Lab 07 – Chi-squared test

ENVX1002 Handbook

Semester 1, 2025

Learning Outcomes

- Learn to use R to calculate a chi-squared test for:
  - Test of proportions
  - Test of independence
- Learn how to interpret statistical output.

Before we begin

Create your Quarto document and save it as `Lab-07.qmd` or similar. There are no data files to download for this lab.

Quick introduction

The chi-square test is used to compare the *observed* distribution to an *expected* distribution, in a situation where we have **two or more categories** in a discrete data. In other words, it compares multiple observed proportions to expected probabilities.

The formula is:

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

where O_i is the observed frequency, E_i is the expected frequency for each categorym and k is the number of categories.

For more information about the technique, consult your lecture slides and tutorial 7.

Exercise 1: wild tulips (walk-through)

Background

Suppose we collected wild tulips and found that 81 were red, 50 were yellow and 27 were white. Are these colours equally common?

If these colours were equally distributed, the expected proportion would be 1/3 for each of the colour. Therefore, we want to test if the observed proportions are significantly different from the expected proportions.

The data is below.

```
tulip <- c(81, 50, 27)
```

Instructions

Utilise the **HATPC** process and test the hypothesis that the proportion of flower colours of tulips are equally common, assuming that the samples are independent. We can explore the data as we check the assumptions of the test.

HATPC:

- Hypothesis
- Assumptions
- Test (statistic)
- P-value
- Conclusion

> **i** Level of significance
>
> **The level of significance is usually set at 0.05.** This value is generally accepted in the scientific community and is also linked to Type 2 errors, where choosing a lower significance increases the likelihood of failing to reject the null hypothesis when it is false.

Hypotheses

What are the null hypothesis and alternative hypotheses?

Click here to view answer

- H_0: There is no significant difference between the observed and the expected proportions of flower colours.
- H_1: There is a significant difference between the observed and the expected proportions of flower colours.

Assumptions

Recall that the assumptions of the χ^2 test are:

1. No cell has expected frequencies less than 1
2. No more than 20% of cells have expected frequencies less than 5

> **i** Note
>
> In the case that the above assumptions are violated then the probability of a type 1 error occurring (rejecting the null hypothesis when it is true, i.e. false positive) increases.

To calculate expected frequencies, we first calculate the total number of tulips and then divide by the number of categories.

```
expected <- rep(sum(tulip) * 1 / 3, 3) #rep function replicated the value we're
calculating inside the brackets
expected
```

```
[1] 52.66667 52.66667 52.66667
```

Does the data satisfy the assumptions of a χ^2 test?

Click here to view answer Yes, as expected frequencies > 5.

Test statistic

The `chisq.test()` function in R is used to calculate the chi-squared test.

```
res <- chisq.test(tulip, p = c(1 / 3, 1 / 3, 1 / 3))
res
```

```

    Chi-squared test for given probabilities

data:  tulip
X-squared = 27.886, df = 2, p-value = 8.803e-07
```

Note that we coud check our assumptions post-analysis by checking the expected frequencies stored in the `expected` object of the output:

```
res$expected
```

```
[1] 52.66667 52.66667 52.66667
```

P-value

Write down how you should report the critical value, p-value and df in a scientific paper?

Click here to view answer $\chi^2 = 27.9, d.f. = 2, p < 0.01$

Conclusions

Based on the p-value, do we accept or reject the null hypothesis?

Click here to view answer We reject the null hypothesis as the p-value is less than 0.05.

Now write a scientific (biological) conclusion based on the outcome.

Click here to view answer There is a significant difference in the proportion of flower colours of tulips ($\chi^2 = 27.9, d.f. = 2, p < 0.01$).

Exercise 2: hermit crabs

Background

In a study of hermit crab behaviour at Point Lookout, North Stradbroke Island, a random sample of 3 types of gastropod shells was collected. Each shell was then scored as being either occupied by a hermit crab or empty. Do hermit crabs prefer a certain shell?

Shell species	Occupied	Empty
Austrochochlea	47	42
Bembicium	10	41
Cirithid	125	49

The data is stored in a `table` object in R below. Note that it is different from a `data.frame` object. You can verify this by using the `str()` or `class()` functions.

```
crabs <- as.table( #Make a table with values for each row
  rbind(
    Aus = c(47, 42),
    Bem = c(10, 41),
    Cir = c(125, 49)
  )
)

colnames(crabs) <- c("Occupied", "Empty") #Add column names to table
str(crabs)
```

```
 'table' num [1:3, 1:2] 47 10 125 42 41 49
 - attr(*, "dimnames")=List of 2
  ..$ : chr [1:3] "Aus" "Bem" "Cir"
  ..$ : chr [1:2] "Occupied" "Empty"
```

```
crabs
```

```
    Occupied Empty
Aus       47    42
Bem       10    41
Cir      125    49
```

