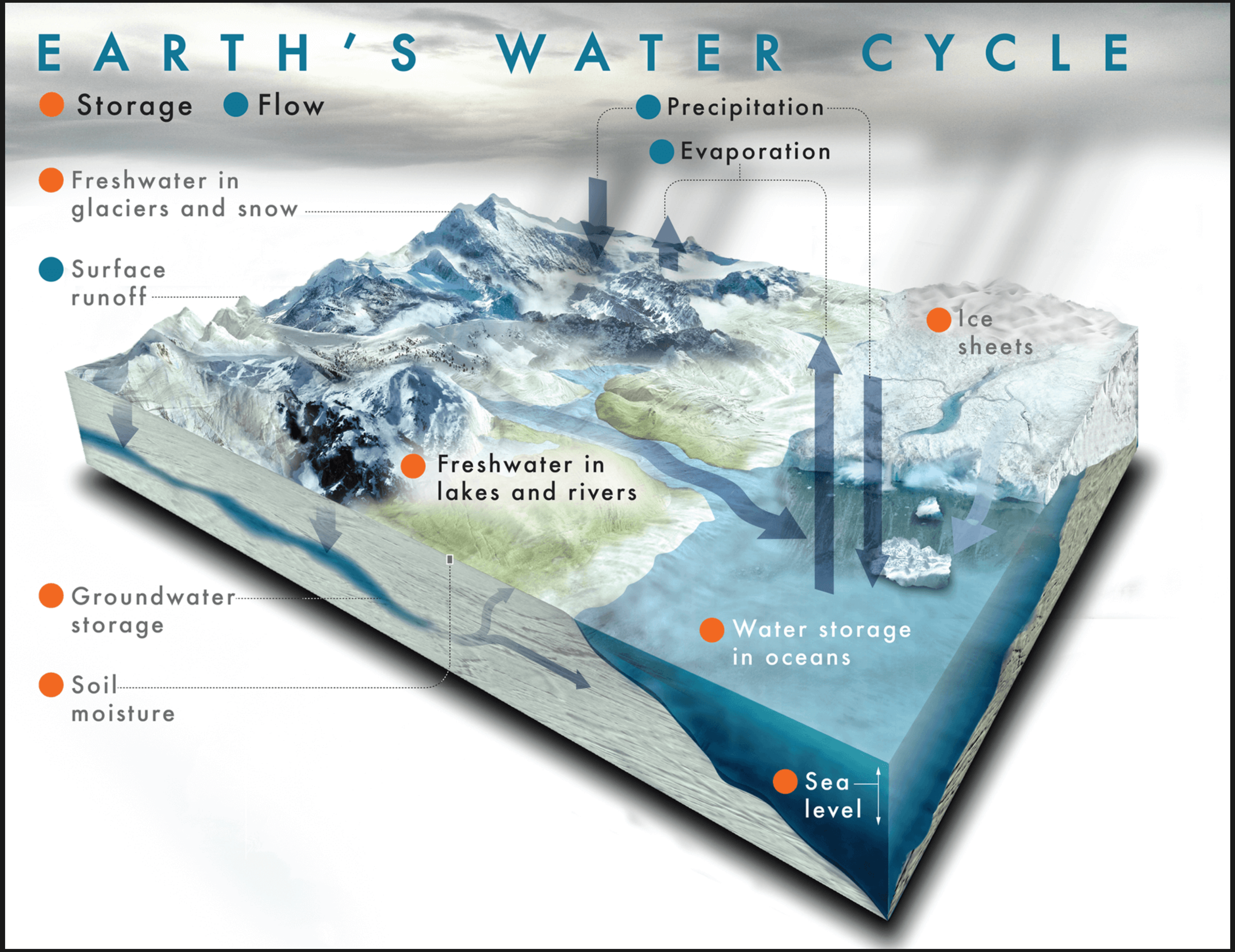
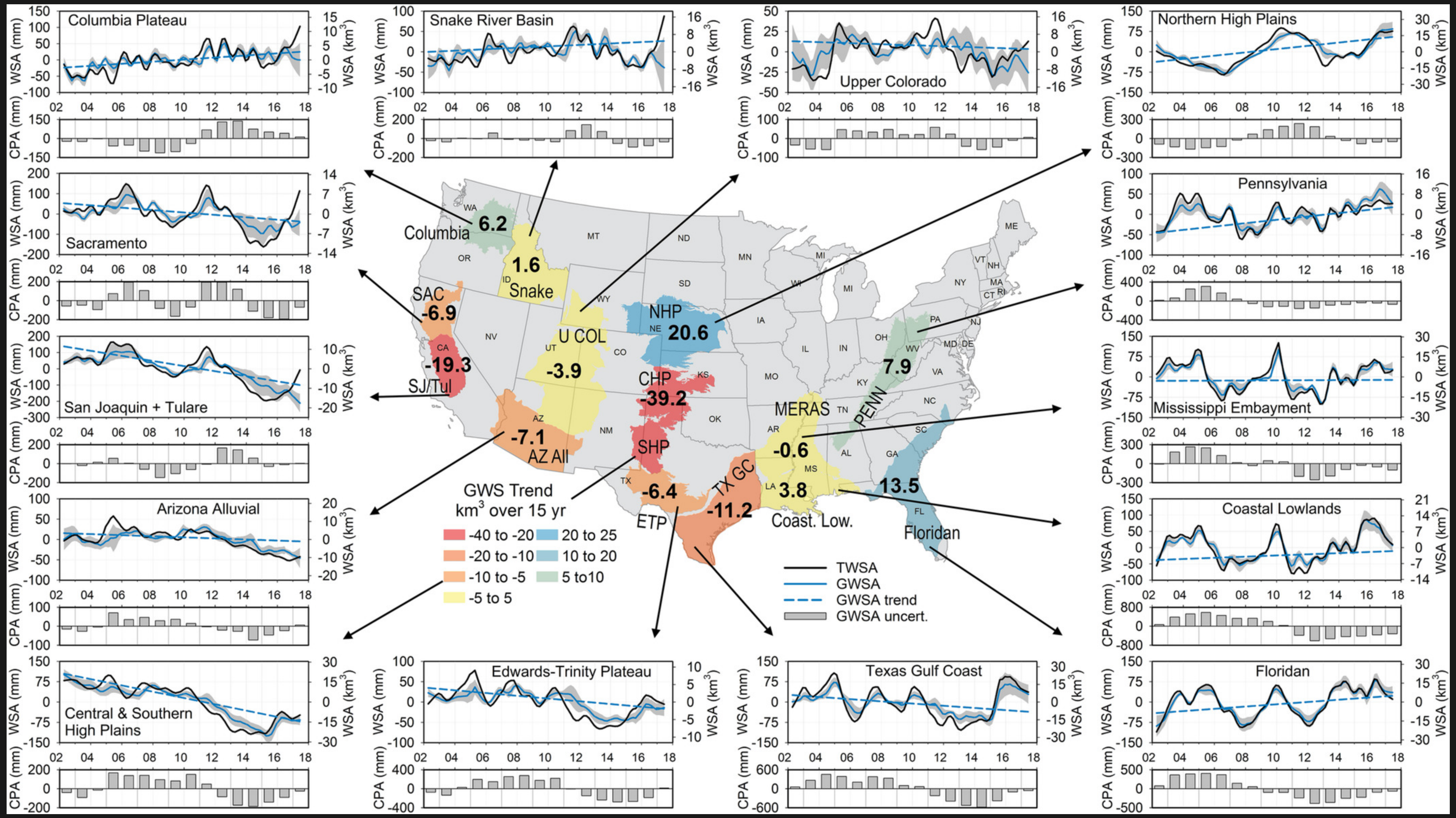
Gravity and the Water Cycle I

Part of EOAfrica R & D



Source: JPL/Caltech

Showcase: US Groundwater changes from GRACE/GRACE-FO



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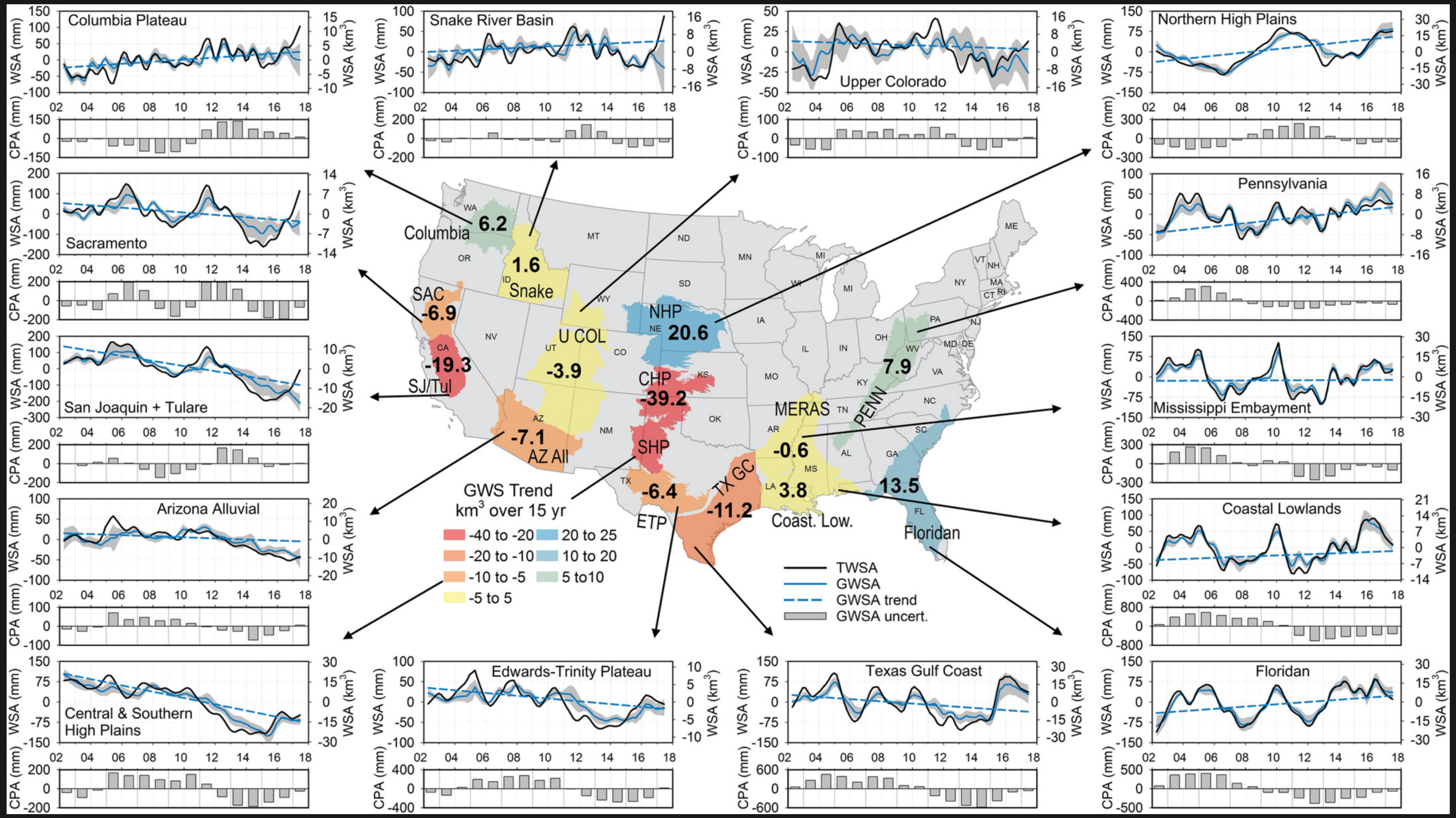

