

Tutorial 10: PCA and Factor Analysis

ENVX2001 – Applied Statistical Methods

Semester 1

PCA and Factor Analysis

Students in class gave “Characteristics of a good ecology lecturer”. They scored from 1-10 on a range of characteristics:

- accessible
- attractive
- Wide range of topics
- Do teaching research
- Dress well
- Complex issues
- Example exam questions
- Notes before
- humour
- Easy marking
- Move around lecture
- Detailed lecture notes
- Clear objective to lecture
- Solicit questions
- Use textbook closely
- Focus on primary research
- Use their own research
- Teach in team
- Understand principles of teaching
- Use pointers
- respected

The students also indicated whether they were:

- a gender (male/female)
- studied mainly a system (marine/terrestrial)
- studied mainly an organismal group (plants/animals)

We are going to use principal components analysis and factor analysis to interpret this dataset.

Setup

Load these packages to make the plots look better:

```
CODE
library(EFAtools)
library(ggplot2)
library(ggfortify)
```

Load the data

First open your dataset:

```
CODE
lecturers <- read.csv("data/lecturers.csv", header = TRUE)
```

Correlation matrix

Let's first do a correlation matrix:

```
CODE
cor(lecturers[, 8:28])
```

```
OUTPUT
```

	accessible	attractive	wide_range
accessible	1.00000000	-0.61876084	0.190062066
attractive	-0.61876084	1.00000000	-0.011570169
wide_range	0.19006207	-0.01157017	1.000000000
teaching.based_research	0.24021638	-0.21382943	0.462870058
Dress_nicely	-0.53198037	0.81887024	-0.022848836
focus_on_complex_issues	0.28252341	-0.04447540	0.567443460
gives_example	-0.23669223	0.23435837	0.065815857
gives_out_note	-0.31173854	0.36819405	-0.187661524
sense_of_humour	-0.29068273	0.33747896	0.081503385
mark_easily	-0.34976811	0.44361281	-0.158545138
moves_around	-0.23371225	0.18475768	-0.087292839
offers_lecture_notes	-0.25762452	0.28256570	-0.070931605
clear_objective	-0.21819811	0.14978520	-0.176518448
solicit_questions	0.10349600	-0.07667948	-0.022123679
text_book	0.03170769	0.02849978	-0.241108100
primary_research	0.14147306	-0.06238516	-0.009628388
own_research	-0.01089281	0.15175696	0.157013029
teach_in_team	0.09004589	-0.13904104	0.005163136
understand_principles	-0.08521319	-0.01080764	-0.097173242
use_a_pointer	-0.51717009	0.25526667	-0.131935123
well_respected	-0.12789662	0.04222022	0.037698097
teaching.based_research	0.24021638	-0.21382943	0.462870058
Dress_nicely	-0.53198037	0.81887024	-0.022848836
focus_on_complex_issues	0.28252341	-0.04447540	0.567443460

gives_example	-0.23626944	0.207509283	
gives_out_note	-0.13154529	0.349298764	
sense_of_humour	-0.37115541	0.326981372	
mark_easily	-0.26802282	0.293513890	
moves_around	0.35915941	0.175612992	
offers_lecture_notes	-0.26062206	0.304730151	
clear_objective	-0.10137698	0.162176313	
solicit_questions	0.06997014	-0.047353486	
text_book	-0.02511112	0.065661794	
primary_research	0.08627931	-0.125765210	
own_research	0.24450347	0.131114596	
teach_in_team	0.05621459	-0.081891992	
understand_principles	0.13360929	0.003830791	
use_a_pointer	-0.40503932	0.246189390	
well_respected	-0.21574299	0.082434616	
	focus_on_complex_issues	gives_example	gives_out_note
accessible	0.28252341	-0.23669223	-0.31173854
attractive	-0.04447540	0.23435837	0.36819405
wide_range	0.56744346	0.06581586	-0.18766152
teaching_based_research	0.36612054	-0.23626944	-0.13154529
Dress_nicely	-0.07452266	0.20750928	0.34929876
focus_on_complex_issues	1.00000000	-0.15606162	-0.31138949
gives_example	-0.15606162	1.00000000	0.41459959
gives_out_note	-0.31138949	0.41459959	1.00000000
sense_of_humour	-0.07005663	0.22759892	0.25028755
mark_easily	-0.18475242	0.18880641	0.39259669
moves_around	0.06116819	-0.15496603	0.26746010
offers_lecture_notes	0.04438636	0.67139954	0.32650197
clear_objective	-0.05740760	0.22152350	0.49329825
solicit_questions	0.12547982	-0.10034117	0.37306647
text_book	-0.14904151	0.23189356	0.41705241
primary_research	0.33824172	-0.05502391	-0.00258233
own_research	0.21007669	-0.03858742	0.11486585
teach_in_team	-0.05016868	0.22073735	0.07101795
understand_principles	0.12582565	0.06367067	0.09564742
use_a_pointer	-0.19722690	0.29421612	0.42798936
well_respected	-0.15321258	0.20022401	0.24197769
	sense_of_humour	mark_easily	moves_around
accessible	-0.29068273	-0.349768108	-0.233712249
attractive	0.33747896	0.443612809	0.184757685
wide_range	0.08150339	-0.158545138	-0.087292839
teaching_based_research	-0.37115541	-0.268022817	0.359159406
Dress_nicely	0.32698137	0.293513890	0.175612992
focus_on_complex_issues	-0.07005663	-0.184752419	0.061168190
gives_example	0.22759892	0.188806413	-0.154966026
gives_out_note	0.25028755	0.392596691	0.267460102
sense_of_humour	1.00000000	0.279195853	0.147981262
mark_easily	0.27919585	1.000000000	-0.102624026
moves_around	0.14798126	-0.102624026	1.000000000
offers_lecture_notes	0.40350521	0.004540766	0.116586008
clear_objective	0.30494267	0.241714172	0.328029643
solicit_questions	0.22196924	0.221507736	0.357039548
text_book	0.11515826	0.493885950	0.018944976
primary_research	-0.11402253	0.013988221	0.201361759
own_research	0.19400676	-0.136905883	0.521788320
teach_in_team	-0.13889755	-0.029214646	-0.002012914
understand_principles	0.08973153	0.029511600	0.281044416
use_a_pointer	0.60147995	0.200483903	0.247735287
well_respected	0.31777004	0.204840208	0.107275533
	offers_lecture_notes	clear_objective	solicit_questions
accessible	-0.257624516	-0.21819811	0.103495996
attractive	0.282565698	0.14978520	-0.076679483
wide_range	-0.070931605	-0.17651845	-0.022123679
teaching_based_research	-0.260622062	-0.10137698	0.069970142
Dress_nicely	0.304730151	0.16217631	-0.047353486

