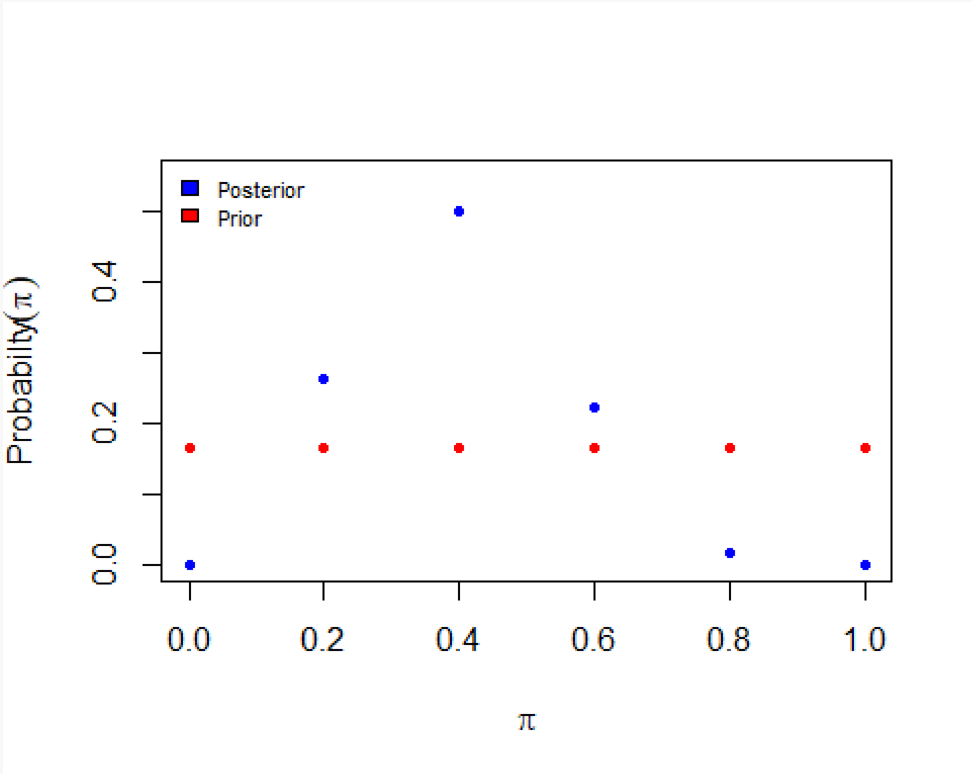
MA5750: APPLIED STATISTICS
ASSIGNMENT 2
E Naveen
ME16B077

6.1 Suppose we have 8 independent trials and each has one of two possible either success or failure. The probability of success remains constant for each trial. In that case, $Y|\pi$ is binomial ($n = 8, \pi$). Suppose we only allow that there are 6 possible values of π, 0, .2, .4, .6, .8, and 1. In that case we say that we have a discrete distribution for π. Initially we have no reason to favour one possible value over another. In that case our we would give all the possible values of π probability equal to 5.

R Code:

```
# Load the dependent libraries
library(Bolstad);

##
## Attaching package: 'Bolstad'

## The following objects are masked from 'package:stats':
##
##     IQR, sd, var

pi_values = seq(from = 0, to = 1, by = 0.2);
pi_prior = seq(from = 1/6, to = 1/6, length.out = length(pi_values));
output_1 = binodp(x = 3, n = 8, pi = pi_values, pi.prior = pi_prior);
```




```
## Conditional distribution of x given pi and  n:
##
##          0      1      2      3      4      5      6      7      8
## 0   1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
```

```
## 0.2 0.1678 0.3355 0.2936 0.1468 0.0459 0.0092 0.0011 0.0001 0.0000
## 0.4 0.0168 0.0896 0.2090 0.2787 0.2322 0.1239 0.0413 0.0079 0.0007
## 0.6 0.0007 0.0079 0.0413 0.1239 0.2322 0.2787 0.2090 0.0896 0.0168
## 0.8 0.0000 0.0001 0.0011 0.0092 0.0459 0.1468 0.2936 0.3355 0.1678
## 1   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
##
## Joint distribution:
##
##           0      1      2      3      4      5      6      7      8
## [1,] 0.1667 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## [2,] 0.0280 0.0559 0.0489 0.0245 0.0076 0.0015 0.0002 0.0000 0.0000
## [3,] 0.0028 0.0149 0.0348 0.0464 0.0387 0.0206 0.0069 0.0013 0.0001
## [4,] 0.0001 0.0013 0.0069 0.0206 0.0387 0.0464 0.0348 0.0149 0.0028
## [5,] 0.0000 0.0000 0.0002 0.0015 0.0076 0.0245 0.0489 0.0559 0.0280
## [6,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1667
##
## Marginal distribution of x:
##
##           0      1      2      3      4      5      6      7      8
## [1,] 0.1975 0.0722 0.0908 0.0931 0.0927 0.0931 0.0908 0.0722 0.1975
##
##
##         Prior Likelihood Posterior
## 0   0.1666667 0.00000000 0.0000000
## 0.2 0.1666667 0.14680064 0.2628337
## 0.4 0.1666667 0.27869184 0.4989733
## 0.6 0.1666667 0.12386304 0.2217659
## 0.8 0.1666667 0.00917504 0.0164271
## 1   0.1666667 0.00000000 0.0000000
```

