ENVX1002 TUTORIAL 4

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> **Learning outcomes**
>
> At the end of this computer practical, students should be able to:
>
> - calculate tail, interval and inverse probabilities associated with the Normal distribution
> - calculate probabilities associated sampling distribution of the sample mean by using simulation in R and using R commands.
> - Understand how simulation links to the central limit theorem.

- ENVX1002_Data4.xlsx
- Alternatively download from Canvas

Normal distribution

Example 1 - Rainfall

Summary statistics and histogram

I have downloaded the daily rainfall at Sydney's Botanical Gardens from 1970-2021.

http://www.bom.gov.au/climate/data/

We are going to do some calculations on the total annual rainfall. To do this we are going to us the `dplyr` package to calculate the total annual rainfall from daily rainfall first. Then we will calculate:

- mean
- standard deviation
- histogram

```r
library(readxl)
rain <- read_excel("data/ENVX1002_Data4.xlsx", sheet = "Sydney_rain")
```

```r
## Calculate total annual rain using dplyr
library(dplyr)
annual_rain <- rain %>%
  group_by(Year) %>%
  summarise(Annual_rain = sum(Precip_mm))
```

```r
mean_rain <- round(mean(annual_rain$Annual_rain), 0)
mean_rain
```

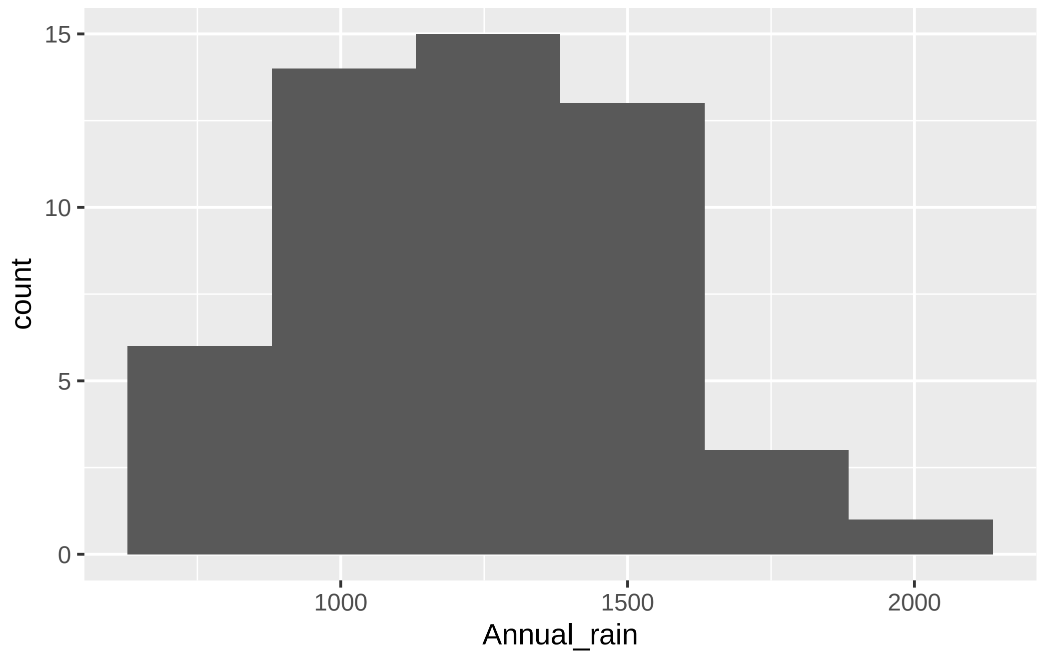
```
[1] 1254
```

```r
sd_rain <- round(sd(annual_rain$Annual_rain), 0)
sd_rain
```

```
[1] 298
```

- Our mean is 1254 mm
- Our standard deviation is 298mm

```r
library(ggplot2)
ggplot(annual_rain, aes(Annual_rain)) +
  geom_histogram(bins = 6)
```



Our distribution looks to be fairly symmetrical albeit a little right skewed. Later in the course you will learn to test if a dataset is normal, we will assume this dataset is. Therefore we can say that

