MCE 431 - HW5 - Elias Newall-Vuillemot

5.7: Sketch the root locus w/ respect to K for the equation $1+KL(s)=0$. Give asymptotes, arrival, & departure angles @ any complex zero or pole. Verify in MATLAB

a) $L(s)=\frac{s+3}{s(s+10)(s^2+2s+2)} \rightarrow \begin{matrix} m=1 & \text{zeros: } -3 \\ n=4 & \text{Poles: } 0, -10, -1\pm j \end{matrix}$

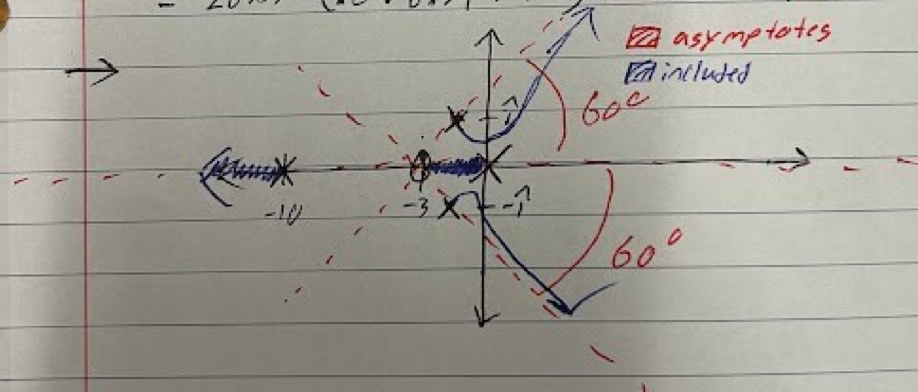
$\rightarrow = \frac{s+3}{s^4+12s^3+22s^2+20s}$

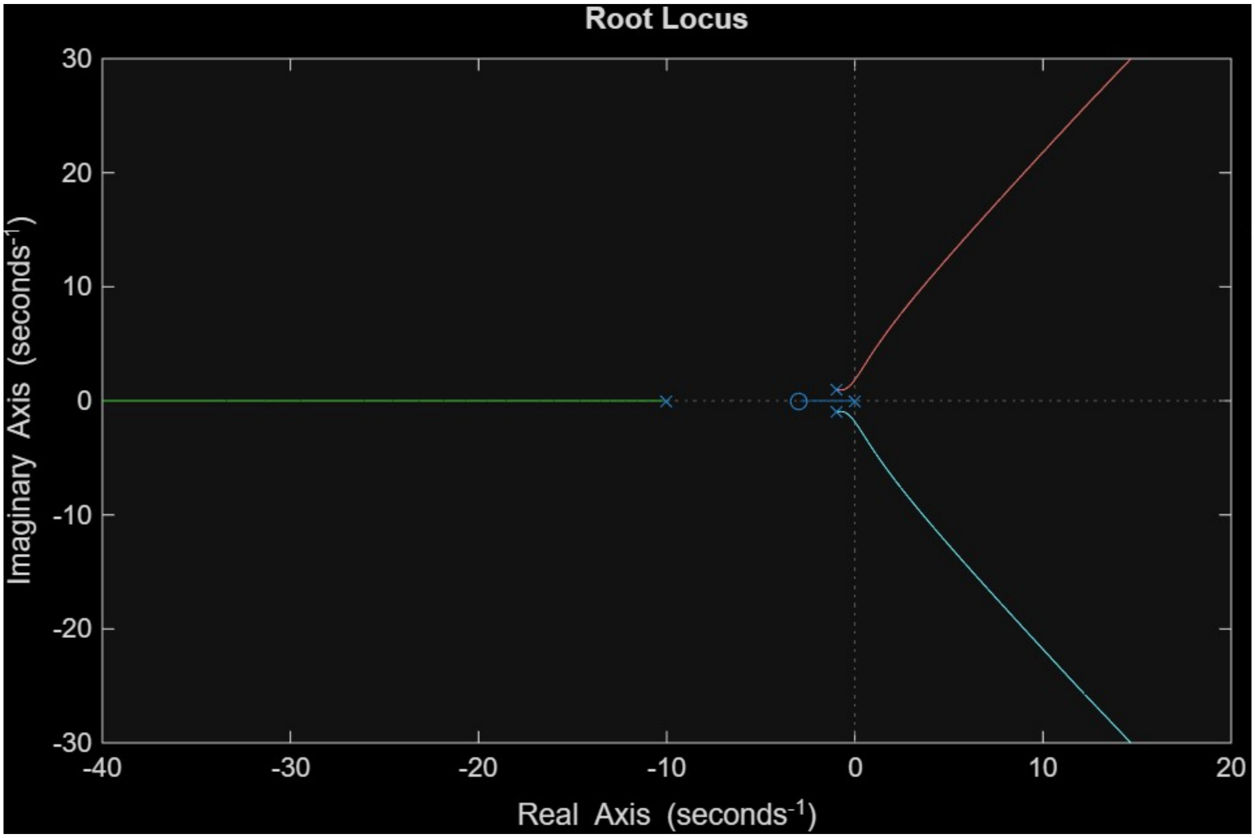
$n-m=3 \rightarrow$ asymptotes @ $180°$ & $\pm 60°$

