1) $X_B^W = \begin{bmatrix} .433 \\ .250 \\ -.866 \end{bmatrix}, Y_B^W = \begin{bmatrix} .177 \\ .918 \\ .353 \end{bmatrix}, Z_B^W = \begin{bmatrix} .884 \\ -.306 \\ .353 \end{bmatrix}$

For R_B^W of the form:

$$R_B^W = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

where: $X_B^W = R_B^W \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, Y_B^W = R_B^W \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, &

$Z_B^W = R_B^W \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

a)

$$\rightarrow R_B^W = \begin{bmatrix} X_B^W & Y_B^W & Z_B^W \end{bmatrix} = \boxed{\begin{bmatrix} .433 & .177 & .884 \\ .250 & .918 & -.306 \\ -.866 & .353 & .353 \end{bmatrix}}$$

$$R_B^W R_B^{W^T} = \sim I_3$$

$\rightarrow$ Orthagonal $\rightarrow$ Valid ref. frame

1) $X_B^W = \begin{bmatrix} .433 \\ .250 \\ -.866 \end{bmatrix}, Y_B^W = \begin{bmatrix} .177 \\ .918 \\ .353 \end{bmatrix}, Z_B^W = \begin{bmatrix} .884 \\ -.306 \\ .353 \end{bmatrix}$

$$R_B^W = \begin{bmatrix} .433 & .177 & .884 \\ .250 & .918 & -.306 \\ -.866 & .353 & .353 \end{bmatrix}$$

$$A^B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad C^W = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

b) $A^W = R_B^W A^B = \boxed{\begin{bmatrix} 3.439 \\ 1.168 \\ 0.959 \end{bmatrix}}$

c) $C^B = R_W^B C^W = R_B^{W^T} C^W = \boxed{\begin{bmatrix} .493 \\ 2.720 \\ 2.392 \end{bmatrix}}$

2) $P_B^A = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$, $P_C^B = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$, $R_B^A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & .707 & -.707 \\ 0 & .707 & .707 \end{bmatrix}$

$R_B^C = \begin{bmatrix} .5 & 0 & -.866 \\ 0 & 1 & 0 \\ .866 & 0 & .5 \end{bmatrix}$ $\quad D^C = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\rightarrow \tilde{D}^A = H_B^A H_C^B \tilde{D}^C$

$\rightarrow H_B^A = \left[\begin{array}{ccc:c} & R_B^A & & P_B^A \\ \hdashline 0 & 0 & & 1 \end{array}\right]$, $H_C^B = \left[\begin{array}{ccc:c} & R_B^{C^T} & & P_C^B \\ \hdashline 0 & 0 & & 1 \end{array}\right]$

& $\tilde{D}^C = \left[\begin{array}{c} D^C \\ \hdashline 1 \end{array}\right]$

$\rightarrow \tilde{D}^A = \begin{bmatrix} 2.366 \\ .552 \\ 2.448 \\ 1 \end{bmatrix}$

$\rightarrow D^A = \begin{bmatrix} 2.366 \\ .552 \\ 2.448 \end{bmatrix}$

3) drone D has RPY $\frac{\pi}{6}, \frac{\pi}{4}, \& \frac{\pi}{3}$

$\& \; P_D^w = \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix}$

a) $R_D^w = R_z(\theta) R_y(\alpha) R_x(\beta)$

$$= \begin{bmatrix} c\frac{\pi}{6} & -s\frac{\pi}{6} & 0 \\ s\frac{\pi}{6} & c\frac{\pi}{6} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\frac{\pi}{4} & 0 & s\frac{\pi}{4} \\ 0 & 1 & 0 \\ -s\frac{\pi}{4} & 0 & c\frac{\pi}{4} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\frac{\pi}{3} & -s\frac{\pi}{3} \\ 0 & s\frac{\pi}{3} & c\frac{\pi}{3} \end{bmatrix}$$

$$= \begin{bmatrix} .612 & .739 & .280 \\ -.354 & .573 & -.739 \\ -.707 & .354 & .612 \end{bmatrix}$$

$$\rightarrow A_D^w = \begin{bmatrix} R_D^w & \vdots & P_D^w \\ \cdots & & \cdots \\ 0\;0\;0 & & 1 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} .612 & .739 & .280 & 1 \\ -.354 & .573 & -.739 & 1 \\ -.707 & .354 & .612 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

3) drone D has RPY $\frac{\pi}{6}, \frac{\pi}{4},$ & $\frac{-\pi}{2}$

& $P_D^W = \begin{bmatrix} 1 \\ 1 \\ 10 \end{bmatrix}$

b) $$A_W^D = \left[\begin{array}{c|c} R_D^{W^T} & -R_D^{W^T} P_D^W \\ \hline 0\,0\,0 & 1 \end{array}\right]$$

$$= \begin{bmatrix} .612 & -.354 & -.707 & 6.812 \\ .739 & .573 & .354 & -4.848 \\ .280 & -.739 & .612 & -5.665 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$