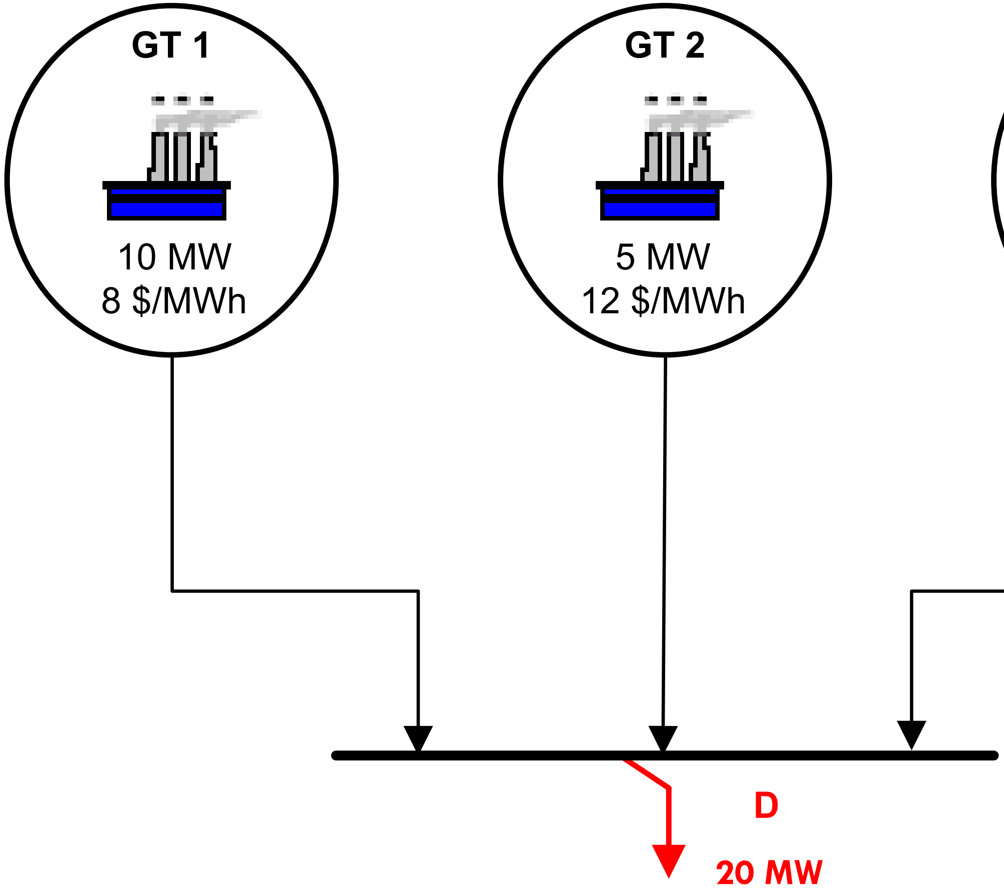
Monitoring load and generation to ensure balance of supply and demand (plus losses)

- Actual load
- Maintaining required system frequency
- Keeping voltage levels within reliability ranges
- Taking corrective action
  - Limiting new power flow schedules
  - Curtailing existing power flow schedules
  - Changing dispatch
  - Load shedding

Let's see an example…

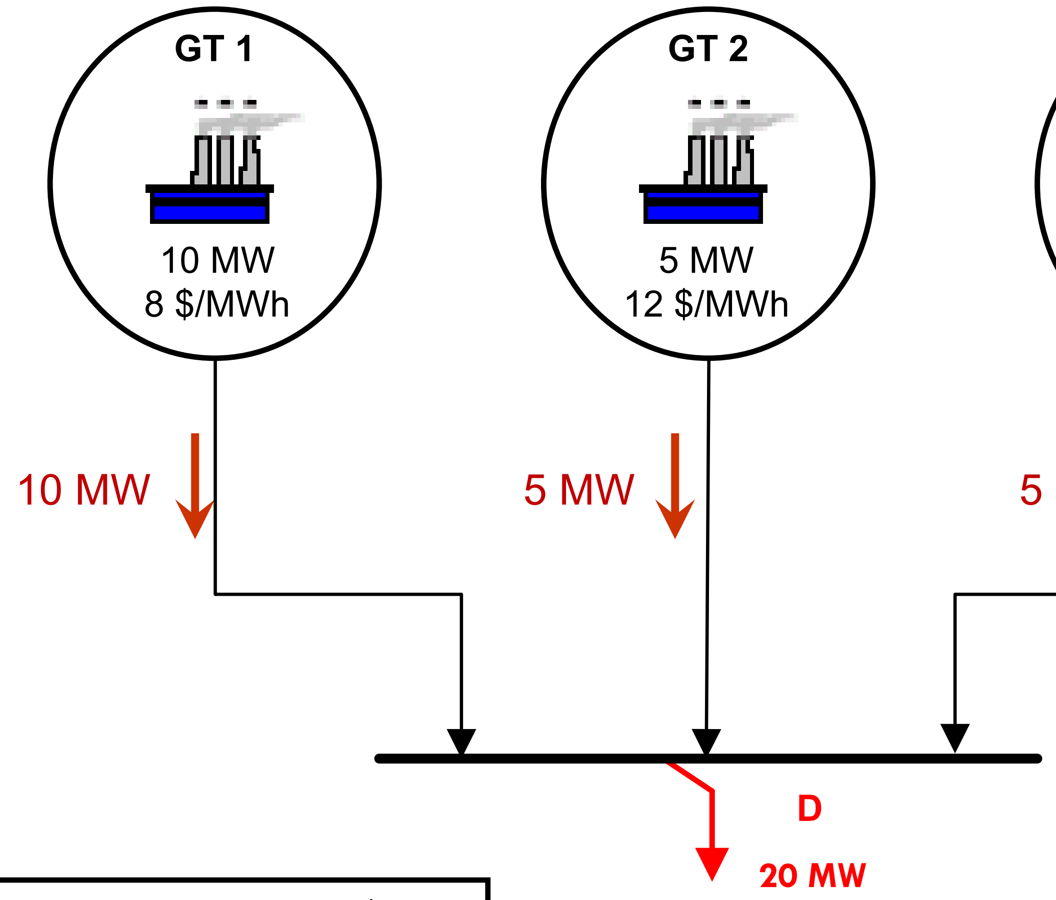
- A power supplier owns three thermal units
  - Cost of each thermal unit ($/MW) as a constant
  - Each plant has a maximum generation capacity
- Need to meet demand
- Assumptions
  - No transmission constraints, no losses
  - No operational constraints other than maximum generation
  - Only one time step
- What is the optimal dispatch of these three units?

Example 1: Thermal Scheduling Problem



How would you dispatch these plants?

Example 1: Thermal Scheduling Problem



How would you dispatch these plants?

Cost = 10*8 + 5*12 + 5*15 = $215

Merit order

Example 1: Thermal Scheduling Problem

- Our problem formulation would be

$$
\begin{aligned}
\min_{P_1,P_2,P_3} \quad & 8P_1 + 12P_2 + 15P_3 \\
s.t. \quad & P_1 + P_2 + P_3 = 20 \\
& P_1 \leq 10 \\
& P_2 \leq 5 \\
& P_3 \leq 20
\end{aligned}
$$

- Should we add more constraints?

What constraints should be added?

- We already have generator capacity, but it could go both ways

  $$P_i^{min} \leq P_i \leq P_i^{max}$$

- Generator ramp limits

  $$P_i - P_{i0} \leq UR_i$$ **(Ramping Up limit in MW)**

  $$P_{i0} - P_i \leq DR_i$$ **(Ramping down limit in MW)**

  where P_{i0} is the initial state, i.e., how much plant i is already producing

- Transmission lines losses
  - More constraints and change on RHS of meet demand constraint