$\alpha = \frac{b_1 - a_1}{n-m} = \frac{3-12}{3} = -3$

takeoff angle $= \phi = \sum_{i=1}^{m} \angle(P_k - z_i) - \sum_{j=1}^{n} \angle(P_k - P_j) + 180$

$= 26.57 - (90 + 6.34 + 135) + 180 = -24.8$




Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
30
20
10
0
-10
-20
-30
-40
-30
-20
-10
0
10
20

3) $L(s) = \frac{(s+3)(s^2+4s+68)}{s^2(s+10)(s^2+4s+85)} \Rightarrow$ $m=3$ zeros: -3 & $-2 \pm 8j$
$n=5$, poles: $\underset{\times 2}{0}$, -10, & $-2 \pm 9j$

$\Rightarrow = \frac{s^3+7s^2+80s+204}{s^5+14s^4+125s^3+850s^2}$

$n-m=2 \Rightarrow$ asymptotes @ $\pm 90°$

$\alpha = \frac{b_1-a_1}{n-m} = \frac{7-14}{2} = -3.5$

departure angle: $\phi = \sum_{i=1}^{m} \angle(p_k - z_i) - \sum_{j=1}^{n} \angle(p_k - p_j) + 180$

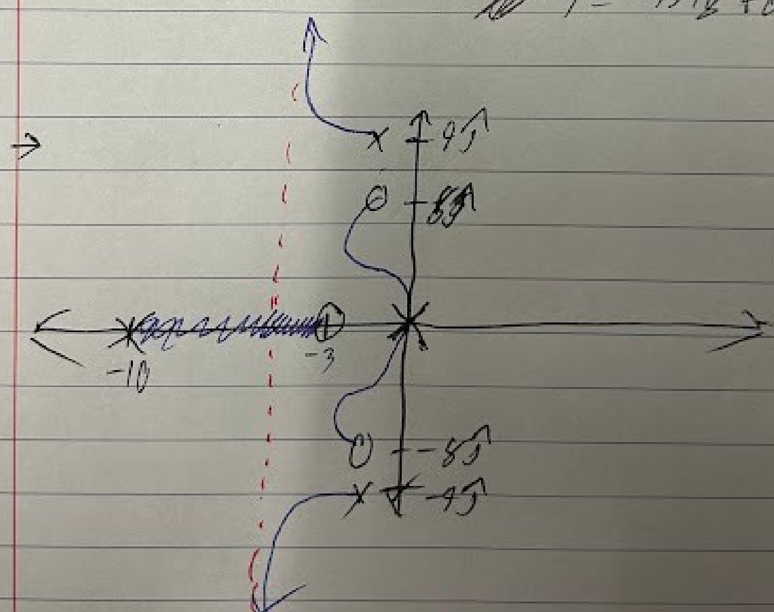
$= (83.66 + 90 + 90) - (102.53 + 48.37 + 90 + 102.53) + 180$

