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# CH5350: APPLIED TIME SERIES ANALYSIS

## ASSIGNMENT – 2

## 1.

$$v[k] = Acos^2(2\pi fk + \phi),$$

$$\phi \text{ is a constant}, A \text{ is a random variable with zero mean and unit variance.}$$

$$E(v[k]) = E(Acos^2(2\pi fk + \phi)) = cos^2(2\pi fk + \phi) \times E(A) = 0$$

$$Covariance = E((v[k] - \mu_k) \times (v[k-l] - \mu_{k-l})) = E(v[k] \times v[k-l])$$

$$= E\big(Acos^2(2\pi fk + \phi) \times Acos^2(2\pi f(k-l) + \phi)\big) = E(\{A \times \cos(2\pi fk + \phi) \times \cos(2\pi f(k-l) + \phi)\}^2)$$

$$= E(\{A \times (\cos(2\pi fk + \phi + 2\pi f(k-l) + \phi) + \cos(2\pi fk + \phi - 2\pi f(k-l) - \phi))\}^2)$$

$$= E(\{A \times (\cos(2\pi f(2k+l) + 2\phi) + \cos(2\pi fl))\}^2) = (\cos(2\pi f(2k+l) + 2\phi) + \cos(2\pi fl))^2 \times E(A^2)$$

We know, $Var(A) = E(A^2) - E(A)^2 = E(A^2) = 1$

$$Covariance = (\cos(2\pi f(2k+l) + 2\phi) + \cos(2\pi fl))^2$$

As the covariance depends on the value k it is not covariance stationary.

## 2.

(a)

$$v[k] = \phi_1 v[k-1] + e[k]$$

$$e[k] \sim N(0, \sigma_e^2), \qquad |\phi| < 1$$

$$corr(v[k], v[k-l]) = \frac{Cov(v[k], v[k-l])}{\sigma(v[k]) \times \sigma(v[k-l])}$$

$$Cov(v[k], v[k-l]) = E(v[k]v[k-l]) - E(v[k]) \times E(v[k-l])$$

$$Assuming\ v[0] = h,$$

$$then\ v[k] = \phi_1^k.h + linear\ combination\ of\ e[k]$$

$$So, E(v[k]) = \phi_1^k.h$$

$$var(v[k]) = E(v[k]^2) - \left(\phi_1^k.h\right)^2$$

$$Cov(v[k], v[k-l]) = E(v[k]v[k-l]) = \phi_1 E(v[k-1]v[k-l]) + E(e[k]v[k-l]) - \phi_1^k.h \times \phi_1^{k-l}.h$$

$$= \phi_1 Cov(v[k-1], v[k-l]) + \sigma_{ev}[l] \ - \phi_1^{2k-l}.h^2$$

$$\sigma_{ev}[0] = \sigma_e^2;\ \sigma_{ev}[l] = 0\ \forall\ l \in \{1,2,\dots\}$$

(c)

$$With\ corr(v[k], v[k-l]) = \phi_1^l \times \left[\frac{var(v[k-l])}{var(v[k])}\right]^{1/2} and\ E(v[k]) = \phi_1^k.h$$

$$at\ large\ times\ k,$$

$$\phi_1^k\ vanishes\ given\ |\phi_1| < 1 \Rightarrow E(v[k]) = \phi_1^k.h = 0$$

$$and\ at\ large\ values\ of\ k,$$

$$k - l \cong k \Rightarrow var(v[k-l]) = var(v[k]) \Rightarrow corr(v[k], v[k-l]) = \phi_1^l$$

So, asymptotically the mean and correlation become invariant with time and hence process becomes stationary.

(d)

$$We\ have\ derived\ earlier\ that\ E(v[k]) = \phi_1^k.h$$

$$If\ v[0] = h = 0, then\ E(v[k]) = 0$$

$$And\ consequently\ we\ have\ \rho_{vv}[l] = (\phi_1)^{|l|}\ (derived\ in\ class)$$

With the initial condition $v[0] = 0$ mean and ACF of the process now become invariant with time and hence process is stationary.

# 3.

(a)

For a MA(2) process $v[k] = e[k] + c_1 e[k-1] + c_2 e[k-2]$.

$f[l] = 0\ \forall l \in \{3,4,\dots\}. Let\ f[1] = \alpha, f[2] = \beta\ and\ we\ know\ f[0] = 1.$

We know, $\rho[0] = 1;\ \rho[1] = \frac{c_1 + c_1 c_2}{1+c_1^2+c_2^2};\ \rho[2] = \frac{c_2}{1+c_1^2+c_2^2}$ and $\rho[k] = 0\ \forall k > 2$.

$$\frac{c_1 + c_1 c_2}{1 + c_1^2 + c_2^2} = \alpha;\ \frac{c_2}{1 + c_1^2 + c_2^2} = \beta$$

We could write conditions for the compatibility of the above two equations to arrive at the conditions for $\alpha\ and\ \beta$ of $f[l]$.

(b)

$$H(q^{-1}) = \frac{1}{1 - 1.4q^{-1} + 0.45q^{-2}}$$

$$With\ v[k] = H(q^{-1}) \times e[k],$$

$$v[k] - 1.4v[k-1] + 0.45v[k-2] = e[k] \Rightarrow v[k] = 1.4v[k-1] - 0.45v[k-2] + e[k]$$

$$\sigma_{vv}[l] = E\big((v[k] - 0)(v[k-l] - 0)\big)$$

$$= E((1.4v[k-1] - 0.45v[k-2] + e[k]) \times (v[k-l]))$$

$$= 1.4 \times \sigma_{vv}[l-1] - 0.45 \times \sigma_{vv}[l-2] + \sigma_{ev}[l]$$

$$ACVF = \{$$

$$1.4 \times \sigma_{vv}[l-1] - 0.45 \times \sigma_{vv}[l-2], \qquad for\ l \neq 0,$$

$$1.4 \times \sigma_{vv}[l-1] - 0.45 \times \sigma_{vv}[l-2] + \sigma_e^2, \qquad for\ l = 0$$

$$\}$$

$$With\ initial\ conditions, we\ have$$

$$\sigma_{vv}[0] = \left(\frac{1 + 0.45}{1 - 0.45}\right) \times (\frac{\sigma_e^2}{(1 + 0.45)^2 - 1.4^2})$$

$$\sigma_{vv}[1] = (\frac{1.4}{1 + 0.45})\left(\frac{1 + 0.45}{1 - 0.45}\right) \times (\frac{\sigma_e^2}{(1 + 0.45)^2 - 1.4^2})$$