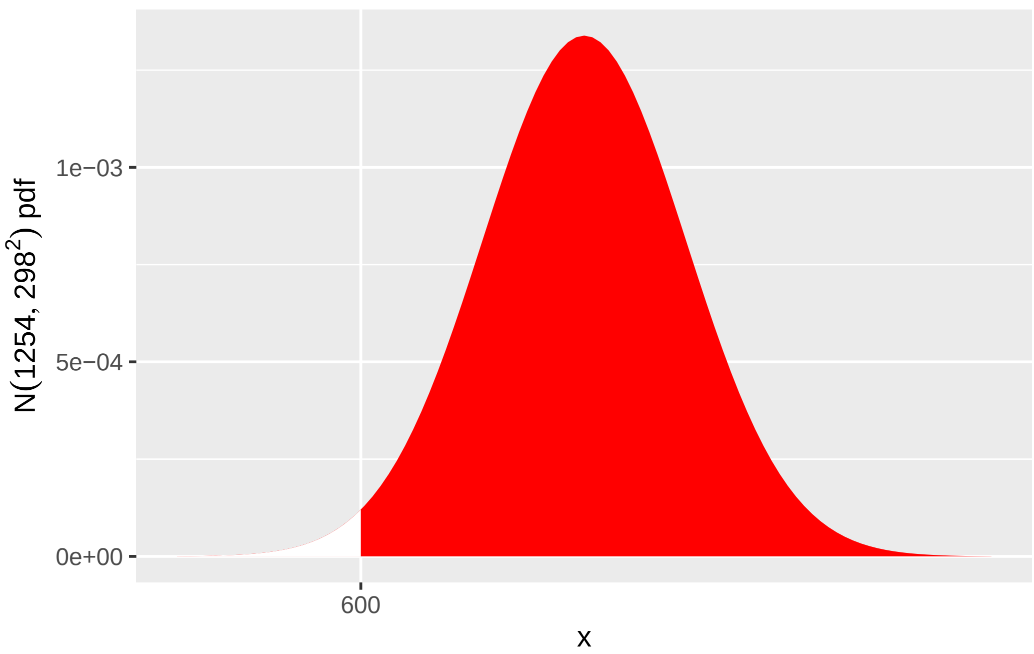
$X = Annual.rainfall \sim N(1254, 298^2)$

Probability calculations

Say we are interested in really dry periods in Sydney as these may result in spikes in water demand. So we want to determine what is the probability that annual Sydney rainfall is less than 600 mm as this may be a threshold to trigger water restrictions?

We want to calculate P(X < 600), we can plot this as follows

```
ggplot(data.frame(x = c(1254-4*298, 1254+4*298)), aes(x = x)) +
  stat_function(fun = dnorm, args = list(mean = 1254, sd = 298),
                geom = "area", fill = "red") +
  stat_function(fun = dnorm, args = list(mean = 1254, sd=298),
                xlim = c(1254-4*298, 600), geom = "area",
                fill = "white") +
  xlab("x") +
  ylab(expression(N(1254,298^2)~pdf)) +
  scale_x_continuous(breaks = 600)
```



To calculate the probability we can use pnorm

```
pnorm(600, 1254, 298)
```

```
[1] 0.01409504
```

So there is a 1.4% chance that less then 600mm of rain will fall in anyone year.

We can also standardise the variable and calculate the probability for the standard normal curve (or the standard normal tables in the old days https://en.wikipedia.org/wiki/Standard_normal_table)