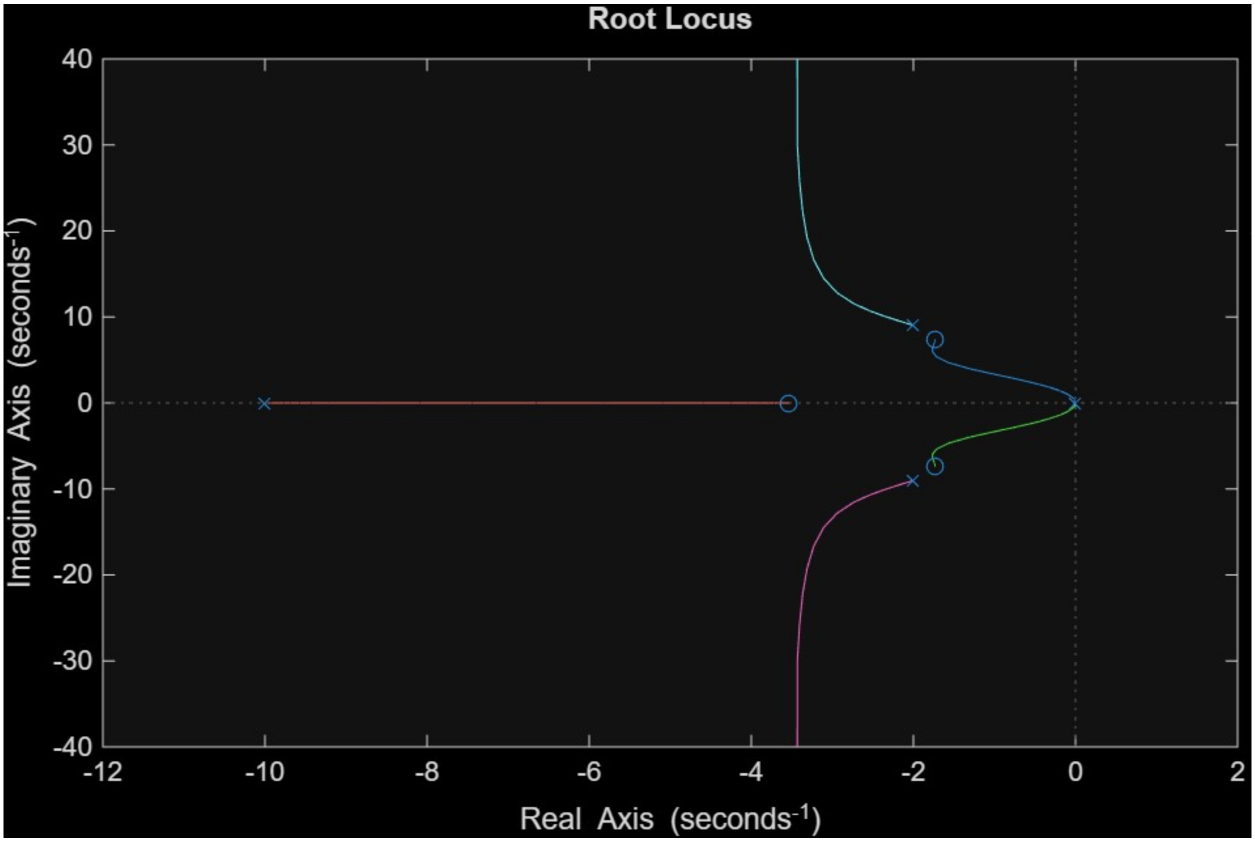
$= 100.2°$ for $-2+9j$

arrival angle: $\psi = -\sum_{i=1}^{m} \angle(z_k - z_i) + \sum_{j=1}^{n} \angle(z_k - p_j) + 180$

$= -(82.87+90) + (104.04 + 104.04 + 45 - 90 + 90) + 180 = 260.2 = -99.8$

$\psi = -99.8$ for $-2+8j$




Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
40
30
20
10
0
-10
-20
-30
-40
-12
-10
-8
-6
-4
-2
0
2

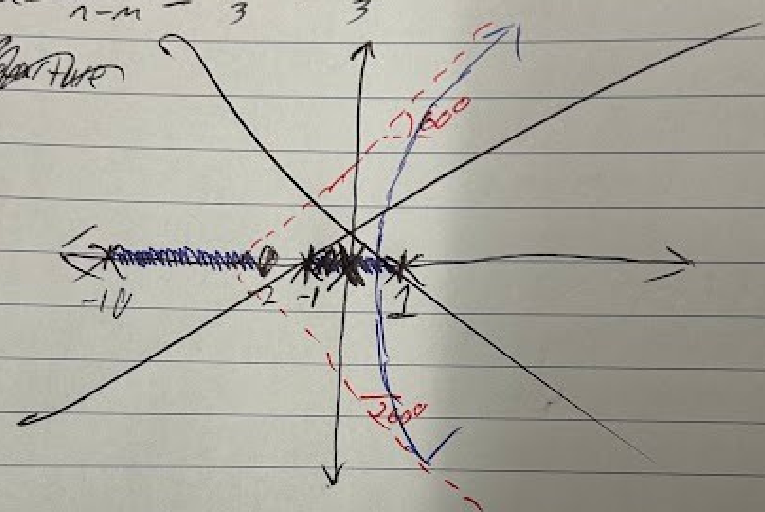
5.8; b) $L(s) = \frac{s+2}{s(s+10)(s^2-1)} \Rightarrow \begin{matrix} m=1 & \text{zeros: } -2 \\ n=4 & \text{poles: } 0, -10, \pm 1 \end{matrix}$