$$\sigma_{vv}[l] = 1.4\sigma_{vv}[l-1] - 0.45\sigma_{vv}[l-2]$$

$$We\ have,$$

$$\rho[0] = 1$$

$$\rho[1] = \frac{1.4}{1 + 0.45} 0.9655$$

$$\rho[2] = \frac{1.4^2}{1 + 0.45} - 0.45 = 0.9017$$

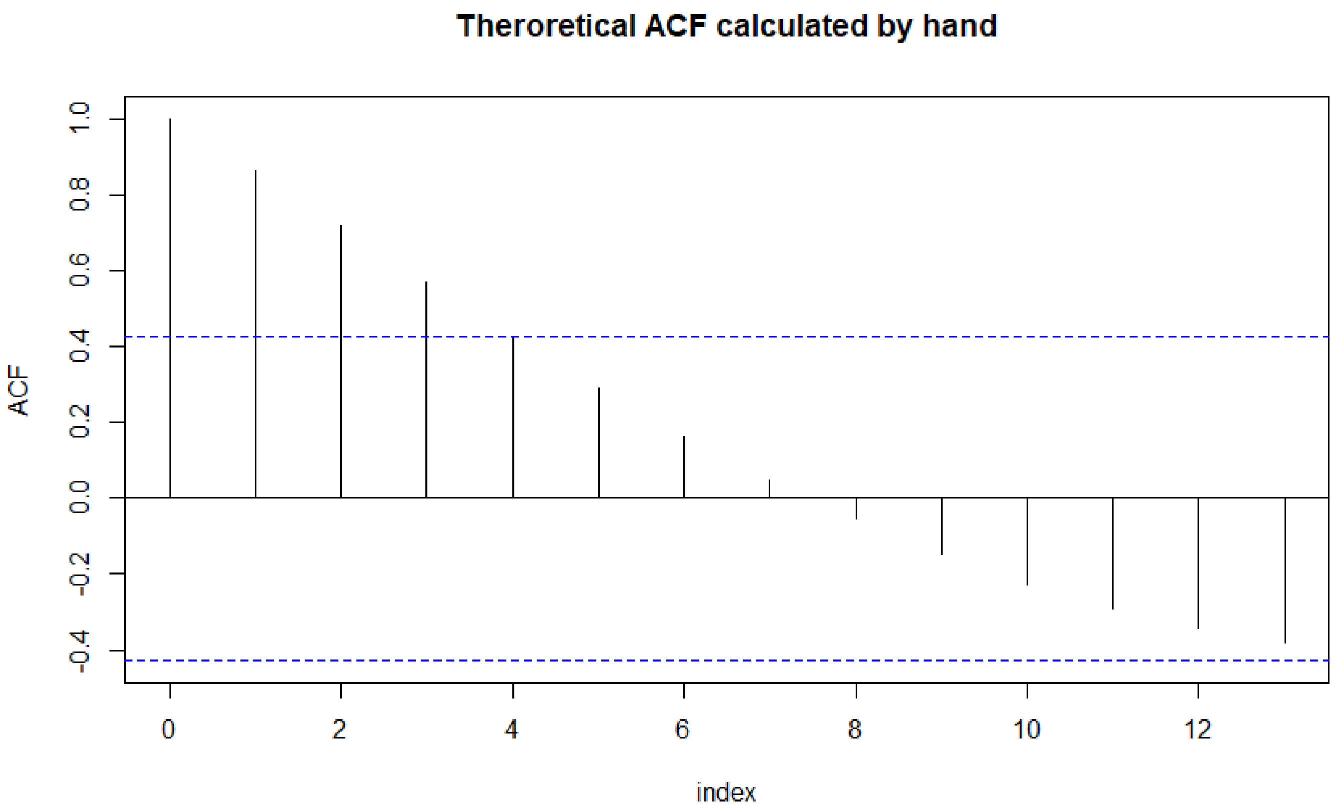
$$\rho[l] = 1.4\rho[l-1] - 0.45\rho[l-2] \; \forall \; l \in \{3,4, \dots\}$$

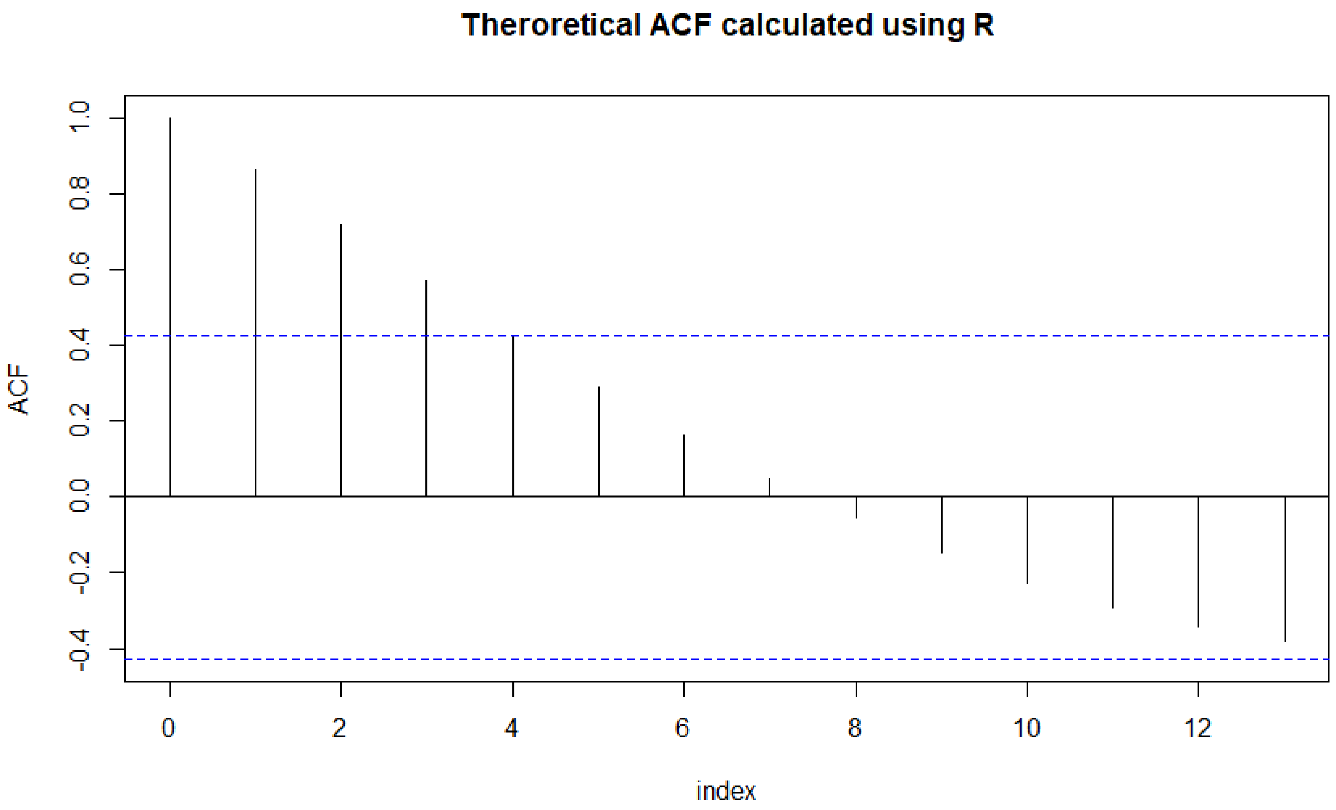
**R Code:**

```
# Calculation of ACF done by hand
th_acf = c(1:21)
th_acf[1] = 1
th_acf[2] = 1.4/1.45
th_acf[3] = ((1.4^2)/1.45)-0.45
for (i in 4:21) {
  th_acf[i] = 1.4*th_acf[i-1]-0.45*th_acf[i-2]
}

# Calculating ACF using R
r_acf = ARMAacf(ar=c(1.4, -0.45), lag.max=20)

acf(th_acf,  xlab='index', ylab='ACF', main = 'Theroretical ACF calculated by hand')
acf(r_acf,  xlab='index', ylab='ACF', main = 'Theroretical ACF calculated using R')
```