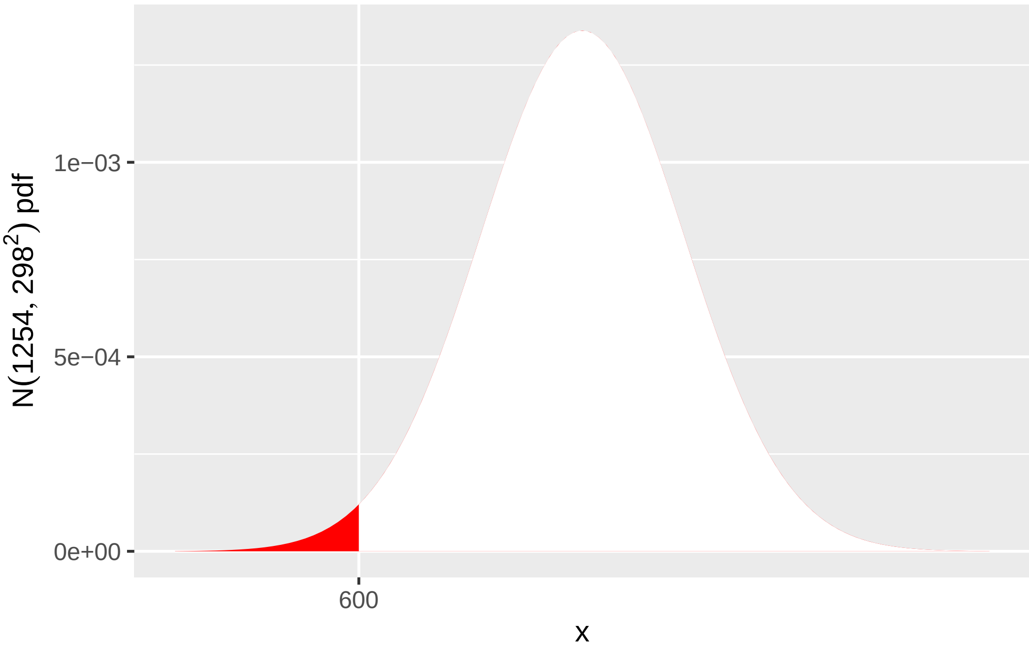
$P(X < 600) = P(\frac{X-1254}{298} < \frac{600-1254}{298}) = P(Z < -2.194631)$

```
pnorm(-2.194631)
```

```
[1] 0.01409504
```

NOTE: To plot P(X>600) we can make some slight modifications by changing the sign in xlim = c(1254-4*298, 600) to a +

```
ggplot(data.frame(x = c(1254-4*298, 1254+4*298)), aes(x = x)) +
  stat_function(fun = dnorm, args = list(mean = 1254, sd = 298),
                geom = "area", fill = "red") +
  stat_function(fun = dnorm, args = list(mean = 1254, sd=298),
                xlim = c(1254+4*298, 600), geom = "area",
                fill = "white") +
  xlab("x") +
  ylab(expression(N(1254,298^2)~pdf)) +
  scale_x_continuous(breaks = 600)
```



Inverse probability calculations

What is the 95% percentile of rainfall?

First we need to calculate the value of x at the 95th percentile

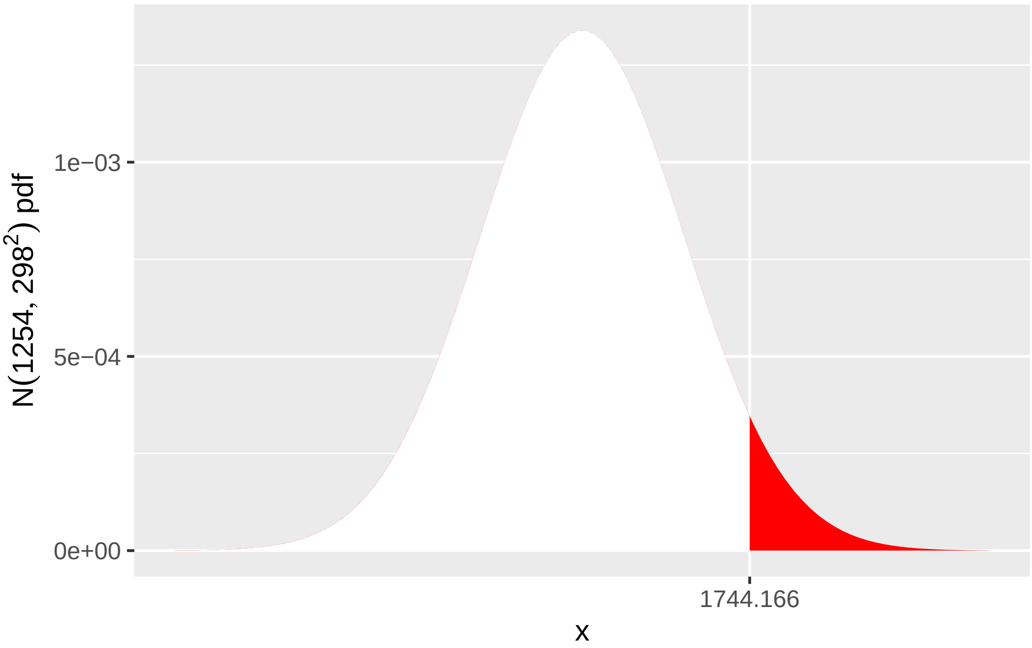
```
qnorm(0.95, 1254, 298)
```

```
[1] 1744.166
```