(a) Identify the matrix of conditional probabilities from the output. Relate these conditional probabilities to the binomial probabilities in Table B. 1.

The following is the matrix of conditional probabilities generated by the 'binodp' R routine. The values generated are verified to be similar as provided in table B.1 in appendix D of the textbook.

```
##          0      1      2      3      4      5      6      7      8
## 0   1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## 0.2 0.1678 0.3355 0.2936 0.1468 0.0459 0.0092 0.0011 0.0001 0.0000
## 0.4 0.0168 0.0896 0.2090 0.2787 0.2322 0.1239 0.0413 0.0079 0.0007
## 0.6 0.0007 0.0079 0.0413 0.1239 0.2322 0.2787 0.2090 0.0896 0.0168
## 0.8 0.0000 0.0001 0.0011 0.0092 0.0459 0.1468 0.2936 0.3355 0.1678
## 1   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
```

(b) What column in the matrix contains the likelihoods?

The highlighted (bold) column in the below table contains the likelihood for y=3 and n=8.

```
##          0      1      2      3      4      5      6      7      8
## 0   1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## 0.2 0.1678 0.3355 0.2936 0.1468 0.0459 0.0092 0.0011 0.0001 0.0000
## 0.4 0.0168 0.0896 0.2090 0.2787 0.2322 0.1239 0.0413 0.0079 0.0007
## 0.6 0.0007 0.0079 0.0413 0.1239 0.2322 0.2787 0.2090 0.0896 0.0168
## 0.8 0.0000 0.0001 0.0011 0.0092 0.0459 0.1468 0.2936 0.3355 0.1678
## 1   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
```

(c) Identify the matrix of joint probabilities from the output. How are these joint probabilities found?

Below is the matrix of joint probabilities from the output. They are found by multiplying each column of the previous table element wise with the discrete prior that we passed to the function earlier. The previous matrix is found by computing $f(y|\pi)$.

```
##           0      1      2      3      4      5      6      7      8
## [1,] 0.1667 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## [2,] 0.0280 0.0559 0.0489 0.0245 0.0076 0.0015 0.0002 0.0000 0.0000
## [3,] 0.0028 0.0149 0.0348 0.0464 0.0387 0.0206 0.0069 0.0013 0.0001
## [4,] 0.0001 0.0013 0.0069 0.0206 0.0387 0.0464 0.0348 0.0149 0.0028
## [5,] 0.0000 0.0000 0.0002 0.0015 0.0076 0.0245 0.0489 0.0559 0.0280
## [6,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.1667
```

(d) Identify the marginal probabilities of Y from the output. How are these found?

The marginal probabilities of Y are as shown below. They are found by summing the joint probabilities under each column in Joint Probabilities matrix.

```
##           0      1      2      3      4      5      6      7      8
## [1,] 0.1975 0.0722 0.0908 0.0931 0.0927 0.0931 0.0908 0.0722 0.1975
```

(e) How are the posterior probabilities found?

The posterior probabilities are found by dividing each element of the column corresponding to the observed data (here y=3) by the corresponding marginal probability of Y.

```
##         Prior Likelihood Posterior
## 0   0.1666667 0.00000000 0.0000000
## 0.2 0.1666667 0.14680064 0.2628337
## 0.4 0.1666667 0.27869184 0.4989733
## 0.6 0.1666667 0.12386304 0.2217659
## 0.8 0.1666667 0.00917504 0.0164271
## 1   0.1666667 0.00000000 0.0000000
```

It can be retrieved using the following few lines of code from the variable storing the output of the function.