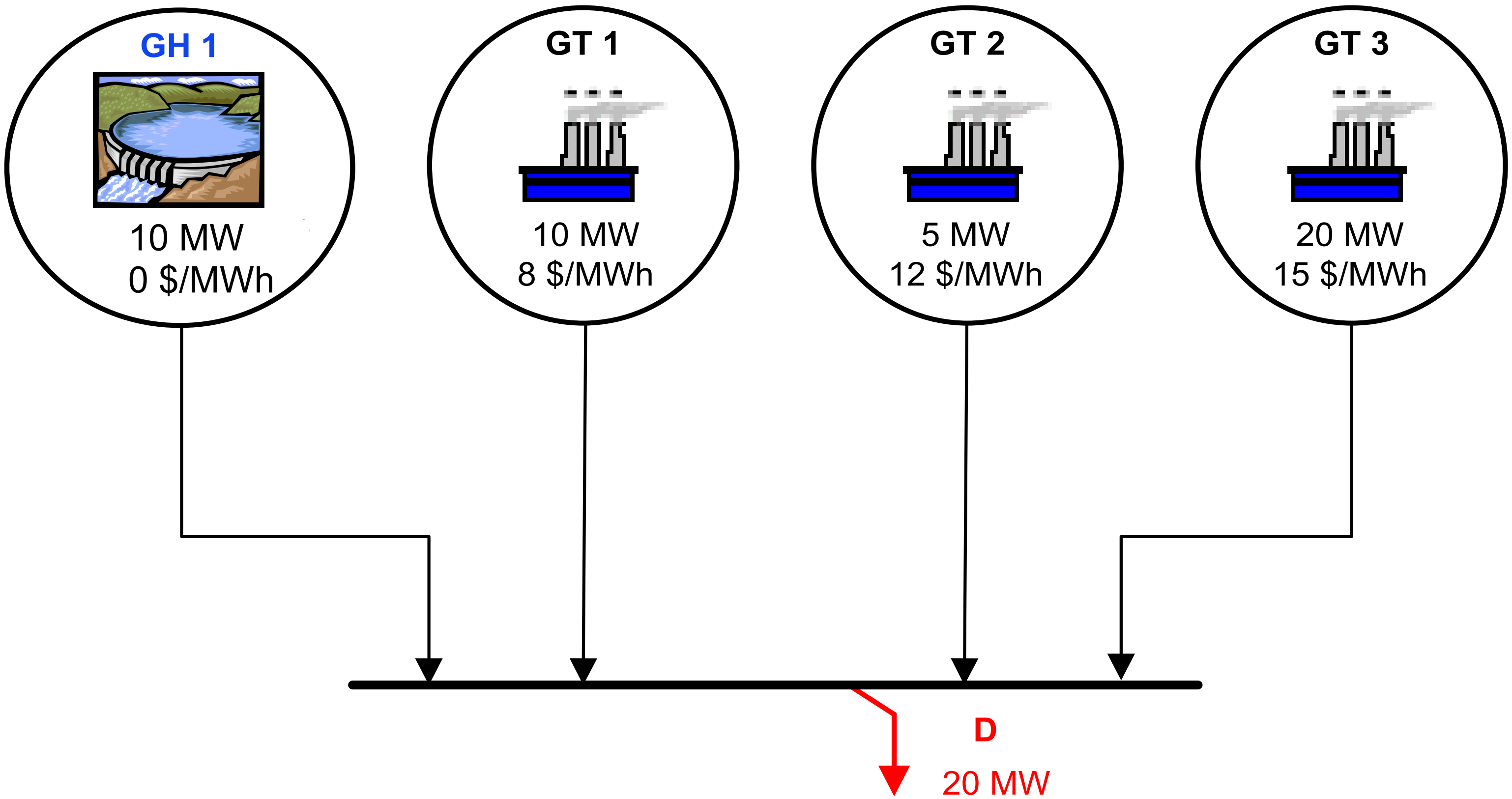
Now let's look at another example…

- We will include one hydro plant with enough water in its reservoir to generate up to 10 MW

- Hydro plants have low operational cost because there is no fuel cost, so we consider no cost at all to generate from the hydro plant

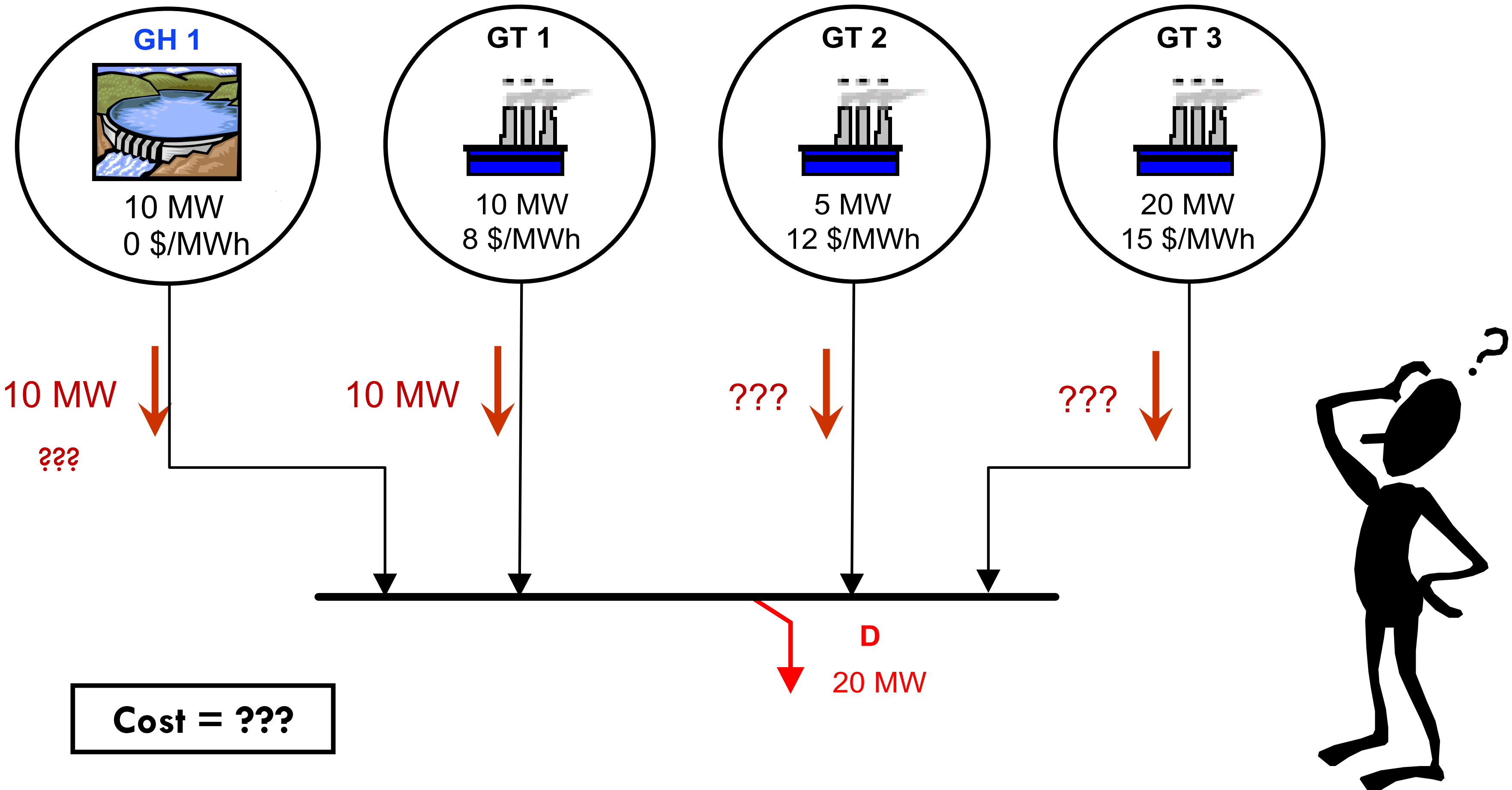
We are now at the hydro-thermal economic dispatch

Example 2: Hydrothermal Scheduling



- Try to come up with the best solution
- Now think about the next time stage. Is your solution still optimal??

Example 2: Hydrothermal Scheduling (2 stages)



$Cost_{t1} = 10*0 + 10*8 = \$80$

$Cost_{t2} = 10*8 + 5*12 + 5*15 = \$215$

$Cost_{t1} = 5*0 + 10*8 + 5*12 = \$140$

$Cost_{t2} = 5*0 + 10*8 + 5*12 = \$140$

What constraints should be added?