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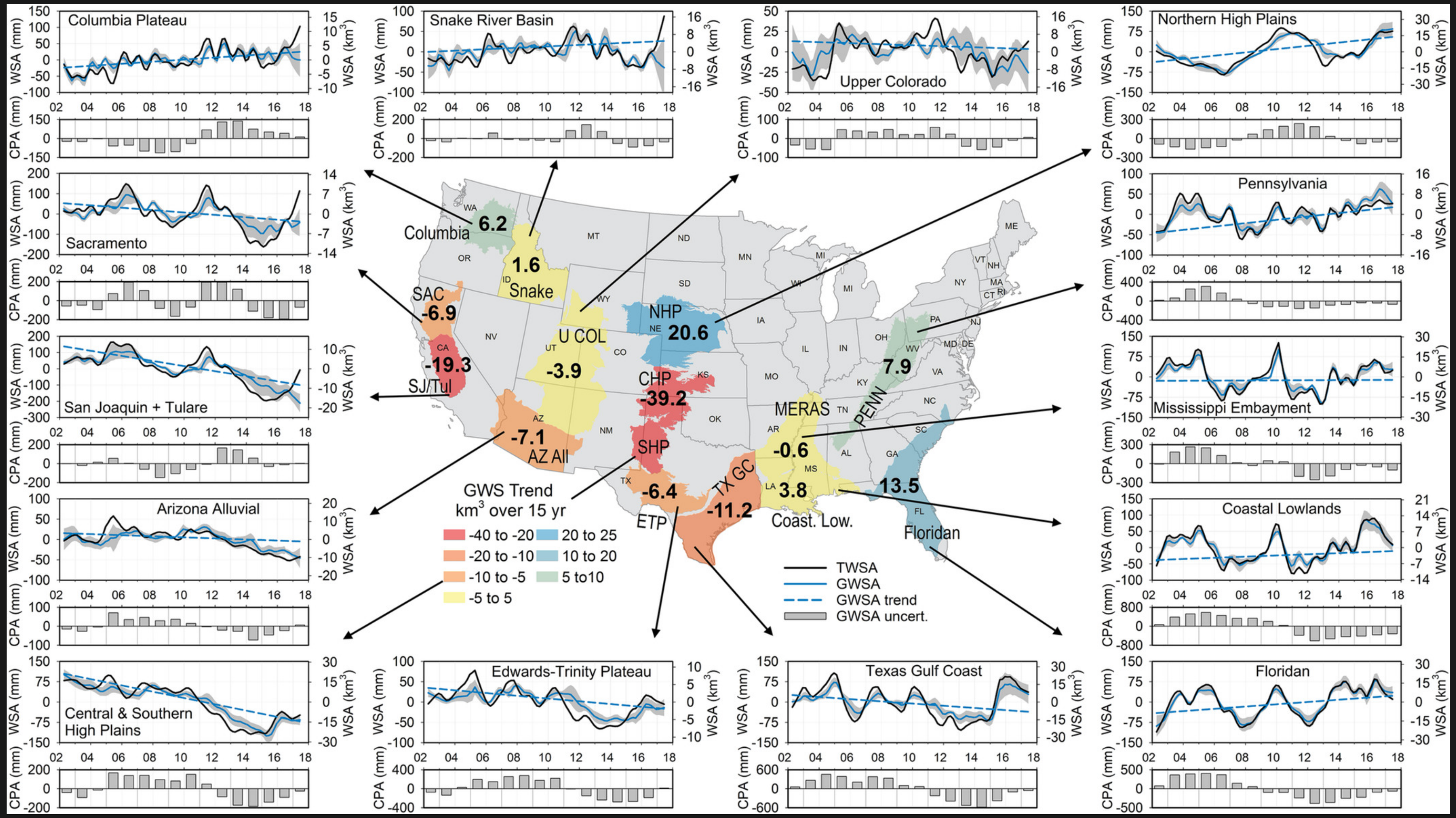

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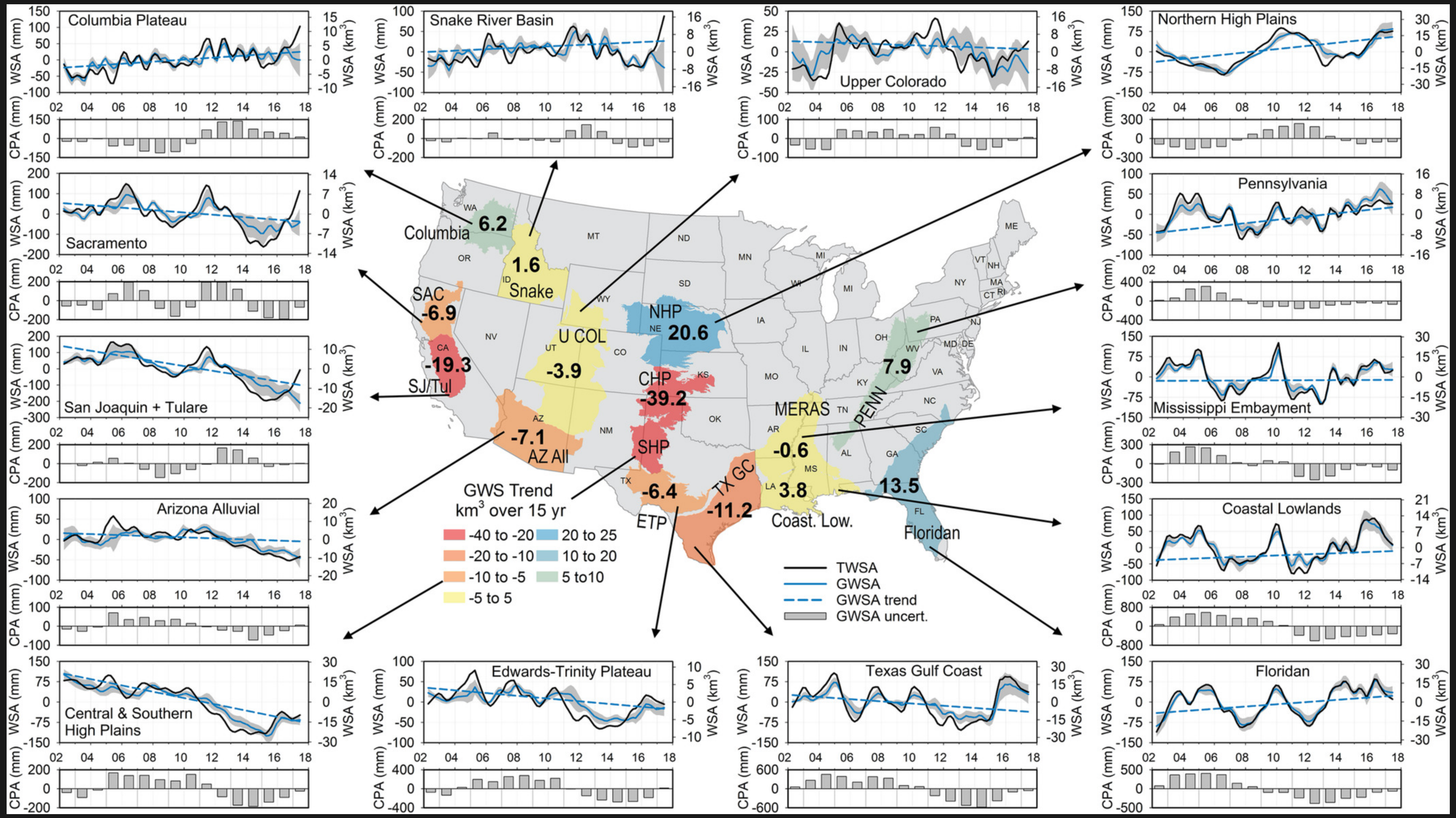

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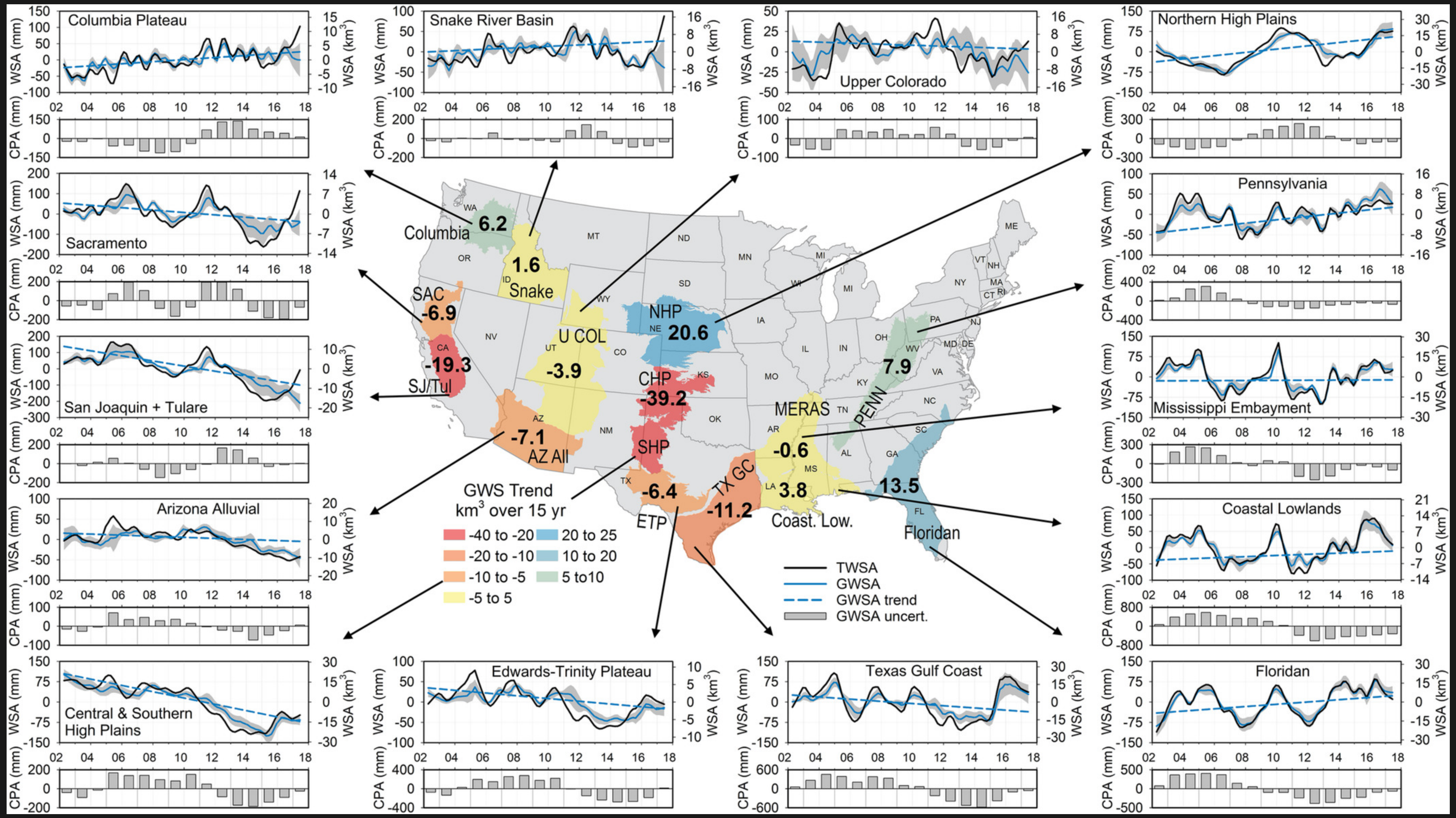

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- Net US decrease 90 km^3 over 15 years, but signs vary per aquifer
- Groundwater peaks (Blue) are damped relative to total water storage (black), why?
- Gravity is sensitive to mass (≠volume!)

Who am I?

