

Lecture 03a – *t*-Tests

ENVX2001 Applied Statistical Methods

Januar Harianto

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History

William Gosset (1908)



Figure 1: William Gosset

- Worked as a statistician at Guinness Brewery in Dublin
- Published under the pseudonym “**Student**” (company policy prevented use of real name)
- Developed the t -distribution and t -tests for **small sample sizes**
- Needed to compare barley varieties with limited data

Ronald Fisher (1925)

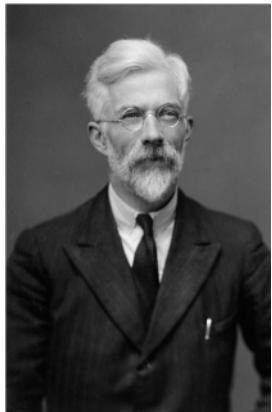


Figure 2: Ronald Fisher

- Extended and formalised Gosset's work
- Introduced the concept of **degrees of freedom**
- Developed **Analysis of Variance (ANOVA)** – next lecture!
- Established many foundations of modern statistical inference

Cattle data

The problem

- Weights of two breeds of cattle are to be compared
- 12 samples were randomly taken from Breed 1, and 15 from Breed 2
- **Are there differences in the mean weight between the two breeds?**

```
cattle ← read.csv("data/cattle.csv")
cattle
```

	Breed1	Breed2
1	187.6	148.1
2	180.3	146.2
3	198.6	152.8
4	190.7	135.3
5	196.3	151.2
6	203.8	146.3
7	190.2	163.5
8	201.0	146.6
9	194.7	162.4
10	221.1	140.2
11	186.7	159.4

12	203.1	181.8
13	NA	165.1
14	NA	165.0
15	NA	141.6

Reshaping the data

The data is in **wide** format – we need to convert it to **long** format for analysis.