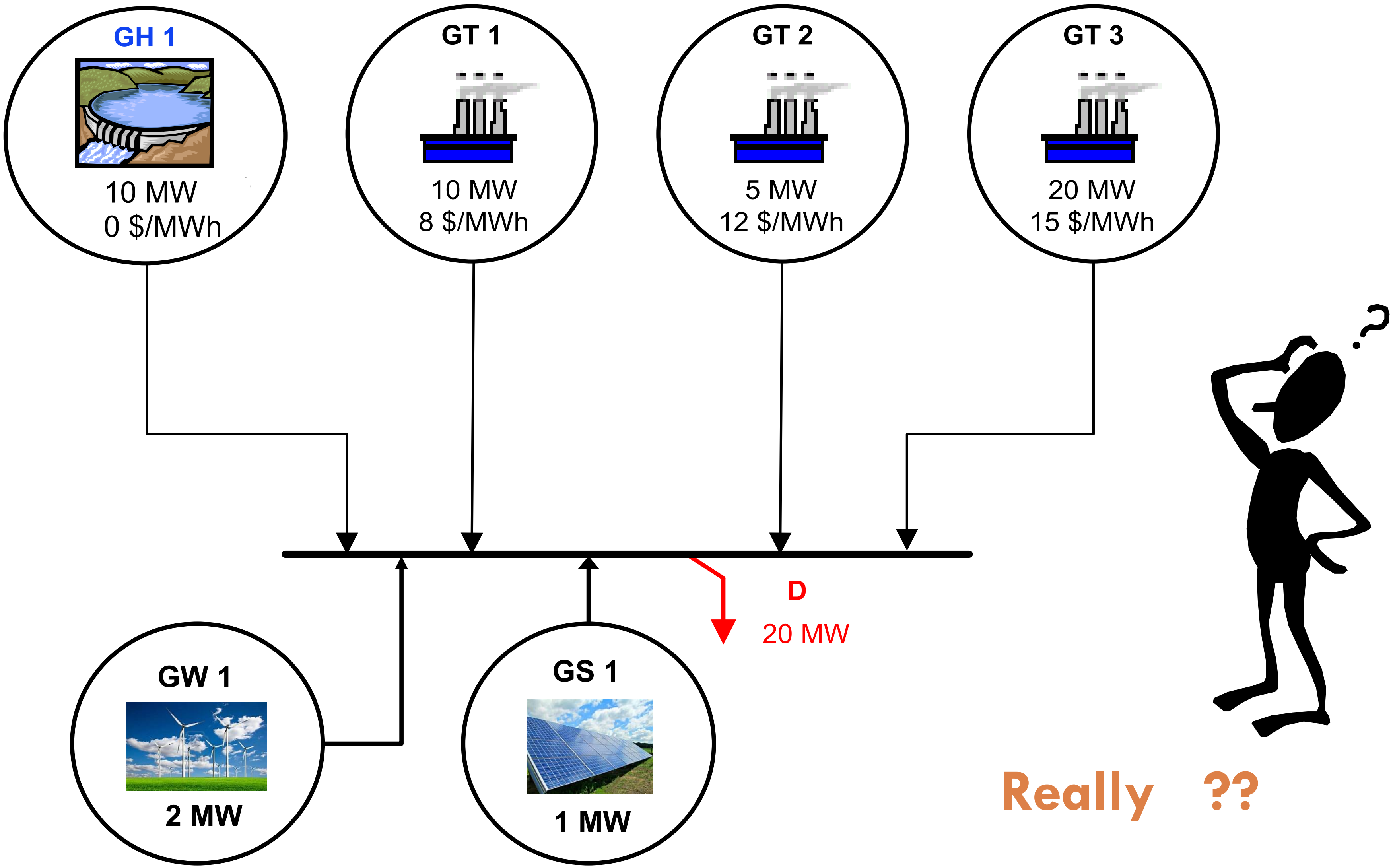
- Hydro power generation as a function of water discharge rate and storage volume
  - This will transform inflow volume $[m^3/s]$ into electricity in $[MW]$
- Power generation limits
- Water balance constraint
- Reservoir storage volume limits
- Water discharge rate limits
- Note that these may differ for hydro plant with reservoir and run-of-river

We will go over this constraints formulation later…

Now let’s make it more complicated!

- Imagine we add a solar and a wind farm to this system…
- That’s the third variation of economic dispatch we will cover
  - Hydro-thermal system plus renewables (W & S)
- Similar to what we did with hydro, we will consider zero cost to generate electricity with solar and wind

Example 3: Hydrothermal with Solar and Wind



What constraints should be added?

- Wind power generation as a function of wind speed
- Solar power generation as a function of solar radiation
- If these plants have storage we can model then similarly as to hydro with reservoir
  - Remember the state of charge equations for the residential storage management
- If they don’t have storage we can model them as run-of-river plants

Main Takeaway for now

- When we have renewables and storage capacity, **our current decision will impact future cost**

- So we need to look ahead to make better decisions today

- But if we look ahead we find…

Uncertainty !!

Uncertainty Modeling

Sources of Uncertainty in ED

- Generation availability
- Load requirements
- Fuel prices
- Forces of nature such as extreme weather

Usually economic dispatch consider **forecasted values** for the uncertain parameters and solve economic dispatch as a **determinist problem**.

But, there are other ways to deal with uncertainty.

They become more and more relevant when we **introduce more renewables** to the economic dispatch problem.

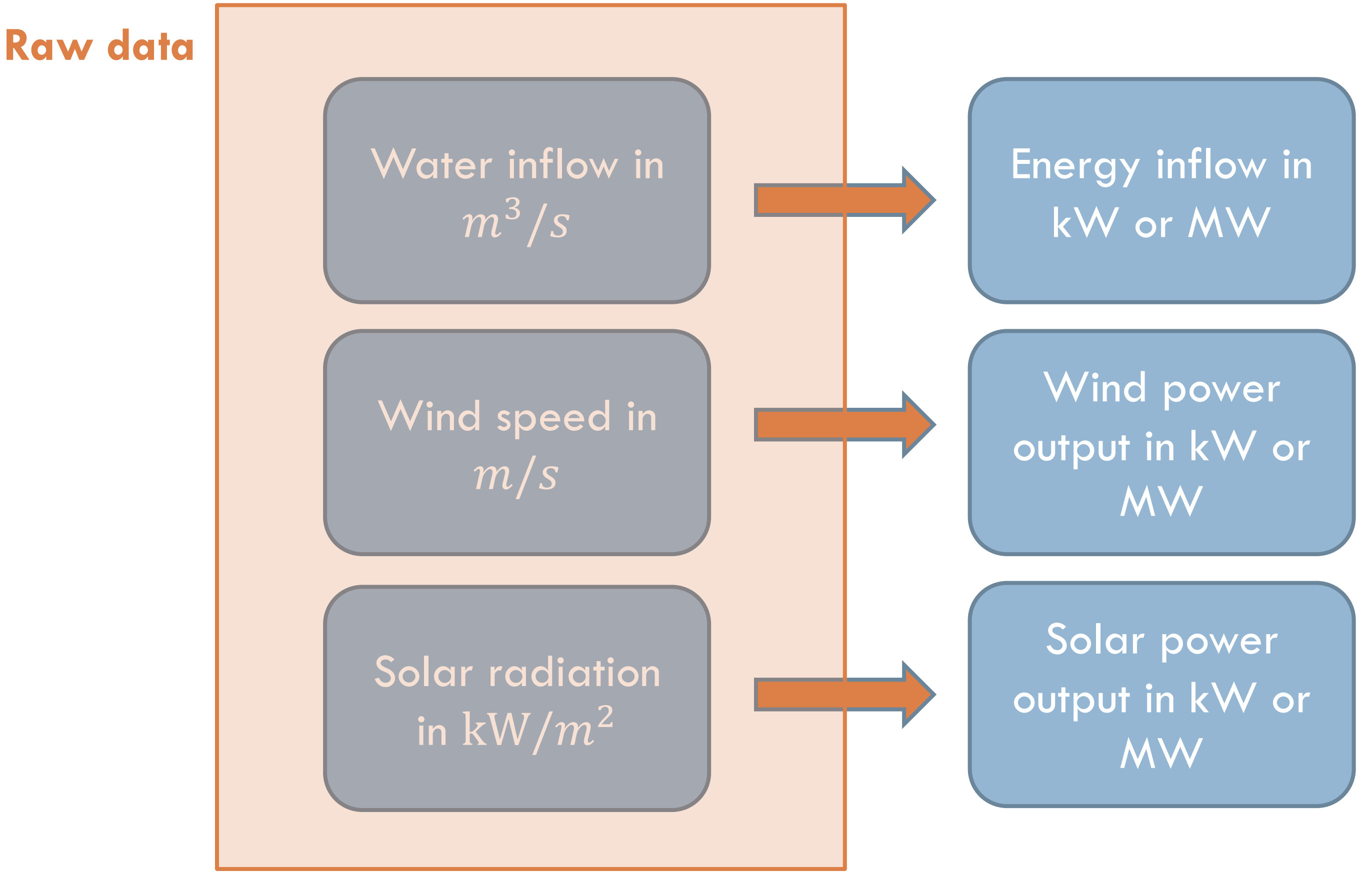
Uncertainty Modeling Techniques

Common methods to deal with uncertainty parameters in the optimization problem:

- **Probabilistic approach**: assumes that uncertain input parameters are random variables with a known probability density function (pdf)
- **Robust optimization**: uses uncertainty sets to describe uncertain parameters. Optimal solution will be optimal even for the worst case scenario
- **Interval analysis**: uncertain parameter takes values from a known interval. Finds the bounds of output variables

