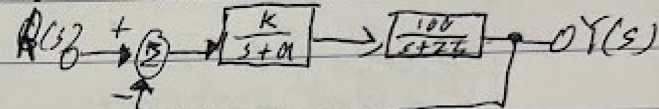
~~19. Suppose you desire the peak time of a given second-order system to be less than t_p'. Draw the region in the s-plane that corresponds to values of the poles that meet the specification $t_p < t_p'$~~

1. For the feedback control system shown in Fig. 3.55 spec. the gain & pole location of the compensator so that the overall closed-loop response to a unit-step input has an overshoot of no more than 25%, & a 1% settling time of no more than 0.1 sec.

$M_p \le 0.25 \quad T_s \le 0.1$



~~$M_p \le 0.25 \to \zeta \ge 0.5$~~

$M_p \le e^{-\pi\zeta/\sqrt{1-\zeta^2}}$

$\to \zeta = 403.7 \cdot 10^{-3}$

$e^{-\zeta\omega_n t_s} = 0.01$

$\to$ for $\zeta = 403.7\cdot10^{-3}$ & $t_s = 0.1$

$\omega_n = 114.$

$\to 2\zeta\omega_n = a + 25 = 92$

$\boxed{\to a = 67}$

$$\frac{\left(\frac{K}{s+a}\right)\left(\frac{100}{s+25}\right)}{1+\left(\frac{K}{s+a}\right)\left(\frac{100}{s+25}\right)} = \frac{\frac{100K}{s^2+(a+25)s+25a}}{1+\frac{100K}{s^2+(a+25)s+25a}}$$

$$= \frac{100K}{s^2+\underbrace{(a+25)}_{2\zeta\omega_n}s+\underbrace{25a+100K}_{\omega_n^2}} = G(s)$$

& $\omega_n^2 = 25a + 100K = 13\cdot10^3 = 25(67) + 100K \to \boxed{K = 113.5}$

$\boxed{\begin{array}{l}\text{Pole: } 67 \\ \text{Gain: } 113.5\end{array}}$

3.30

A feedback system has the following response specification

$\% M_p \le 16\%$, $t_s \le 6.9$ sec, $t_p \le 1.5$ sec

a) sketch the region of acceptable closed-loop poles in the s-plane for the system assuming the TF can be approximated as simple second order

M_p ~~[illegible]~~ $\le e^{-\pi\zeta/\sqrt{1-\zeta^2}}$ where $M_p = 0.10$

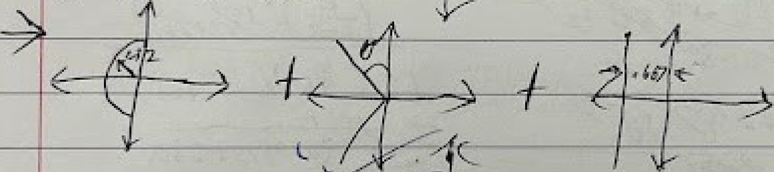
$\rightarrow \zeta = 503.9 \cdot 10^{-3}$

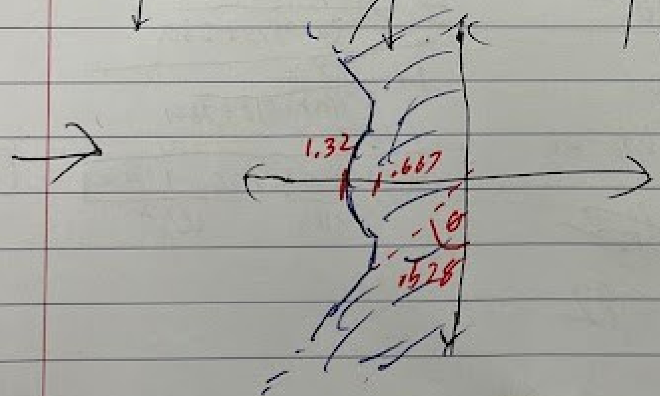
$e^{-\zeta\omega_n t_s} = .01$ where $\zeta = .514$ & $t_s = 6.9$ sec

$\Rightarrow \omega_n = 1.32$

$\sigma = \zeta\omega_n = 0.667$

$\sin^{-1}(\zeta) = .528$ rad $= \theta$





3.45: If a step input is applied to this plant, what do you estimate the rise-time, settling time, & overshoot to be? state your reasoning.

$$\frac{Y(s)}{R(s)} = T_1(s) = \frac{2}{s^2 + \underbrace{2s}_{2\zeta\omega_n} + \underbrace{2}_{\omega_n^2}}$$

$$\Rightarrow \omega_n^2 = 2 \Rightarrow \omega_n = \sqrt{2}$$

$$2\zeta\omega_n = 2 = 2\zeta\sqrt{2} \Rightarrow \zeta = \sqrt{2}/2$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = e^{-\pi\frac{\sqrt{2}}{2}/\sqrt{1-\left(\frac{\sqrt{2}}{2}\right)^2}} = \boxed{.043 = \text{overshoot}}$$

$$e^{-\zeta\omega_n t_s} = .01 = e^{-\frac{\sqrt{2}}{2}\sqrt{2}\,t_s} \rightarrow e^{-t_s} = .01 \Rightarrow t_s = 4.6 \text{ sec}$$

$$t_r = \frac{1.8}{\omega_n} = \frac{1.8}{\sqrt{2}} = \cancel{2.5\text{ sec}}\ 1.3 \text{ sec}$$