As both the plots are similar the theoretical calculations are verified.

## 4.

(a)

$$v[k] - 1.3v[k-1] + 0.24v[k-2] = e[k]$$

$$\phi_{vv}[1] = \rho_{vv}[1] = \frac{1.3}{1+0.24} = 1.0483$$

$$\rho_{vv}[2] = \frac{1.3^2}{1+0.24} - 0.24 = 1.1229$$

$$\phi_{vv}[2] = \frac{\rho_{vv}[2] - \rho_{vv}[1]^2}{1-\rho_{vv}[1]^2} = -0.2422$$

$$v[k] = e[k] + 0.6e[k-1]$$

$$\phi_{vv}[1] = \rho_{vv}[1] = \frac{0.6}{1+0.6^2} = 0.4411$$

$$\phi_{vv}[2] = -\frac{0.6^2(1-0.6^2)}{1-0.6^{2*(2+1)}} = -0.2416$$

**R Code:**

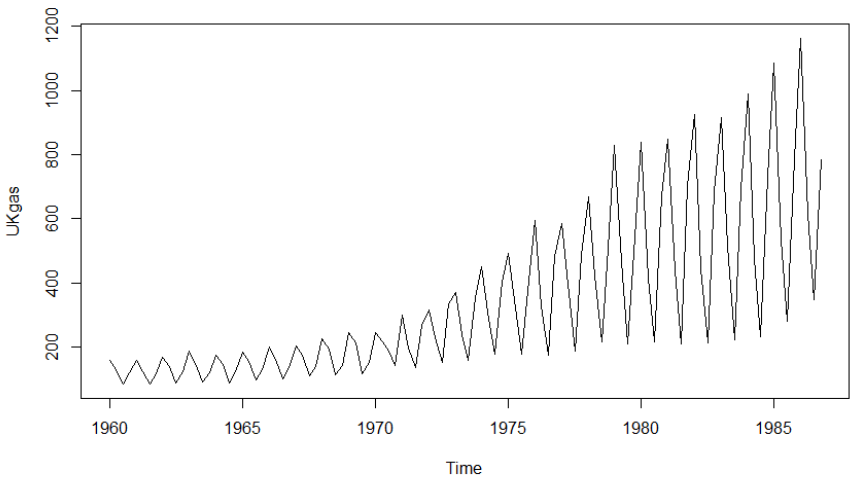
```
# AR(2) process
r_pacf_1 = ARMAacf(ar=c(1.3, -0.24), lag.max=20, pacf=TRUE)
print('For AR(2) process')
r_pacf_1[1]
r_pacf_1[2]

# MA(1) process
r_pacf_2 = ARMAacf(ma=c(0.6), lag.max=20, pacf=TRUE)
print('For MA(1) process')
r_pacf_2[1]
r_pacf_2[2]
```

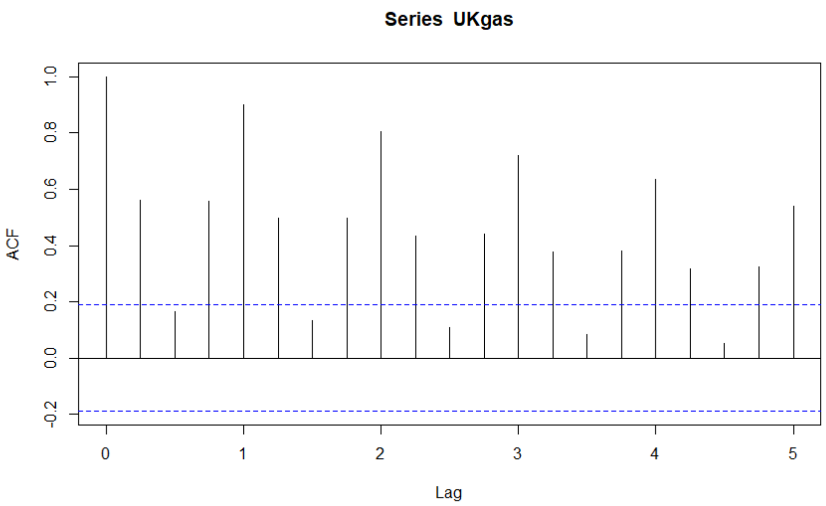
**Output:**
'For AR(2) process'
l=1 1.048387
l=2 -0.24
'For MA(1) process'
l=1 0.4411765
l=2 -0.2416756

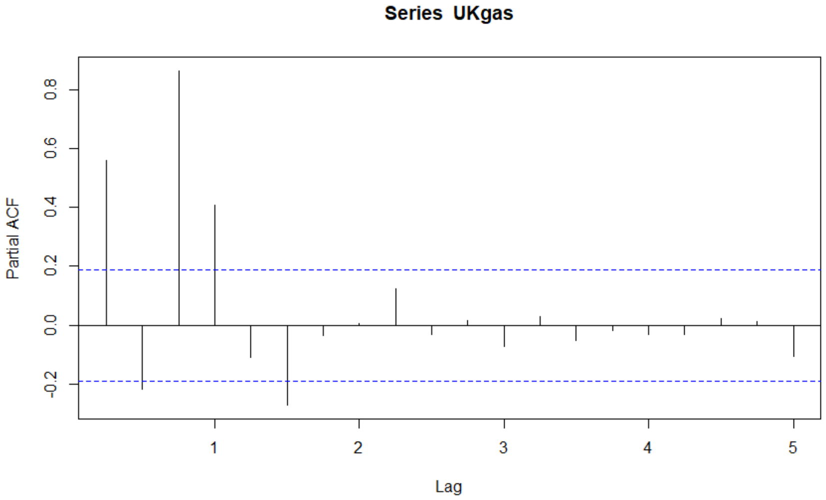
The PACF values obtained using R and by hand calculation are similar.