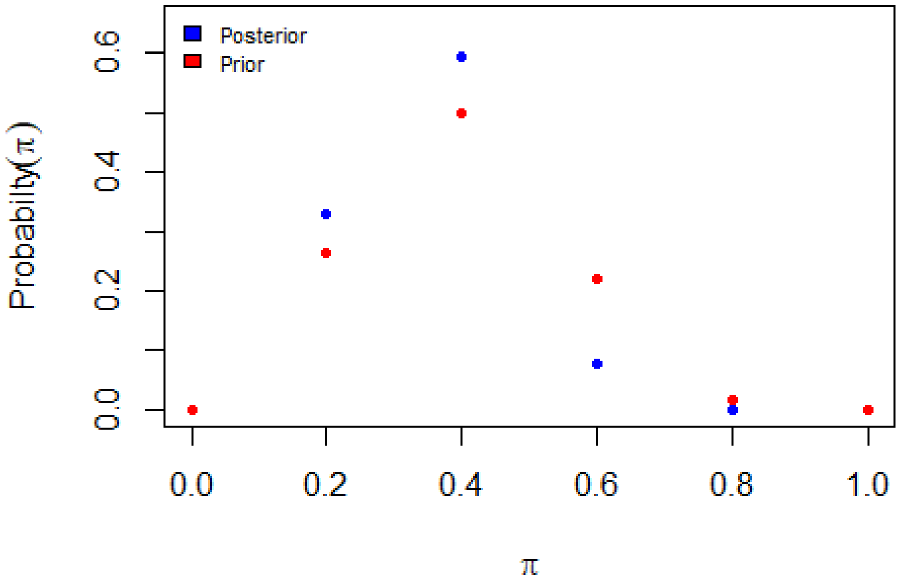
```
# Print the discrete pi values followed by posterior probabilities.
output_1$param.x;

## [1] 0.0 0.2 0.4 0.6 0.8 1.0

output_1$posterior;

## [1] 0.0000000 0.2628337 0.4989733 0.2217659 0.0164271 0.0000000
```

6.2 Suppose we take an additional 7 trials, and achieve 2 successes.

(a) Let the posterior after the 8 trials and 3 successes in the previous problem be the prior and use BinoDPmac or the equivalent R function to find the new posterior distribution for π.

R Code:

```
new_prior = output_1$posterior;
output_2 = binodp(x = 2, n = 7, pi = pi_values, pi.prior = new_prior);
```



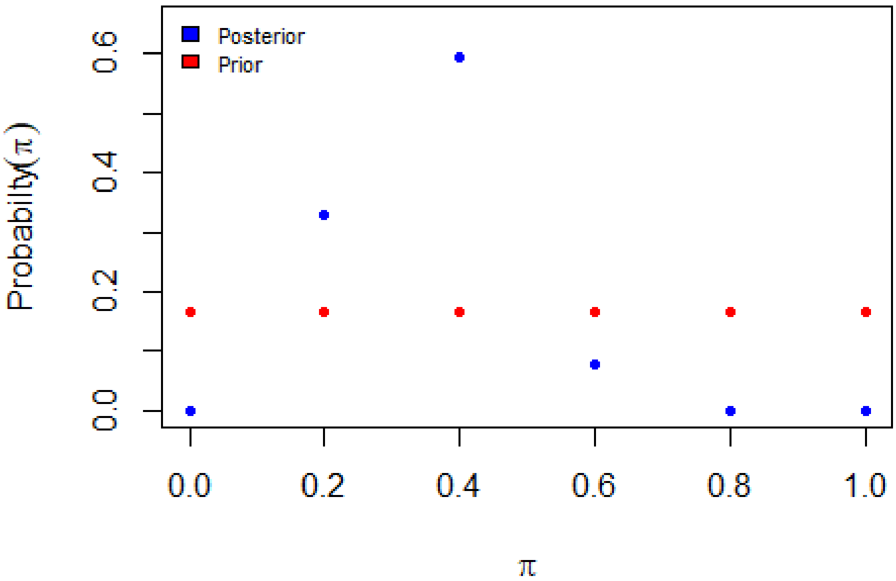

```
## Conditional distribution of x given pi and  n:
##
##           0      1      2      3      4      5      6      7
## 0   1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## 0.2 0.2097 0.3670 0.2753 0.1147 0.0287 0.0043 0.0004 0.0000
## 0.4 0.0280 0.1306 0.2613 0.2903 0.1935 0.0774 0.0172 0.0016
## 0.6 0.0016 0.0172 0.0774 0.1935 0.2903 0.2613 0.1306 0.0280
## 0.8 0.0000 0.0004 0.0043 0.0287 0.1147 0.2753 0.3670 0.2097
## 1   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
##
## Joint distribution:
##
##            0      1      2      3      4      5      6      7
## [1,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## [2,] 0.0551 0.0965 0.0723 0.0301 0.0075 0.0011 0.0001 0.0000
## [3,] 0.0140 0.0652 0.1304 0.1449 0.0966 0.0386 0.0086 0.0008
## [4,] 0.0004 0.0038 0.0172 0.0429 0.0644 0.0579 0.0290 0.0062
## [5,] 0.0000 0.0000 0.0001 0.0005 0.0019 0.0045 0.0060 0.0034
## [6,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
##
## Marginal distribution of x:
##
##            0      1    2      3      4      5      6      7
## [1,] 0.0695 0.1655 0.22 0.2184 0.1704 0.1022 0.0437 0.0105
##
##
##         Prior Likelihood    Posterior
## 0   0.0000000  0.0000000 0.0000000000
## 0.2 0.2628337  0.2752512 0.3289134253
## 0.4 0.4989733  0.2612736 0.5927126725
## 0.6 0.2217659  0.0774144 0.0780526976
## 0.8 0.0164271  0.0043008 0.0003212045
## 1   0.0000000  0.0000000 0.0000000000

# Print the discrete pi values followed by posterior probabilities.
output_2$param.x;
```

```
## [1] 0.0 0.2 0.4 0.6 0.8 1.0

output_2$posterior;

## [1] 0.0000000000 0.3289134253 0.5927126725 0.0780526976 0.0003212045
## [6] 0.0000000000
```