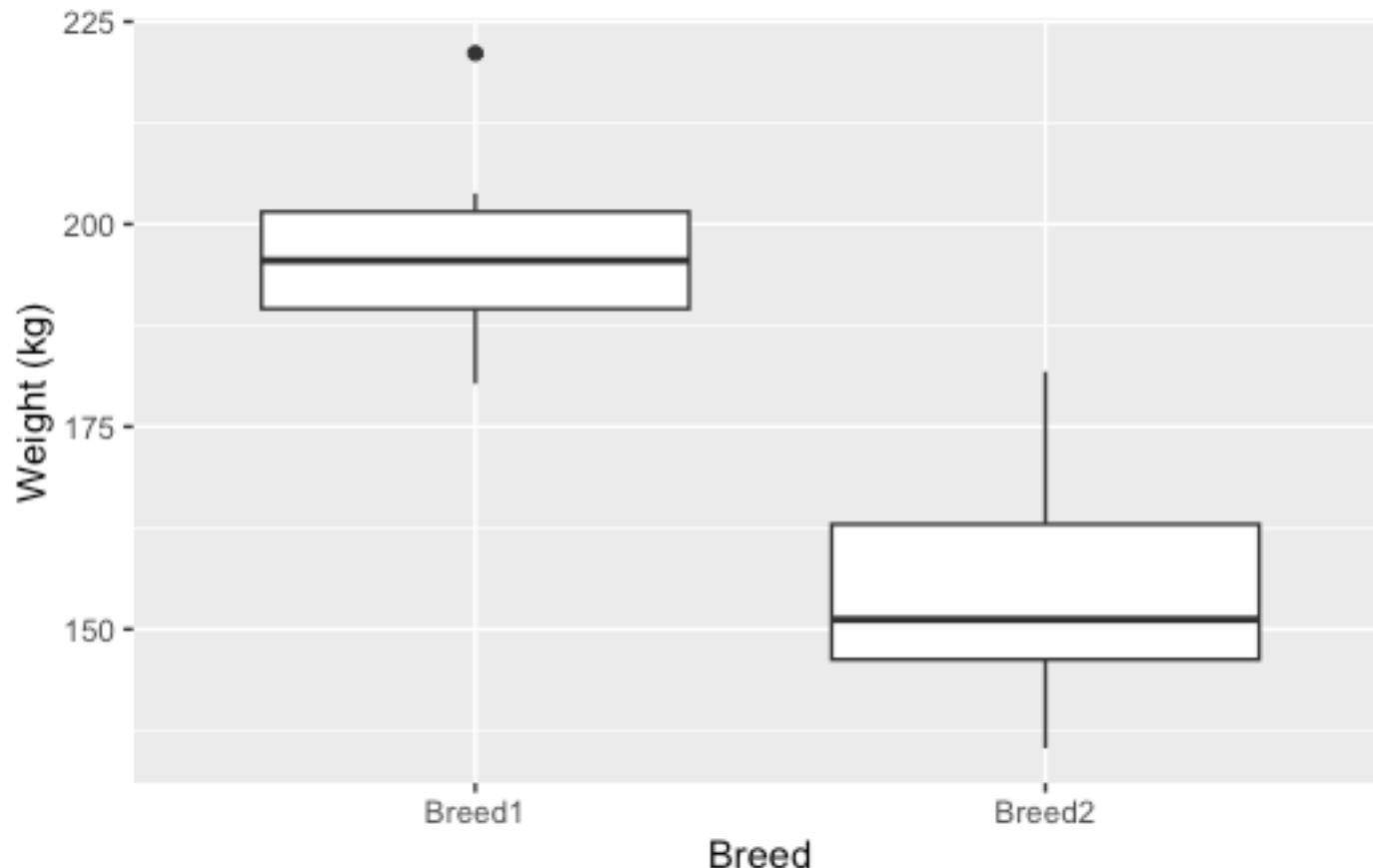
```
cattle ← read.csv("data/cattle.csv") |>  
  pivot_longer(  
    cols = everything(),  
    names_to = "breed",  
    values_to = "weight"  
  ) |>  
  mutate(breed = as.factor(breed)) |>  
  drop_na()  
  
cattle
```

```
# A tibble: 27 × 2  
  breed   weight  
  <fct>   <dbl>  
1 Breed1    188.  
2 Breed2    148.  
3 Breed1    180.
```

```
4 Breed2    146.  
5 Breed1    199.  
6 Breed2    153.  
7 Breed1    191.  
8 Breed2    135.  
9 Breed1    196.  
10 Breed2   151.  
# i 17 more rows
```

Visualising the data

```
ggplot(cattle, aes(breed, weight)) +  
  geom_boxplot() +  
  labs(x = "Breed", y = "Weight (kg)")
```



Model and hypotheses

Two-sample t -test model

Observed data = Group Mean + Random Error (residuals)

$$y_{ij} = \mu_i + \varepsilon_{ij}$$

- $i = 1, 2$ (group); $j = 1, 2, \dots, n_i$ (replicate)

In the cattle example:

- μ_1 = mean body weight (kg) for Breed 1
- μ_2 = mean body weight (kg) for Breed 2

Hypotheses

- **Null hypothesis:** $H_0 : \mu_1 = \mu_2$
- **Alternative hypothesis:** $H_1 : \mu_1 \neq \mu_2$

Test statistic:

$$t = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{s^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{\bar{y}_1 - \bar{y}_2}{SE(\bar{y}_1 - \bar{y}_2)} = \frac{\Delta \text{ in mean}}{SE \text{ of the } \Delta \text{ in mean}}$$