Field trip Speulderbos, June 2021

- Roelof Rietbroek, main field: geodesy
- since august 2020 assistant prof at the WRS department
- Before that
    - Aerospace Engineering at the TU Delft (NL), MSc 2007
    - Researcher/Phd candidate at GFZ Potsdam & University of Bonn (Germany)
    - PhD 2014 title: "Retrieval of Sea Level and Surface Loading Variations from Geodetic Observations and Model Simulations"

What will you learn

We're not talking about the Hollywood version of 'gravity'

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We're not talking about the Hollywood version of 'gravity'

- The basic principles of how geodesy/gravity relates to the water cycle
- The measurement principles of satellite gravimetry and terrestrial gravimetry
- How gravity measurements can contribute to improving hydrological models

Classical definition of Geodesy

Friedrich Robert Helmert

Helmert's desk (GFZ, Potsdam)

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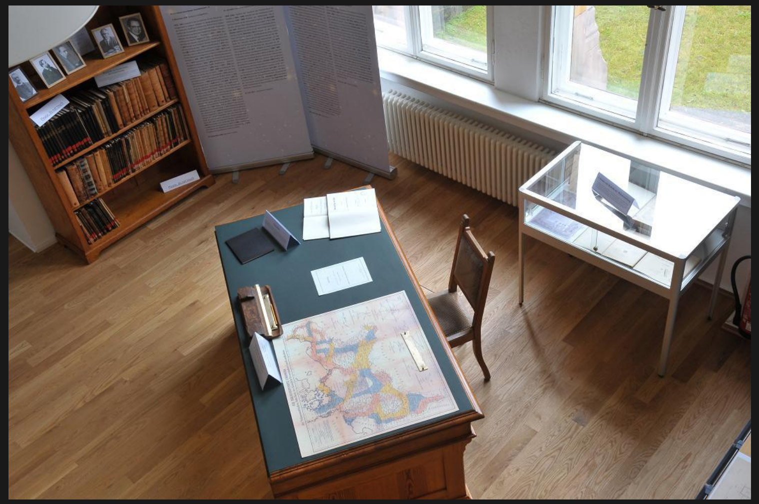

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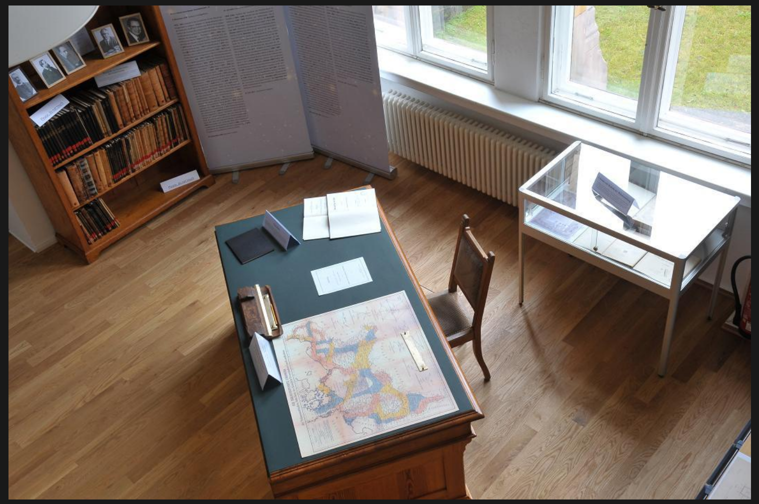

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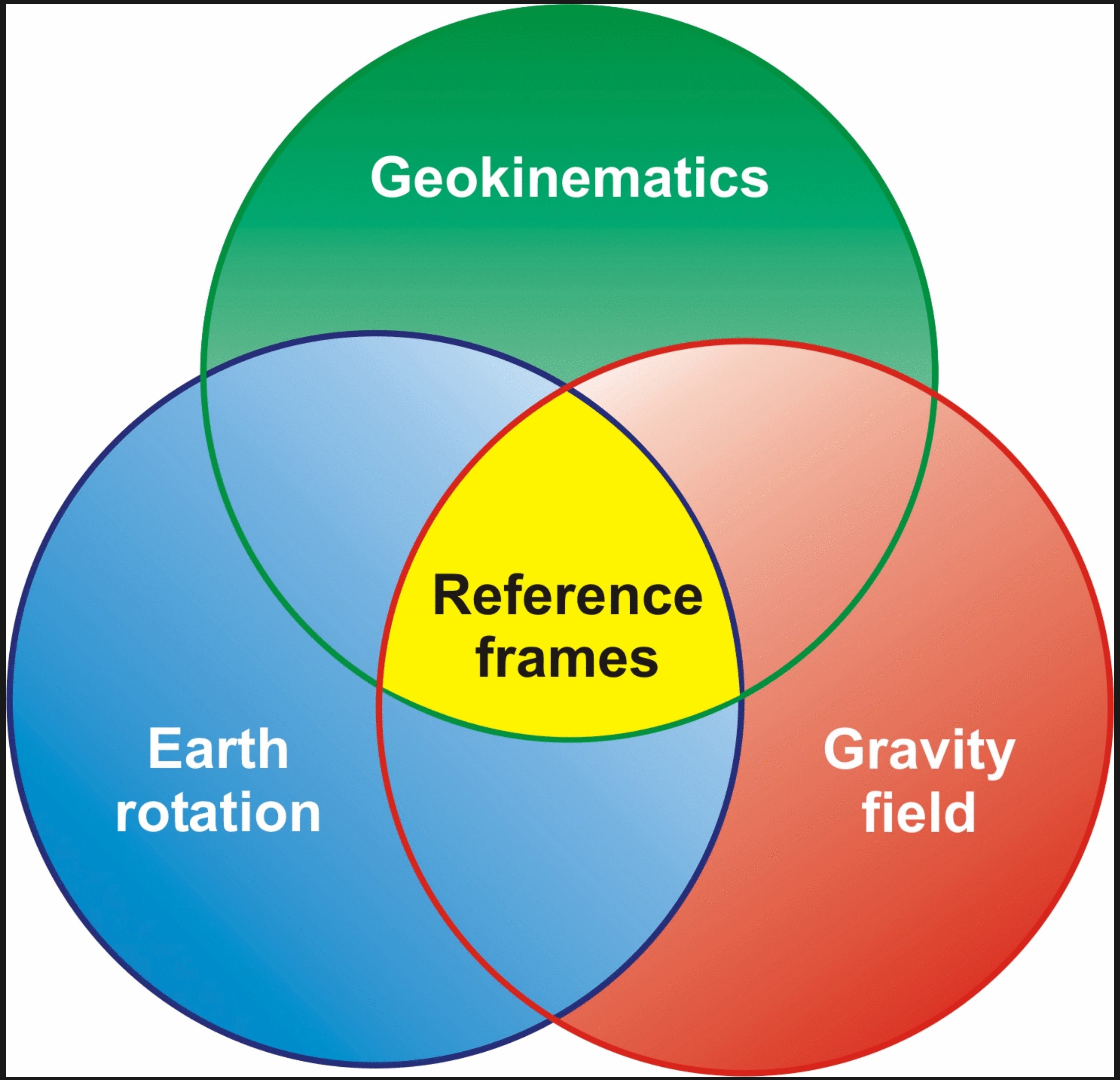
- Still relevant and well ahead of his time

The three pillars of Geodesy



Source: www.iag-ggos.org

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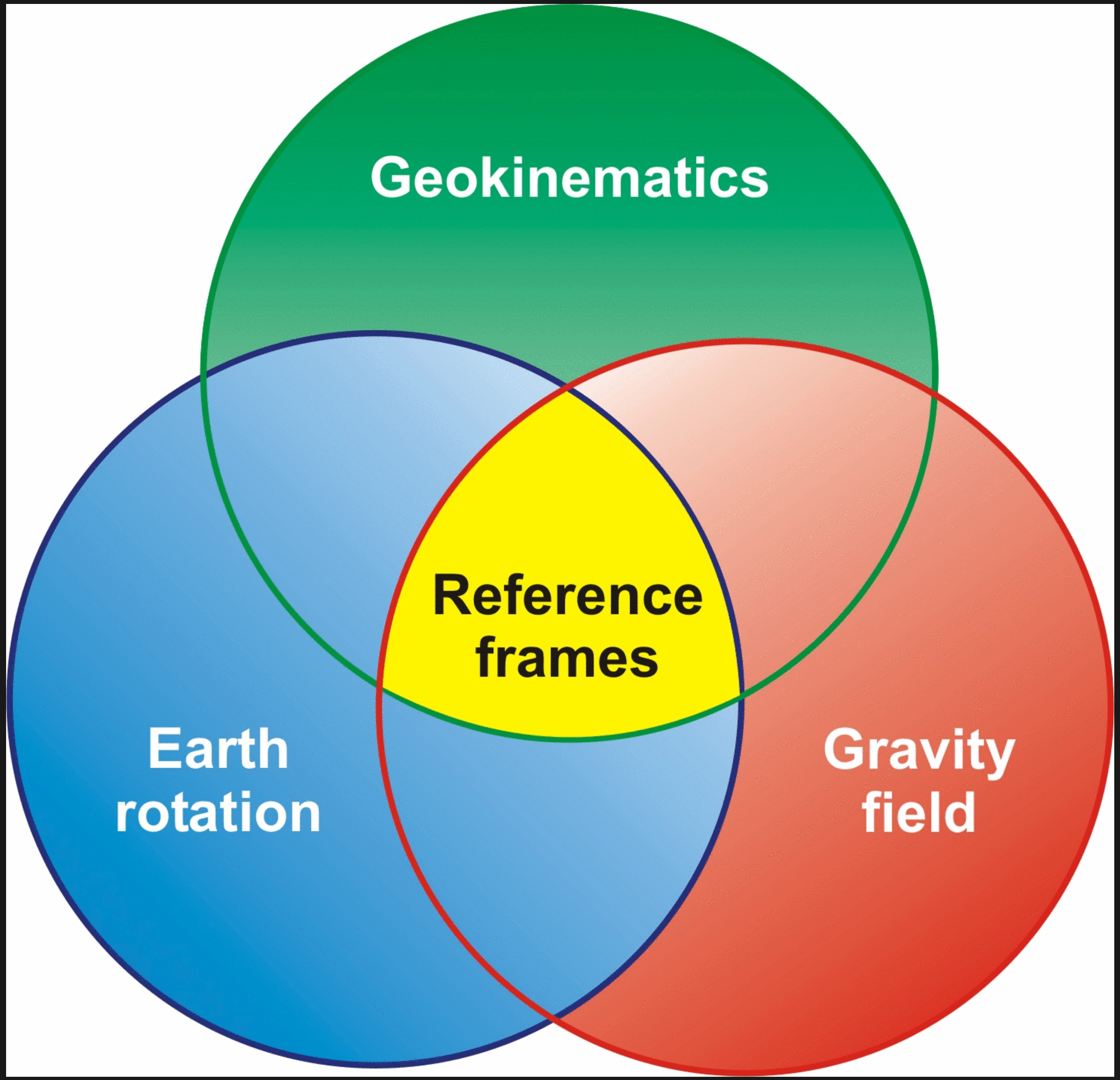