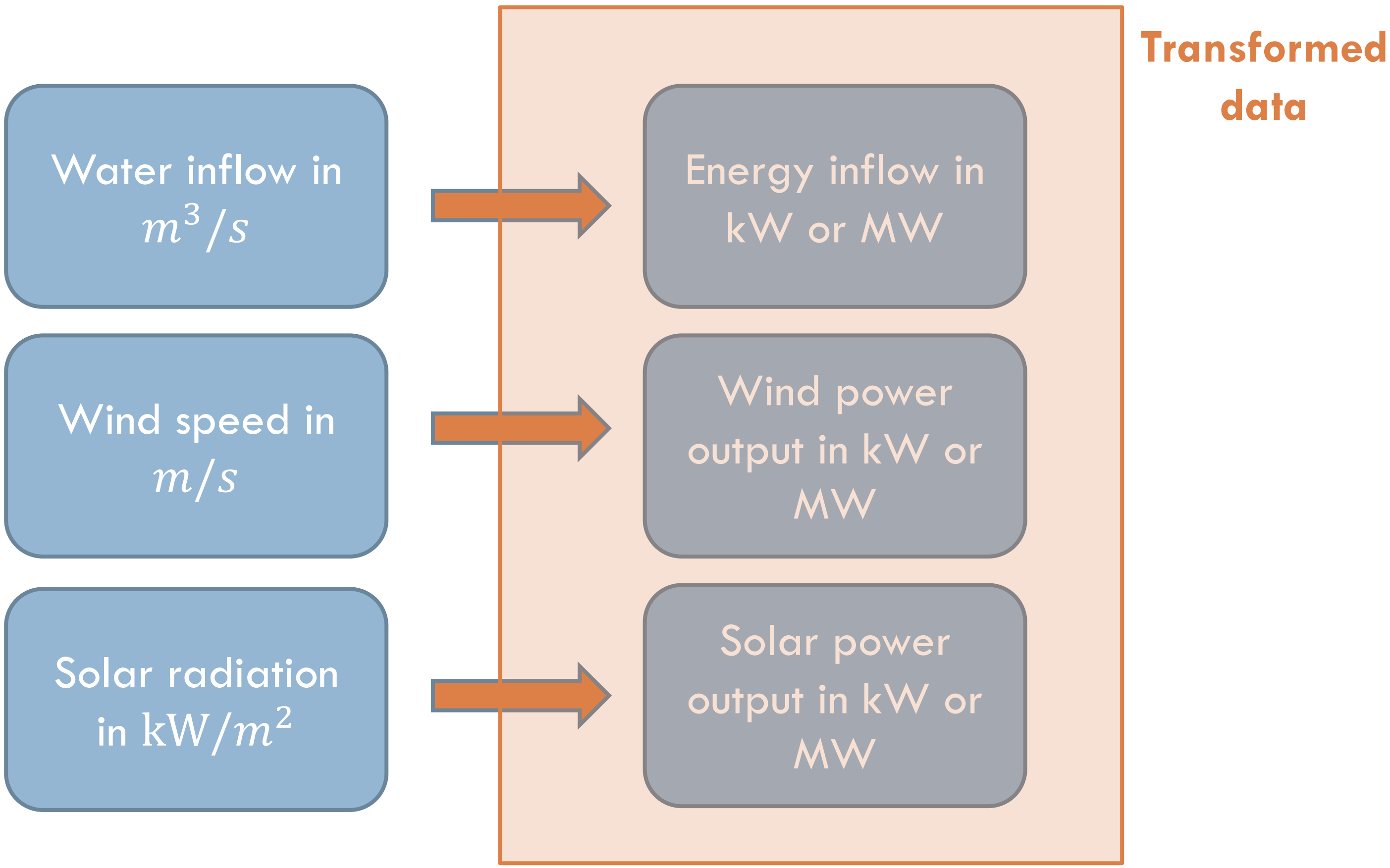
How do we incorporate uncertainty?

- Approach 1- Consider uncertainty on natural resources



How do we incorporate uncertainty?

- Approach 2 - Consider uncertainty on energy output



How do we incorporate uncertainty?

- Which one is the better approach?
- From a forecasting perspective, ideally we should model the raw data
- From a optimization perspective, having the forecasted energy output eliminates the need to incorporate the transformation equations as constraints

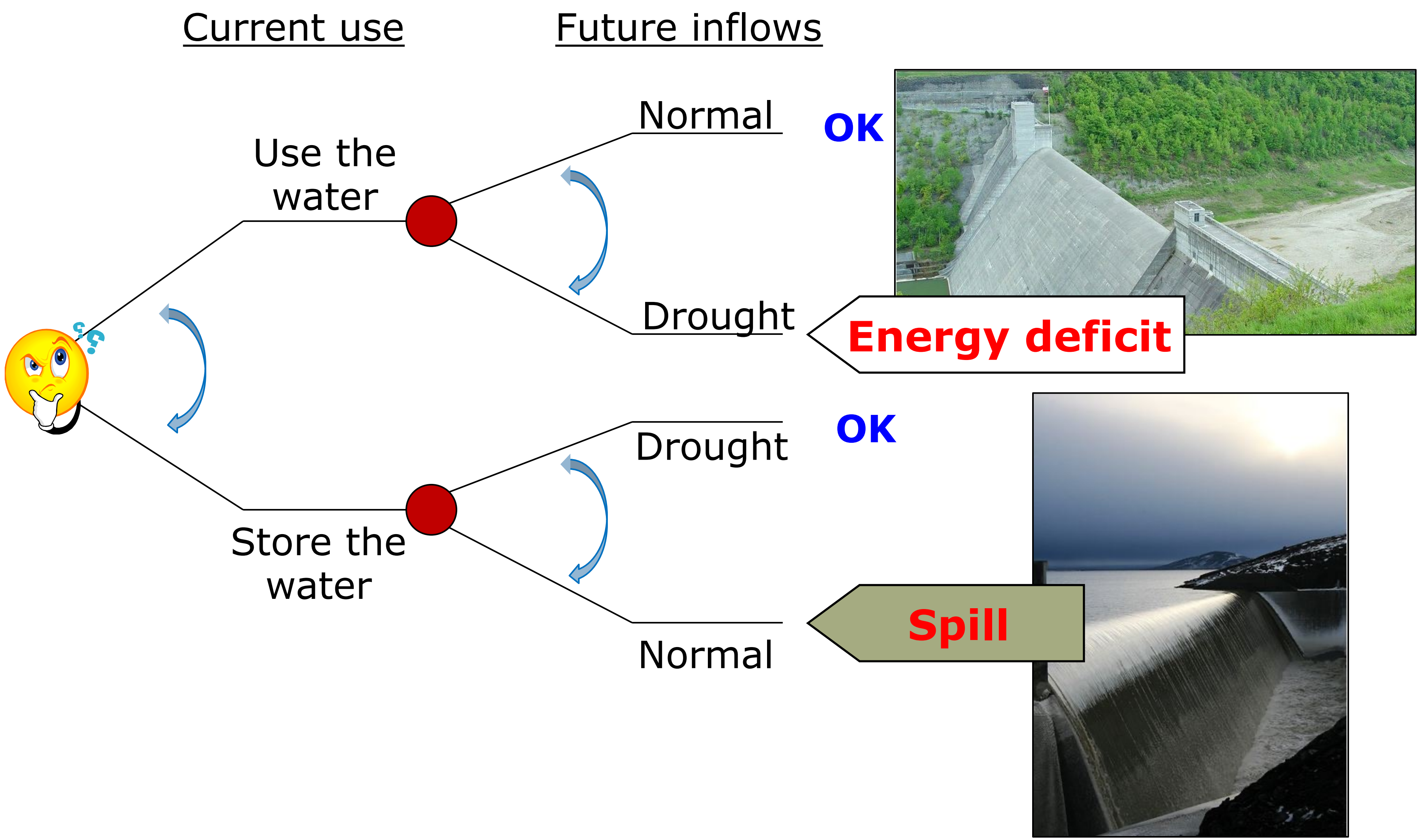
Closer Look at Hydrothermal Scheduling Problem

Hydrothermal Scheduling Problem

- Find the sequence of hydro releases and thermal plant dispatches for a planning horizon in order to match system demand
  - Resource management
  - Input variable forecasting
  - Operational aspects
- Basic economic criterion is the same
  - Minimize operational costs (present + future)

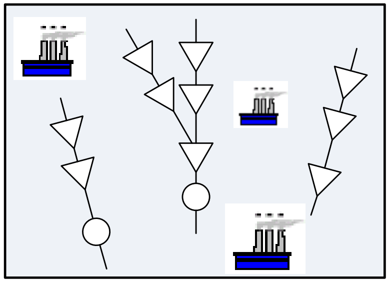
But now we worry about future cost!

Decision Tree

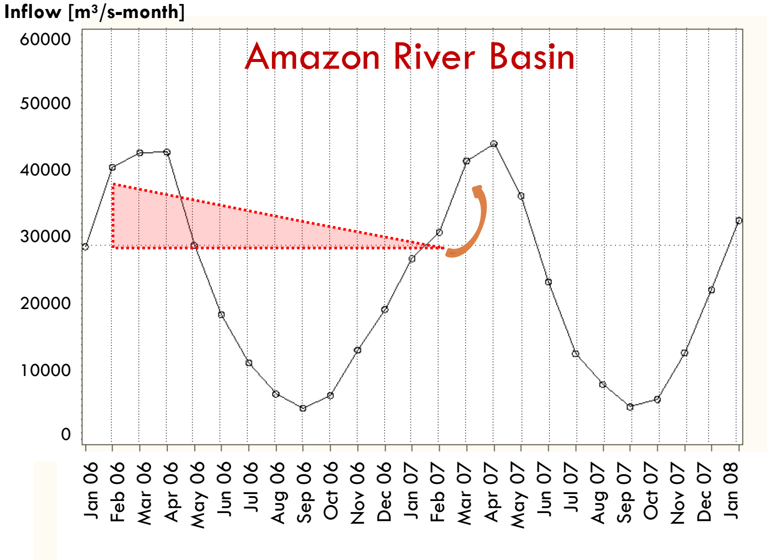


Problem Characteristics

- Temporal coupling: relation between a decision today and the consequences in the future