Degrees of freedom: $n_1 + n_2 - 2$

t-distribution

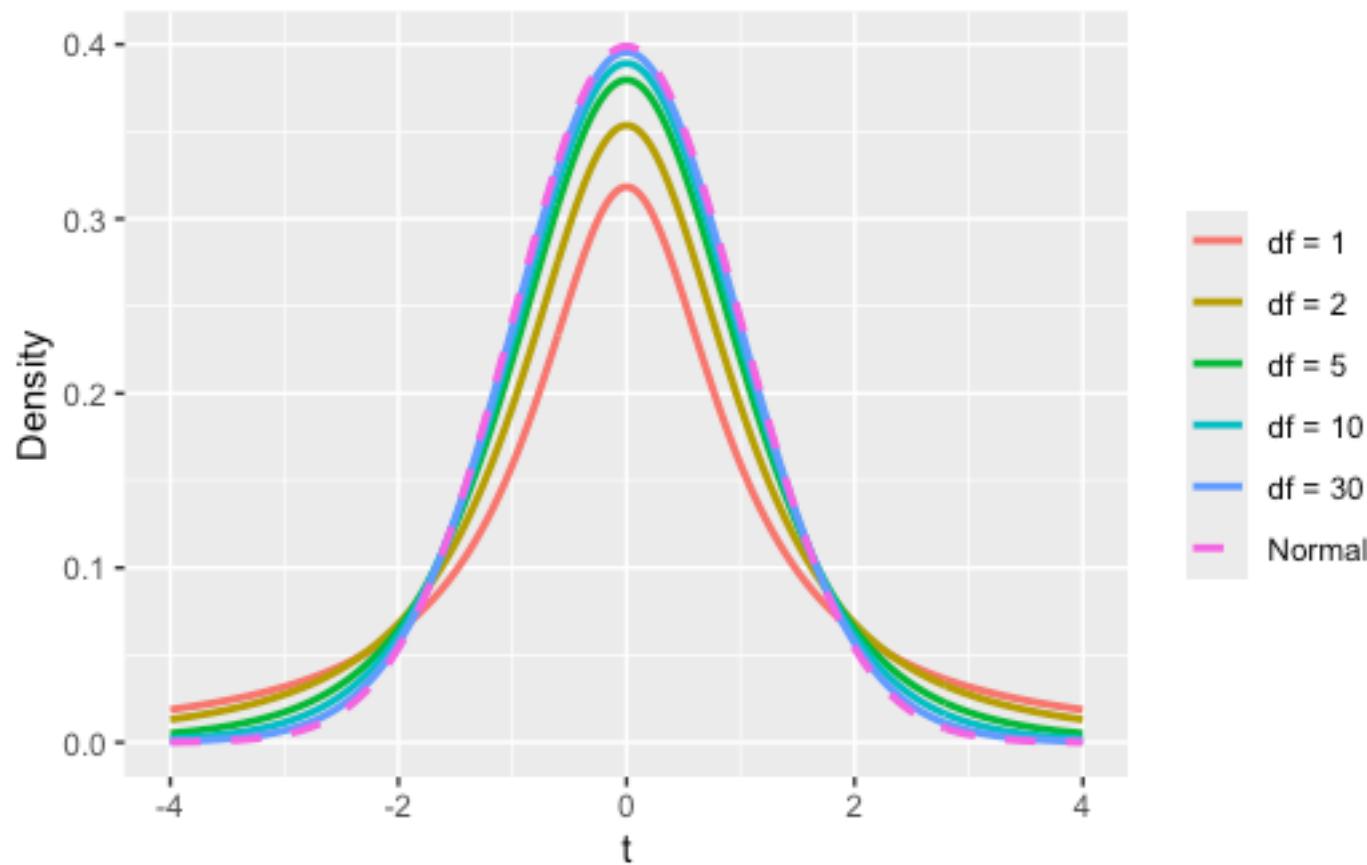
Shape changes with degrees of freedom

```
x ← seq(-4, 4, length.out = 300)

t_data ← map_dfr(c(1, 2, 5, 10, 30), function(df) {
  tibble(x = x, density = dt(x, df), df = paste("df =", df))
})

normal_data ← tibble(x = x, density = dnorm(x), df = "Normal")

bind_rows(t_data, normal_data) |>
  mutate(df = fct_relevel(df, "df = 1", "df = 2", "df = 5", "df = 10", "df = 30", "Normal")) |>
  ggplot(aes(x, density, colour = df, linetype = df)) +
  geom_line(linewidth = 1) +
  scale_linetype_manual(values = c(rep("solid", 5), "dashed")) +
  labs(x = "t", y = "Density", colour = NULL, linetype = NULL) +
  theme(legend.position = "right")
```