(b) In total, we have taken 15 trials and achieved 5 successes. Go back to the original prior and use BinoDPmac or the equivalent R function to find the posterior after the 15 trials and 5 successes.

R Code:

```
pi_values = seq(from = 0, to = 1, by = 0.2);
pi_prior = seq(from = 1/6, to = 1/6, length.out = length(pi_values));
output_1plus2 = binodp(x = 5, n = 15, pi = pi_values, pi.prior = pi_prior);
```




```
## Conditional distribution of x given pi and  n:
##
##          0      1      2      3      4      5      6      7      8      9
## 0   1.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## 0.2 0.0352 0.1319 0.2309 0.2501 0.1876 0.1032 0.0430 0.0138 0.0035 0.0007
## 0.4 0.0005 0.0047 0.0219 0.0634 0.1268 0.1859 0.2066 0.1771 0.1181 0.0612
## 0.6 0.0000 0.0000 0.0003 0.0016 0.0074 0.0245 0.0612 0.1181 0.1771 0.2066
## 0.8 0.0000 0.0000 0.0000 0.0000 0.0000 0.0001 0.0007 0.0035 0.0138 0.0430
## 1   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
##         10     11     12     13     14     15
## 0   0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## 0.2 0.0001 0.0000 0.0000 0.0000 0.0000 0.0000
## 0.4 0.0245 0.0074 0.0016 0.0003 0.0000 0.0000
## 0.6 0.1859 0.1268 0.0634 0.0219 0.0047 0.0005
## 0.8 0.1032 0.1876 0.2501 0.2309 0.1319 0.0352
## 1   0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
##
## Joint distribution:
##
##           0      1      2      3      4      5      6      7      8      9
## [1,] 0.1667 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
```

```
## [2,] 0.0059 0.0220 0.0385 0.0417 0.0313 0.0172 0.0072 0.0023 0.0006 0.0001
## [3,] 0.0001 0.0008 0.0037 0.0106 0.0211 0.0310 0.0344 0.0295 0.0197 0.0102
## [4,] 0.0000 0.0000 0.0000 0.0003 0.0012 0.0041 0.0102 0.0197 0.0295 0.0344
## [5,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0001 0.0006 0.0023 0.0072
## [6,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
##          10     11     12     13     14     15
## [1,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## [2,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000
## [3,] 0.0041 0.0012 0.0003 0.0000 0.0000 0.0000
## [4,] 0.0310 0.0211 0.0106 0.0037 0.0008 0.0001
## [5,] 0.0172 0.0313 0.0417 0.0385 0.0220 0.0059
## [6,] 0.0000 0.0000 0.0000 0.0000 0.0000 0.1667
## 
## Marginal distribution of x:
## 
##           0      1      2      3      4      5      6      7      8      9
## [1,] 0.1726 0.0228 0.0422 0.0525 0.0536 0.0523 0.0519 0.0521 0.0521 0.0519
##          10     11     12     13     14     15
## [1,] 0.0523 0.0536 0.0525 0.0422 0.0228 0.1726
## 
## 
##         Prior  Likelihood    Posterior
## 0   0.1666667 0.000000000 0.0000000000
## 0.2 0.1666667 0.103182294 0.3289134253
## 0.4 0.1666667 0.185937845 0.5927126725
## 0.6 0.1666667 0.024485642 0.0780526976
## 0.8 0.1666667 0.000100764 0.0003212045
## 1   0.1666667 0.000000000 0.0000000000
```

```
# Print the discrete pi values followed by posterior probabilities.
output_1plus2$param.x;
```

```
## [1] 0.0 0.2 0.4 0.6 0.8 1.0
```

```
output_1plus2$posterior;
```

```
## [1] 0.0000000000 0.3289134253 0.5927126725 0.0780526976 0.0003212045
## [6] 0.0000000000
```