Therefore, 95% of annual total rainfall values are less than 1744.166 mm of rain i.e. 5% of the time we can expect more than 1744.166 mm of rain.

```
ggplot(data.frame(x = c(1254-4*298, 1254+4*298)), aes(x = x)) +
  stat_function(fun = dnorm, args = list(mean = 1254, sd = 298),
                geom = "area", fill = "red") +
  stat_function(fun = dnorm, args = list(mean = 1254, sd=298),
```

```
                    xlim = c(1254-4*298, 1744.166), geom = "area",
                    fill = "white") +
    xlab("x") +
    ylab(expression(N(1254,298^2)~pdf)) +
    scale_x_continuous(breaks = 1744.166)
```



Sampling distribution

Example - simulation of the central limit theorem

Olson and Miller reported the inter-orbital widths (in mm), distance between the eyes, of domestic pigeons in their 1958 book, *Morphological Integration*.

The following is a subset of the inter-orbital width (mm) of 40 domestic pigeons and the measurements are are as follows

```
IOW <- data.frame(width = c(12.2, 12.9, 11.8, 11.9, 11.6, 11.1, 12.3, 12.2, 11.8, 11.8, 10.7, 11.5, 11.3, 11.2, 11.6,
```

The mean and standard deviation of the inter-orbital width are as follows:

```
mean(IOW$width)
```

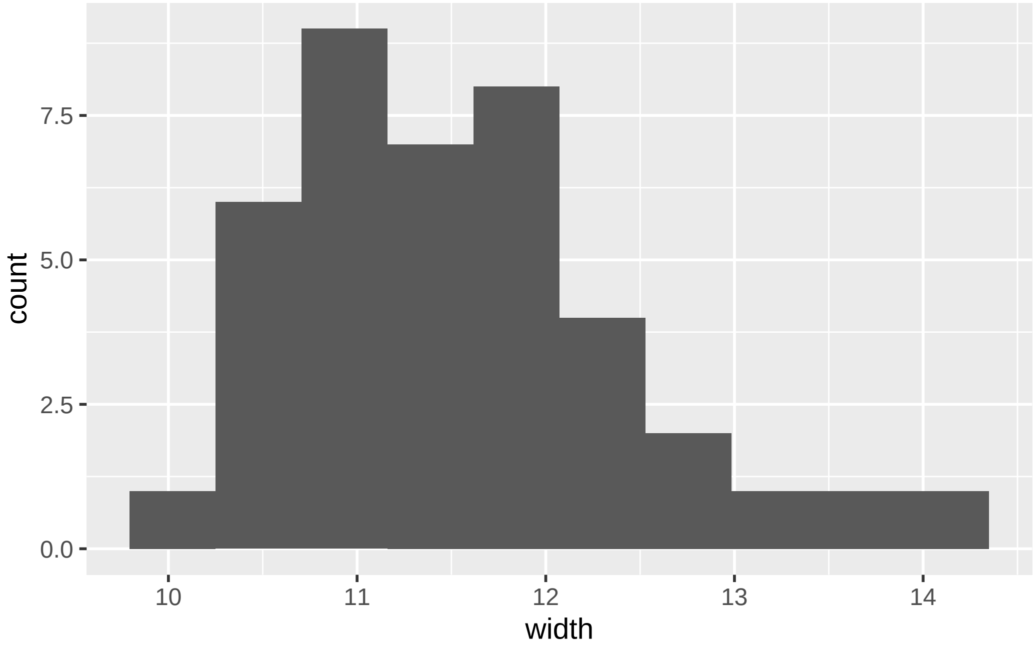
```
[1] 11.595
```

```
sd(IOW$width)
```

```
[1] 0.9122949
```

We can use the `ggplot2` package to plot a histogram of the inter-orbital width:

```r
ggplot(IOW, aes(width))+
  geom_histogram(bins = 10)
```