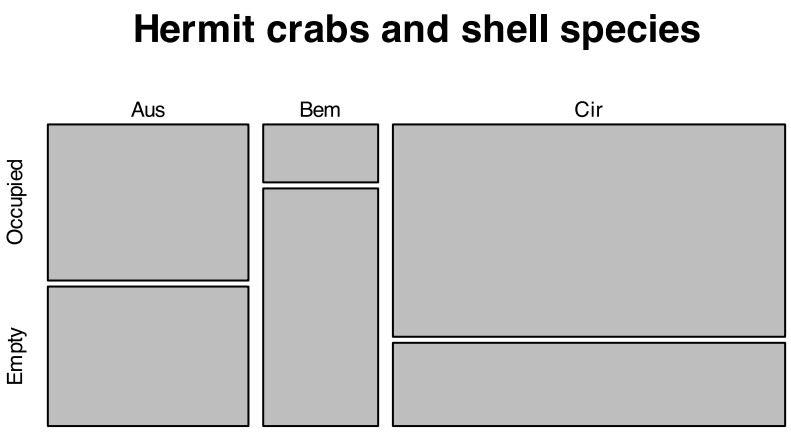
Data exploration

Since we have a multi-dimensional dataset, we can try to plot the data to visualise it.

A mosaic plot is a graphical representation of the data in a two-way contingency table. It is a way of visualising the relationship between two categorical variables.

Try the code below. Can you interpret the plot?

```r
# mosaic plot of crabs
mosaicplot(crabs, main = "Hermit crabs and shell species")
```



Click here to view interpretation The plot shows that the distribution of hermit crabs in the different shell species is not equal. The majority of hermit crabs are found in the *Cirithid* shell species, followed by *Austrochochlea* and *Bembicium*. This can be observed by the **width** of the boxes in the plot.

There are also differences in the number of empty shells in the different shell species. The *Bembicium* shell species has the highest number of empty shells, followed by *Austrochochlea* and *Cirithid*. This can be observed by the **height** of the boxes in the plot.

HATPC

Now it is your turn to test the hypothesis that the three shell species are equally preferred by hermit crabs. Follow the HATPC process with the following questions in mind (but you don't have to answer them individually):

1. What are the null hypothesis and alternative hypotheses?
2. Does the data satisfy the assumptions of a χ^2 test?
3. How should you report the critical value, p-value and df in a scientific paper?
4. Based on the p-value, do we accept or reject the null hypothesis?
5. Write a scientific (biological) conclusion based on the outcome.

Use the markdown structure below to answer the questions (if you wish):

```
## Exercise 2: hermit crabs
### Hypothesis
### Assumptions
### Test statistic
### P-value
### Conclusions
```

Take your time, and when you are ready, check your answers with your demonstrators.

Answer to Exercise 2

Hypothesis

$\mathbf{H_0}$: There is no significant difference between the observed and the expected use of different shell species by hermit crabs.

$\mathbf{H_1}$: There is a significant difference between the observed and the expected use of different shell species by hermit crabs.

Assumptions

To check whether the expected frequencies are greater than 5, we can either perform the calculation manually or check the output of the `chisq.test()` function.

```
fit <- chisq.test(crabs)
fit$expected
```

```
     Occupied    Empty
Aus  51.58599 37.41401
Bem  29.56051 21.43949
Cir 100.85350 73.14650
```

From the output, we can see that all expected frequencies are greater than 5. The other assumption where no cell has expected frequencies less than 1 is also satisfied.

Test statistic

If you have not already done so, perform the chi-squared test using the `chisq.test()` function in R.

```
fit <- chisq.test(crabs)
fit
```

```

    Pearson's Chi-squared test
```

```
data:  crabs
X-squared = 45.512, df = 2, p-value = 1.31e-10
```

We reject the null hypothesis as the p-value is less than 0.05 and conclude that there is a significant difference between the observed and the expected use of different shell species by hermit crabs ($\chi^2 = 45.5, d.f. = 2, p < 0.01$).

Done!

This is the end of the lab. Remember to save your work, render your document and ask your demonstrators for feedback if you are unsure about your answers.

Bonus take home exercises

Exercise 1: National RSPCA statistics

The RSPCA releases statistics on the number of animals they receive, reclaim and rehome every year. In the 2023-24 financial year, the RSPCA received 17468 dogs, 26704 cats, and 37497 other animals. The "other" category includes horses, small animals, livestock and wildlife.