(c) What does this show?

The posterior probability computed by combining 3 successes in 8 tries and 2 successes in 7 tries into 5 successes in 15 tries produces the same result as the end result when done sequentially. This is in accordance to the theory that we learnt in the Chapter.

6.3 The Minitab macro PoisDPmac or the equivalent R function is used to find the posterior distribution when the observation distribution of Y|μ is Poisson (μ) and we have a discrete prior distribution for μ. Details for invoking PoisDRmac are in Appendix C. The details for the equivalent R function are in Appendix D. Suppose there are six possible values μ = 1, ..., 6 and the prior probabilities are given. Suppose the first observation is Y1 = 2. Use PoisDPmac or the equivalent R function to find the posterior distribution g(μ|y).

R Code:

```
# Load the dependent libraries
library(Bolstad)
```

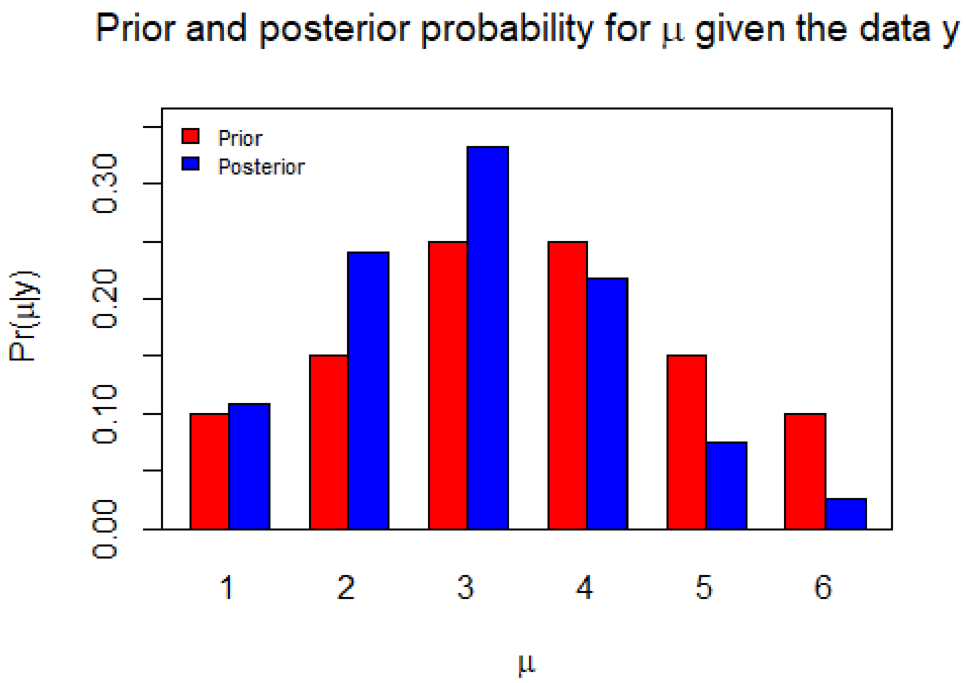
```
## 
## Attaching package: 'Bolstad'
```

```
## The following objects are masked from 'package:stats':
##
##     IQR, sd, var
```

```
mu_values = seq(from = 1, to = 6, by = 1);
mu_prior = c(0.1, 0.15, 0.25, 0.25, 0.15, 0.1);
output = poisdp(y.obs = 2, mu = mu_values, mu.prior = mu_prior)
```

```
## Prior
## -----
##      mu Pr(mu)
## [1,]  1   0.10
## [2,]  2   0.15
## [3,]  3   0.25
## [4,]  4   0.25
## [5,]  5   0.15
## [6,]  6   0.10
##
## k1:   0.9995
## k2:   6
##
## Conditional probability of y1 given mu
## -------------------------------------
##             0          1          2          3          4           5
## 1 0.367879441 0.36787944 0.18393972 0.06131324 0.01532831 0.003065662
## 2 0.135335283 0.27067057 0.27067057 0.18044704 0.09022352 0.036089409
## 3 0.049787068 0.14936121 0.22404181 0.22404181 0.16803136 0.100818813
## 4 0.018315639 0.07326256 0.14652511 0.19536681 0.19536681 0.156293452
## 5 0.006737947 0.03368973 0.08422434 0.14037390 0.17546737 0.175467370
## 6 0.002478752 0.01487251 0.04461754 0.08923508 0.13385262 0.160623141
##              6            7            8            9           10
## 1 0.0005109437 7.299195e-05 9.123994e-06 1.013777e-06 1.013777e-07
## 2 0.0120298030 3.437087e-03 8.592716e-04 1.909493e-04 3.818985e-05
## 3 0.0504094067 2.160403e-02 8.101512e-03 2.700504e-03 8.101512e-04
## 4 0.1041956346 5.954036e-02 2.977018e-02 1.323119e-02 5.292477e-03
## 5 0.1462228081 1.044449e-01 6.527804e-02 3.626558e-02 1.813279e-02
## 6 0.1606231410 1.376770e-01 1.032577e-01 6.883849e-02 4.130309e-02
##             11           12           13           14           15
## 1 9.216156e-09 7.680130e-10 5.907792e-11 4.219851e-12 2.813234e-13
## 2 6.943609e-06 1.157268e-06 1.780413e-07 2.543447e-08 3.391262e-09
## 3 2.209503e-04 5.523758e-05 1.274713e-05 2.731529e-06 5.463057e-07
## 4 1.924537e-03 6.415123e-04 1.973884e-04 5.639669e-05 1.503912e-05
## 5 8.242177e-03 