focus_on_complex_issues	0.044386363	-0.05740760	0.125479819
gives_example	0.671399539	0.22152350	-0.100341166
gives_out_note	0.326501972	0.49329825	0.373066471
sense_of_humour	0.403505211	0.30494267	0.221969236
mark_easily	0.004540766	0.24171417	0.221507736
moves_around	0.116586008	0.32802964	0.357039548
offers_lecture_notes	1.000000000	0.51135418	0.134906607
clear_objective	0.511354179	1.000000000	0.667857876
solicit_questions	0.134906607	0.66785788	1.000000000
text_book	0.241763186	0.50985104	0.514856144
primary_research	0.027238866	-0.05933422	0.008953955
own_research	0.014724611	0.04037952	0.313649567
teach_in_team	0.134589361	0.09431599	0.129502715
understand_principles	0.204933977	0.52773521	0.525180942
use_a_pointer	0.475656687	0.48740938	0.245720968
well_respected	0.173541917	0.20806935	0.217669413
text_book primary_research own_research			
accessible	0.031707686	0.141473062	-0.01089281
attractive	0.028499784	-0.062385161	0.15175696
wide_range	-0.241108100	-0.009628388	0.15701303
teaching_based_research	-0.025111119	0.086279306	0.24450347
Dress_nicely	0.065661794	-0.125765210	0.13111460
focus_on_complex_issues	-0.149041509	0.338241715	0.21007669
gives_example	0.231893556	-0.055023907	-0.03858742
gives_out_note	0.417052411	-0.002582330	0.11486585
sense_of_humour	0.115158256	-0.114022533	0.19400676
mark_easily	0.493885950	0.013988221	-0.13690588
moves_around	0.018944976	0.201361759	0.52178832
offers_lecture_notes	0.241763186	0.027238866	0.01472461
clear_objective	0.509851042	-0.059334220	0.04037952
solicit_questions	0.514856144	0.008953955	0.31364957
text_book	1.000000000	-0.092231864	-0.11384268
primary_research	-0.092231864	1.000000000	0.35299282
own_research	-0.113842676	0.352992824	1.000000000
teach_in_team	0.176402014	-0.085933891	-0.08668898
understand_principles	0.287773155	0.017218953	0.06370238
use_a_pointer	0.001337384	-0.149851010	0.15399945
well_respected	0.274222528	0.016210854	-0.10140227
teach_in_team understand_principles use_a_pointer			
accessible	0.090045886	-0.085213191	-0.517170092
attractive	-0.139041035	-0.010807636	0.255266673
wide_range	0.005163136	-0.097173242	-0.131935123
teaching_based_research	0.056214590	0.133609295	-0.405039321
Dress_nicely	-0.081891992	0.003830791	0.246189390
focus_on_complex_issues	-0.050168682	0.125825652	-0.197226905
gives_example	0.220737347	0.063670673	0.294216117
gives_out_note	0.071017950	0.095647419	0.427989355
sense_of_humour	-0.138897546	0.089731525	0.601479950
mark_easily	-0.029214646	0.029511600	0.200483903
moves_around	-0.002012914	0.281044416	0.247735287
offers_lecture_notes	0.134589361	0.204933977	0.475656687
clear_objective	0.094315990	0.527735214	0.487409381
solicit_questions	0.129502715	0.525180942	0.245720968
text_book	0.176402014	0.287773155	0.001337384
primary_research	-0.085933891	0.017218953	-0.149851010
own_research	-0.086688981	0.063702380	0.153999453
teach_in_team	1.000000000	0.101857142	-0.016777680
understand_principles	0.101857142	1.000000000	0.104631048
use_a_pointer	-0.016777680	0.104631048	1.000000000
well_respected	0.545915684	0.207782811	0.318541502
well_respected			
accessible	-0.12789662		
attractive	0.04222022		
wide_range	0.03769810		
teaching_based_research	-0.21574299		

Dress_nicely	0.08243462
focus_on_complex_issues	-0.15321258
gives_example	0.20022401
gives_out_note	0.24197769
sense_of_humour	0.31777004
mark_easily	0.20484021
moves_around	0.10727553
offers_lecture_notes	0.17354192
clear_objective	0.20806935
solicit_questions	0.21766941
text_book	0.27422253
primary_research	0.01621085
own_research	-0.10140227
teach_in_team	0.54591568
understand_principles	0.20778281
use_a_pointer	0.31854150
well_respected	1.00000000

CODE

```
Corrmatrix <- cor(lecturers[, 8:28])
```

Bartlett's Test of Sphericity

Now we can do the Bartlett's Test of Sphericity. This test compares the correlation matrix to an identity matrix. If it is significant, it is worth doing a PCA.

CODE

```
BARTLETT(Corrmatrix, N = 34, cor_method = c("pearson"))
```

OUTPUT

✓ The Bartlett's test of sphericity was significant at an alpha level of .05.
These data are probably suitable for factor analysis.

$\chi^2(210) = 355.07, p < .001$

Principal Components Analysis

Note: to make this a PCA based on a correlation matrix, we have to scale the variables, hence `scale = TRUE`. There are two main principal components functions, but they are very similar. Note `prcomp` calls the loadings "rotations", not to be confused with rotations below.

CODE

```
pca1 <- prcomp(lecturers[, 8:28], scale = TRUE)
pca2 <- princomp(lecturers[, 8:28], cor = TRUE)
```

CODE

```
summary(pca1)
```

OUTPUT

Importance of components:

	PC1	PC2	PC3	PC4	PC5	PC6	PC7
Standard deviation	2.2206	1.6616	1.5324	1.29807	1.19561	1.12098	1.08311
Proportion of Variance	0.2348	0.1315	0.1118	0.08024	0.06807	0.05984	0.05586
Cumulative Proportion	0.2348	0.3663	0.4781	0.55833	0.62641	0.68624	0.74211
	PC8	PC9	PC10	PC11	PC12	PC13	PC14
Standard deviation	1.0737	0.95681	0.79545	0.73119	0.7055	0.65580	0.50887
Proportion of Variance	0.0549	0.04359	0.03013	0.02546	0.0237	0.02048	0.01233
Cumulative Proportion	0.7970	0.84060	0.87073	0.89619	0.9199	0.94037	0.95270
	PC15	PC16	PC17	PC18	PC19	PC20	PC21
Standard deviation	0.49817	0.47309	0.42640	0.36970	0.32493	0.24020	0.19872
Proportion of Variance	0.01182	0.01066	0.00866	0.00651	0.00503	0.00275	0.00188
Cumulative Proportion	0.96452	0.97518	0.98384	0.99034	0.99537	0.99812	1.00000

CODE

```
summary(pca2)
```

OUTPUT

Importance of components:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5
Standard deviation	2.2205654	1.6615887	1.5324030	1.29806969	1.19560985
Proportion of Variance	0.2348053	0.1314703	0.1118219	0.08023738	0.06807062
Cumulative Proportion	0.2348053	0.3662756	0.4780975	0.55833484	0.62640546
	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
Standard deviation	1.12097809	1.08311410	1.07371597	0.95680927	0.79544930
Proportion of Variance	0.05983771	0.05586363	0.05489838	0.04359448	0.03013046
Cumulative Proportion	0.68624317	0.74210679	0.79700517	0.84059965	0.87073010
	Comp.11	Comp.12	Comp.13	Comp.14	Comp.15
Standard deviation	0.73119377	0.70551609	0.65579857	0.50886890	0.49817027
Proportion of Variance	0.02545925	0.02370252	0.02047961	0.01233084	0.01181779
Cumulative Proportion	0.89618936	0.91989188	0.94037149	0.95270232	0.96452011
	Comp.16	Comp.17	Comp.18	Comp.19	
Standard deviation	0.4730918	0.426395520	0.369702631	0.324933440	
Proportion of Variance	0.0106579	0.008657769	0.006508573	0.005027702	
Cumulative Proportion	0.9751780	0.983835781	0.990344355	0.995372057	
	Comp.20	Comp.21			
Standard deviation	0.240198751	0.198724365			
Proportion of Variance	0.002747402	0.001880542			
Cumulative Proportion	0.998119458	1.000000000			

Loadings

Let's look at the loadings. Called "rotations" in `prcomp` and "loadings" in `princomp`. They are the Pearson's correlation between that variable and that Principal Component.