Using the HATPC framework, test whether these animals were received in equal proportions.

```
received <- c(17468, 26704, 37497)
```

Solution

- Hypothesis

  H_0 : There is no significant difference between the observed and the expected number of animals received from each category

  H_1 : There is a significant difference betwwen the observed and the expected number of animals received from each category

- Assumptions

```
expected_received <- rep(sum(received)*1/3, 3)
expected_received
```

```
[1] 27223 27223 27223
```

1) No cell has expected frequencies less than 1 and 2) No more than 20% of cells have expected frequencies less than 5 so the assumptions are met

- Test (statistic)

```
chisq.test(received)
```

```
    Chi-squared test for given probabilities

data:  received
X-squared = 7382.9, df = 2, p-value < 2.2e-16
```

- P-value > p < 0.05, so we reject the null hypothesis
- Conclusion

> There is a significant difference betwwen the observed and the expected number of animals received from each category

Exercise 2: UC Berkeley Admissions

For the two exercises we're going to use simplified versions of the inbuilt dataset 'UCBAdmissions', which has data on student admissions for Berkeley. The dataset shows how many students were rejected and admitted to the university by both department and gender.

2.1 Admissions by department

Did every department at UC Berkeley admit students in equal proportions?

```
dept_admissions <- c(601, 370, 322, 269, 147, 46)
```

Solution

- Hypothesis

> H_0:There is no significant difference between the expected and observed number of students admitted by each department

> H_1: There is a significant difference between the expected and observed number of students admitted by each department

- Assumptions

```
rep(sum(dept_admissions)*1/6, 6)
```

```
[1] 292.5 292.5 292.5 292.5 292.5 292.5
```

1) No cell has expected frequencies less than 1 and 2) No more than 20% of cells have expected frequencies less than 5 so the assumptions are met

- Test (statistic)

```
chisq.test(dept_admissions)
```

```
    Chi-squared test for given probabilities

data:  dept_admissions
X-squared = 630.88, df = 5, p-value < 2.2e-16
```

- P-value

  $p < 0.05$, so we reject the null hypothesis

- Conclusion

  There is a significant difference between the expected and observed number of students admitted by each department

2.2

Are male and female students admitted and rejected in the same proportion?

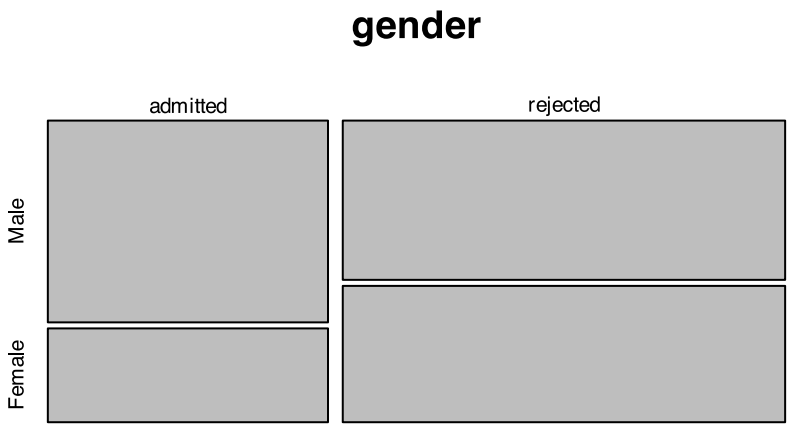
```
gender <- as.table( #Make a table with values for each row
  rbind(
    admitted = c(1198, 557),
    rejected = c(1493, 1278)
  )
)

colnames(gender) <- c("Male", "Female") #Add column names to table
```

Solution

Can make a mosaic plot to look at what the data looks like

```
mosaicplot(gender)
```



- Hypothesis

  H_0:There is no significant difference between the expected and observed number of students admitted or rejected by gender

  H_1:There is a significant difference between the expected and observed number of students admitted or rejected by gender

- Assumptions

```
#Fit the chisq test to be able to call up the expected
fit_gender <-chisq.test(gender)

fit_gender$expected
```

```
             Male    Female
admitted 1043.461  711.5389
rejected 1647.539 1123.4611
```

  1) No cell has expected frequencies less than 1 and 2) No more than 20% of cells have expected frequencies less than 5 so the assumptions are met

- Test (statistic)

```
#call up the test we ran above
fit_gender
```

```
    Pearson's Chi-squared test with Yates' continuity correction

data:  gender
X-squared = 91.61, df = 1, p-value < 2.2e-16
```

- P-value

  $p < 0.05$, so we reject the null hypothesis

- Conclusion

  There is a significant difference between the expected and observed number of students admitted or rejected by gender