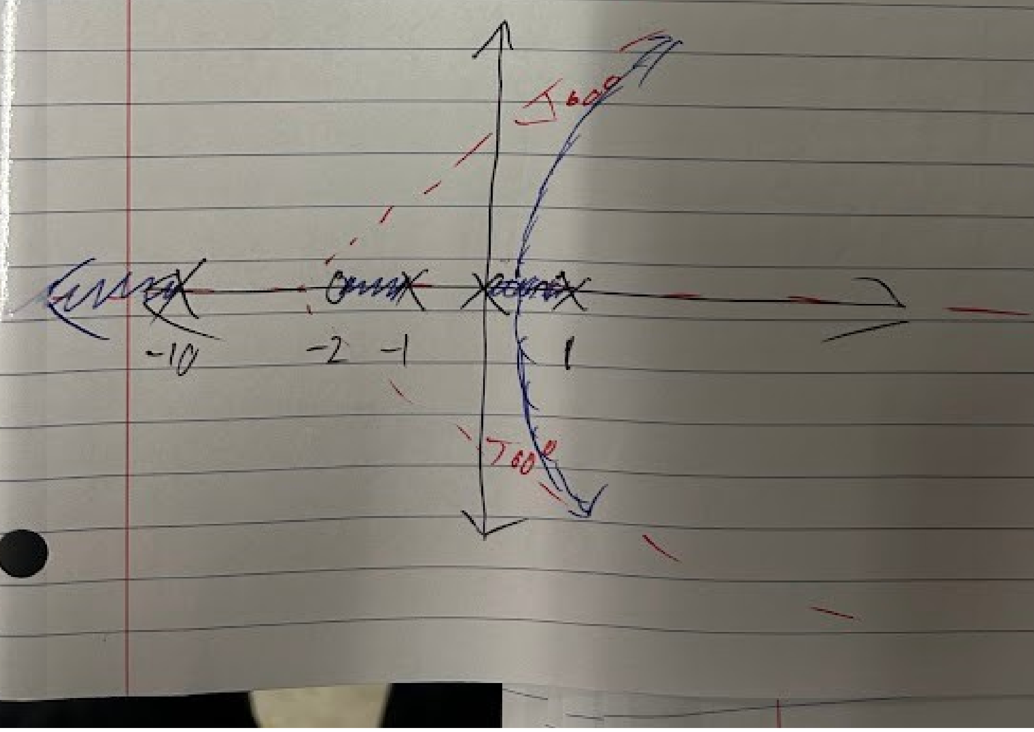
$\Rightarrow = \frac{s+2}{s^4+10s^3-s^2-10s}$