(b)
**UKGas data**
**R Code**:
```
plot(UKgas)
plot(acf(UKgas))
plot(pacf(UKgas))
```



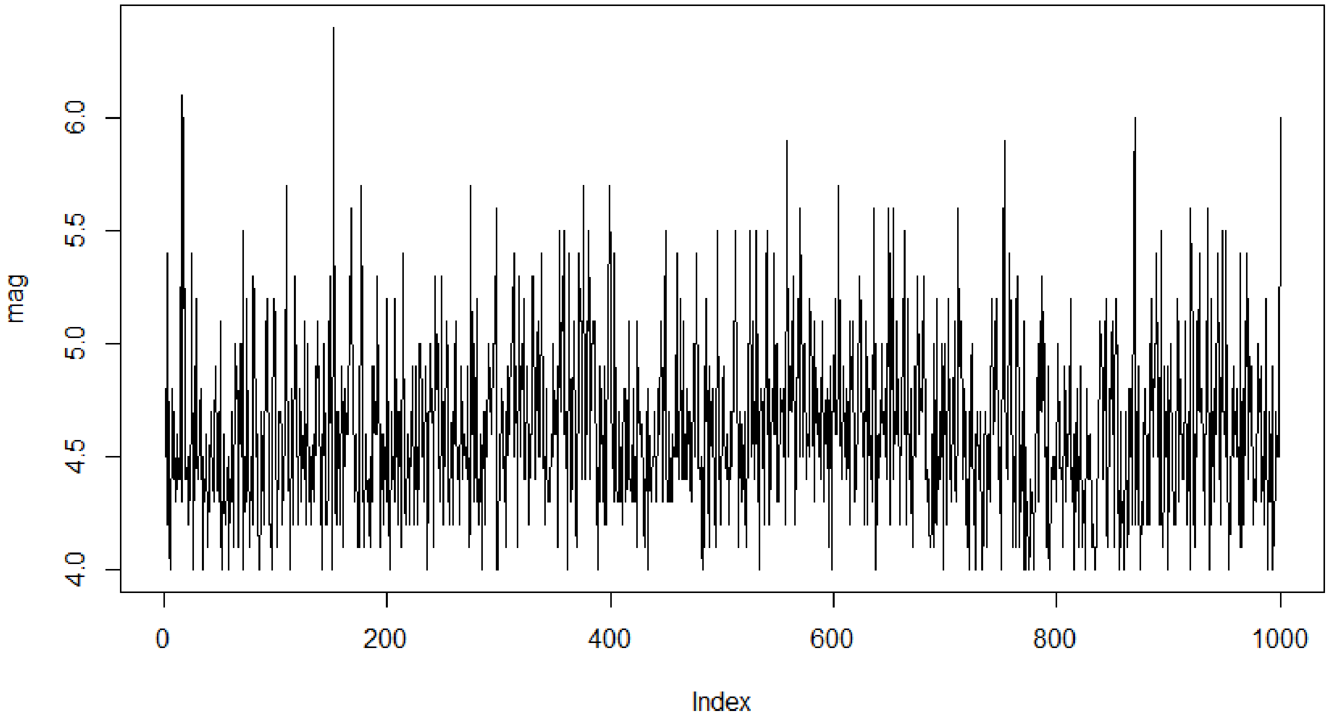

The process is non-stationary. There seems to be a periodicity and an Integrating effect.