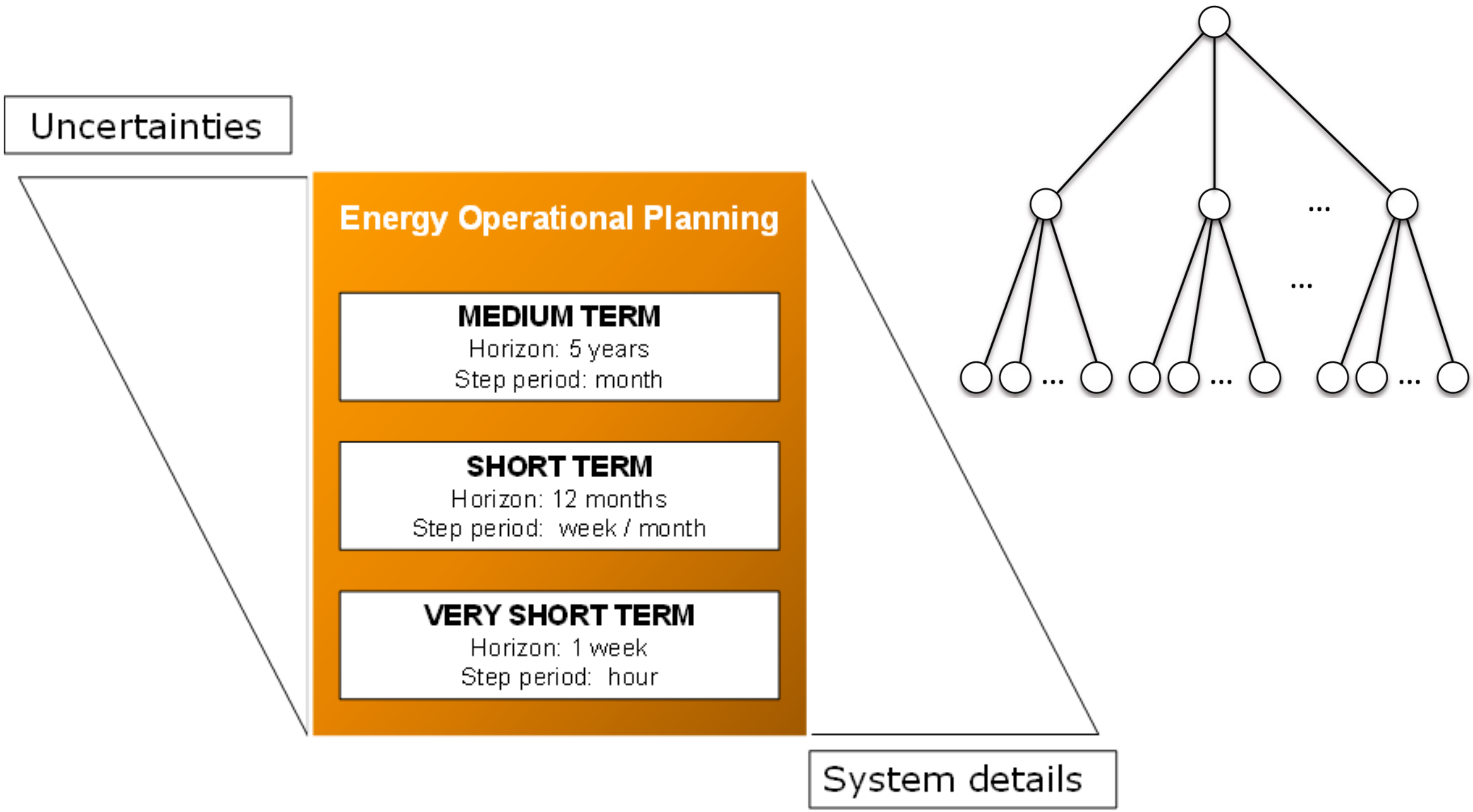
- Spatial coupling: the decision for one plant can influence other plants in the cascade



- Stochastic nature of the water inflows

- Large problem: multi-period with many decision variables and constraints

Problem Characteristics (*cont'd*)



Hydrothermal Scheduling Problem Formulation

Example 1- Thermal only system

- LP formulation

$$
\begin{aligned}
\min_{P_1,P_2,P_3} \quad & 8P_1 + 12P_2 + 15P_3 \\
s.t. \quad & P_1 + P_2 + P_3 = 20 \\
& P_1 \le 10 \\
& P_2 \le 5 \\
& P_3 \le 20 \\
& P_1, P_2, P_3 \ \ge 0
\end{aligned}
$$

Example 2 – Hydrothermal system

Stage 1

$$\min_{P} \; z = \; 8P_{G1} + 12P_{G2} + 15P_{G3}$$

$$\begin{aligned} s.t.\quad & P_{G1} + P_{G2} + P_{G3} + \boldsymbol{P_H} = 20 \\ & P_{G1} \le 10 \\ & P_{G2} \le 5 \\ & P_{G3} \le 20 \\ & \boldsymbol{P_H \le 10} \\ & \text{All } P \ge 0 \end{aligned}$$

Stage 2

$$\min_{P} \; z = \; 8P_{G1} + 12P_{G2} + 15P_{G3}$$

$$\begin{aligned} s.t.\quad & P_{G1} + P_{G2} + P_{G3} + P_H = 20 \\ & P_{G1} \le 10 \\ & P_{G2} \le 5 \\ & P_{G3} \le 20 \\ & P_H \le ? \\ & \text{All } P \ge 0 \end{aligned}$$

How to optimize the two stages simultaneously?

- Could use the multi-period LP
- Introduce one more index to the decision variables: t

$$\min_{P} z = \; 8(P_{G1}^1 + P_{G1}^2) + 12(P_{G2}^1 + P_{G2}^2) + 15(P_{G3}^1 + P_{G3}^2)$$

$$\begin{aligned}
s.t. \quad & P_{G1}^1 + P_{G2}^1 + P_{G3}^1 + P_H^1 = 20 \\
& P_{G1}^2 + P_{G2}^2 + P_{G3}^2 + P_H^2 = 20 \\
& P_{G1}^1, P_{G1}^2 \le 10 \\
& P_{G2}^1, P_{G2}^2 \le 5 \\
& P_{G1}^1, P_{G1}^2 \le 20 \\
& \boldsymbol{P_H^1 + P_H^2 \le 10} \\
& \text{All } P \ge 0
\end{aligned}$$

How to implement this model in R?
Look at file *EconDispatch.R* on Sakai

What if we have more than 2 stages?

- General form: Multi-stage problem

$$\min_{P} z = \sum_{t} \sum_{i} C_{Gi} P_{Gi}^{t}$$

$$s.t. \quad \sum_{i} P_{Gi}^{t} + P_{H}^{t} = D \qquad \forall t$$

$$P_{Gi}^{t} \leq \overline{GT_i} \qquad \forall t, i$$

$$\sum_{t} P_{H}^{t} \leq \overline{GH}$$

$$P_{Gi}^{t}, P_{H}^{t} \geq 0 \quad \forall t, i$$

Note that we are still considering everything deterministic. And there is no incoming inflow from one stage to the other

If you want it more to be realistic, need to incorporate water balance equation and transformation of inflow into energy

(more on that later)

Example 3: Hydro-thermal + wind + solar

- Let's look at the multistage formulation

$$\min_{P} \ z = \sum_{t} \sum_{i} C_{Gi} P_{Gi}^{t}$$

$$s.t. \quad \sum_{i} P_{Gi}^{t} + P_{H}^{t} + P_{S}^{t} + P_{W}^{t} = D \qquad \forall t$$

$$P_{Gi}^{t} \leq \overline{GT_i} \qquad \forall t, i$$

$$P_{W}^{t} \leq \overline{GW}$$

$$P_{S}^{t} \leq \overline{GS}$$

$$\sum_{t} P_{H}^{t} \leq \overline{GH}$$

$$P_{Gi}^{t}, P_{H}^{t}, P_{W}^{t}, P_{S}^{t} \geq 0 \qquad \forall t, i$$

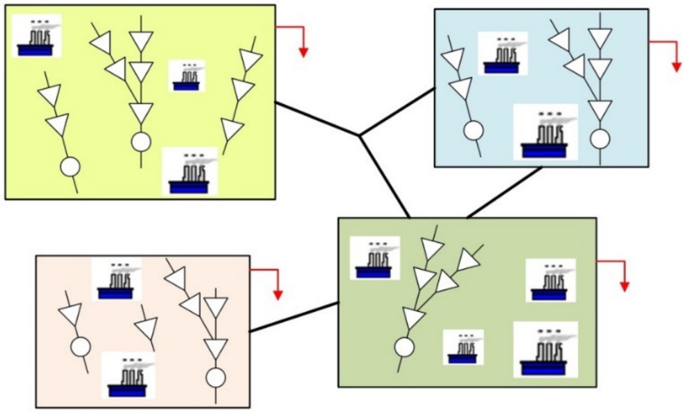
$$\boldsymbol{P_{S}^{t} + P_{W}^{t} \leq 0.3D}$$

Here we incorporated wind and sun in a very simplistic approach. Because we know that we don't have the same 2MW of wind and 1MW of sun at all time stages

Some markets are limiting the penetration from sun and wind at a percentage (0.3, 0.5, etc) of the demand level, just to maintain reliability of the system!