CODE

```
pca1
```

OUTPUT

Standard deviations (1, .., p=21):

```
[1] 2.2205654 1.6615887 1.5324030 1.2980697 1.1956099 1.1209781 1.0831141
[8] 1.0737160 0.9568093 0.7954493 0.7311938 0.7055161 0.6557986 0.5088689
[15] 0.4981703 0.4730918 0.4263955 0.3697026 0.3249334 0.2401988 0.1987244
```

Rotation (n x k) = (21 x 21):

PC1	PC2	PC3	PC4
-----	-----	-----	-----

accessible	-0.27105784	0.21481141	0.22284903	-0.109380126
attractive	0.25953819	-0.19001404	-0.35322829	0.047967149
wide_range	-0.11796047	0.14523503	-0.23536689	-0.465316942
teaching.based_research	-0.17807186	0.33523672	-0.16973235	-0.005540905
Dress_nicely	0.24737662	-0.15550598	-0.32565089	0.019172145
focus_on_complex_issues	-0.13467331	0.28837561	-0.24412817	-0.297540766
gives_example	0.22797978	-0.11739164	0.06984178	-0.437201342
gives_out_note	0.32056620	0.04438472	0.02749136	0.100499687
sense_of_humour	0.27564472	-0.03120757	-0.13957405	-0.145704311
mark_easily	0.23686285	-0.11243202	0.08113616	0.196601185
moves_around	0.13236700	0.32827473	-0.27087444	0.213497862
offers_lecture_notes	0.28307512	0.02357872	-0.01627849	-0.372830743
clear_objective	0.31573398	0.26378859	0.12826236	0.066194937
solicit_questions	0.18991164	0.43659484	0.13380240	0.160366349
text_book	0.21299001	0.16712512	0.33485831	0.124899445
primary_research	-0.05608411	0.19206706	-0.16387602	0.001745603
own_research	0.04318371	0.28387549	-0.37836892	0.056442158
teach_in_team	0.04818137	0.11021896	0.29523823	-0.327405642
understand_principles	0.14556568	0.33936721	0.10777339	0.053017840
use_a_pointer	0.32019196	-0.03811643	-0.08101559	-0.080227102
well_respected	0.19384039	0.06977679	0.22029921	-0.263146206
	PC5	PC6	PC7	PC8
accessible	-0.006862732	0.20175545	0.075192964	0.13471983
attractive	-0.304217305	-0.05278985	-0.006014137	-0.01346850
wide_range	-0.251311171	0.08634713	-0.331523146	0.07096692
teaching.based_research	-0.307853355	-0.22290900	-0.040540304	-0.27328747
Dress_nicely	-0.299180706	-0.12573075	-0.046516486	-0.12380132
focus_on_complex_issues	-0.170654964	0.29340602	-0.050872762	0.07474147
gives_example	-0.075647764	0.06855598	0.364935010	-0.17131172
gives_out_note	-0.125013318	-0.08366732	0.223406707	0.02145305
sense_of_humour	0.216056466	0.21464296	-0.328598912	0.27397871
mark_easily	-0.421810061	0.15983061	-0.046406386	0.38018642
moves_around	0.166526391	-0.35024271	0.055092321	-0.07707096
offers_lecture_notes	0.146235397	0.18757528	0.287000528	-0.27400475
clear_objective	0.060342195	0.17777324	-0.032682046	-0.21837583
solicit_questions	-0.005900941	0.14588578	-0.182965113	0.09074671
text_book	-0.356204508	0.15361788	0.084096939	0.02338965
primary_research	0.039054189	0.06897414	0.592731543	0.45913721
own_research	0.156844077	-0.09496398	0.160942737	0.20826388
teach_in_team	-0.096207102	-0.54758870	0.037625674	0.05473642
understand_principles	0.010243773	0.05372028	-0.114475017	-0.27603888
use_a_pointer	0.411769112	0.01616452	-0.180288938	0.05917537
well_respected	0.013159486	-0.40377696	-0.183055505	0.39655405
	PC9	PC10	PC11	PC12
accessible	0.23711499	0.41979852	0.03945146	0.148881566
attractive	-0.16438427	0.20365082	0.06904042	-0.107432225
wide_range	0.22155409	-0.26438749	-0.03259079	-0.018048262
teaching.based_research	0.19889412	-0.24510749	-0.11135378	0.247899071
Dress_nicely	-0.10406494	0.38639604	0.08377430	0.098387712
focus_on_complex_issues	-0.28979913	0.00432150	0.36226348	-0.066562002
gives_example	0.19510676	-0.15342441	-0.35973985	-0.159175284
gives_out_note	0.41895610	-0.21289394	0.20038838	0.006019332
sense_of_humour	0.09730489	0.19787809	-0.29410062	0.309619454
mark_easily	-0.04288340	-0.25885827	-0.05713892	-0.143735223
moves_around	0.02114182	-0.04788447	0.02682634	0.353171999
offers_lecture_notes	-0.07931457	0.16874959	0.11255136	0.207814735
clear_objective	-0.04989256	-0.08180409	0.29595319	0.009127372
solicit_questions	0.10754501	0.13028684	0.14132328	-0.276862219
text_book	0.12628767	0.17521009	-0.12180066	0.267854799
primary_research	-0.30348754	-0.18921946	0.01954047	0.138689169
own_research	0.28543090	0.27973695	-0.29907250	-0.427500246
teach_in_team	-0.07446630	0.20427502	0.21413563	-0.322896702
understand_principles	-0.49690454	-0.08895650	-0.50033599	-0.189676003
use_a_pointer	0.09757553	-0.27912944	0.22992547	-0.115886008
well_respected	-0.19000184	-0.03498965	-0.08926599	0.276488059