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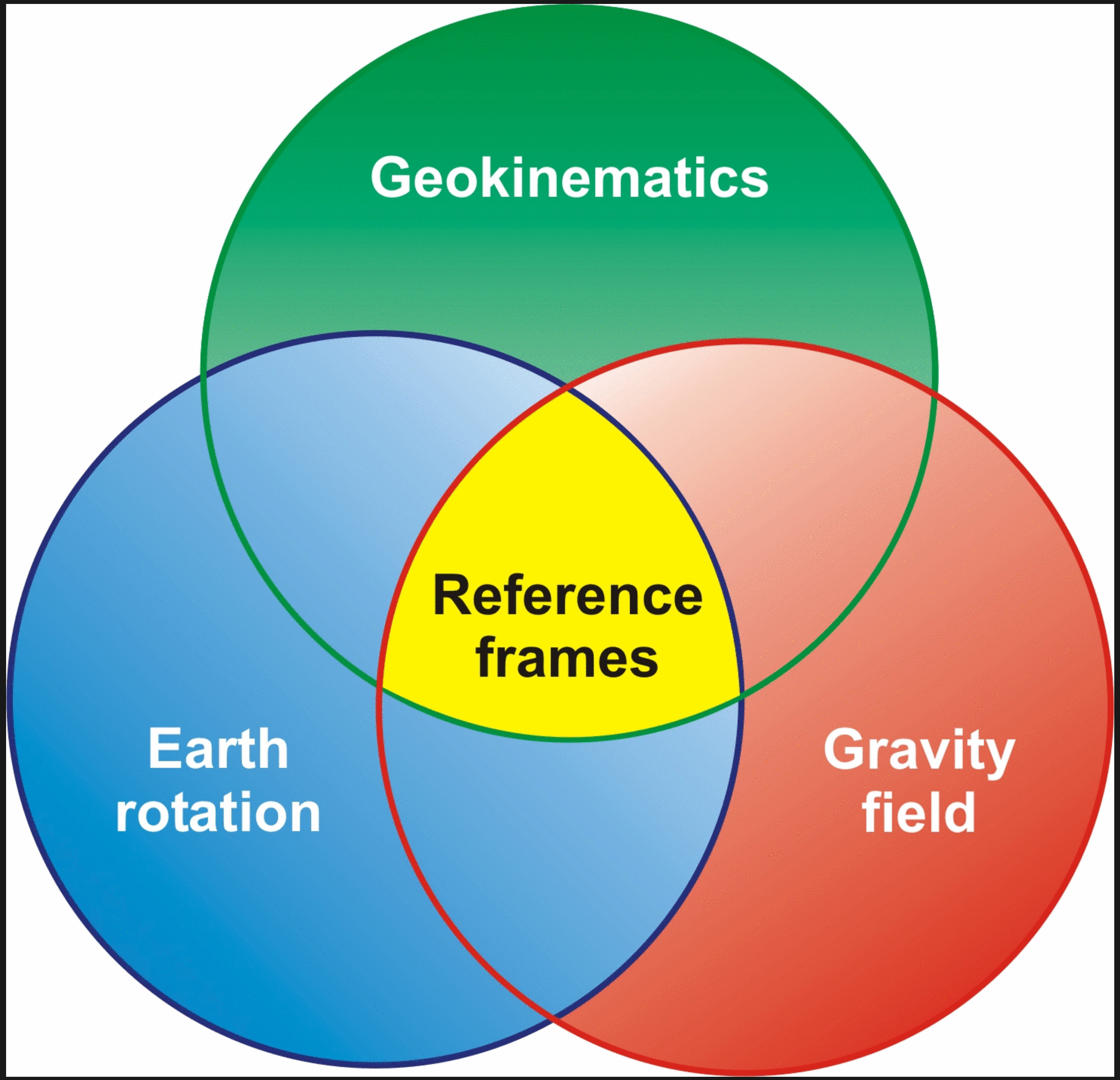

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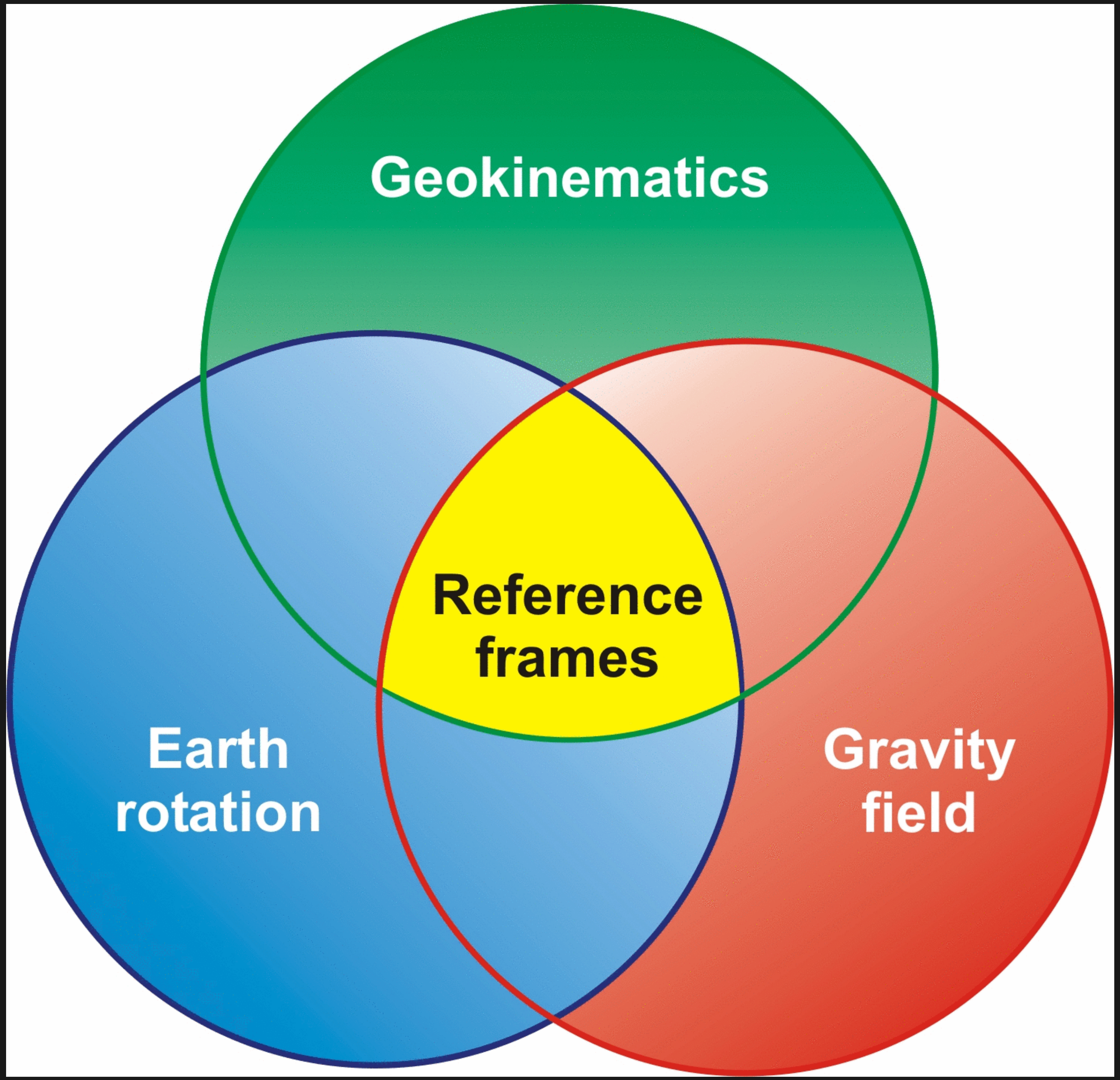

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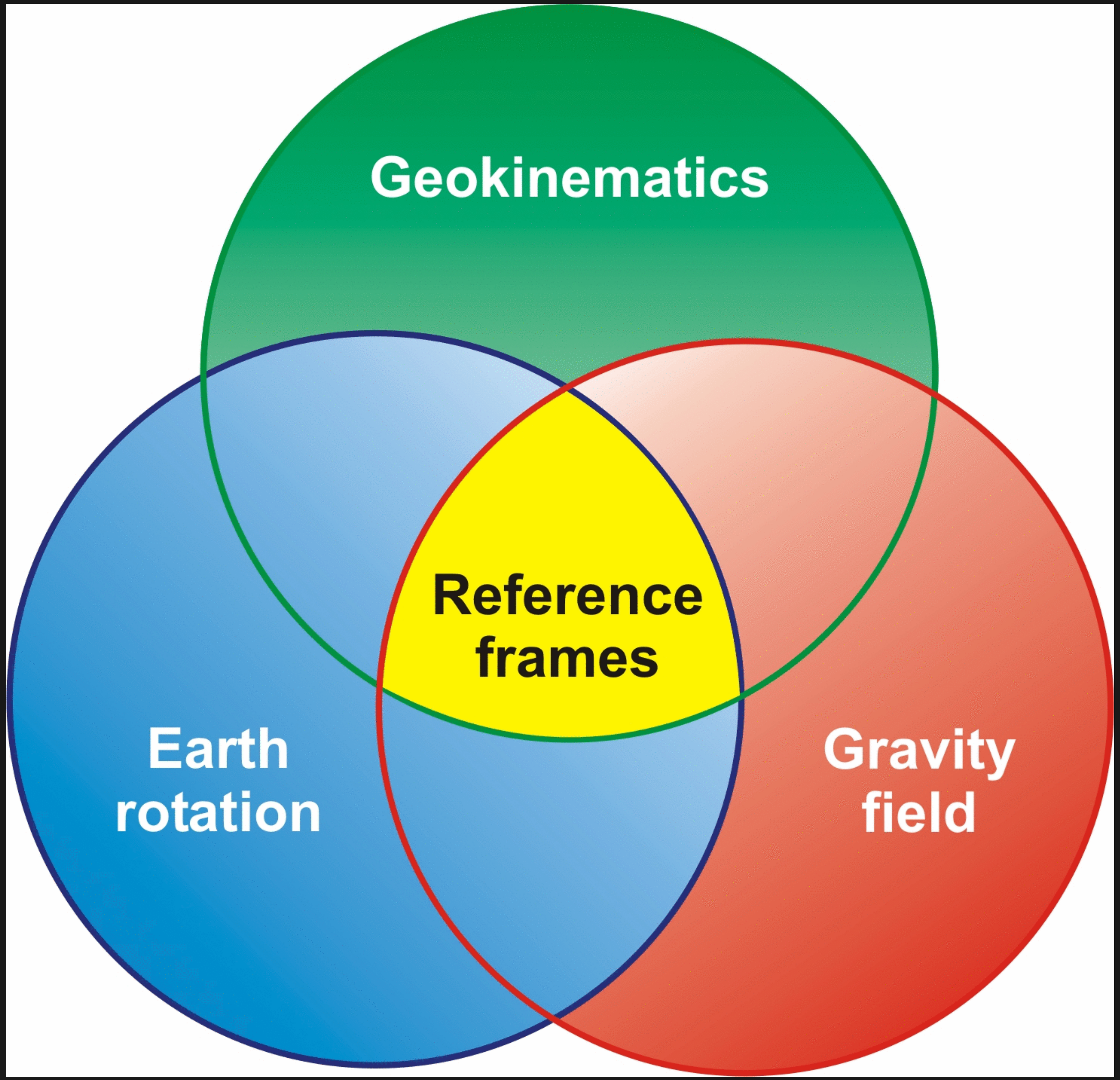

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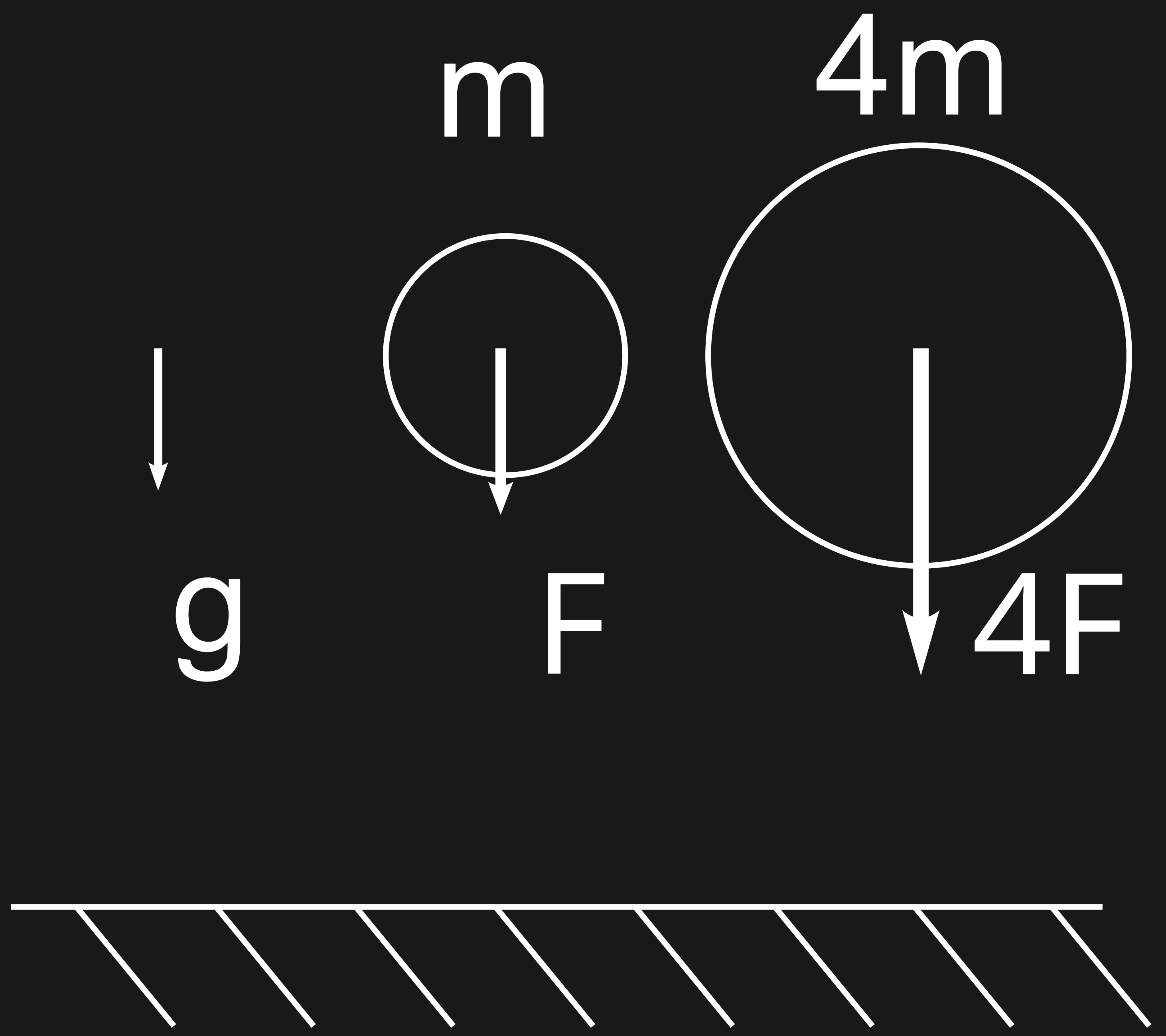
- **Reference frames:** Definition and realization of Celestial and or Earth-fixed reference systems/frames