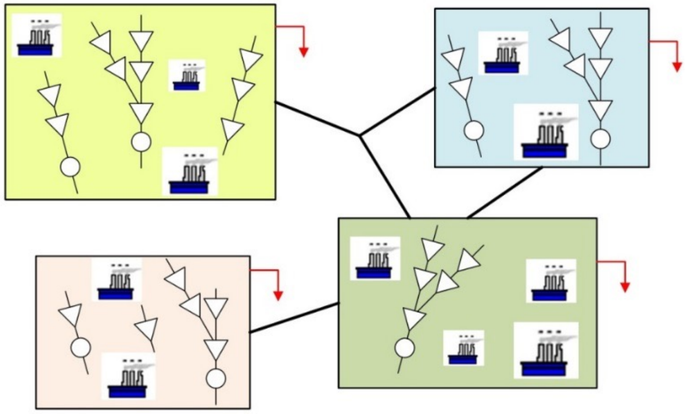
General HTSP: Variables & Parameters

- Sets:
  - Set of **hydro power plants:** $i \in I$
  - Set of **thermal power plants:** $\ell \in L$
  - Set of time stages: $t \in T$
  - Set of electrical **subsystems:** $r \in R$
  - Set of **curtailment levels:** $k \in K$
  - Subset of upstream reservoirs: M_i
- Decision variables:
  - **Hydro generation:** GH_i^t
  - **Spilled volumes:** S_i^t
  - **Water volume storage:** x_i^t
  - **Thermal generation:** GT_ℓ^t
  - **Energy transfers between regions:** $F_{r\,r'}^t$
  - **Load curtailment:** GD_k^t



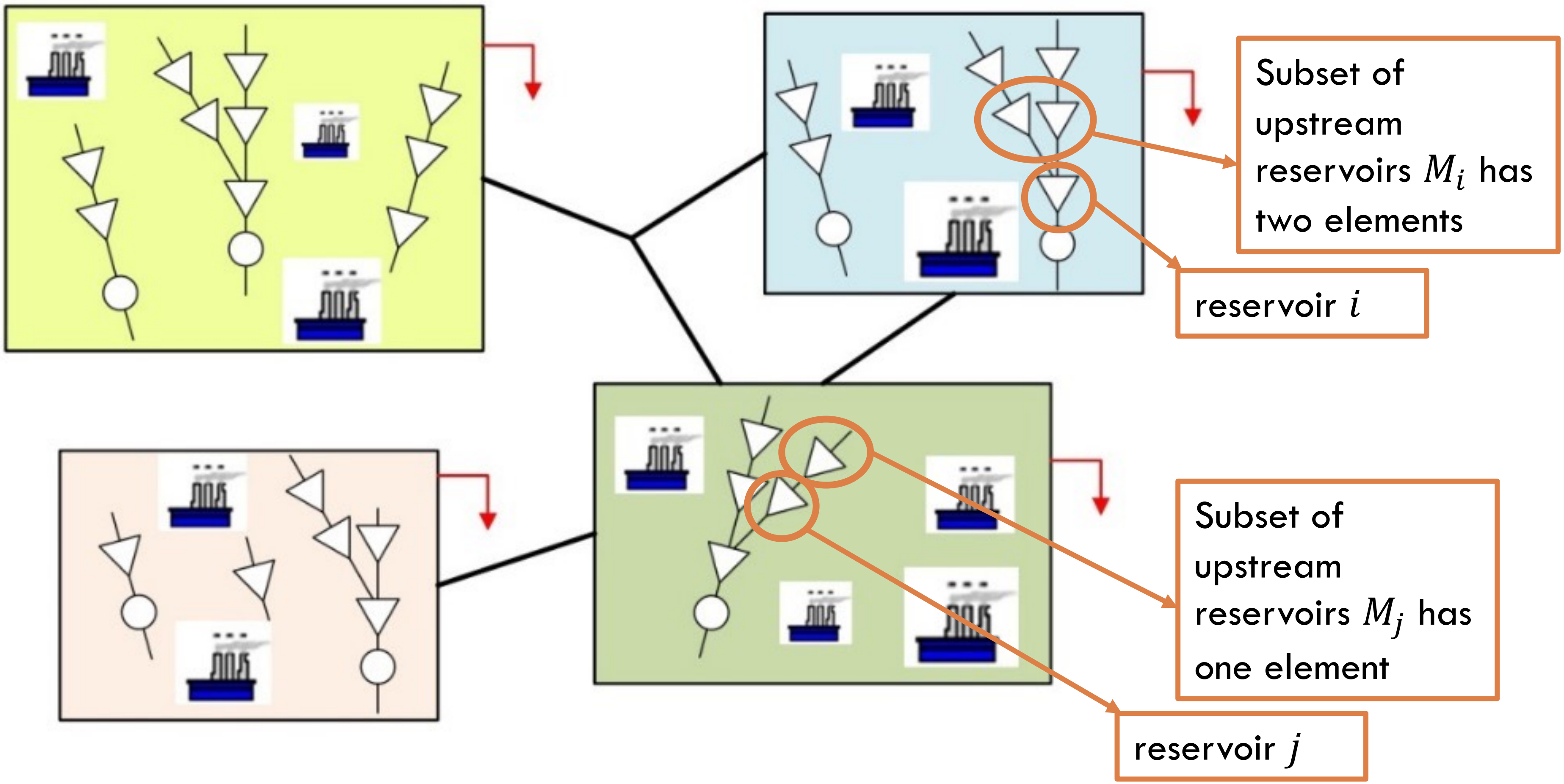
- Parameters:
  - Future water inflows: $b_t, b_{t+1}, \dots, b_T$ (uncertainty)
  - Electricity demand at region r: D_{tr}
  - Bound limits: $\underline{x}$, $\overline{x}$

Aggregated Reservoir Scheme



System aggregated in four regions

Examples: Subset of upstream reservoirs



HTSP Model Formulation for Stage-t

$$h_t(x^{t-1}, b_t^{\omega}) = \min \overbrace{\sum_{\ell \in L} c_\ell^t GT_\ell^t + \sum_{k \in K} u_k^t GD_k^t}^{\text{Present Cost}} + \overbrace{\frac{1}{(1+\beta)} \mathbb{E}_{b_{t+1}|b_1,\dots,b_t} h_{t+1}(x^t, b_{t+1})}^{\text{Expected Future Cost}}$$

Cost of stage t depends on decisions at previous states and inflow at stage t for scenario ω

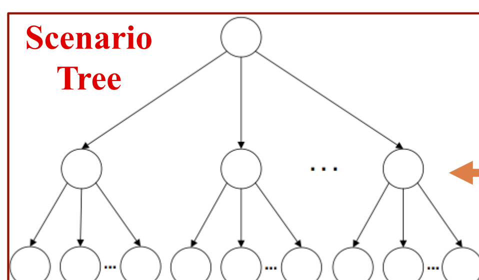
Cost of dispatching thermal plants

Cost of curtailing load

Discount rate

Expected value of cost at t+1, why expected ??

Because I have a number of possible realizations for inflow b_{t+1}



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Water Balance s.t. $x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} (GH_j^t + S_j^t) = x_i^{t-1} + b_{i,t}^{\omega} \quad \forall i \in I$

Water volume of reservoir left at stage t

Hydro generation at t

Water spilled at t

Set of reservoirs upstream on the cascade

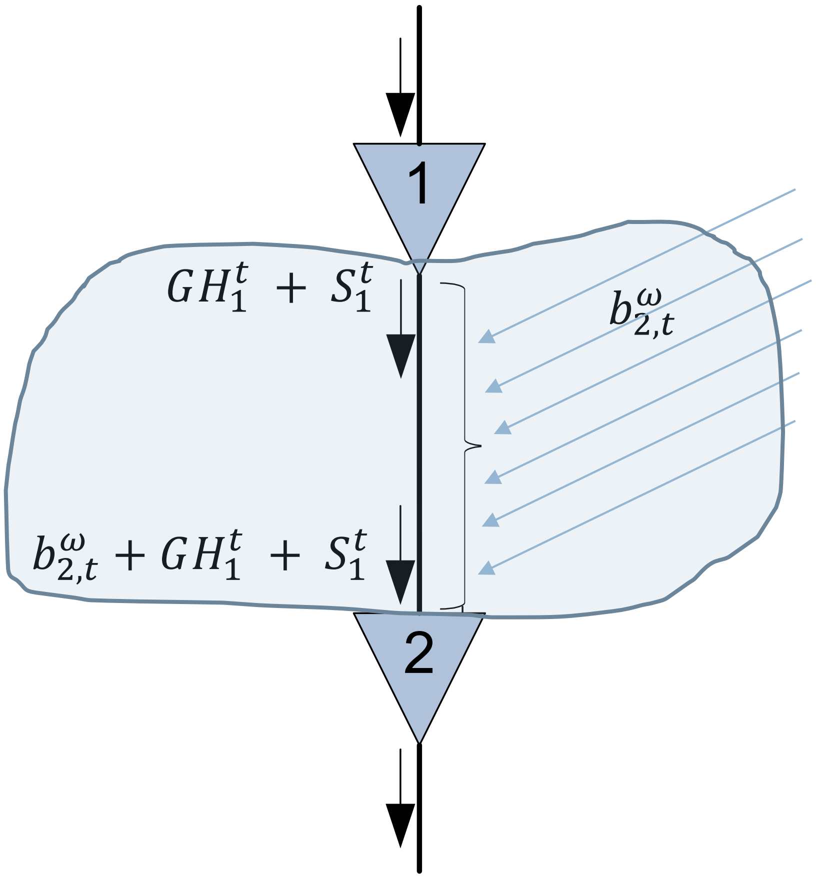
Generation and spillage upstream on the cascade for reservoir j

