

$$1) \theta_A = \frac{\pi}{6} \rightarrow R_A = \begin{bmatrix} \cos \theta_A & -\sin \theta_A \\ \sin \theta_A & \cos \theta_A \end{bmatrix}$$

$$P_A = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\rightarrow P_B = R_B^A P_A = \begin{bmatrix} \cos \frac{\pi}{6} & -\sin \frac{\pi}{6} \\ \sin \frac{\pi}{6} & \cos \frac{\pi}{6} \end{bmatrix} \begin{bmatrix} 4 \\ 4 \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$

$$2) \theta_A = \frac{\pi}{4} \rightarrow R_A = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$P_B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\rightarrow P_A = R_A^B P_B = \begin{bmatrix} 4.95 \\ 0.71 \end{bmatrix}$$

$$3) R_A^B = \begin{bmatrix} \cos \frac{\pi}{6} & -\sin \frac{\pi}{6} \\ \sin \frac{\pi}{6} & \cos \frac{\pi}{6} \end{bmatrix} \theta_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad R_A^B = \begin{bmatrix} \cos \frac{\pi}{6} & \sin \frac{\pi}{6} \\ -\sin \frac{\pi}{6} & \cos \frac{\pi}{6} \end{bmatrix} = [R_B^A]^{-1}$$

$$\rightarrow H_B^A = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} & 3 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow P_B = H_B^A P_A = \begin{bmatrix} 1.866 \\ 1.232 \\ 1 \end{bmatrix}$$

$$H_A^B = \begin{bmatrix} [R_A^B] & [R_B^A] \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & -3.6 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_B = [H_A^B] [P_A] = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & -3.6 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.866 \\ 1.232 \\ 1 \end{bmatrix}$$

$$\rightarrow x = 1.866 \quad y = 1.232$$

$$4. \quad O_B^A = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \theta_B^A = \frac{\pi}{4} \quad \rho_B^A = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$H_B^A = \begin{bmatrix} \cos \frac{\pi}{4} & -\sin \frac{\pi}{4} & 3 \\ \sin \frac{\pi}{4} & \cos \frac{\pi}{4} & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\rho^A = H_B^A \rho^B = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 3 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.29 \\ 6.45 \\ 1 \end{bmatrix}$$

$$x = 2.24 \quad y = 6.45$$