MIE 431 – HW 1 – Elias N.V

1. $f_{ss} = \lim_{s \to 0} sF(s) = \frac{10}{s(s+1)} \cdot s = 10$

~~$\mathcal{L}[e^{-at}] = \frac{1}{s+a} \to \mathcal{L}\left[\frac{10}{s(s+1)}\right]$~~

$\mathcal{L}[1-e^{-at}] = \frac{1}{s(s+a)} \Rightarrow \mathcal{L}^{-1}\left[\frac{10}{s(s+1)}\right] = 10(1-e^{-t})$

$\lim_{t \to \infty} 10(1-e^{-t}) = 10$ ✓

2. $\lim_{s \to \infty} sF(s) = \lim_{s \to \infty} s \frac{1}{(s+2)^2} = 0 \to f(0) = 0$

$\mathcal{L}[\dot{f}(t)] = sF(s) + f(0)$

$\to \dot{f}(0) = \lim_{s \to \infty} s\left(\frac{s}{(s+2)^2} + 0\right) = 1$

$\to f(0) = 0$ & $\dot{f}(0) = 1$

3. $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 2 \\ 5 \end{bmatrix}}_{B} u$ ~~where~~

$y = \underbrace{\begin{bmatrix} 1 & 2 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{3}_{D} u$

$sX(s) = AX(s) + BU(s) \to (sI-A)X(s) = BU(s) \Rightarrow X(s) = (sI-A)^{-1}BU(s)$

$\to Y(s) = CX(s) + DU(s)$

$\to Y(s) = (C(sI-A)^{-1}B + D)U(s)$ $\quad \det\left(\begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}\right) =$

$\to \frac{Y(s)}{U(s)} = [1, 2]\left(sI - \begin{bmatrix} -5, & -1 \\ 3, & -1 \end{bmatrix}\right)^{-1} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 3$ $\quad s^2 + 6s + 8 \neq 0$

$\to [1, 2]\left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -5, & -1 \\ 3 & -1 \end{bmatrix}\right)^{-1} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 3 \to [1, 2] \begin{bmatrix} \frac{s+1}{s^2+6s+8} & \frac{-1}{s^2+6s+8} \\ \frac{3}{s^2+6s+8} & \frac{s+5}{s^2+6s+8} \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 3 \to$

~~Actuation~~ ~~Spring~~

$$\left[\frac{s+7}{s^2+6s+8}, \frac{2s+9}{s^2+6s+8}\right]\begin{bmatrix}2\\5\end{bmatrix}+3 \Rightarrow \frac{2s+14+10s+45}{s^2+6s+8}+3$$

$$=\frac{12s+59}{s^2+6s+8}+3=\frac{12s+59}{(s+4)(s+2)}+3=G(s)=\frac{Y(s)}{U(s)}$$

3.3: $\mathcal{L}[8\sin(.75t)]=8\left(\frac{.75}{s^2+.75^2}\right)$

$\mathcal{L}[\cos(1.5t)+4\sin(1.5t)+1.7e^{-.5t}\cos(1.5t)]=\frac{s}{s^2+1.5^2}+\frac{4\cdot 1.5}{s^2+1.5^2}+1.7\frac{s+.5}{(s+.5)^2+1.5^2}$

$\mathcal{L}[.4t^3+1.8e^{t}\sin(2.2t)]=.4\frac{3!}{s^4}+1.8\frac{2.2}{(s-1)^2+2.2^2}$

3.5(b): $\mathcal{L}[2+3\sin^2(4t)+5\cos^2(t)]=\frac{2}{s}+3\frac{2\cdot 4^2}{4\cdot 4^2 s+s^3}+5\frac{2+s^2}{4s+s^3}$

3.8(c): $\mathcal{L}^{-1}\left[\frac{2(s^2+s+1)}{s(s+1)^2}\right]=2\mathcal{L}^{-1}\left[\frac{s^2}{s(s+1)^2}+\frac{s}{s(s+1)^2}+\frac{1}{s(s+1)^2}\right]$

$$=2\left((1-t)e^{-t}+te^{-t}+1-e^{-t}(1+t)\right)$$