Force versus Acceleration



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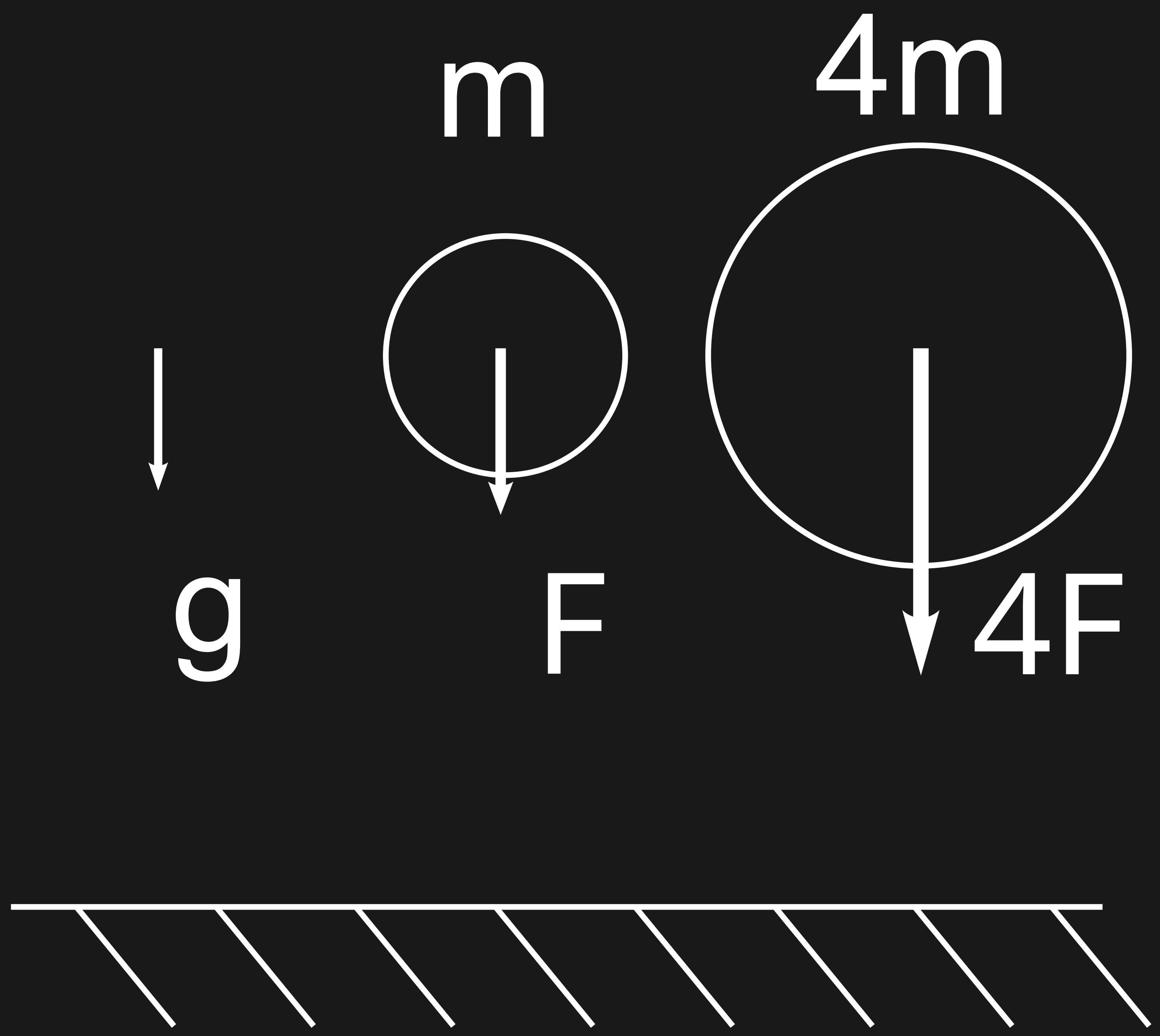

- Gravitational force for: objects in free fall:
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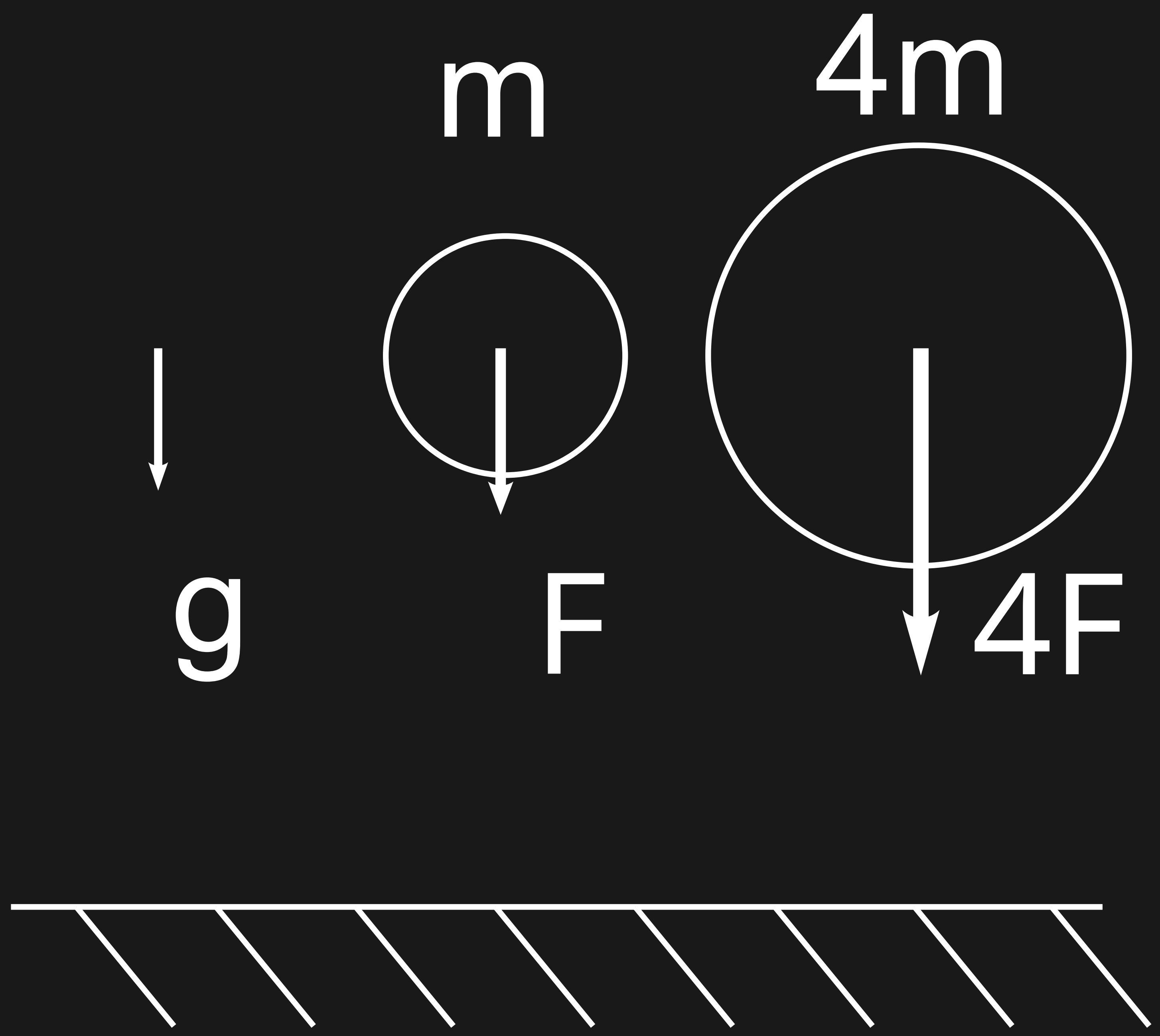

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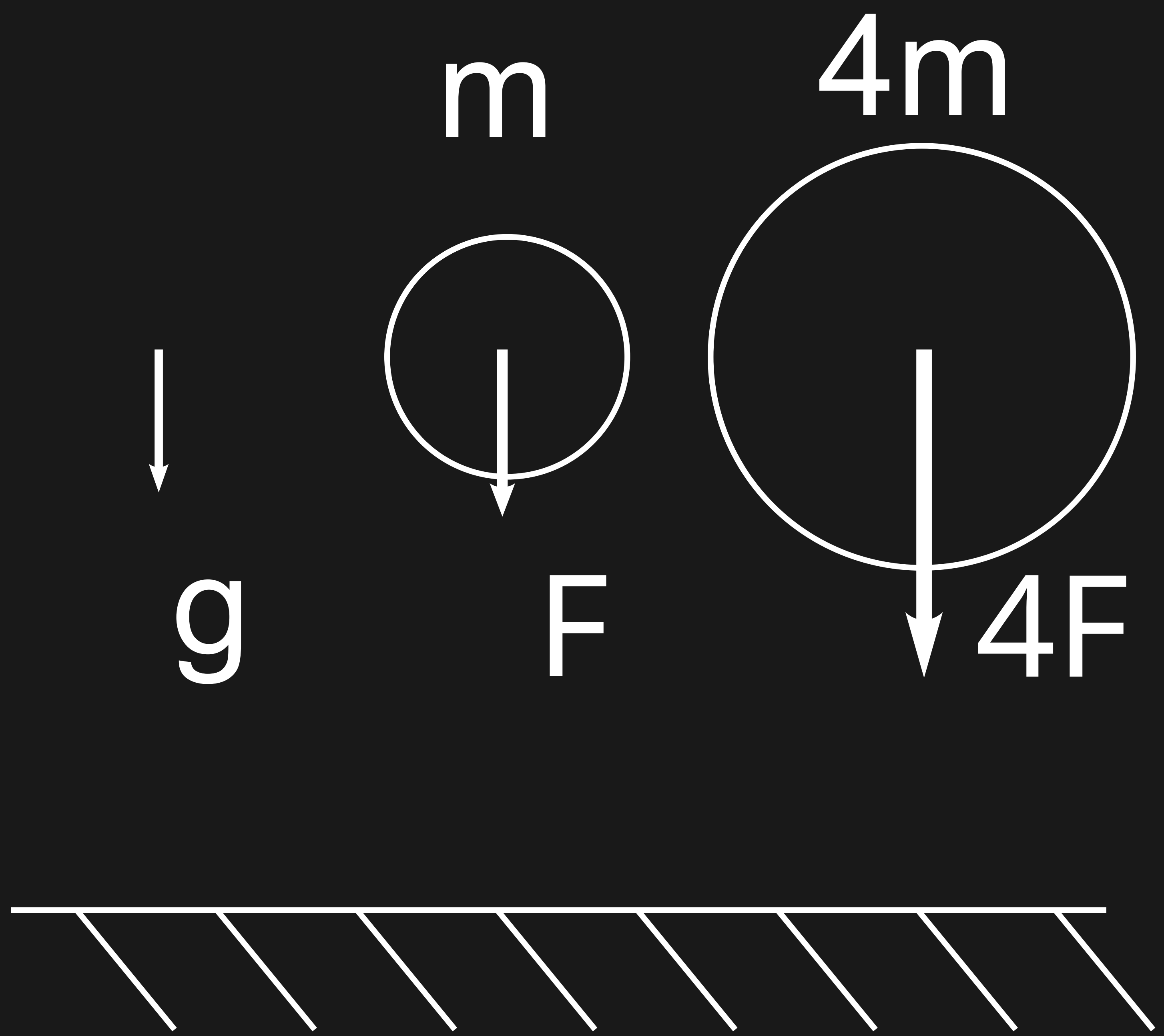