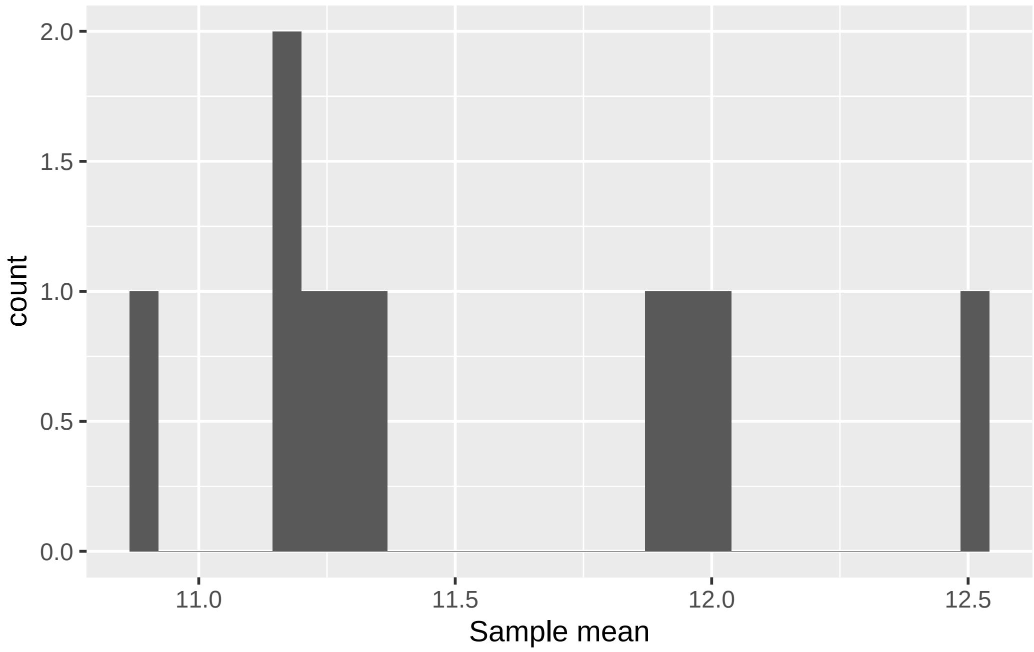


We can use the `sample` function to simulate the sampling distribution of the sample mean. We will take 5 samples of the inter-orbital width and calculate the sample mean. We will repeat this 10, 100, 1000 and 10000 times and plot the sampling distribution of the sample mean.

10

```r
set.seed(1)
sample_means <- data.frame(Sample_mean = vector(length = 10))
for(i in 1:10){
        sample_means$Sample_mean[i] <- mean(sample(IOW$width, 5, replace = TRUE))
}
ggplot(sample_means, aes(Sample_mean))+
  geom_histogram() +
  xlab("Sample mean")
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



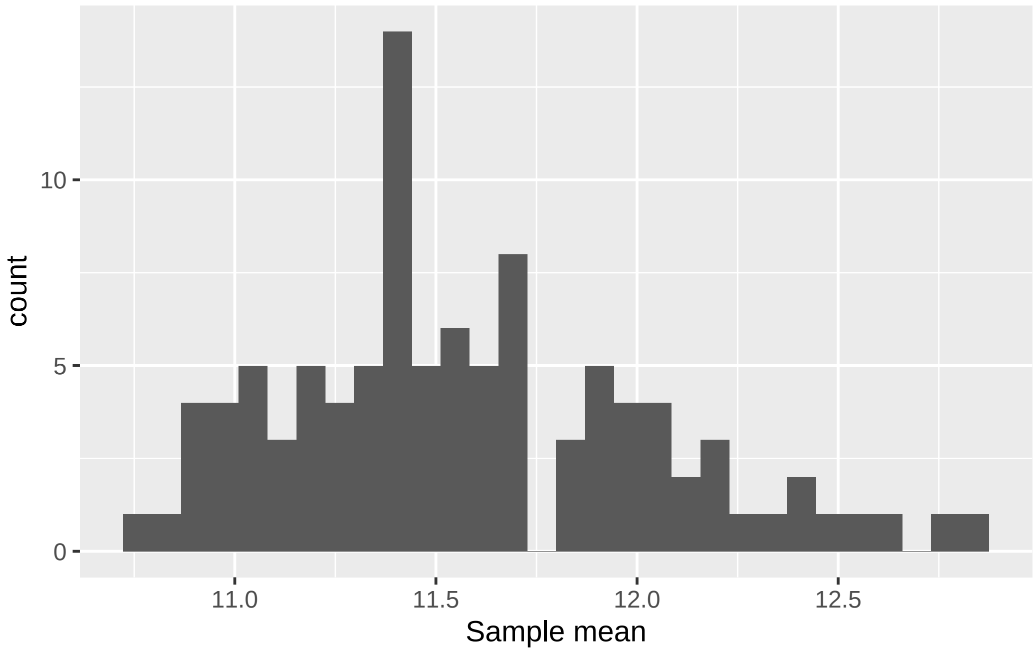