3.434240e-03 1.320862e-03 4.717363e-04 1.572454e-04
## 6 2.252896e-02 1.126448e-02 5.198991e-03 2.228139e-03 8.912556e-04
##             16
## 1 1.758271e-14
## 2 4.239078e-10
## 3 1.024323e-07
## 4 3.759779e-06
## 5 4.913920e-05
## 6 3.342208e-04
##
##
## Joint probability of y1 and mu
## ------------------------------
##                 0           1           2           3           4
## [1,] 0.0367879441 0.036787944 0.018393972 0.006131324 0.001532831
## [2,] 0.0203002925 0.040600585 0.040600585 0.027067057 0.013533528
## [3,] 0.0124467671 0.037340301 0.056010452 0.056010452 0.042007839
## [4,] 0.0045789097 0.018315639 0.036631278 0.048841704 0.048841704
```

```
## [5,] 0.0010106920 0.005053460 0.012633651 0.021056084 0.026320105
## [6,] 0.0002478752 0.001487251 0.004461754 0.008923508 0.013385262
##                 5            6            7            8            9
## [1,] 0.0003065662 5.109437e-05 7.299195e-06 9.123994e-07 1.013777e-07
## [2,] 0.0054134113 1.804470e-03 5.155630e-04 1.288907e-04 2.864239e-05
## [3,] 0.0252047034 1.260235e-02 5.401008e-03 2.025378e-03 6.751260e-04
## [4,] 0.0390733630 2.604891e-02 1.488509e-02 7.442545e-03 3.307798e-03
## [5,] 0.0263201055 2.193342e-02 1.566673e-02 9.791706e-03 5.439837e-03
## [6,] 0.0160623141 1.606231e-02 1.376770e-02 1.032577e-02 6.883849e-03
##                10           11           12           13           14
## [1,] 1.013777e-08 9.216156e-10 7.680130e-11 5.907792e-12 4.219851e-13
## [2,] 5.728478e-06 1.041541e-06 1.735902e-07 2.670619e-08 3.815170e-09
## [3,] 2.025378e-04 5.523758e-05 1.380940e-05 3.186783e-06 6.828822e-07
## [4,] 1.323119e-03 4.811342e-04 1.603781e-04 4.934710e-05 1.409917e-05
## [5,] 2.719918e-03 1.236327e-03 5.151360e-04 1.981292e-04 7.076045e-05
## [6,] 4.130309e-03 2.252896e-03 1.126448e-03 5.198991e-04 2.228139e-04
##                15           16
## [1,] 2.813234e-14 1.758271e-15
## [2,] 5.086893e-10 6.358616e-11
## [3,] 1.365764e-07 2.560808e-08
## [4,] 3.759779e-06 9.399448e-07
## [5,] 2.358682e-05 7.370880e-06
## [6,] 8.912556e-05 3.342208e-05
##
##
## Marginal probability of y1
## -------------------------
##            0            1            2            3            4
## 7.537248e-02 1.395852e-01 1.687317e-01 1.680301e-01 1.456213e-01
##            5            6            7            8            9
## 1.123805e-01 7.850256e-02 5.024339e-02 2.971521e-02 1.633535e-02
##           10           11           12           13           14
## 8.381623e-03 4.026637e-03 1.815945e-03 7.705889e-04 3.083602e-04
##           15           16
## 1.166092e-04 4.175858e-05
##
##
##   Mu Prior Likelihood  Posterior
## 1  1  0.10 0.18393972 0.10901314
## 2  2  0.15 0.27067057 0.24062217
## 3  3  0.25 0.22404181 0.33194980
## 4  4  0.25 0.14652511 0.21709779
## 5  5  0.15 0.08422434 0.07487420
## 6  6  0.10 0.04461754 0.02644289
```