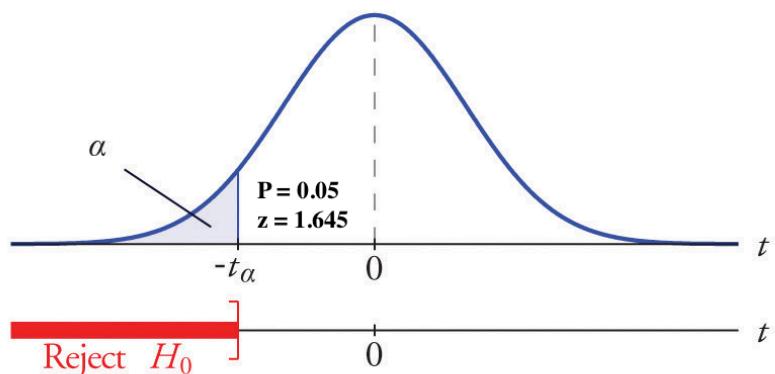


i Note

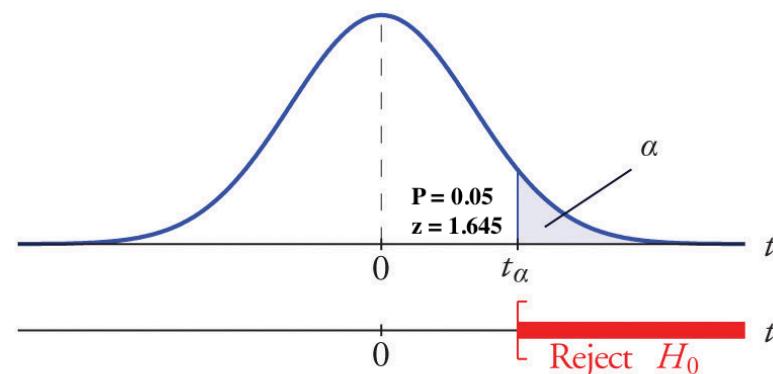
As degrees of freedom increase, the t -distribution approaches the standard normal distribution.

One-tailed vs two-tailed tests

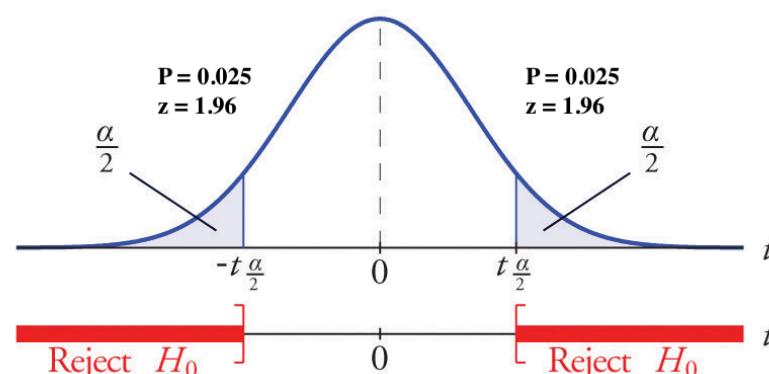
$$H_a : \mu < \mu_0$$



$$H_a : \mu > \mu_0$$



$$H_a : \mu \neq \mu_0$$



- **Two-tailed**: tests for a difference in *either* direction ($\mu_1 \neq \mu_2$)
- **One-tailed**: tests for a difference in a *specific* direction ($\mu_1 > \mu_2$ or $\mu_1 < \mu_2$)
- In most cases, we use **two-tailed** tests unless we have a strong *a priori* reason for one direction.