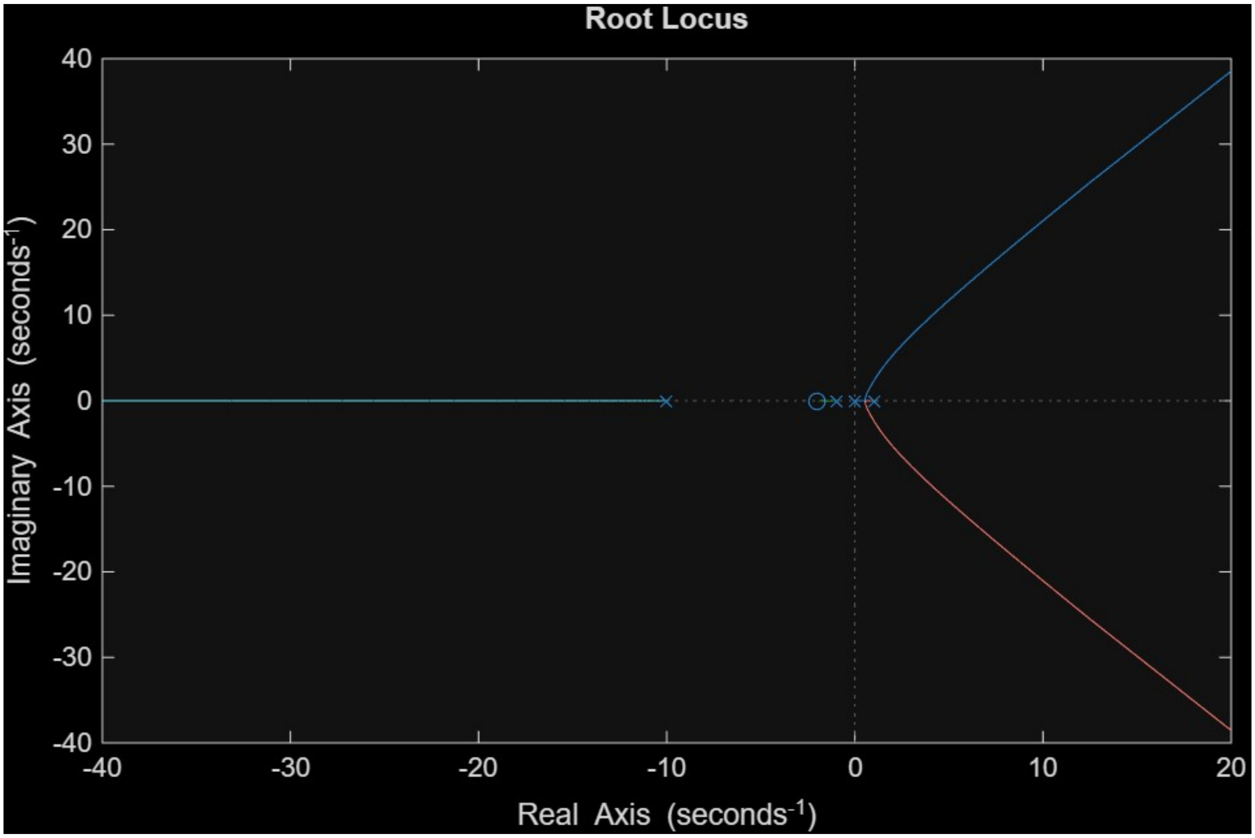
$n-m=3 \Rightarrow$ asymptotes @ 180^0 & $\pm 60^0$

$\alpha = \frac{b_1 - a_1}{n-m} = \frac{2-10}{3} = \frac{-8}{3}$

Departure







Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
40
30
20
10
0
-10
-20
-30
-40
-40
-30
-20
-10
0
10
20

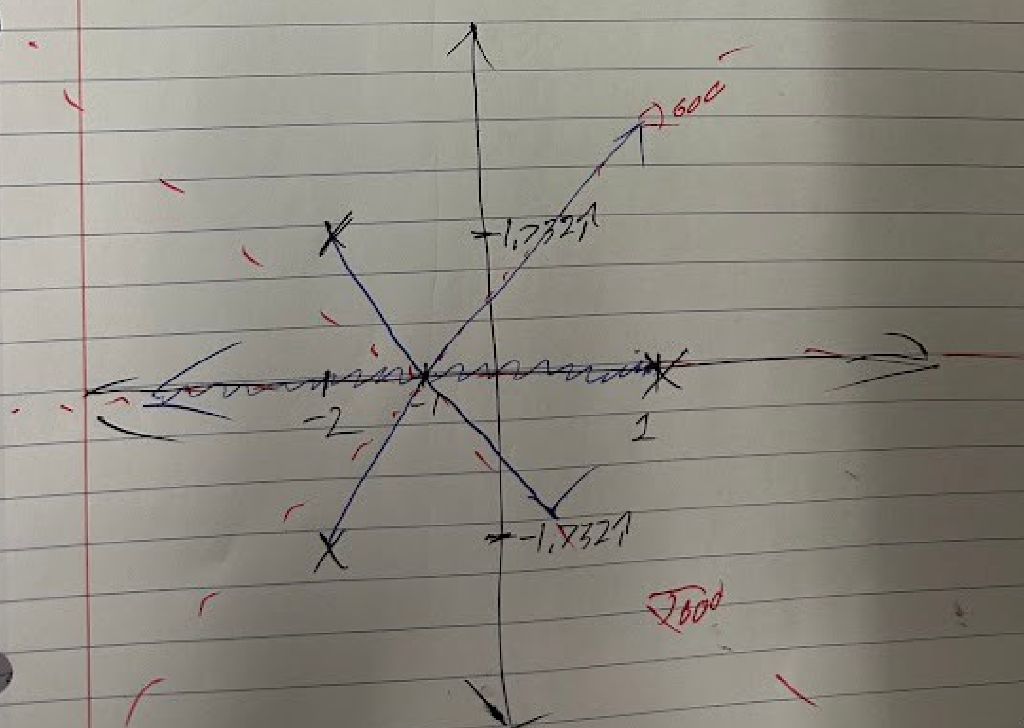
(f) $\frac{1}{(s-1)[(s+2)^2+3]} = \frac{1}{s^3+3s^2+3s-7}$ = m=0 zeros none
= n=3 poles: 1 & $-2 \pm 1.732j$