	PC13	PC14	PC15	PC16
accessible	0.29231904	0.29580819	-0.222218786	0.095682695
attractive	0.10512874	0.20004636	0.069413634	-0.326657310
wide_range	0.14339377	-0.15578216	-0.042633744	-0.346118318
teaching.based_research	-0.16129758	-0.08569727	-0.291561886	0.304862272
Dress_nicely	0.22024728	-0.29025496	-0.196064822	0.331319175
focus_on_complex_issues	-0.11651678	0.21121417	0.380088085	0.275036237
gives_example	-0.02596566	0.13003779	0.073844050	-0.048615414
gives_out_note	0.52366782	0.22815294	0.150396690	0.187752138
sense_of_humour	-0.09102492	0.24787583	-0.269328590	0.077361879
mark_easily	-0.30618081	0.26528469	-0.239118638	0.067806707
moves_around	-0.19413727	0.41519824	0.163481854	-0.233951595
offers_lecture_notes	-0.18626080	-0.01943147	-0.008128194	-0.028997162
clear_objective	0.02299500	-0.21161788	-0.392005665	-0.377275931
solicit_questions	0.07959209	-0.06601352	0.067535709	-0.098815936
text_book	-0.31596633	-0.28217199	0.346092878	0.063696048
primary_research	0.11269387	-0.18656539	-0.263215477	-0.012502540
own_research	-0.14671997	-0.23213427	0.082887276	0.001508984
teach_in_team	-0.23420220	0.19002827	-0.271548974	0.056217287
understand_principles	0.29771815	0.14826763	-0.020146024	0.175848894
use_a_pointer	-0.10543967	-0.11603812	-0.008762183	0.433779829
well_respected	0.22357927	-0.22236358	0.251201419	-0.060389007
	PC17	PC18	PC19	PC20
accessible	-0.487523780	-0.092943639	0.11623660	-0.08184741
attractive	-0.106937105	-0.195239242	0.45086924	-0.09762660
wide_range	0.009616636	0.068156323	0.05963088	-0.34244915
teaching.based_research	0.006376281	0.018683190	0.22919635	0.23688191
Dress_nicely	-0.124236446	0.174946567	-0.38290347	-0.11111538
focus_on_complex_issues	0.057689702	-0.197551507	-0.21480326	0.19658872
gives_example	-0.206970055	0.030539850	-0.40647428	-0.01600380
gives_out_note	0.321857496	-0.107588372	0.08121002	0.10250939
sense_of_humour	0.430141107	-0.113398698	-0.08853507	0.02641527
mark_easily	-0.269651092	0.167631153	0.01976224	0.19779113
moves_around	-0.207902119	0.101506396	-0.24054414	-0.20539739
offers_lecture_notes	0.024256519	0.371837654	0.41986631	0.20173823
clear_objective	-0.097333800	-0.419634434	-0.21848656	0.23314508
solicit_questions	0.094290088	0.606166863	-0.05758082	-0.04550053
text_book	0.072364215	-0.254638772	0.07071928	-0.35855399
primary_research	0.130404141	0.041767487	0.01625913	-0.25697656
own_research	-0.068753059	-0.196939862	0.08026083	0.24801010
teach_in_team	0.247146896	-0.095095343	0.00645715	-0.19776650
understand_principles	-0.001605969	-0.075193184	0.12987739	-0.13337875
use_a_pointer	-0.352414119	-0.113600108	0.19774694	-0.34608373
well_respected	-0.231089402	0.008802903	0.03736378	0.36974689
	PC21			
accessible	-0.002932735			
attractive	0.405148279			
wide_range	-0.308435374			
teaching.based_research	0.335322467			
Dress_nicely	-0.150318850			
focus_on_complex_issues	0.045604055			
gives_example	0.338040434			
gives_out_note	-0.184254445			
sense_of_humour	0.144536843			
mark_easily	-0.277239849			
moves_around	-0.151918843			
offers_lecture_notes	-0.289119982			
clear_objective	-0.008051690			
solicit_questions	0.367111373			
text_book	-0.026709294			
primary_research	0.123144467			
own_research	-0.206478451			
teach_in_team	-0.055864220			
understand_principles	-0.183080778			

use_a_pointer	0.121125177
well_respected	0.079347988

CODE

loadings(pca2)

OUTPUT

Loadings:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8
accessible	0.271	0.215	0.223	0.109		0.202		0.135
attractive	-0.260	-0.190	-0.353		0.304			
wide_range	0.118	0.145	-0.235	0.465	0.251		-0.332	
teaching_based_research	0.178	0.335	-0.170		0.308	-0.223		-0.273
Dress_nicely	-0.247	-0.156	-0.326		0.299	-0.126		-0.124
focus_on_complex_issues	0.135	0.288	-0.244	0.298	0.171	0.293		
gives_example	-0.228	-0.117		0.437			0.365	-0.171
gives_out_note	-0.321			-0.100	0.125		0.223	
sense_of_humour	-0.276		-0.140	0.146	-0.216	0.215	-0.329	0.274
mark_easily	-0.237	-0.112		-0.197	0.422	0.160		0.380
moves_around	-0.132	0.328	-0.271	-0.213	-0.167	-0.350		
offers_lecture_notes	-0.283			0.373	-0.146	0.188	0.287	-0.274
clear_objective	-0.316	0.264	0.128			0.178		-0.218
solicit_questions	-0.190	0.437	0.134	-0.160		0.146	-0.183	
text_book	-0.213	0.167	0.335	-0.125	0.356	0.154		
primary_research		0.192	-0.164				0.593	0.459
own_research		0.284	-0.378		-0.157		0.161	0.208
teach_in_team		0.110	0.295	0.327		-0.548		
understand_principles	-0.146	0.339	0.108				-0.114	-0.276
use_a_pointer	-0.320				-0.412		-0.180	
well_respected	-0.194		0.220	0.263		-0.404	-0.183	0.397
	Comp.9	Comp.10	Comp.11	Comp.12	Comp.13	Comp.14	Comp.15	
accessible	0.237	0.420		0.149	0.292	0.296	0.222	
attractive	-0.164	0.204		-0.107	0.105	0.200		
wide_range	0.222	-0.264			0.143	-0.156		
teaching_based_research	0.199	-0.245	-0.111	0.248	-0.161		0.292	
Dress_nicely	-0.104	0.386			0.220	-0.290	0.196	
focus_on_complex_issues	-0.290		0.362		-0.117	0.211	-0.380	
gives_example	0.195	-0.153	-0.360	-0.159		0.130		
gives_out_note	0.419	-0.213	0.200		0.524	0.228	-0.150	
sense_of_humour		0.198	-0.294	0.310		0.248	0.269	
mark_easily		-0.259		-0.144	-0.306	0.265	0.239	
moves_around				0.353	-0.194	0.415	-0.163	
offers_lecture_notes		0.169	0.113	0.208	-0.186			
clear_objective			0.296			-0.212	0.392	
solicit_questions	0.108	0.130	0.141	-0.277				
text_book	0.126	0.175	-0.122	0.268	-0.316	-0.282	-0.346	
primary_research	-0.303	-0.189		0.139	0.113	-0.187	0.263	
own_research	0.285	0.280	-0.299	-0.428	-0.147	-0.232		
teach_in_team		0.204	0.214	-0.323	-0.234	0.190	0.272	
understand_principles	-0.497		-0.500	-0.190	0.298	0.148		
use_a_pointer		-0.279	0.230	-0.116	-0.105	-0.116		
well_respected	-0.190			0.276	0.224	-0.222	-0.251	
	Comp.16	Comp.17	Comp.18	Comp.19	Comp.20	Comp.21		
accessible		0.488		0.116				
attractive	-0.327	0.107	0.195	0.451		-0.405		
wide_range	-0.346				0.342	0.308		
teaching_based_research	0.305			0.229	-0.237	-0.335		
Dress_nicely	0.331	0.124	-0.175	-0.383	0.111	0.150		
focus_on_complex_issues	0.275		0.198	-0.215	-0.197			
gives_example		0.207		-0.406		-0.338		
gives_out_note	0.188	-0.322	0.108		-0.103	0.184		