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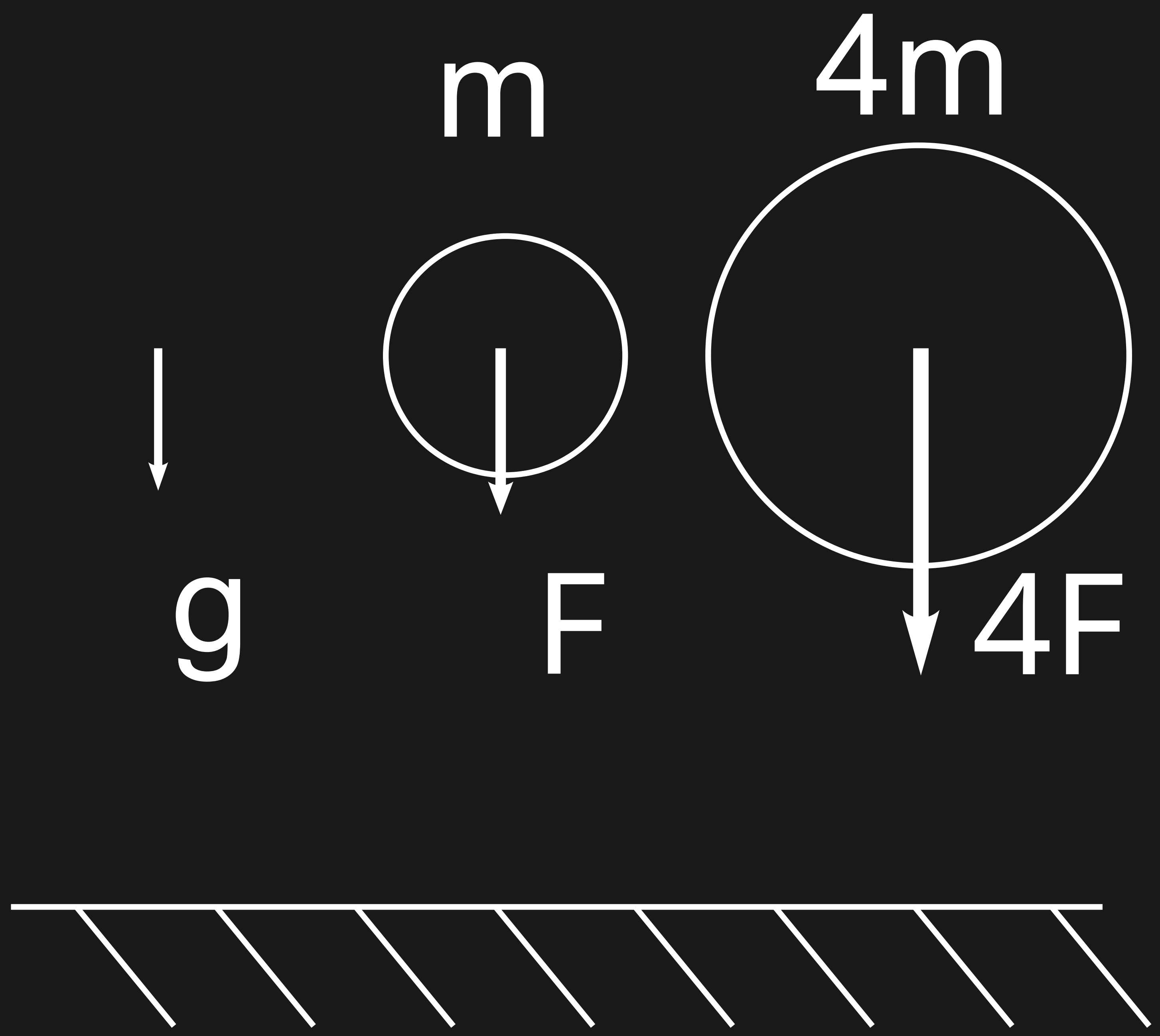

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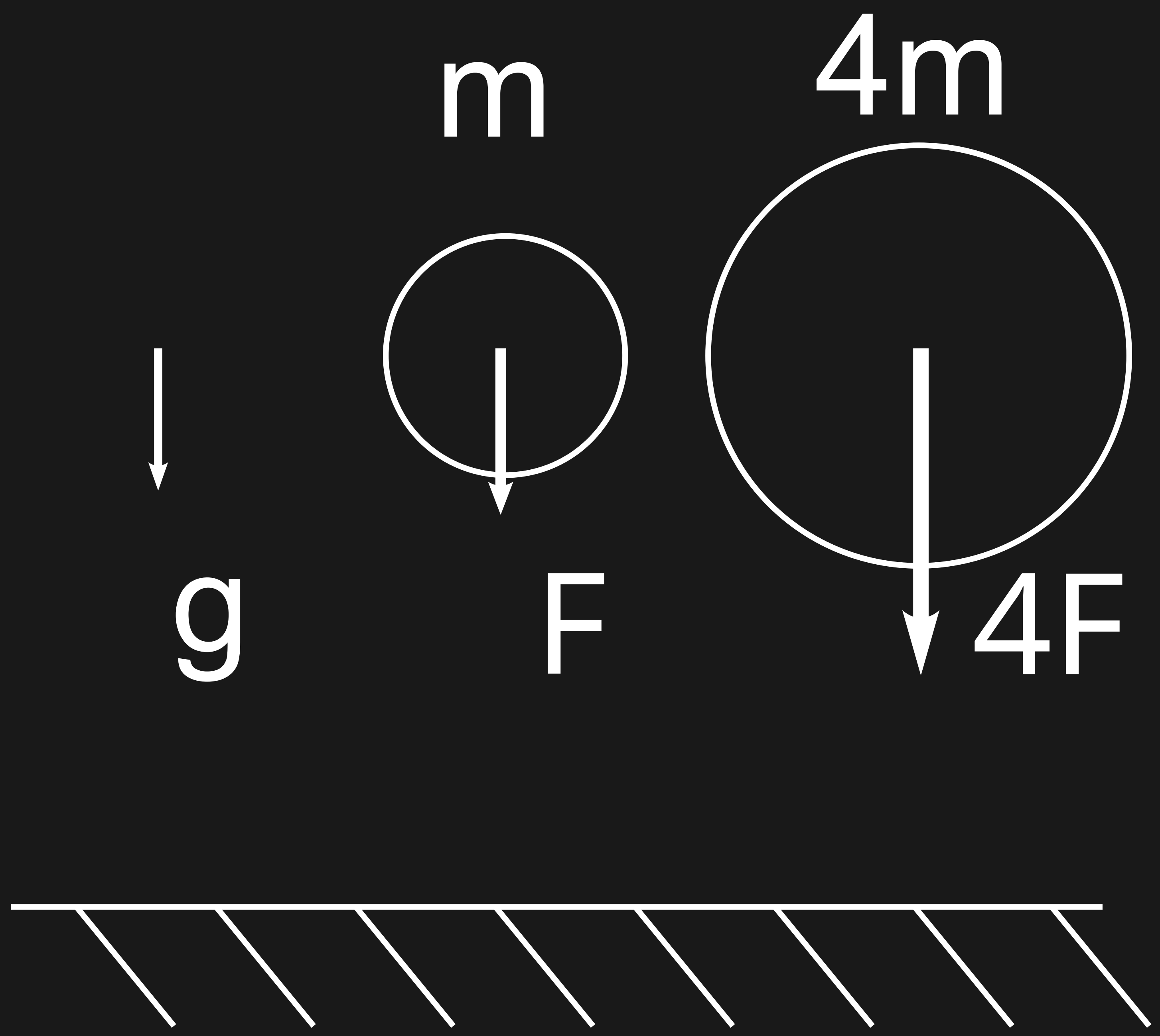

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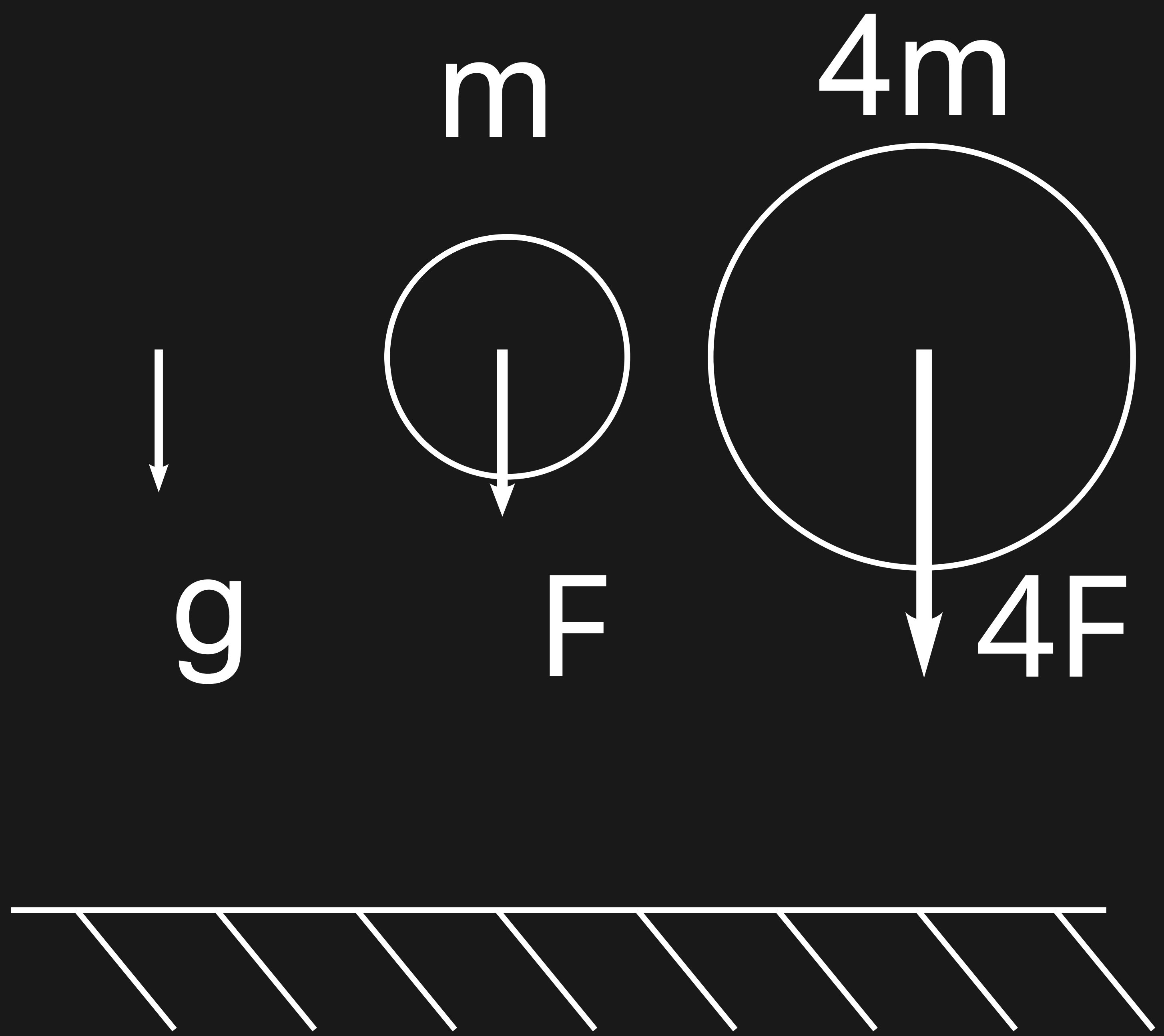