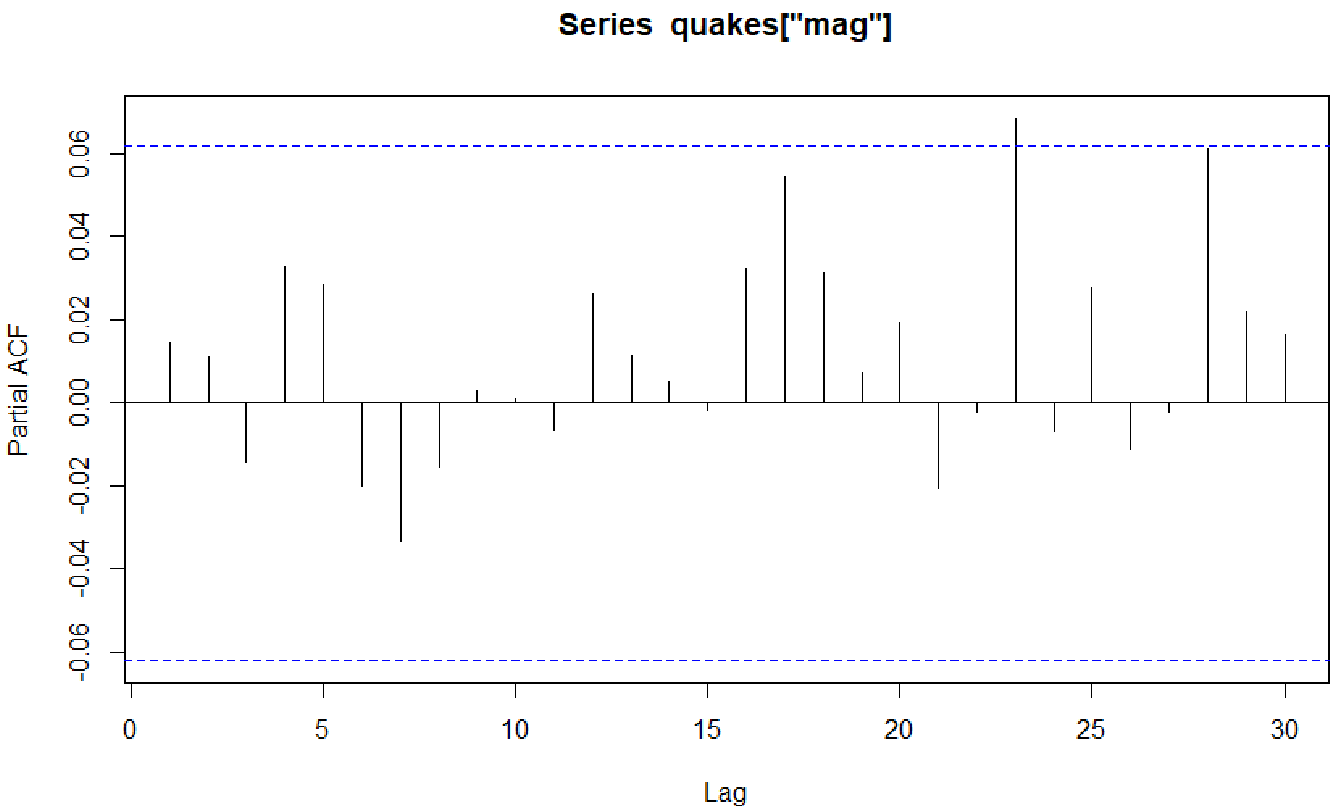


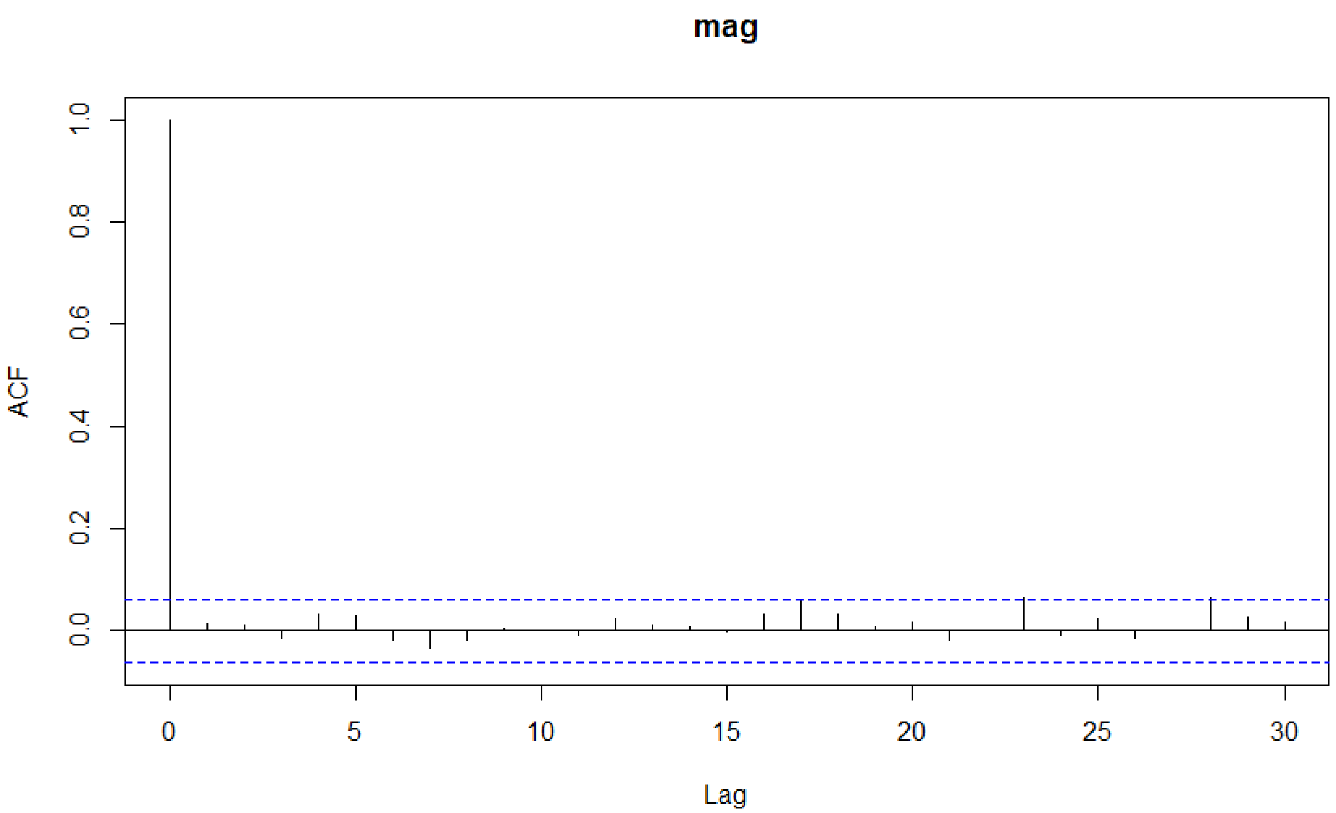

**quakes (mag)**
**R code:**

```
with(quakes,plot(mag, type='l'))
plot(acf(quakes['mag']))
plot(pacf(quakes['mag']))
```




The process seems to be stationary with mean greater than zero. There seems to be no periodicity and integrating effect.




From the ACF and PACF plots it is evident that the process seems to closely resemble a White noise process.