sense_of_humour		-0.430	0.113					-0.145	
mark_easily		0.270	-0.168				-0.198	0.277	
moves_around	-0.234	0.208	-0.102	-0.241	0.205	0.152			
offers_lecture_notes			-0.372	0.420	-0.202	0.289			
clear_objective	-0.377		0.420	-0.218	-0.233				
solicit_questions			-0.606					-0.367	
text_book			0.255			0.359			
primary_research		-0.130				0.257	-0.123		
own_research			0.197			-0.248	0.206		
teach_in_team		-0.247				0.198			
understand_principles	0.176			0.130	0.133	0.183			
use_a_pointer	0.434	0.352	0.114	0.198	0.346	-0.121			
well_respected		0.231				-0.370			

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9
SS loadings	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Proportion Var	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
Cumulative Var	0.048	0.095	0.143	0.190	0.238	0.286	0.333	0.381	0.429

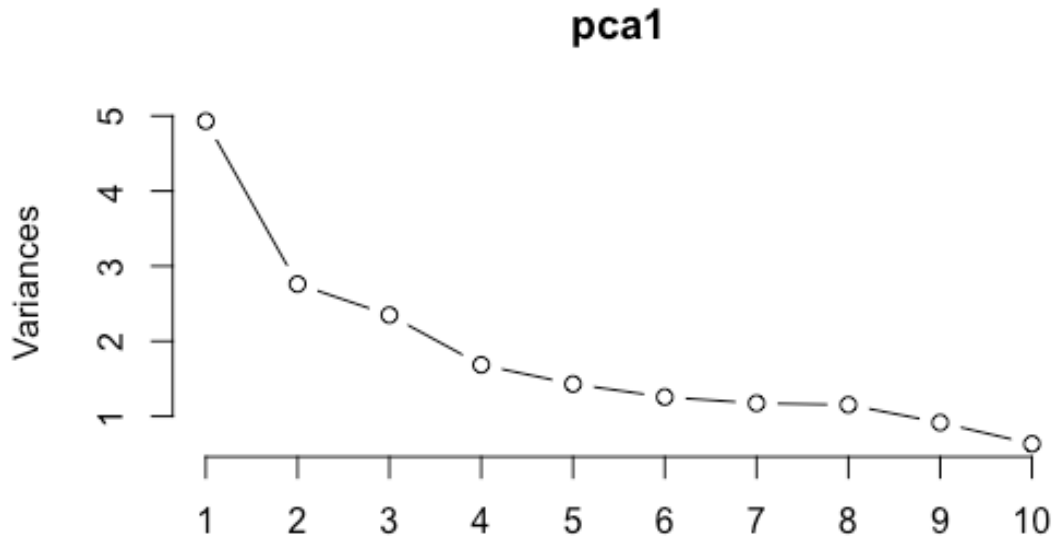
	Comp.10	Comp.11	Comp.12	Comp.13	Comp.14	Comp.15	Comp.16	Comp.17
SS loadings	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Proportion Var	0.048	0.048	0.048	0.048	0.048	0.048	0.048	0.048
Cumulative Var	0.476	0.524	0.571	0.619	0.667	0.714	0.762	0.810

	Comp.18	Comp.19	Comp.20	Comp.21
SS loadings	1.000	1.000	1.000	1.000
Proportion Var	0.048	0.048	0.048	0.048
Cumulative Var	0.857	0.905	0.952	1.000

Screeplot

To do a screeplot, follow the commands below. Note this is the standard deviations, which are just square root of the variances or eigenvalues.

```
CODE
screeplot(pca1, type = "lines")
```



Principal Component Scores

To get your principal components scores for plotting and analysis, do the following:

CODE

```
pca1$x
```

OUTPUT

	PC1	PC2	PC3	PC4	PC5	PC6
[1,]	0.834364400	-0.44633674	-0.7389322	0.6505980	0.67162071	-0.059426515
[2,]	-0.057591233	2.43259604	0.3643921	0.1044273	-1.35364156	-1.513379283
[3,]	-1.414356172	-0.85486631	0.2591048	-0.7950895	0.39445964	-0.362282532
[4,]	-3.007651543	-1.54426710	1.8013306	-1.4495208	-0.06900951	-0.378437639
[5,]	-0.913357854	0.15572053	1.9852660	0.2647465	0.05455313	-0.190589811
[6,]	2.715375811	-0.61458292	1.2581973	0.6291478	1.66664928	0.980101358
[7,]	-3.260274779	-1.46022078	-1.3182136	-1.5445289	2.59801832	1.373084699
[8,]	1.690020920	4.10059174	-0.8286511	-2.7204201	-0.89839668	0.477869956
[9,]	-2.017591018	-0.19055562	1.2282190	-0.1591693	-1.81724286	-0.504760633
[10,]	2.684027310	-0.28238921	-2.7053968	-0.6373616	1.24913045	0.114945320
[11,]	1.602080176	1.73848677	-0.7560946	-0.1669273	1.46214681	-1.241067456
[12,]	3.864282071	0.91902758	-3.0506154	-0.3116270	0.14774890	-1.707816527
[13,]	-1.537400621	0.79553155	0.2734490	2.2991348	-1.33509038	1.020459263
[14,]	-3.180580364	-1.55370059	-1.1043790	0.3994455	-0.21820422	-0.416321249
[15,]	0.251424671	-0.87357501	0.8416185	1.3248859	1.62509470	-1.332946815
[16,]	2.892895033	-0.54107588	-0.4987349	0.5314171	-0.40738282	-0.008629769
[17,]	2.025139659	0.87994104	2.1794942	1.4021183	-1.10315491	-1.185333328
[18,]	-4.509205462	2.93089784	-2.9570995	3.4152682	0.56608967	1.421754123
[19,]	-0.363910804	0.59536235	1.0533136	-0.1849003	-0.12823425	-0.808435775
[20,]	0.513833400	1.34820539	1.7227901	-1.3132714	-1.04480943	0.987729132