Manual calculations

Group means and standard deviations

```
cattle_summary ← cattle |>
  group_by(breed) |>
  summarise(
    n = n(),
    mean_wt = mean(weight, na.rm = TRUE),
    sd_wt = sd(weight, na.rm = TRUE)
  )
```

```
cattle_summary
```

```
# A tibble: 2 × 4
  breed      n  mean_wt   sd_wt
  <fct> <int>   <dbl>   <dbl>
1 Breed1     12    196.   10.6
2 Breed2     15    154.   12.3
```

Computing the t -statistic

```
# Extract values
n1 ← cattle_summary$n[1]
n2 ← cattle_summary$n[2]
m1 ← cattle_summary$mean_wt[1]
m2 ← cattle_summary$mean_wt[2]
s1 ← cattle_summary$sd_wt[1]
s2 ← cattle_summary$sd_wt[2]

# Pooled variance
sp2 ← ((n1 - 1) * s1^2 + (n2 - 1) * s2^2) / (n1 + n2 - 2)

# t-statistic
t_stat ← (m1 - m2) / sqrt(sp2 * (1/n1 + 1/n2))
t_stat
```

```
[1] 9.462409
```

```
# Degrees of freedom  
df ← n1 + n2 - 2  
  
# p-value (two-tailed)  
p_val ← 2 * pt(-abs(t_stat), df)  
p_val
```

```
[1] 9.663383e-10
```

Running `t.test()` in R

The easy way

```
t.test(weight ~ breed, data = cattle, var.equal = TRUE)
```

Two Sample t-test

```
data: weight by breed
t = 9.4624, df = 25, p-value = 9.663e-10
alternative hypothesis: true difference in means between group Breed1 and group Breed2 is not equal
to 0
95 percent confidence interval:
 33.23011 51.71989
sample estimates:
mean in group Breed1 mean in group Breed2
      196.175          153.700
```



Tip

`var.equal = TRUE` performs the **pooled** (Student's) *t*-test, which assumes equal variances. This matches our manual calculation.

Assumptions

Three key assumptions

1. **Equal variances:** $\sigma_1^2 \approx \sigma_2^2$
2. **Normality:** $\varepsilon_{ij} \sim N(0, \sigma^2)$
3. **Independence** of observations

Checking equal variances

The only difference between the two groups should be the **location of the mean** – all else should remain the same.