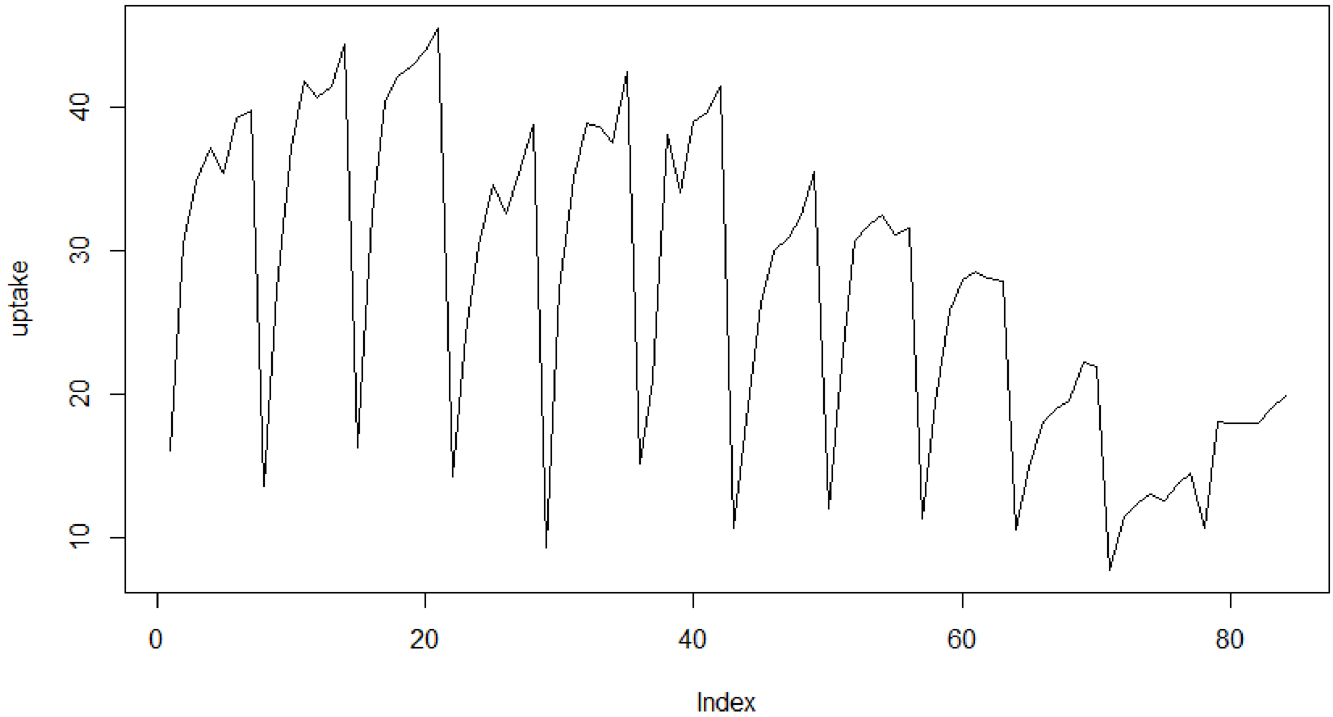
CO2 – uptake and conc

**R Code:**

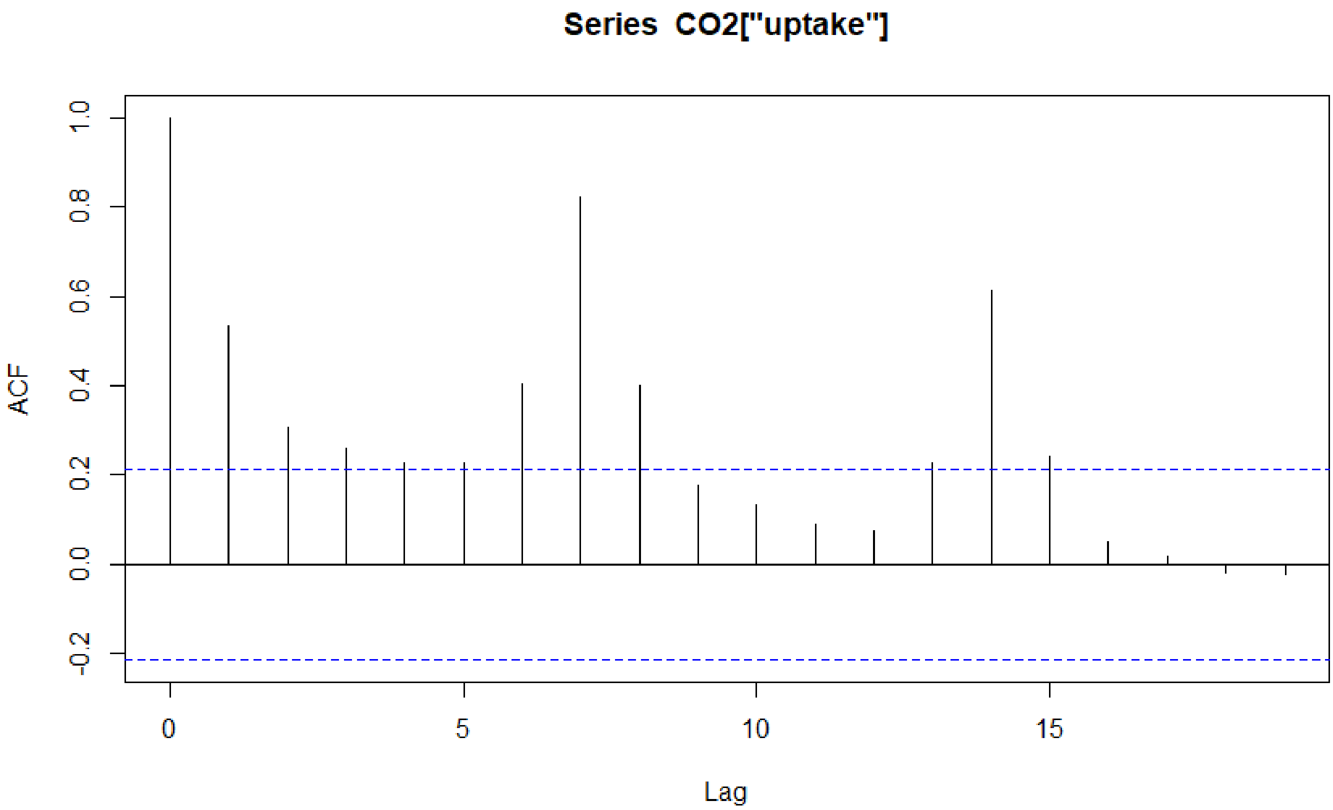
```
with(CO2,plot(conc, type='l'))
plot(acf(CO2['conc']))
plot(pacf(CO2['conc']))
```