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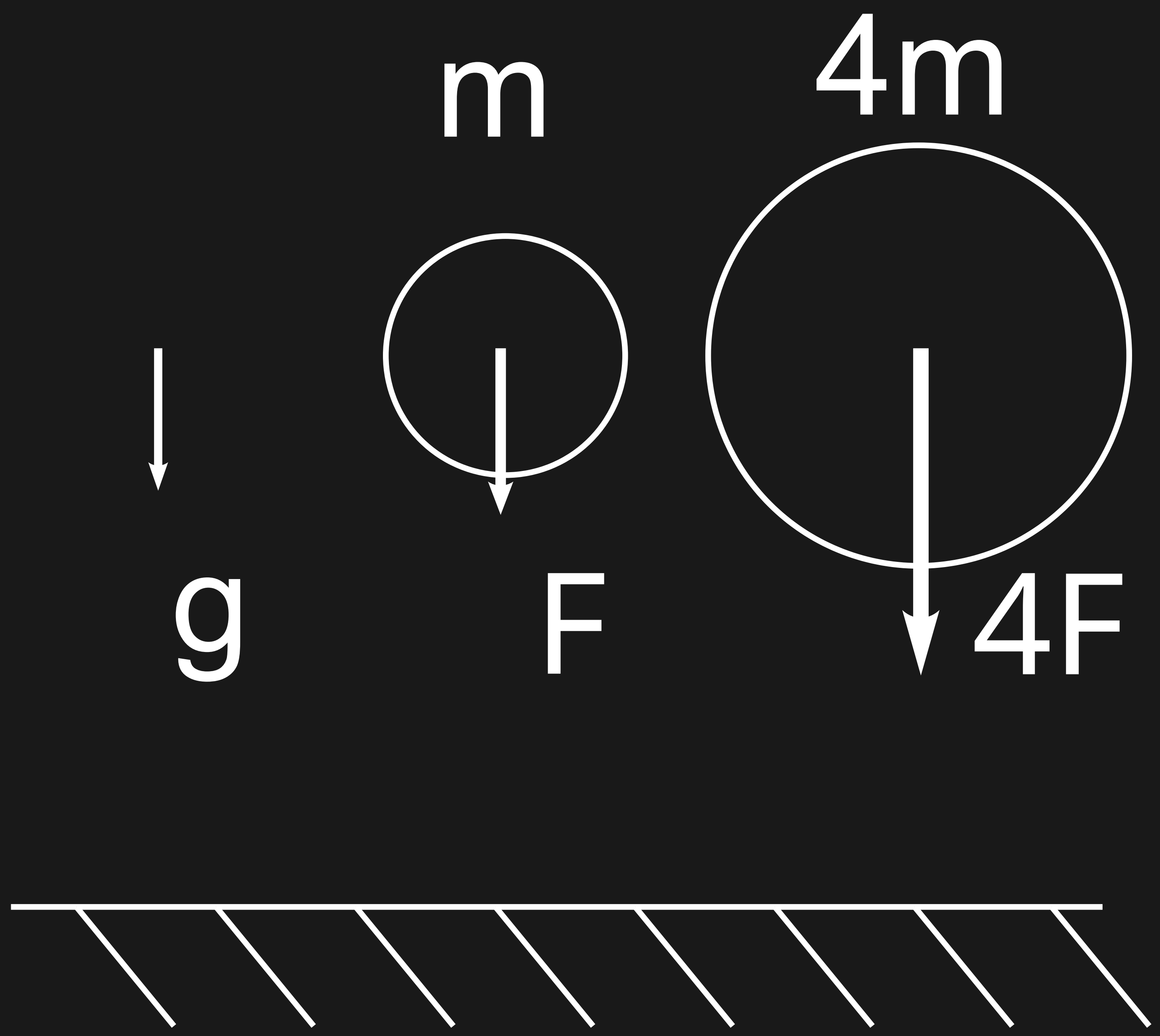

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- How to determine gravity **g**?

What influences gravity?

magnitude of gravity $g = |\vec{g}|$

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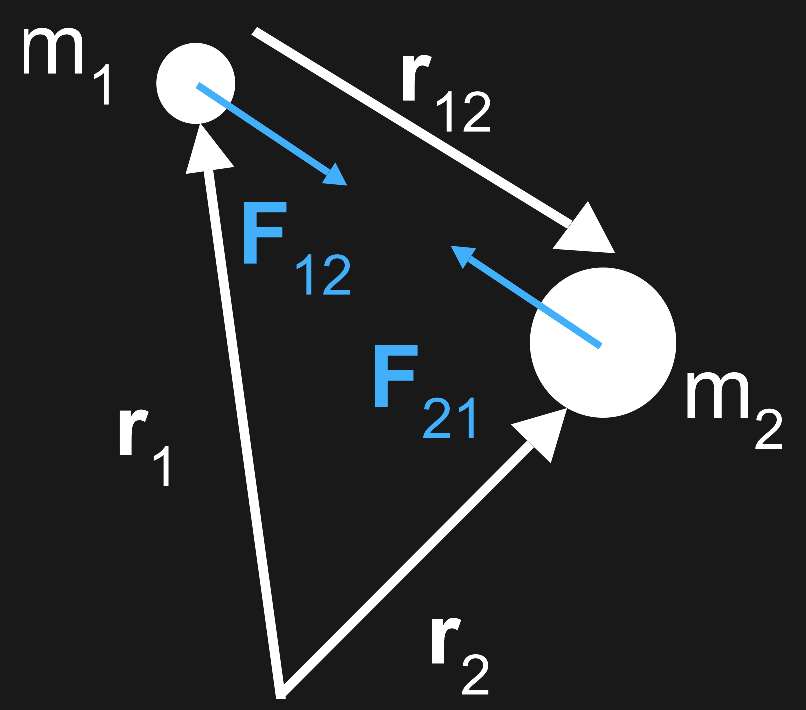
Gal mGal µGal

$$9.807246731\ldots \mathrm{m/s^2}$$

- Relativity, 1mm height change
- Ocean topography, polar motion
- Nearby large buildings, hydrology
- Earth and Ocean tides, 1m height change
- Large reservoirs
- Internal mass distribution
- Mountain, ocean trench, 1km height
- Flattening and rotation

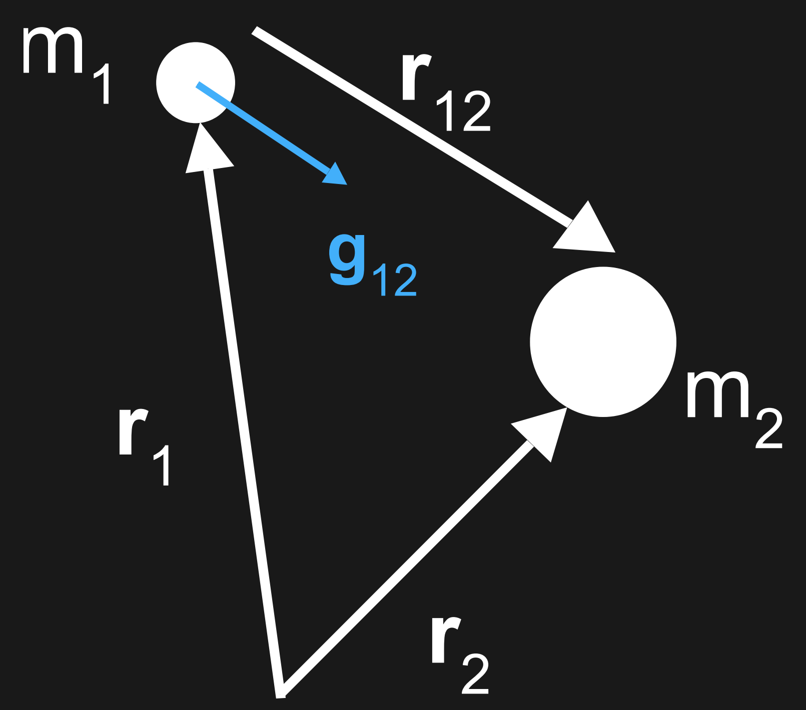
Newton’s law of gravitation

- Gravitational force → $\mathbf{F}_{12} = \frac{G m_1 m_2 \mathbf{r}_{12}}{|\mathbf{r}_{12}|^3}$
  - proportional to both masses
  - Inversely-proportional to square of distance
- Action = Reaction → $\mathbf{F}_{12} = -\mathbf{F}_{21}$
- Gravititational constant → $G = 6.67384 \times 10^{-11} \frac{m^3}{kg \cdot s}$



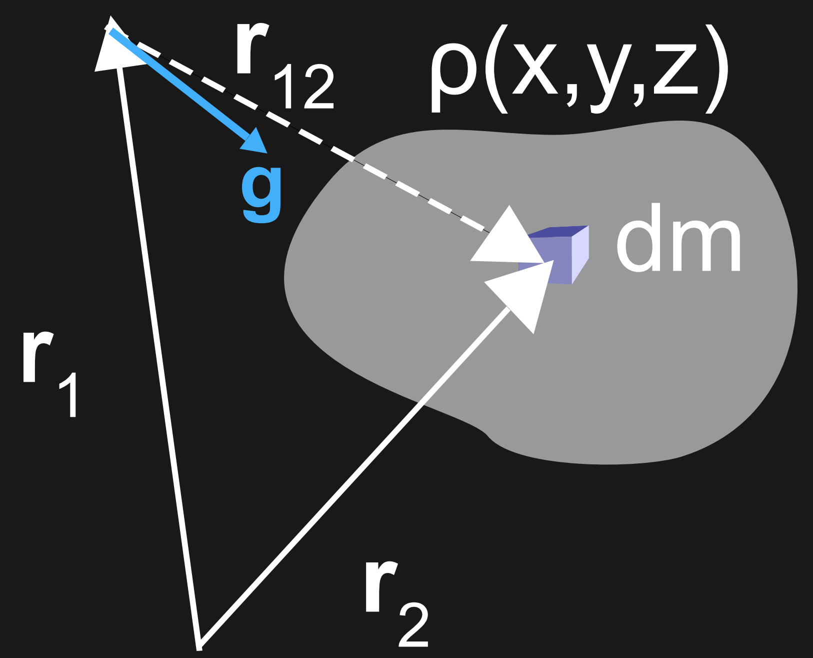
Describe the gravitational field of a central body in terms of acceleration

- Acceleration for a point mass $\rightarrow$ $\mathbf{g}_{12} = \frac{Gm_2\mathbf{r}_{12}}{|\mathbf{r}_{12}|^3}$
- More complex bodies: apply superposition principle
  - Addition of multiple point masses
  - Introduction of a density function



Integrate density to obtain the gravitational field

$$\mathbf{g} = \iiint_{\Omega} \frac{G\mathbf{r}_{12}}{|\mathbf{r}_{12}|^3} \rho(x, y, z) dx dy dz$$