General guide: $\frac{\text{larger SD}}{\text{smaller SD}} < 2.0$

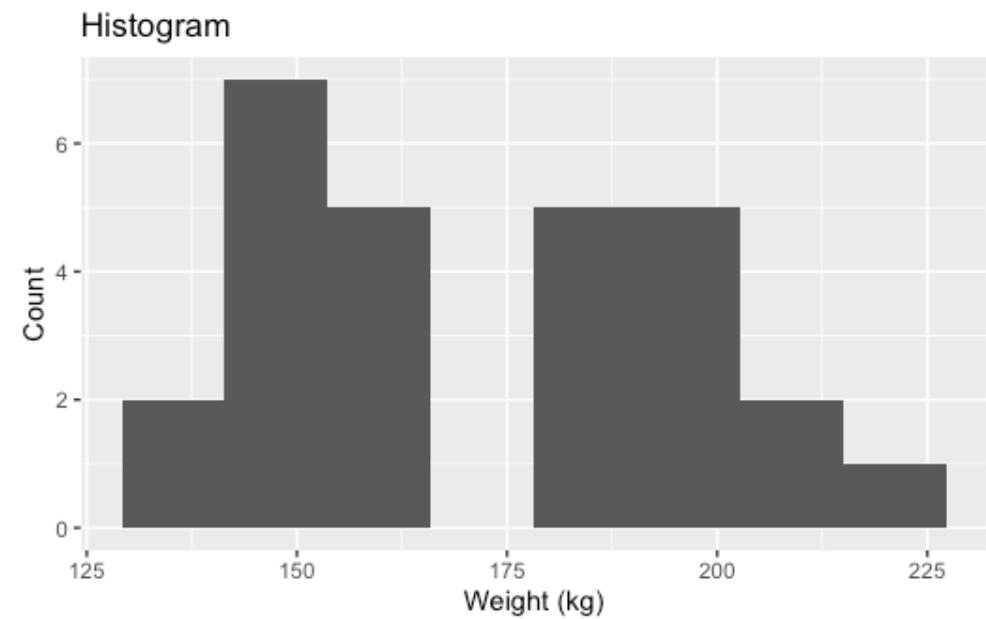
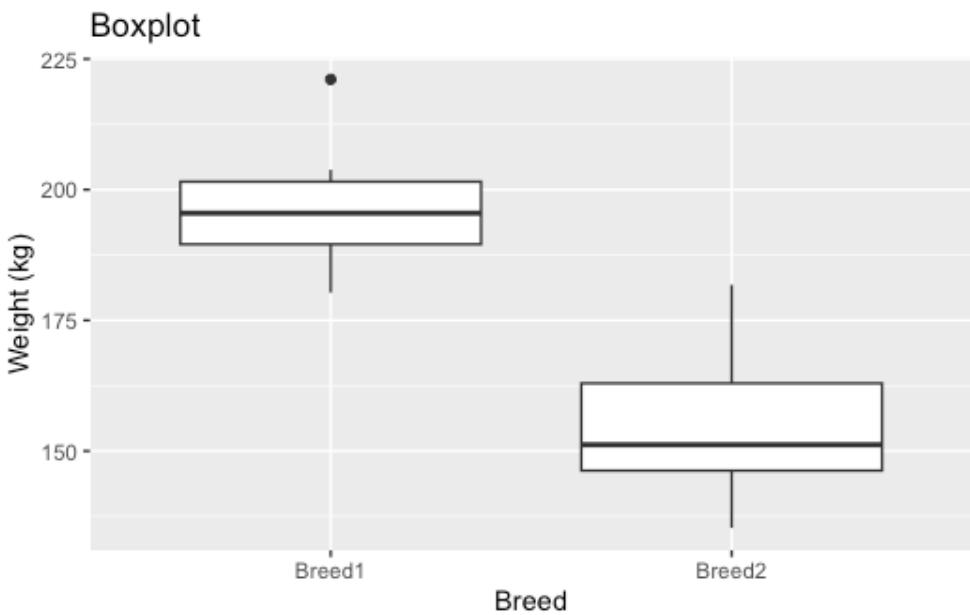
```
max(cattle_summary$sd_wt) / min(cattle_summary$sd_wt)
```

```
[1] 1.158756
```

The ratio is well under 2, so we can assume equal variances.

Checking normality: visual

```
ggplot(cattle, aes(breed, weight)) +  
  geom_boxplot() +  
  labs(title = "Boxplot", x = "Breed", y = "Weight (kg)")  
ggplot(cattle, aes(weight)) +  
  geom_histogram(bins = 8) +  
  labs(title = "Histogram", x = "Weight (kg)", y = "Count")
```



Checking normality: Shapiro-Wilk test

```
shapiro.test(cattle$weight)
```

```
Shapiro-Wilk normality test

data: cattle$weight
W = 0.93704, p-value = 0.103
```

- If $p > 0.05$, the data is **not** significantly different from a normal distribution, *i.e.* we can assume normality.

What if assumptions are not met?

- **Equal variances** not met – use Welch's *t*-test (the default in R's `t.test()` when `var.equal = FALSE`)
- **Normality** not met – if $N > 30$, we can often assume normality anyway (**Central Limit Theorem**)
- **Independence** not met – consider a **paired *t*-test** (e.g. before/after measurements on the same subjects)

Summary

Key points

- The **two-sample *t*-test** compares means of two independent groups
- The test statistic follows a *t*-distribution with $n_1 + n_2 - 2$ degrees of freedom
- Always check assumptions: **equal variances, normality, independence**
- Use `t.test()` in R for quick analysis

Next up

- What if we have **more than two groups**? We need **ANOVA**!

Thanks!

Questions?

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