[21,]	0.671318696	-1.57314535	0.3233421	-0.3775963	1.07805278	0.122220926
[22,]	-1.199789658	-0.75223096	0.2125650	-1.9753352	-0.38988416	2.577574944
[23,]	2.699925571	-4.05301101	-2.2580650	0.1089181	-2.08232998	0.035139868
[24,]	3.671607537	-2.76308860	2.3108314	1.9763565	-0.12222446	1.145347105
[25,]	1.708338444	1.14870366	0.1941423	-0.8978228	0.53891069	1.218476886
[26,]	0.697044049	-0.09475043	-0.9235260	0.5556583	1.51777743	-0.726583470
[27,]	-3.637428809	-1.19350763	-0.2178772	-1.3363292	-0.26449368	-3.193959083
[28,]	0.919217152	1.59671229	1.9213986	0.1968770	0.58065254	0.264448784
[29,]	-0.012679393	2.32518659	0.2155462	0.3501560	-0.47396645	0.658332807
[30,]	-2.075112687	-0.04538414	0.2156446	-1.0632633	0.42197992	0.038464825
[31,]	-2.320188992	-0.57228262	-0.1349819	2.0161446	-0.36632609	-0.152398000
[32,]	0.005054503	-1.40748778	-2.5517407	-0.5548639	-3.02867573	0.937512584
[33,]	0.061169986	-0.15050467	1.6836625	-0.7372732	0.53018221	0.408905305
[1,]	PC7	PC8	PC9	PC10	PC11	PC12
[2,]	0.94163613	1.01611660	0.65086100	0.7049538165	-0.12679861	1.07464589
[3,]	2.49608290	-1.03057653	0.99789650	-0.6955702786	0.62054036	0.43728658
[4,]	-1.14833452	-0.13690605	1.56172193	-1.0103974039	-0.22663448	-0.62688596
[5,]	1.55864777	-2.37965647	-1.88594310	0.1842266066	0.56734997	-0.37865892
[6,]	-0.90420171	-1.46124279	-0.41077774	0.7877696717	0.87274235	-0.40364293
[7,]	-0.48508037	-0.39417790	-1.03586330	-0.5943650505	-0.20101933	0.39114013
[8,]	0.92263413	0.27756052	0.64943390	0.6951735358	0.85851959	-0.18227075
[9,]	1.01619589	1.42931518	-0.35783368	-0.0705759673	0.38608546	-0.45604663
[10,]	-3.06056251	0.01542140	0.97910235	-0.3689951357	-0.81042854	0.29905809
[11,]	-1.15019858	-2.22756355	1.92108719	1.1858684843	0.03051034	-0.44295070
[12,]	-1.38076963	0.10114808	-1.28404266	0.4344189806	0.24591922	-0.18193776
[13,]	0.63745807	-0.33245072	-0.46272040	-0.4907654143	-0.49725530	0.06959804
[14,]	0.50021492	1.64298989	-0.02204408	-0.0008218307	-0.32919047	-1.47757160
[15,]	0.96910614	-0.33466161	1.43687692	-0.8126002209	-1.05968338	-0.42468108
[16,]	0.80890447	-0.52662610	-0.88626822	0.4773237243	-0.83842767	0.73180898
[17,]	0.99711436	1.01401737	-0.40202982	0.1817443327	-0.53794142	-0.60055239
[18,]	0.20967322	-0.58311856	0.70515256	-0.0396632716	0.45009864	0.68839301
[19,]	-0.81537520	-0.85666393	-0.94273181	0.0414522567	0.07186325	0.63521158
[20,]	-0.39788697	0.90638787	-0.41749861	0.8035047424	-0.32339287	-0.65225431
[21,]	-1.71530837	-0.05810388	-0.44792612	0.6080155185	0.04950397	0.22979058
[22,]	-0.17473258	-0.49430919	-0.74983375	-2.3172182635	-0.86711224	-0.89907281
[23,]	0.96335825	0.59588805	-0.69173585	0.7875438802	-1.58060139	0.69952356
[24,]	-0.79226369	0.31761740	-1.80001880	-0.5911334313	0.33709088	0.50027519
[25,]	0.66242150	0.79456398	1.28229789	1.1369072252	0.36494912	0.29993320
[26,]	0.11491096	-0.67752131	1.55591224	-0.9531517638	-0.43306504	0.96727700
[27,]	-0.97171844	2.00236186	0.03915553	0.0979107090	1.01294064	-0.86467646
[28,]	-0.10713776	1.13254478	0.06800519	1.3104870153	-0.78844151	0.38194366
[29,]	-0.11368363	-1.13303668	-0.44515516	-0.0309968966	-0.05286304	-0.17959227
[30,]	0.03995439	-0.06103477	-0.17348145	0.3417751499	-1.22149166	-0.09463653
[31,]	-0.61338026	1.72693492	-0.07276041	-1.2660686396	1.38200547	1.95263797
[32,]	0.79663725	0.35281020	0.08109498	-0.5131215937	0.67712222	-0.13873520
[33,]	-0.05972762	-1.30023954	0.11504776	0.5298008516	1.06058730	-0.29503060
[34,]	0.25541147	0.66221148	0.44501905	-0.5534313394	0.90651818	-1.05932657
[1,]	PC13	PC14	PC15	PC16	PC17	
[2,]	1.04550104	0.30220825	-0.26493487	0.990718773	-0.161522621	
[3,]	-0.51140324	0.23962723	0.04514841	-0.864576510	0.021378514	
[4,]	-0.45367844	0.47042709	0.44536737	0.117387548	1.132044168	
[5,]	0.76623701	0.33414241	-0.06674780	0.005890288	-0.317090528	
[6,]	-0.22473438	1.02668595	-1.11036905	0.253771120	0.573823233	
[7,]	-0.21758119	0.35417045	0.55539524	0.380784887	0.577014348	
[8,]	-1.13189958	-0.40654870	0.30620976	0.183130027	-0.149934970	
[9,]	0.48772693	-0.20754494	0.30040817	0.432882057	-0.190155729	
[10,]	0.47865039	0.06044313	0.29758301	0.065987000	-0.412015158	
[11,]	-0.03562393	0.19804842	0.33881205	-0.396898429	-0.531313384	
[12,]	-0.90180692	-0.34546312	-0.75620771	0.625598888	-0.544476288	
[13,]	0.30509441	0.08813338	-0.58059025	-0.035026694	0.625823661	
[14,]	-0.73594105	0.53970030	-0.79585891	0.555871566	-0.252855797	
[15,]	-0.21555707	-0.91839176	-1.10063830	-0.346724418	-0.207470713	
[16,]	0.72072562	-0.64024769	-0.07151878	-0.333963902	0.392495606	
[17,]	0.13959352	0.68049473	0.84486635	-0.265106061	-0.372669045	
[18,]	-1.02024905	-0.87052303	0.58413566	0.839988817	0.250265142	

```

[18,] 0.77126745 -0.26031481 0.33705965 -0.046675725 0.049282576
[19,] 0.70815779 0.48976813 0.01791635 -0.847195425 -0.125274155
[20,] 0.62144737 -0.75001136 -0.14219657 -0.508865779 0.007346965
[21,] 0.41210426 -0.10629725 0.27929458 0.308950842 -0.697452881
[22,] -0.59841689 0.14146264 0.02533706 -0.177525294 0.552971887
[23,] -0.77530358 -0.21170311 -0.19235313 -0.271578336 -0.009956336
[24,] 0.33558108 -0.22233178 -0.25908437 -0.300951777 -0.380190914
[25,] 0.64085526 0.78994926 -0.57225157 0.361706690 -0.160414591
[26,] 0.13507319 0.08748313 0.02580035 -0.785383761 0.456930349
[27,] -0.10431199 0.13635919 0.47578097 0.572230799 -0.118992564
[28,] -1.18521749 -0.05682740 0.35960572 -0.270080750 -0.621317543
[29,] -0.75999067 -0.33173999 0.05271739 -0.414813487 0.312481307
[30,] -0.34096948 0.30426194 -0.35124709 -0.539719364 -0.359452130
[31,] 0.15560337 0.44934089 0.86956307 0.051061891 -0.041440723
[32,] 0.33869730 -0.18428438 0.18487185 0.420292080 0.248554770
[33,] 1.13036896 -1.18047718 -0.08187461 0.238832439 0.453583546

      PC18      PC19      PC20      PC21
[1,] 0.512534055 0.403637255 -0.2288274019 -0.180597545
[2,] 0.929440714 0.048122004 0.2051146098 -0.271152883
[3,] 0.403516656 -0.074179505 -0.2401958070 -0.046316428
[4,] -0.380413260 -0.016801083 0.0236080284 0.075154120
[5,] -0.121436088 0.216713273 -0.2532722027 0.163180806
[6,] 0.016193332 -0.176022291 0.8096079044 -0.052570563
[7,] -0.041349452 0.491356049 0.1502223084 -0.205615503
[8,] -0.169826040 -0.182268894 -0.0271317045 0.331935144
[9,] -0.170416032 0.311027931 0.4390800414 -0.032482679
[10,] -0.228929235 -0.629795038 -0.1373141246 0.072909759
[11,] 0.223418908 -0.399313078 0.1457129024 -0.346000339
[12,] -0.361282411 0.136577651 0.1213875759 0.000801914
[13,] 0.048394699 -0.297878346 0.0857017322 -0.076965204
[14,] -0.556266739 0.183329956 0.1575056075 0.005626377
[15,] 0.003248088 -0.275646771 -0.0488034493 -0.010278083
[16,] -0.569143575 0.610124017 0.0001699299 -0.165370989
[17,] -0.876273716 0.145493206 -0.3407035497 -0.133229363
[18,] 0.139800530 0.266416571 -0.0612808299 0.138282533
[19,] -0.288699113 -0.005113258 0.1267147465 -0.263464768
[20,] 0.261595957 0.070258777 -0.3355360658 -0.432480871
[21,] 0.478432361 -0.146679334 -0.3985086902 -0.071646635
[22,] -0.090751620 -0.213675088 -0.1815364089 -0.142428632
[23,] 0.266768773 0.195774478 -0.1770461524 0.191720088
[24,] 0.354865033 -0.239914062 0.0937794098 0.244839284
[25,] -0.161337294 0.208473109 -0.0508044281 0.105092550
[26,] -0.040945263 0.377943641 -0.1672829144 0.091569499
[27,] 0.419873948 -0.083337959 0.0482601803 0.312179849
[28,] 0.322418197 0.609269862 -0.0166547989 0.337744117
[29,] 0.091695107 -0.301130070 -0.0161094691 0.322694249
[30,] -0.412731002 -0.414631089 0.0294813311 0.084790553
[31,] -0.298987891 -0.721677547 -0.2130771535 -0.111634161
[32,] 0.187128445 -0.059177362 0.2716590271 -0.102730154
[33,] 0.109463929 -0.037277006 0.1860798156 0.166443959