```
mean(sample_means$Sample_mean)
```

```
[1] 11.552
```

100

```
set.seed(1)
sample_means <- data.frame(Sample_mean = vector(length = 100))
for(i in 1:100){
        sample_means$Sample_mean[i] <- mean(sample(IOW$width, 5, replace = TRUE))
}
ggplot(sample_means, aes(Sample_mean))+
  geom_histogram() +
  xlab("Sample mean")
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



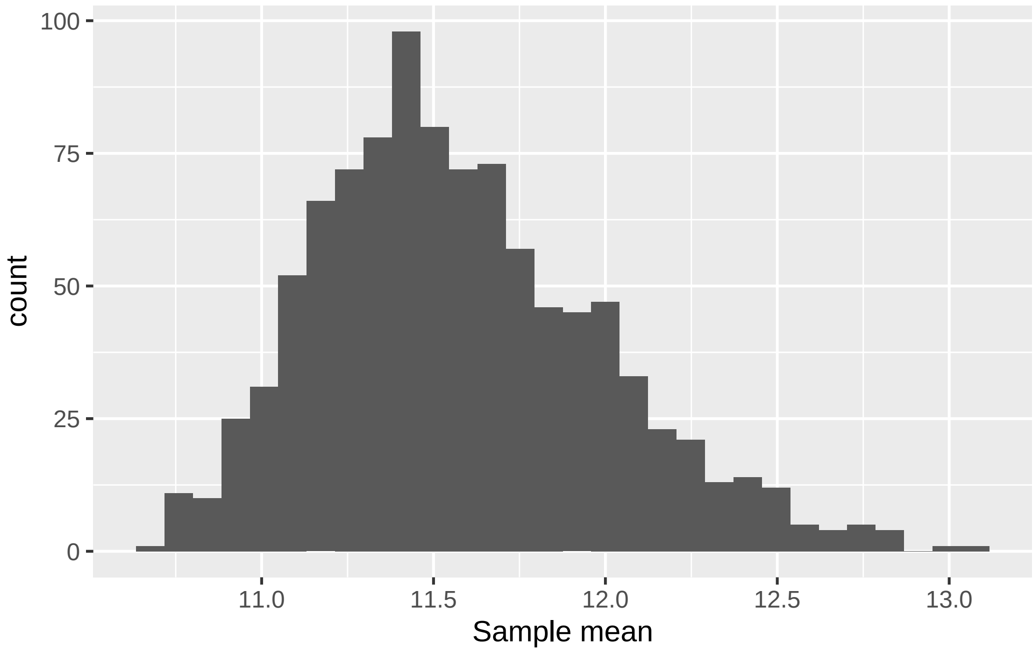

```
mean(sample_means$Sample_mean)
```

```
[1] 11.581
```