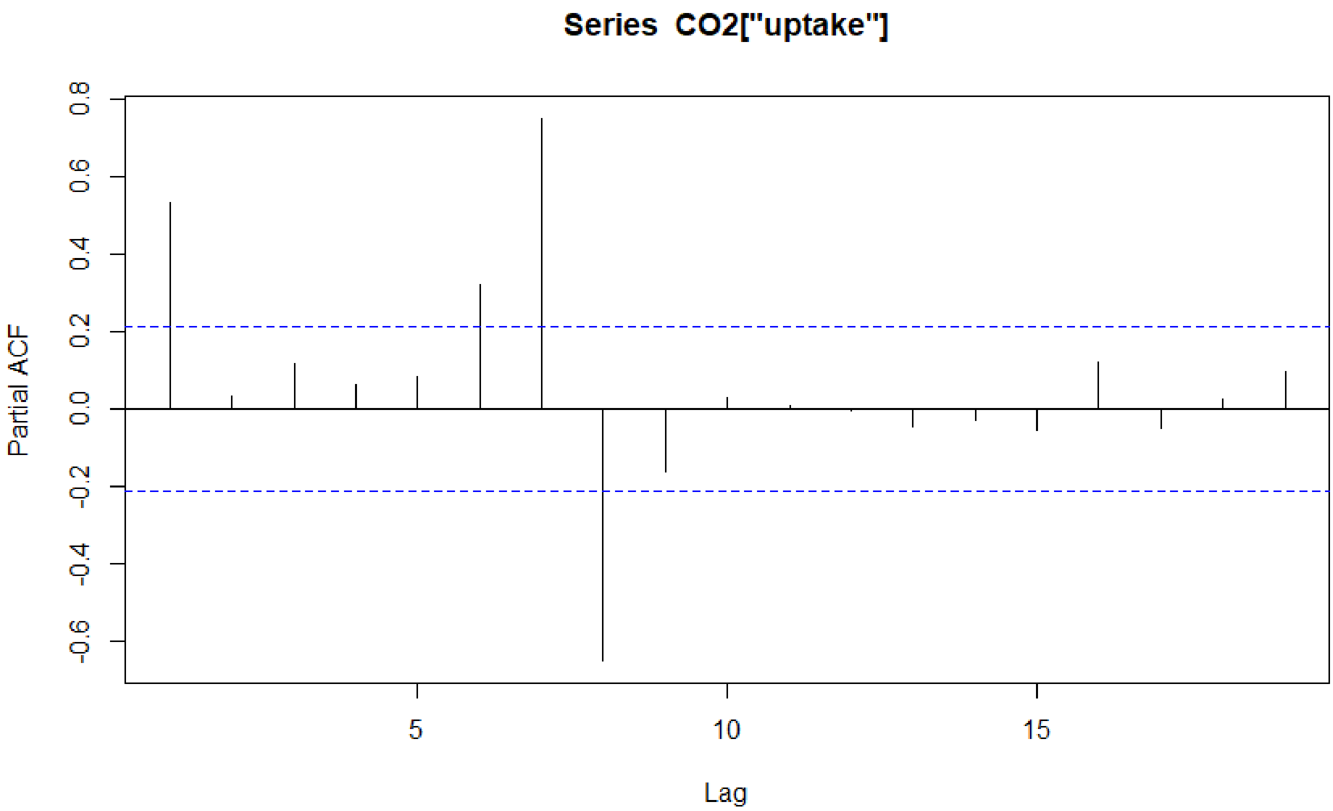
```
with(CO2,plot(uptake, type='l'))
plot(acf(CO2['uptake']))
plot(pacf(CO2['uptake']))
```

Uptake plots:



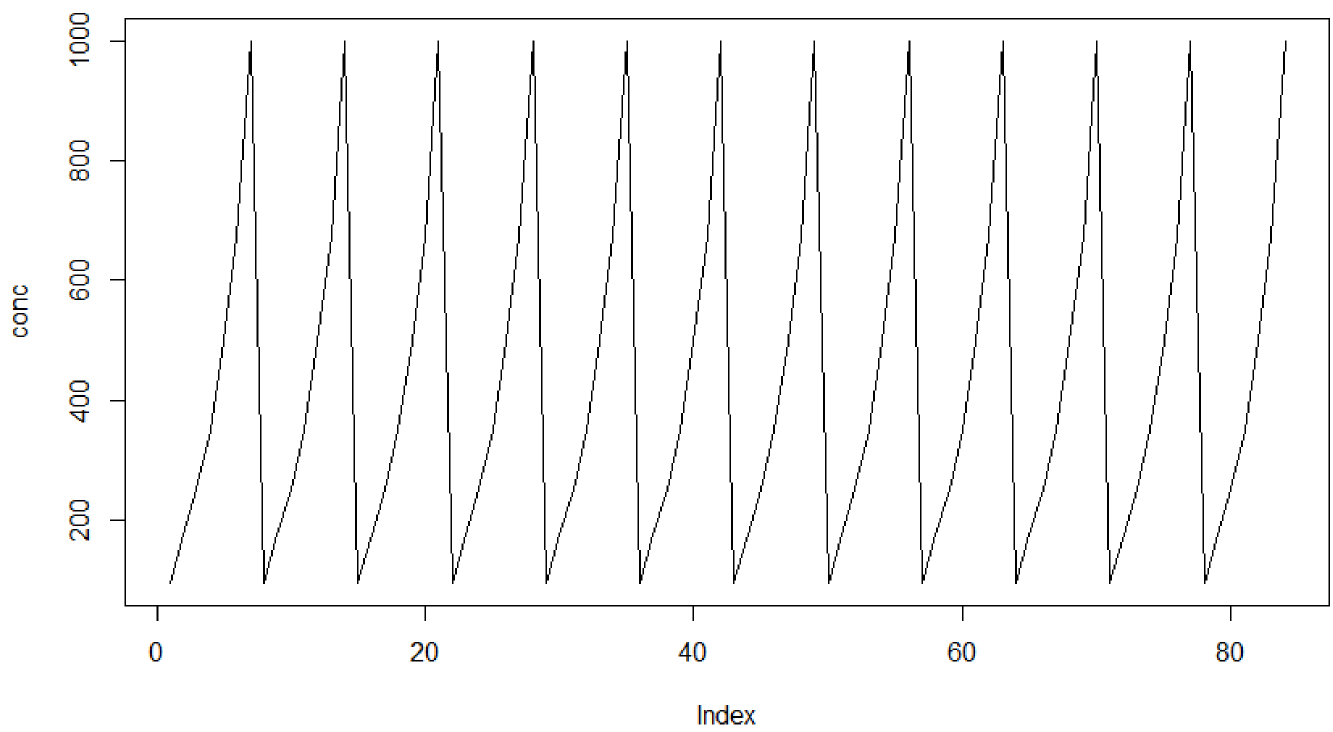

The process is non-Stationary. There is a periodicity and an Integrating effect.