```

So to combine them with the original variables, do this:

```

CODE
lecturerpcscores ← cbind(lecturers, pca1$x)

```

Biplot

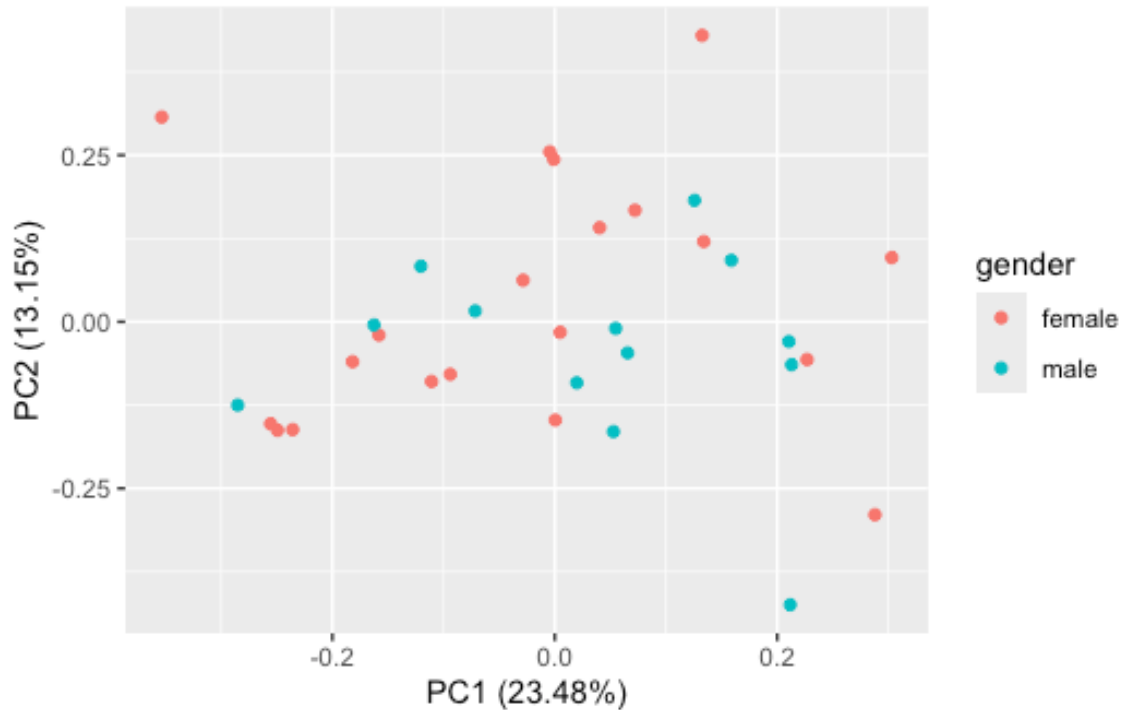
We can produce a biplot in ggplot2, using the factor gender as the colour:

CODE

```
autoplot(pca1, data = lecturerpcscores, colour = 'gender')
```

OUTPUT

```
Warning: `aes_string()` was deprecated in ggplot2 3.0.0.  
i Please use tidy evaluation idioms with `aes()``.  
i See also `vignette("ggplot2-in-packages")` for more information.  
i The deprecated feature was likely used in the ggfortify package.  
Please report the issue at <https://github.com/sinhrks/ggfortify/issues>.
```



We could add the loadings, but they are a bit messy, best to look at loadings or rotations tables.