1000

```
set.seed(1)
sample_means <- data.frame(Sample_mean = vector(length = 1000))
for(i in 1:1000){
        sample_means$Sample_mean[i] <- mean(sample(IOW$width, 5, replace = TRUE))
}
ggplot(sample_means, aes(Sample_mean))+
  geom_histogram() +
  xlab("Sample mean")
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```



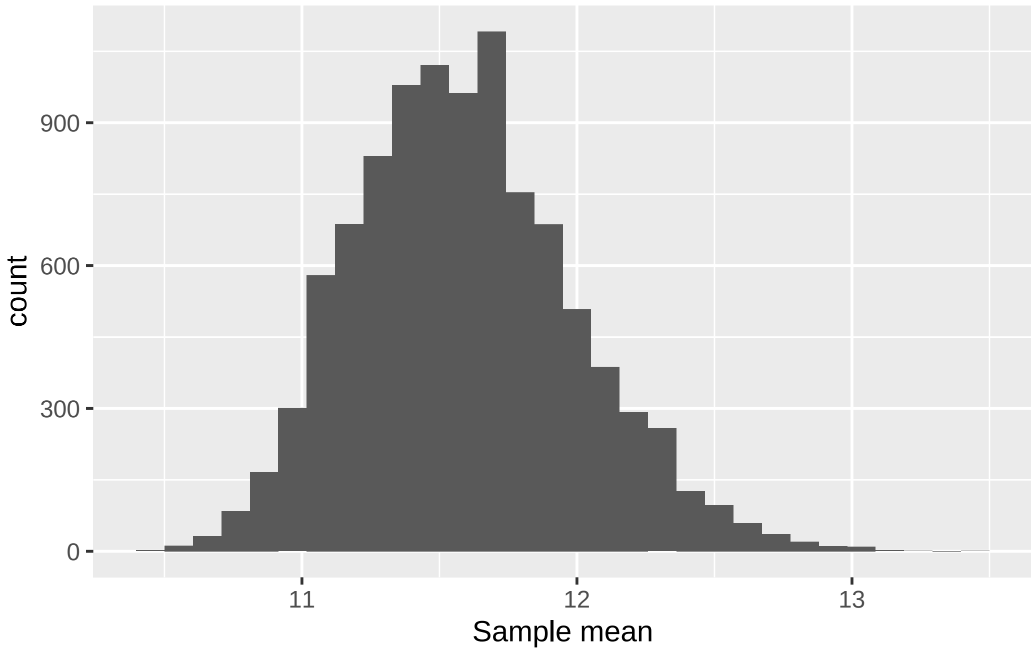

```r
mean(sample_means$Sample_mean)
```

```
[1] 11.57442
```

10000

```r
set.seed(1)
sample_means <- data.frame(Sample_mean = vector(length = 10000))
for(i in 1:10000){
        sample_means$Sample_mean[i] <- mean(sample(IOW$width, 5, replace = TRUE))
}
ggplot(sample_means, aes(Sample_mean))+
  geom_histogram() +
  xlab("Sample mean")
```

```
`stat_bin()` using `bins = 30`. Pick better value with `binwidth`.
```




```
mean(sample_means$Sample_mean)
```

```
[1] 11.59095
```

Example 3 - Past exam question

Interorbital widths (in mm) of 5 domestic pigeons are as follows: 12.2 12.9 11.8 11.9 11.2

i) Obtain the sample mean and sample standard deviation of the widths.

ii) Calculate the standard error of the mean. Interpret this briefly but carefully, highlighting the difference in interpretation between the standard deviation and the standard error of the mean.

iii)

-Sample mean:

$$\overline{x} = \frac{12.2 + 12.9 + 11.8 + 11.9 + 11.2}{5} = 12mm$$

- Sample sd:

$$s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{5}(x_i - \overline{x})^2}$$

$$= \sqrt{\frac{1}{5-1}((12.2-12)^2 + (12.9-12)^2 + ... + (11.2-12)^2)} = 0.62mm$$

Using R

```
pigeon <- c(12.2,12.9,11.8,11.9,11.2)
pigeon_mean <- mean(pigeon)
pigeon_mean
```

```
[1] 12
```

```r
pigeon_sd <- sd(pigeon)
pigeon_sd
```

```
[1] 0.6204837
```

ii) The standard error of the mean

$$se(\overline{X}) = \frac{s}{\sqrt{n}} = \frac{0.62}{\sqrt{5}} \approx 0.28mm$$

By r:

```r
pigeon_sem <-pigeon_sd/sqrt(length(pigeon)) ## length(pigeon) gives you n
pigeon_sem
```

```
[1] 0.2774887
```

The standard error (se) gives an estimate of how uncertain we are about the sample mean - it is effectively the standard deviation **sd** of the sample mean. The difference between **sd** and **se** is that **sd** describes the amount of variation in the sample being studied and the **se** describes the uncertainty in the estimate of the mean.