(a) Identify the matrix of conditional probabilities from the output. Relate these conditional probabilities to the Poisson probabilities in Table B.5.

The matrix of conditional probabilities from output are as shown below.

```
## Conditional probability of y1 given mu
## -------------------------------------
##             0          1          2          3          4           5
## 1 0.367879441 0.36787944 0.18393972 0.06131324 0.01532831 0.003065662
## 2 0.135335283 0.27067057 0.27067057 0.18044704 0.09022352 0.036089409
## 3 0.049787068 0.14936121 0.22404181 0.22404181 0.16803136 0.100818813
## 4 0.018315639 0.07326256 0.14652511 0.19536681 0.19536681 0.156293452
## 5 0.006737947 0.03368973 0.08422434 0.14037390 0.17546737 0.175467370
## 6 0.002478752 0.01487251 0.04461754 0.08923508 0.13385262 0.160623141
##              6            7            8            9           10
## 1 0.0005109437 7.299195e-05 9.123994e-06 1.013777e-06 1.013777e-07
## 2 0.0120298030 3.437087e-03 8.592716e-04 1.909493e-04 3.818985e-05
## 3 0.0504094067 2.160403e-02 8.101512e-03 2.700504e-03 8.101512e-04
## 4 0.1041956346 5.954036e-02 2.977018e-02 1.323119e-02 5.292477e-03
## 5 0.1462228081 1.044449e-01 6.527804e-02 3.626558e-02 1.813279e-02
## 6 0.1606231410 1.376770e-01 1.032577e-01 6.883849e-02 4.130309e-02
##             11           12           13           14           15
## 1 9.216156e-09 7.680130e-10 5.907792e-11 4.219851e-12 2.813234e-13
## 2 6.943609e-06 1.157268e-06 1.780413e-07 2.543447e-08 3.391262e-09
## 3 2.209503e-04 5.523758e-05 1.274713e-05 2.731529e-06 5.463057e-07
## 4 1.924537e-03 6.415123e-04 1.973884e-04 5.639669e-05 1.503912e-05
## 5 8.242177e-03 3.434240e-03 1.320862e-03 4.717363e-04 1.572454e-04
## 6 2.252896e-02 1.126448e-02 5.198991e-03 2.228139e-03 8.912556e-04
##             16
## 1 1.758271e-14
## 2 4.239078e-10
## 3 1.024323e-07
## 4 3.759779e-06
```

```
## 5 4.913920e-05
## 6 3.342208e-04
```

(b) What column in the matrix contains the likelihoods?