Water volume of reservoir at stage t-1

Incoming inflow at t for scenario ω

For all hydro plants, or aggregated hydro

Incremental Inflow Definition



$b_{2,t}^{\omega}$ = water captured/received between reservoirs 1 and 2, a.k.a., incremental inflow at reservoir 2

HTSP Model Formulation for Stage-t

$$h_t(x^{t-1}, b_t^{\omega}) = \min \underbrace{\sum_{\ell \in L} c_\ell^t GT_\ell^t + \sum_{k \in K} u_k^t GD_k^t}_{\text{Present Cost}} + \underbrace{\frac{1}{(1+\beta)} \mathbb{E}_{b_{t+1}|b_1,\ldots,b_t} h_{t+1}(x^t, b_{t+1})}_{\text{Expected Future Cost}}$$

Water Balance

$$\text{s.t.} \quad x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} \left(GH_j^t + S_j^t\right) = x_i^{t-1} + b_t^{\omega} \quad \forall i \in I$$

Demand Satisfaction

$$\sum_{i \in I_r} \rho_i GH_i^t + \sum_{\ell \in L_r} GT_\ell^t + \sum_{k \in K_r} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r'\,r}^t = D_{tr} \quad \forall r \in R$$

Productivity of hydro i, coefficient that will transform water into energy

Water used for generation at hydro i

Sum of Thermal generation

Sum of curtailed load

Note that all three sums are over subsets I_r, L_r and K_r

Energy transfer between region r and r'

Energy transfer between region r' and r

Demand at time t and region r

HTSP Model Formulation for Stage-t

$$h_t(x^{t-1}, b_t^{\omega}) = \min \overbrace{\sum_{\ell \in L} c_\ell^t GT_\ell^t + \sum_{k \in K} u_k^t GD_k^t}^{\text{Present Cost}} + \overbrace{\frac{1}{(1+\beta)} \mathbb{E}_{b_{t+1}|b_1,\dots,b_t} h_{t+1}(x^t, b_{t+1})}^{\text{Expected Future Cost}}$$

Water Balance

$$\text{s.t.} \quad x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} \left(GH_j^t + S_j^t\right) = x_i^{t-1} + b_t^{\omega} \quad \forall i \in I$$

Demand Satisfaction

$$\sum_{i \in I_r} \rho_i GH_i^t + \sum_{\ell \in L_r} GT_\ell^t + \sum_{k \in K_r} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r'\,r}^t = D_{tr} \quad \forall r \in R$$

Min and max bounds on reservoir volume

$$\underline{x_i^t} \le x_i^t \le \overline{x}_i^t \qquad \forall i \in I$$

$$0 \le GH_i^t \le \overline{GH_i^t} \quad \forall i \in I$$

Bounds on hydro generation capacity

Nonnegativity on spillage (defy gravity!!)

$$0 \le S_i^t \qquad \forall i \in I$$

$$\underline{GT_\ell^t} \le GT_\ell^t \le \overline{GT_\ell^t} \quad \forall \ell \in L$$

Min and max bounds on thermal generation capacity

Nonnegativity on curtailment

$$0 \le GD_k^t \qquad \forall k \in K$$

$$0 \le F_{r\,r'}^t \le \overline{F_{r\,r'}^t} \quad \forall (r, r') \in R$$

Bounds on energy transfer (transmission constraint)

HTSP Model Formulation for Stage-t

$$h_t(x^{t-1}, b_t^{\omega}) = \min \underbrace{\sum_{\ell \in L} c_\ell^t GT_\ell^t + \sum_{k \in K} u_k^t GD_k^t}_{\text{Present Cost}} + \underbrace{\frac{1}{(1+\beta)} \mathbb{E}_{b_{t+1}|b_1,\ldots,b_t} h_{t+1}(x^t, b_{t+1})}_{\text{Expected Future Cost}}$$

Water Balance $\quad$ s.t. $\quad x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} \left(GH_j^t + S_j^t\right) = x_i^{t-1} + b_t^{\omega} \quad \forall i \in I$

Demand Satisfaction

$$\sum_{i \in I_r} \rho_i GH_i^t + \sum_{\ell \in L} GT_\ell^t + \sum_{k \in K} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t = D_{tr} \quad \forall r \in R$$

Simple Bounds

$$\underline{x_i^t} \leq x_i^t \leq \overline{x}_i^t \qquad \forall i \in I$$

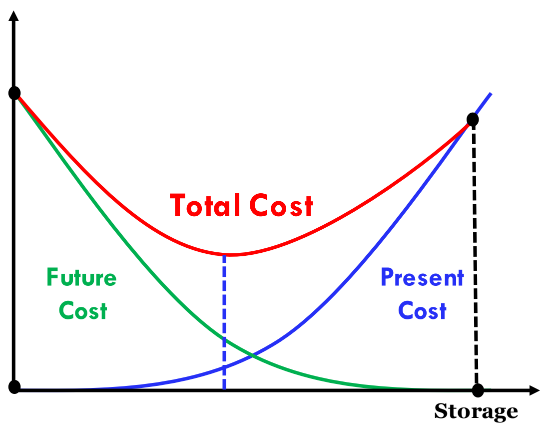
$$0 \leq GH_i^t \leq \overline{GH_i^t} \quad \forall i \in I$$

$$0 \leq S_i^t \qquad \forall i \in I$$

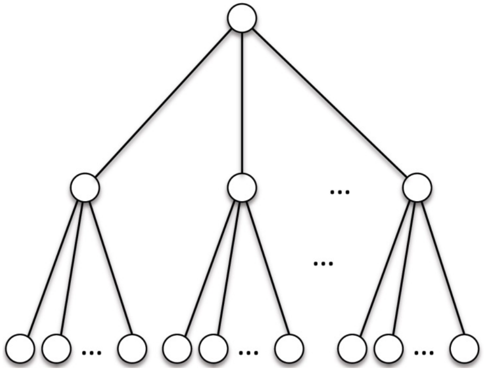
$$\underline{GT_\ell^t} \leq GT_\ell^t \leq \overline{GT_\ell^t} \quad \forall \ell \in L$$

$$0 \leq GD_k^t \qquad \forall k \in K$$

$$0 \leq F_{r\,r'}^t \leq \overline{F_{r\,r'}^t} \quad \forall (r, r') \in R$$



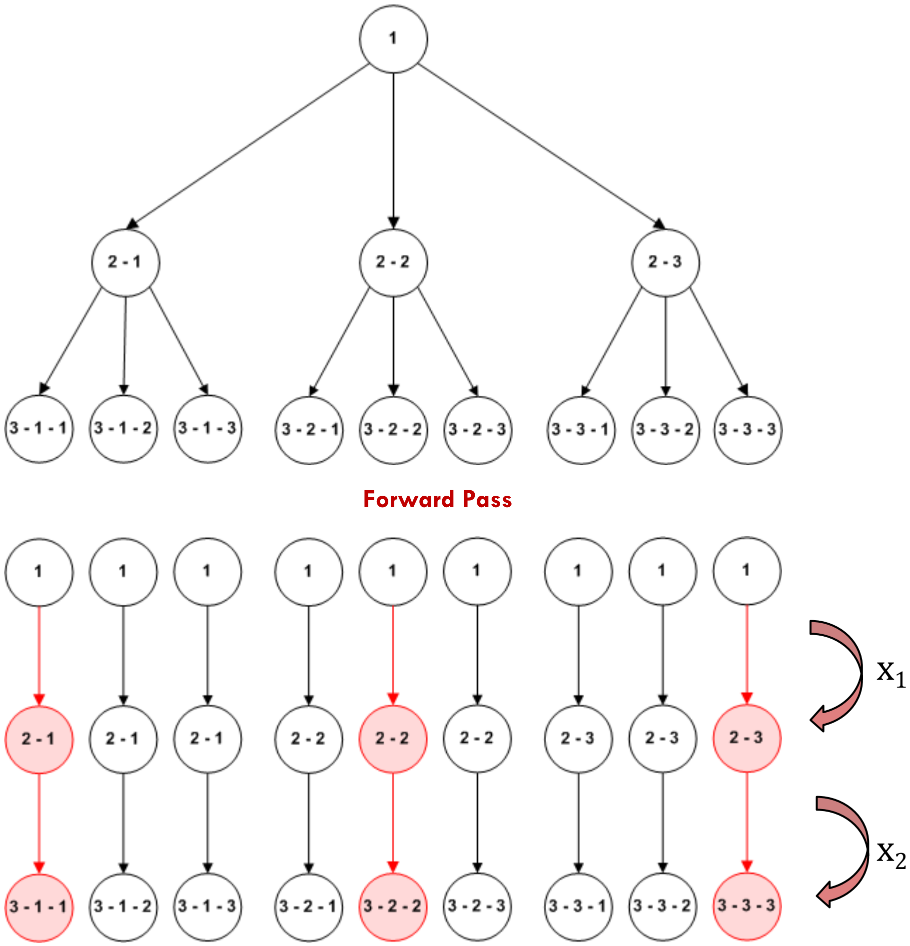
What's next?