You can make the plot using different symbols, change background etc through reading the ggplot2 documentation.

ANOVA on PC scores

You can use this file to do the ANOVAs, you will have to do yourself. For example for differences in PC scores for gender:

CODE

```
AOV1 <- aov(lecturerpcscores$PC1 ~ gender, data = lecturerpcscores)  
summary(AOV1)
```

OUTPUT

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
gender	1	4.6	4.596	0.93	0.342
Residuals	31	153.2	4.942		

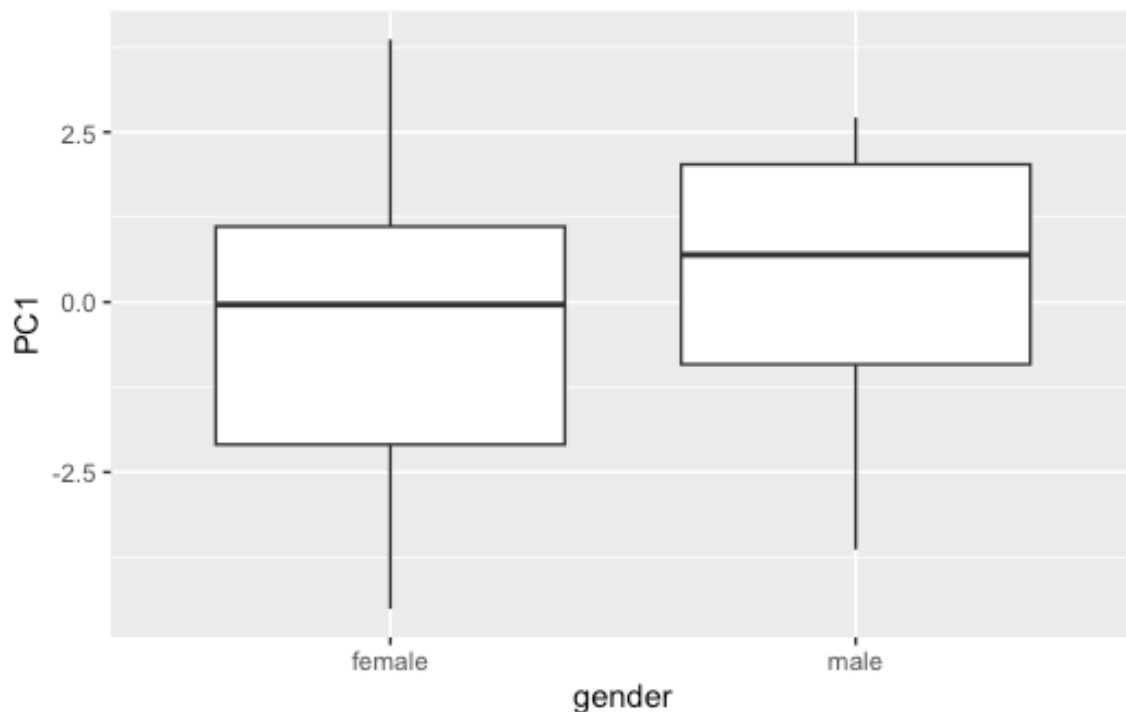
You can do something similar for your projects and regression, using your PC scores as the predictors.

Boxplot

To do a boxplot of variables use ggplot2:

CODE

```
ggplot(lecturerpcscores, aes(x = gender, y = PC1)) +  
  geom_boxplot()
```



Factor Analysis with Varimax Rotation

To do a Varimax rotation on your principal components, follow these commands. Do a factor analysis with the rotation being Varimax:

CODE

```
fa1 ← factanal(lecturers[, 8:28], 10, rotation = "varimax")  
fa1
```

OUTPUT

Call:

```
factanal(x = lecturers[, 8:28], factors = 10, rotation = "varimax")
```

Uniquenesses:

accessible	attractive	wide_range
0.381	0.005	0.005
teaching.based_research	Dress_nicely	focus_on_complex_issues
0.005	0.283	0.332
gives_example	gives_out_note	sense_of_humour
0.005	0.448	0.473
mark_easily	moves_around	offers_lecture_notes
0.005	0.294	0.141
clear_objective	solicit_questions	text_book
0.244	0.005	0.310
primary_research	own_research	teach_in_team
0.310	0.237	0.548
understand_principles	use_a_pointer	well_respected
0.584	0.067	0.005

Loadings:

	Factor1	Factor2	Factor3	Factor4	Factor5	Factor6	Factor7
accessible		-0.583	-0.426	0.222	-0.110		-0.130
attractive		0.961	0.106				0.158
wide_range	-0.177			0.952			
teaching.based_research			-0.356	0.374	-0.114		
Dress_nicely		0.829	0.100				
focus_on_complex_issues	0.116			0.674		-0.163	-0.141
gives_example		0.122	0.112		0.955	0.163	0.132
gives_out_note	0.362	0.290	0.216	-0.196	0.305	0.125	0.261
sense_of_humour	0.127	0.229	0.599	0.125			0.108
mark_easily	0.150	0.298	0.143				0.910
moves_around	0.326	0.244	0.186	-0.102	-0.155		-0.194
offers_lecture_notes	0.362	0.240	0.333		0.667		-0.232
clear_objective	0.768	0.109	0.343		0.169		
solicit_questions	0.880	-0.139	0.130	0.109	-0.156		0.169
text_book	0.611		-0.141	-0.152	0.166	0.202	0.427
primary_research							
own_research	0.100			0.129		-0.112	-0.102
teach_in_team	0.154		-0.103		0.155	0.614	
understand_principles	0.608					0.155	
use_a_pointer	0.164	0.146	0.902	-0.142	0.165		
well_respected	0.133		0.287			0.931	0.101

	Factor8	Factor9	Factor10
accessible			
attractive		-0.121	
wide_range	0.102	0.137	-0.126
teaching.based_research	0.158	0.819	
Dress_nicely			
focus_on_complex_issues			0.369
gives_example			
gives_out_note	0.244		-0.120
sense_of_humour		-0.226	
mark_easily	-0.113	-0.106	
moves_around	0.429	0.444	0.217
offers_lecture_notes	-0.159		0.167
clear_objective			
solicit_questions	0.326		
text_book	-0.146		
primary_research	0.213		0.790
own_research	0.784	0.119	0.255
teach_in_team			
understand_principles		0.110	
use_a_pointer	0.109		


```

well_respected          -0.100

      Factor1 Factor2 Factor3 Factor4 Factor5 Factor6 Factor7 Factor8
SS loadings      2.654  2.398  1.996  1.705  1.667  1.423  1.325  1.137
Proportion Var   0.126  0.114  0.095  0.081  0.079  0.068  0.063  0.054
Cumulative Var   0.126  0.241  0.336  0.417  0.496  0.564  0.627  0.681
      Factor9 Factor10
SS loadings      1.039  0.970
Proportion Var   0.049  0.046
Cumulative Var   0.731  0.777

Test of the hypothesis that 10 factors are sufficient.
The chi square statistic is 28.56 on 45 degrees of freedom.
The p-value is 0.973

```