The highlighted column below contains the likelihood required.

```
## Conditional probability of y1 given mu
## -------------------------------------
##             0          1          2          3          4           5
## 1 0.367879441 0.36787944 0.18393972 0.06131324 0.01532831 0.003065662
## 2 0.135335283 0.27067057 0.27067057 0.18044704 0.09022352 0.036089409
## 3 0.049787068 0.14936121 0.22404181 0.22404181 0.16803136 0.100818813
## 4 0.018315639 0.07326256 0.14652511 0.19536681 0.19536681 0.156293452
## 5 0.006737947 0.03368973 0.08422434 0.14037390 0.17546737 0.175467370
## 6 0.002478752 0.01487251 0.04461754 0.08923508 0.13385262 0.160623141
```

(c) Identify the matrix of joint probabilities from the output. How are these joint probabilities found?

Below is the matrix of joint probabilities from the output. They are found by multiplying each column of the previous table element wise with the discrete prior that we passed to the function earlier. The previous matrix is found by computing $f(y|\mu)$.

```
## Joint probability of y1 and mu
## ------------------------------
##                 0           1           2           3           4
## [1,] 0.0367879441 0.036787944 0.018393972 0.006131324 0.001532831
## [2,] 0.0203002925 0.040600585 0.040600585 0.027067057 0.013533528
## [3,] 0.0124467671 0.037340301 0.056010452 0.056010452 0.042007839
## [4,] 0.0045789097 0.018315639 0.036631278 0.048841704 0.048841704
## [5,] 0.0010106920 0.005053460 0.012633651 0.021056084 0.026320105
## [6,] 0.0002478752 0.001487251 0.004461754 0.008923508 0.013385262
##                 5            6            7            8            9
## [1,] 0.0003065662 5.109437e-05 7.299195e-06 9.123994e-07 1.013777e-07
## [2,] 0.0054134113 1.804470e-03 5.155630e-04 1.288907e-04 2.864239e-05
## [3,] 0.0252047034 1.260235e-02 5.401008e-03 2.025378e-03 6.751260e-04
## [4,] 0.0390733630 2.604891e-02 1.488509e-02 7.442545e-03 3.307798e-03
## [5,] 0.0263201055 2.193342e-02 1.566673e-02 9.791706e-03 5.439837e-03
## [6,] 0.0160623141 1.606231e-02 1.376770e-02 1.032577e-02 6.883849e-03
##                10           11           12           13           14
## [1,] 1.013777e-08 9.216156e-10 7.680130e-11 5.907792e-12 4.219851e-13
## [2,] 5.728478e-06 1.041541e-06 1.735902e-07 2.670619e-08 3.815170e-09
## [3,] 2.025378e-04 5.523758e-05 1.380940e-05 3.186783e-06 6.828822e-07
## [4,] 1.323119e-03 4.811342e-04 1.603781e-04 4.934710e-05 1.409917e-05
## [5,] 2.719918e-03 1.236327e-03 5.151360e-04 1.981292e-04 7.076045e-05
## [6,] 4.130309e-03 2.252896e-03 1.126448e-03 5.198991e-04 2.228139e-04
##                15           16
## [1,] 2.813234e-14 1.758271e-15
## [2,] 5.086893e-10 6.358616e-11
## [3,] 1.365764e-07 2.560808e-08
## [4,] 3.759779e-06 9.399448e-07
## [5,] 2.358682e-05 7.370880e-06
## [6,] 8.912556e-05 3.342208e-05
```

(d) Identify the marginal probabilities of Y from the output. How are these found?

The marginal probabilities of Y are as shown below. They are found by summing the joint probabilities under each column in Joint Probabilities matrix.

```
## Marginal probability of y1
## -------------------------
##            0            1            2            3            4
## 7.537248e-02 1.395852e-01 1.687317e-01 1.680301e-01 1.456213e-01
##            5            6            7            8            9
## 1.123805e-01 7.850256e-02 5.024339e-02 2.971521e-02 1.633535e-02
##           10           11           12           13           14
## 8.381623e-03 4.026637e-03 1.815945e-03 7.705889e-04 3.083602e-04
##           15           16
## 1.166092e-04 4.175858e-05
```

(e) How are the posterior probabilities found?

The posterior probabilities are found by dividing each element of the column corresponding to the observed data (here y=2) in Joint probability by the corresponding marginal probability of Y.

```
##   Mu Prior Likelihood  Posterior
## 1  1  0.10 0.18393972 0.10901314
## 2  2  0.15 0.27067057 0.24062217
## 3  3  0.25 0.22404181 0.33194980
## 4  4  0.25 0.14652511 0.21709779
## 5  5  0.15 0.08422434 0.07487420
## 6  6  0.10 0.04461754 0.02644289
```