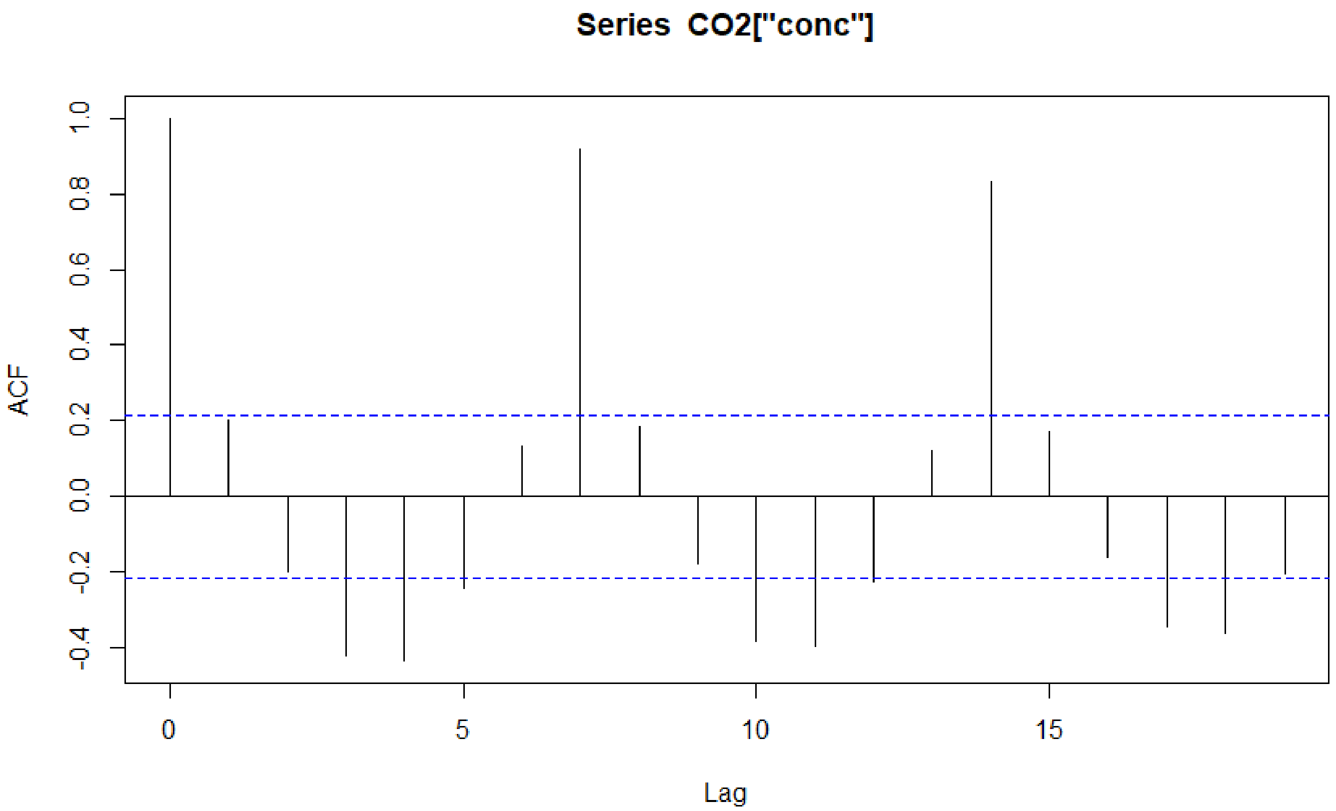


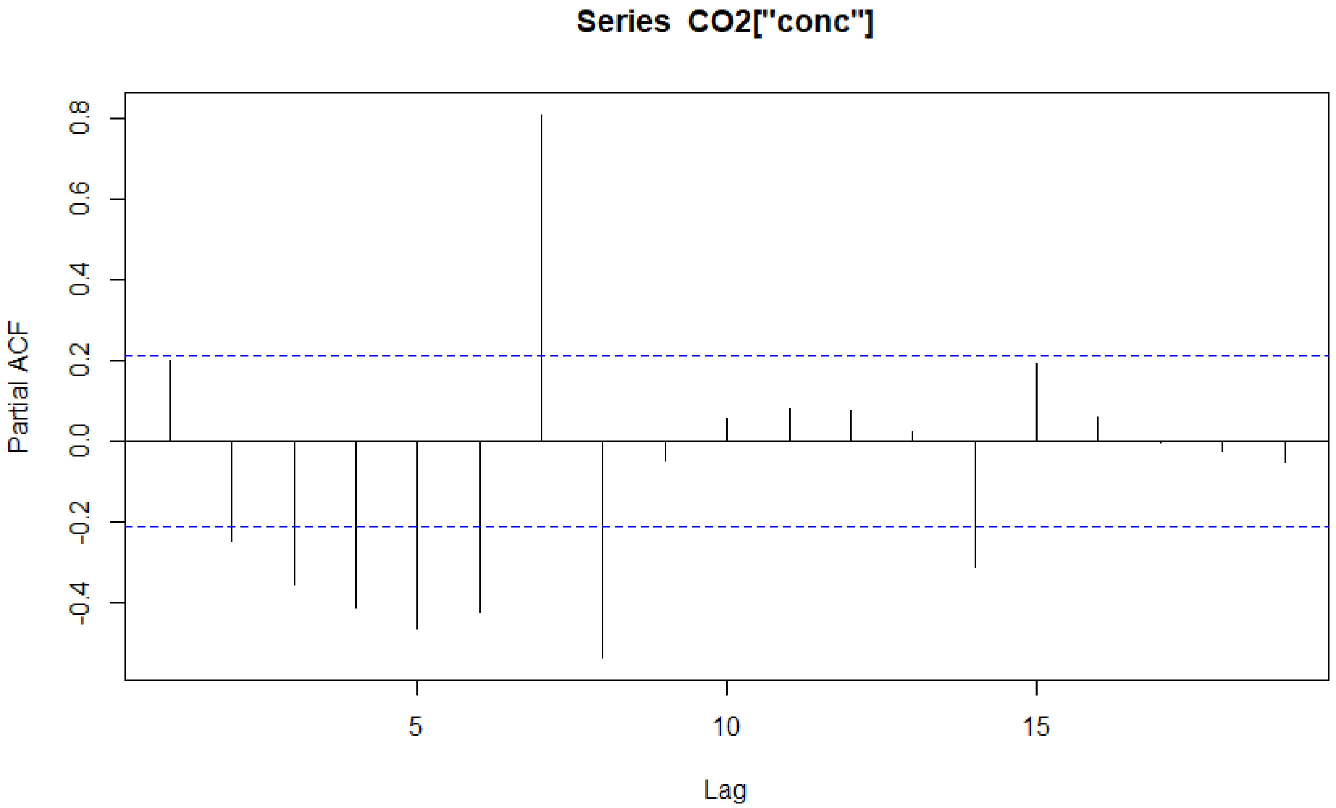

It is difficult to concur if the process is AR or MA because the time period of periodicity is small and the ACF seems to decline in each of the time period. In PACF there is a sharp cut off so we can say it is somewhat like an AR process with periodicity.

Conc plots:




The process is stationary although after the assumption of ergodicity. There is a clear periodicity but no integrating effect.




From the ACF and PACF plots regardless of the values we could say that the process is quite Deterministic.