3.53: Use Routh to determine if the following are stable

a) $KG(s) = \dfrac{4(s+2)}{s(s^3+2s^2+3s+4)}$

$\rightarrow 1 + KG(s) = 0$

$\rightarrow s^4+2s^3+3s^2+4s+4s+8=0=s^4+2s^3+3s^2+8s+8=0$

$\rightarrow a_1, a_2 \ldots = 2, 3, 8, 8$

$$\rightarrow \begin{array}{c|ccc} s^4 & 1 & 3 & 8 \\ s^3 & 2 & 8 & 0 \\ s^2 & -1 & 8 & 0 \\ s^1 & \cancel{16}\,24 & 0 & 0 \\ s^0 & 8 & 0 & 0 \end{array}$$

$\rightarrow$ 2 sign switches $\rightarrow$ unstable

b) $KG(s) = \dfrac{2(s+4)}{s^2(s+1)}$

$\rightarrow s^2(s+1)+2(s+4)=0=s^3+s^2+2s+8=0$

$\rightarrow a_1, a_2, \ldots = 1, 2, 8$

$$\rightarrow \begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 1 & 8 \\ s^1 & -6 & 0 \\ s^0 & 8 & 0 \end{array}$$

$\rightarrow$ 2 sign switches $\rightarrow$ unstable

c) $KG(s) = \dfrac{4(s^3+2s^2+s+1)}{s^2(s^3+2s^2-s-1)}$

$\rightarrow s^5+2s^4-s^3-s^2+4s^3+8s^2+4s+4=s^5+2s^4+3s^3+7s^2+4s+4$

$\rightarrow a_1, a_2, \ldots = 2, 3, 7, 4, 4$

$$\rightarrow \begin{array}{c|ccc} s^5 & 1 & 3 & 4 \\ s^4 & 2 & 7 & 4 \\ s^3 & -.5 & 2 & 0 \\ s^2 & 15 & 4 & 0 \\ s^1 & 2.133 & 0 & 0 \\ s^0 & 4 & 0 & 0 \end{array}$$

$\rightarrow$ 2 sign switches $\rightarrow$ unstable

3.55: Find the range of k for which all roots are in the LHP

$$s^5+5s^4+10s^3+10s^2+5s+k=0$$

Using Routh's: For $a_1, a_2, \ldots = 5, 10, 10, 5, k$

$$\begin{array}{c|ccc}
s^5 & 1 & 10 & 5 \\
s^4 & 5 & 10 & k \\
s^3 & 8 & 5-2k & 0 \\
s^2 & \frac{55+10k}{8} & k & 0 \\
s^1 & \frac{-20k^2-124k+275}{55+10k} & 0 & 0 \\
s^0 & k & 0 & 0
\end{array}$$

For stability:

$$\frac{55+10k}{8} > 0 \quad \& \quad \frac{-20k^2-124k+275}{55+10k} > 0 \quad \& \quad k > 0$$

$\rightarrow k > -5.5 \qquad -7.93 < k < 1.73 \qquad k > 0$

$\rightarrow 0 < k < 1.73$

3.59) $s^3+(6+k)s^2+(5+6k)s+5k=0$

stable for real part < -1

$p = s+1 \rightarrow s = p-1$

$\rightarrow (p-1)^3+(6+k)(p-1)^2+(5+6k)(p-1)+5k=0$

$\rightarrow p^3+(3+k)p^2+(4k-4)p+0=0$

$$\rightarrow \begin{array}{c|cc}
p^3 & 1 & 4k-4 \\
p^2 & 3+k & 0 \\
p^1 & 4k-4 & 0 \\
p^0 & 0 & 0
\end{array}$$

$\rightarrow k > -3 \;\&\; k > 1$

$\rightarrow \boxed{k > 1}$