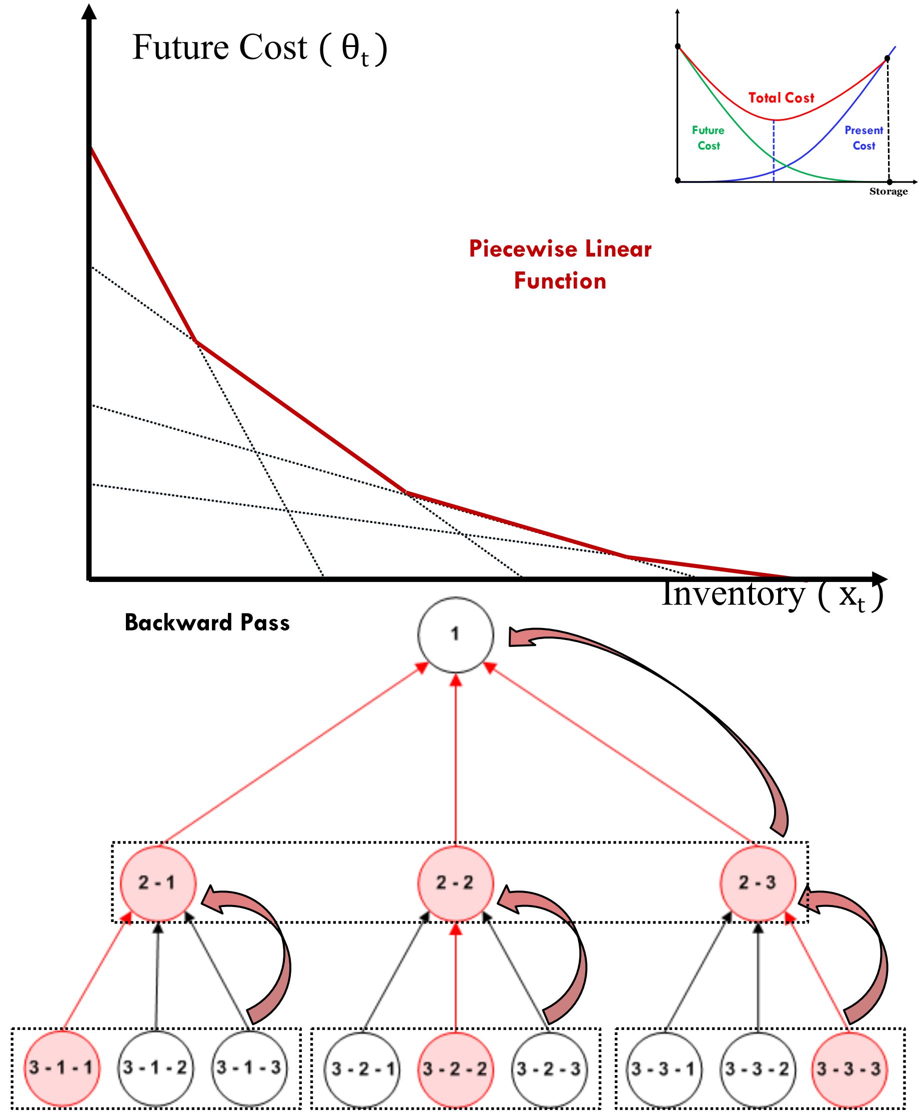


- Now we need to solve it!!!
- Remember that scenario tree??
- The model we just created will be solved for all these nodes
- Do you see the dots … it means that each node may have as many sons as you imagine
- Oh no…
- Can't we just go back to thermal only??

No! Let's save the world with green energy and stochastic optimization!

Sampling-based Decomposition Algorithm





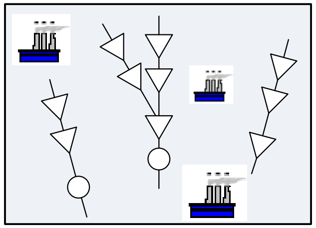
de Queiroz, A.R., Morton, D.P., (2013) Sharing Cuts under Aggregated Forecasts when Decomposing Multi-stage Stochastic Programs, Operations Research Lett, 41(3): 311-316

Let's add wind power…

- **Wind power plants may be considered such as run-of-river hydro plants**

Except that power variability is higher in smaller time intervals for wind

- **Objective:** Minimize operational costs to supply system electricity demand with hydro, thermal and wind plants
- **Constraints:**
  - Water balance
  - Electricity demand satisfaction
  - **Max wind power generation**
  - Electricity exchanges between regions
  - Other operational bounds

HTSP + Wind

Present Cost Future Cost Function

$$h_t(x^{t-1}, b_t^{\omega}) = \min \sum_{\ell \in L} c_\ell^t GT_\ell^t + \sum_{k \in K} u_k^t GD_k^t + \frac{1}{(1+\beta)} \mathbb{E}_{b_{t+1}|b_1,\dots,b_t} h_{t+1}(x^t, b_{t+1})$$

Water Balance

$$\text{s.t.} \quad x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} (GH_j^t + S_j^t) = x_i^{t-1} + b_t^{\omega} \quad \forall i \in I$$

Demand Satisfaction

$$\sum_{i \in I_r} \rho_i GH_i^t + \sum_{\ell \in L} GT_\ell^t + \sum_{k \in K} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t = D_{tr} \quad \forall r \in R$$

$$+ \sum_{v \in N_r} GW_v^t$$

All Simple
Bounds from previous HTSP

Set of wind plants

Maximum wind power generation (transformation of wind speed into energy output)

$$GW_v^t \leq \frac{1}{2} \sigma A_v V_{v,t,\omega}^3 C_P^v \qquad \forall v \in N$$

Air density

Area swept by wind turbine

Wind speed

Efficiency (max 0.59)

Wind speed will depend on scenario

HTSP + Solar

Present Cost

Future Cost Function

$$h_t(x^{t-1}, b_t^{\omega}) = \min \sum_{\ell \in L} c_\ell^t GT_\ell^t + \sum_{k \in K} u_k^t GD_k^t + \frac{1}{(1+\beta)} \mathbb{E}_{\,b_{t+1}|b_1,\dots,b_t} h_{t+1}(x^t, b_{t+1})$$

Water Balance

$$\text{s.t.} \;\; x_i^t + GH_i^t + S_i^t - \sum_{j \in M_i} \left(GH_j^t + S_j^t\right) = x_i^{t-1} + b_t^{\omega} \quad \forall i \in I$$

Demand Satisfaction

$$\sum_{i \in I_r} \rho_i GH_i^t + \sum_{\ell \in L} GT_\ell^t + \sum_{k \in K} GD_k^t - \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t + \sum_{\substack{r' \in R \\ r' \neq r}} F_{r\,r'}^t = D_{tr} \quad \forall r \in R$$

$$+ \sum_{pv \in Q_r} GS_{pv}^t$$

All Simple

Bounds from previous HTSP

$$GS_{pv}^t \leq I_{T,\omega} \eta_{pv} A_{pv} \qquad \forall pv \in Q$$

Solar radiation

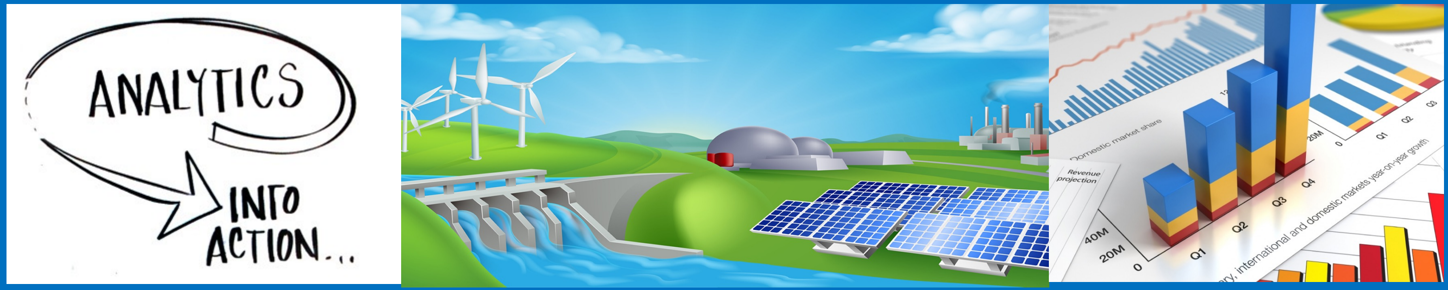
System efficiency

Array surface area

Set of solar plants

Maximum solar power generation (transformation of solar radiation into energy output)

Solar radiation will depend on scenario



THANK YOU !

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