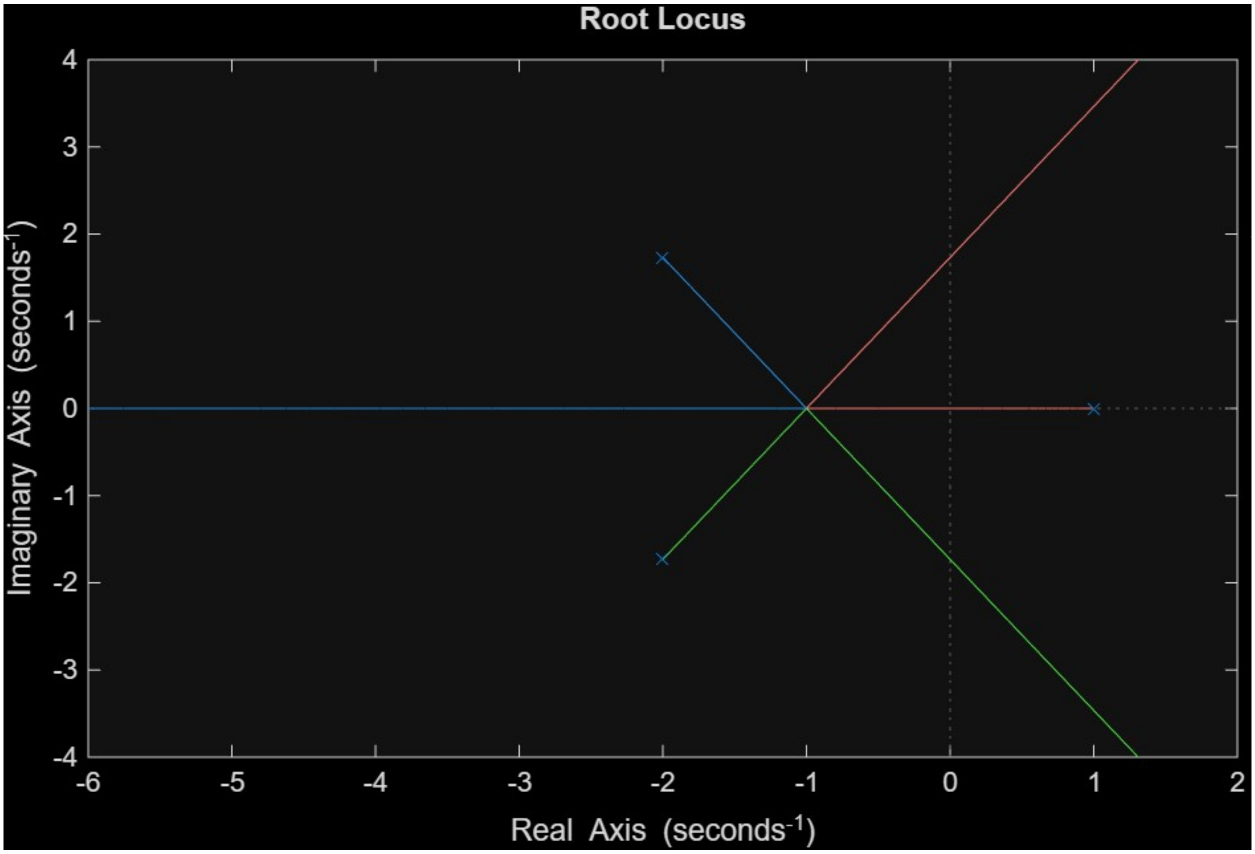
$n-m=3 \Rightarrow$ asymptotes @ 180^0 & $\pm 60^0$

departure angle $= \phi = 180 - \sum_{j=1}^{2}(P_K - P_j)$

$= 180 - (150+90) = -60^0$ for $-2+1.732j$

$\alpha = \frac{b_1 - a_1}{n-m} = \frac{0-3}{3} = -1$




Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
4
3
2
1
0
-1
-2
-3
-4
-6
-5
-4
-3
-2
-1
0
1
2