

3.53°

$$a) \quad s^4 + 5.2s^3 + 18.9s^2 + 43.2s + 45.4$$

	a_1	a_2	a_3	a_4
s^4	1	18.9	43.2	45.4
s^3	5.2	43.2	0	
s^2	10.6	45.4	0	
s^1	20.9	0	0	
s^0	45.4	0	0	

No sign switches \rightarrow 0

$$b) \quad s^5 + 1.02s^4 + 1.123s^3 + 0.686s^2 + 0.154s + 2$$

s^5	1	1.123	0.154		
s^4	0.102	0.686	2		
s^3	-5.6	-14.5	0		
s^2	0.3	2	0		
s^1	14.3	0	0		
s^0	2	0	0		

2 sign switches \rightarrow 2

$$c) \quad s^4 + 152s^3 + 12s^2 - 1932s - 4921$$

s^4	1	12	-4921	
s^3	152	-1932	0	
s^2	24.7	-4921	0	
s^1	28338	0	0	
s^0	-4921	0	0	

1 sign switch \rightarrow 1

d) $99s^3 - s^2 - 6s - 7$

$$\begin{array}{c|cc} s^3 & 1 & -\frac{6}{99} \\ s^2 & -\frac{1}{99} & -\frac{7}{99} \\ s^1 & -7.1 & 0 \\ s^0 & -\frac{7}{99} & 0 \end{array}$$

1 sign switch $\rightarrow \boxed{1}$

e) $s^4 + 8s^2 + 36$

$$s^4 \mid 1 \ 8 \ 36 \quad (1s^4 + 8s^2 + 36) \frac{s}{s} = [4, 16, 0]$$

$$\begin{array}{c|ccc} s^3 & 0 & 0 & 0 & \text{row of zeros} \end{array}$$

$$s^2$$

$$s^1$$

$$s^0$$

$$\begin{array}{c|ccc} s^4 & 1 & 8 & 36 \end{array}$$

$$\rightarrow s^3 \mid 4 \ 16 \ 0$$

$$s^2 \mid 4 \ 36 \ 0$$

$$s^1 \mid -20 \ 0 \ 0$$

$$s^0 \mid 36 \ 0 \ 0$$

2 sign switches $\rightarrow \boxed{2}$

2.55: $KG(s) = \frac{k(s+6)}{s[(s+7)(s+1.2)(s^2+8s+6)]}$

$$1 + KG(s) = 0 \rightarrow s[(s+7)(s+1.2)(s^2+8s+6)] + k(s+6) = 0$$

$$= s^4 + 2.7s^3 + 8.36s^2 + (12.072+k)s + 11.04$$

→ using Routh's stability criterion

s^4	1	8.36	11.04
s^3	2.7	(12.072+k)	0
s^2	$\frac{10.5-k}{2.7}$	11.04	0
s^1	$\frac{-k^2-1.572k+46.3}{10.5-k}$	0	0
s^0	11.04	0	0

for the system to be stable;

$$\left(\left(\frac{10.5-k}{2.7} > 0 \right) \& \left(\frac{-k^2-1.572k+46.3}{10.5-k} > 0 \right) \right) = 1$$

$$\Rightarrow k < 10.5 \& \cancel{6.06} -7.64 < k < 6.06$$

$$\Rightarrow -7.64 < k < 6.06$$

3.54:

$$s^4 + (10+k_2)s^3 + (121+k_1)s^2 + (k_1+k_1k_2+110k_2+210)s + (11k_1+110)$$

for all coeffs to be positive:

$$k_2 > -10, k_1 > -121, k_1 > -9.09, \& k_1+k_1k_2+110k_2 > 210$$

$$a_1 = 10+k_2, a_2 = 121+k_1, a_3 = k_1+k_1k_2+110k_2+210 \& a_4 = 11k_1+110$$

for Routh's stability criterion:

$$\begin{array}{c|ccc} s^4 & 1 & a_2 & a_4 \\ s^3 & a_1 & a_3 & 0 \\ s^2 & & & \\ s^1 & & & \\ s^0 & & & \end{array}$$

$$\begin{array}{c|ccc} s^4 & 1 & a_2 & a_4 \\ s^3 & a_1 & a_3 & 0 \\ s^2 & \frac{a_1a_2-a_3}{a_1} & 0 & 0 \\ s^1 & c_1 & 0 & 0 \\ s^0 & a_4 & 0 & 0 \end{array}$$

$$\frac{a_1a_2-a_3}{a_1} = b_1$$

$$\frac{a_3b_1-a_1a_4}{b_1} = c_1$$

$$\rightarrow \text{additionally } \frac{a_1a_2-a_3}{a_1} > 0 \& \frac{a_3b_1-a_1a_4}{b_1} > 0$$

$$\rightarrow k_1 > -9.09 \& k_2 > -1$$