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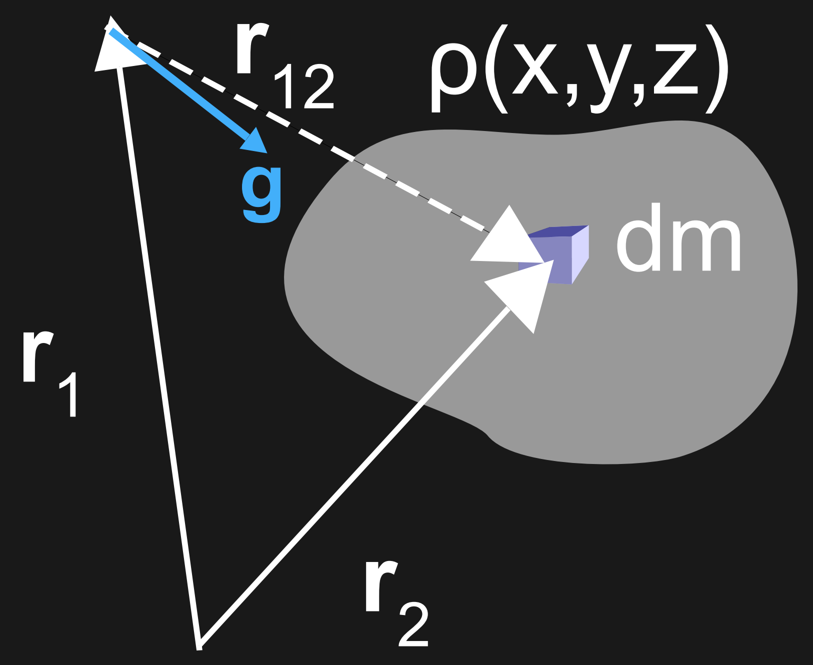

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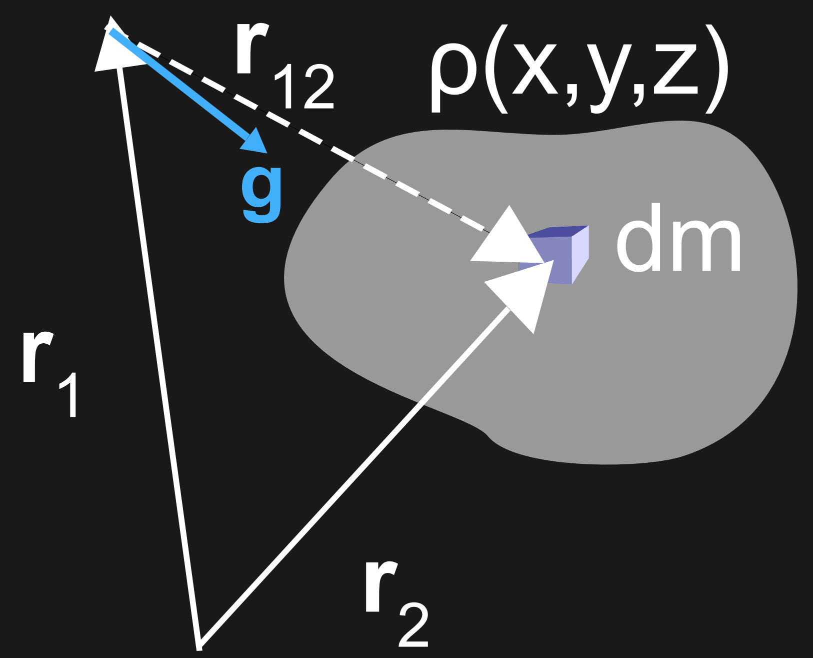

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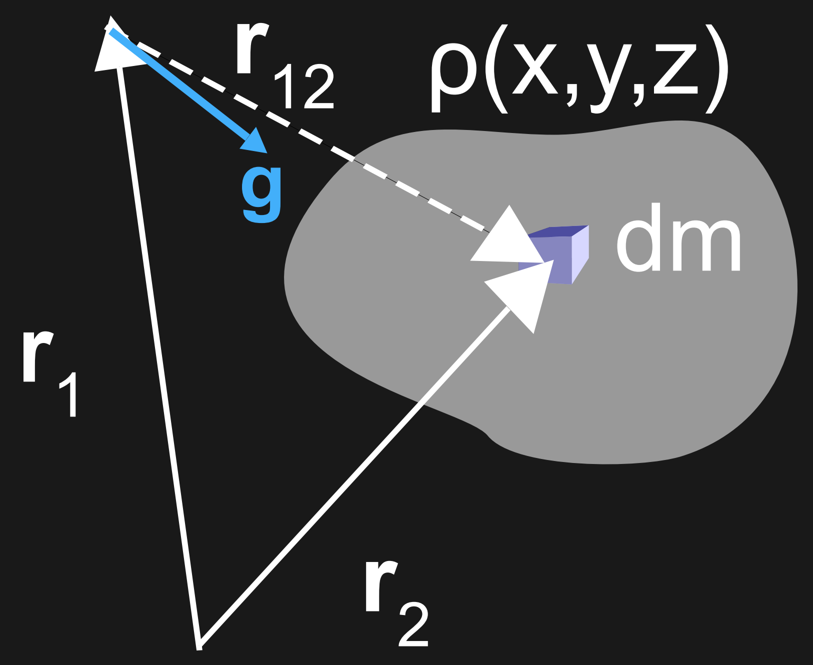

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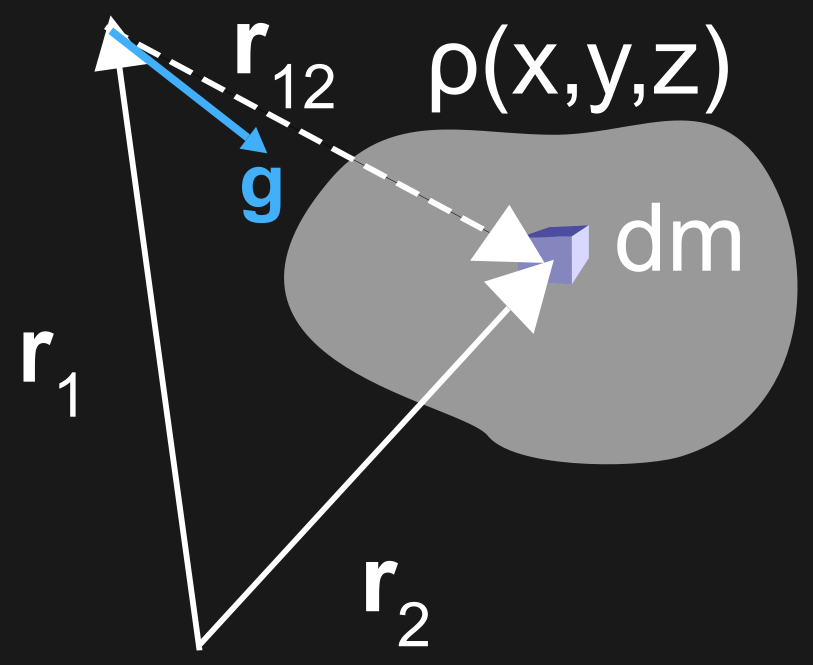
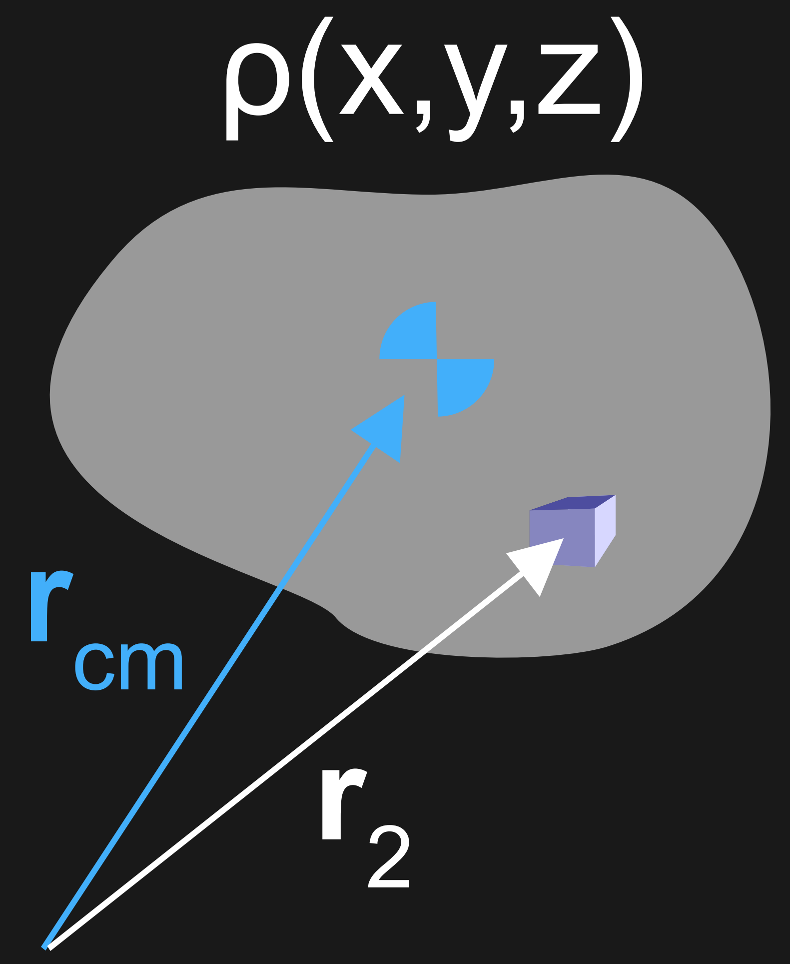
- Is there a better choice for the reference frame origin?



A better origin: Center of mass

$$\mathbf{r}_{cm} = \frac{1}{M}\iiint_{\Omega} \rho(x,y,z)\mathbf{r}_2\, dx\,dy\,dz$$



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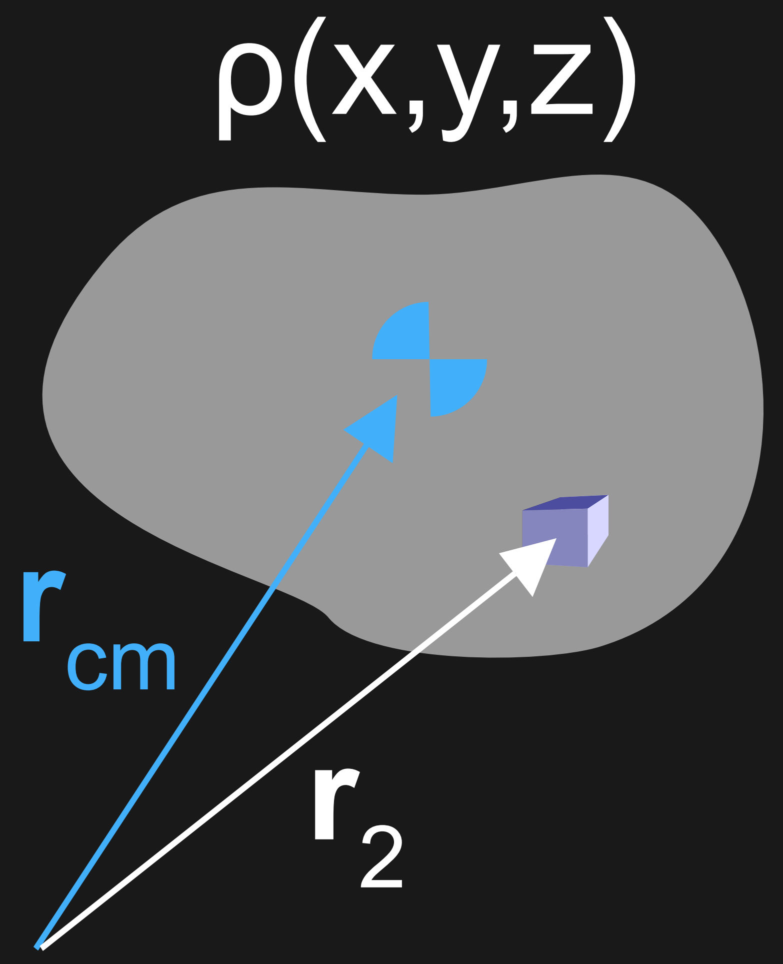

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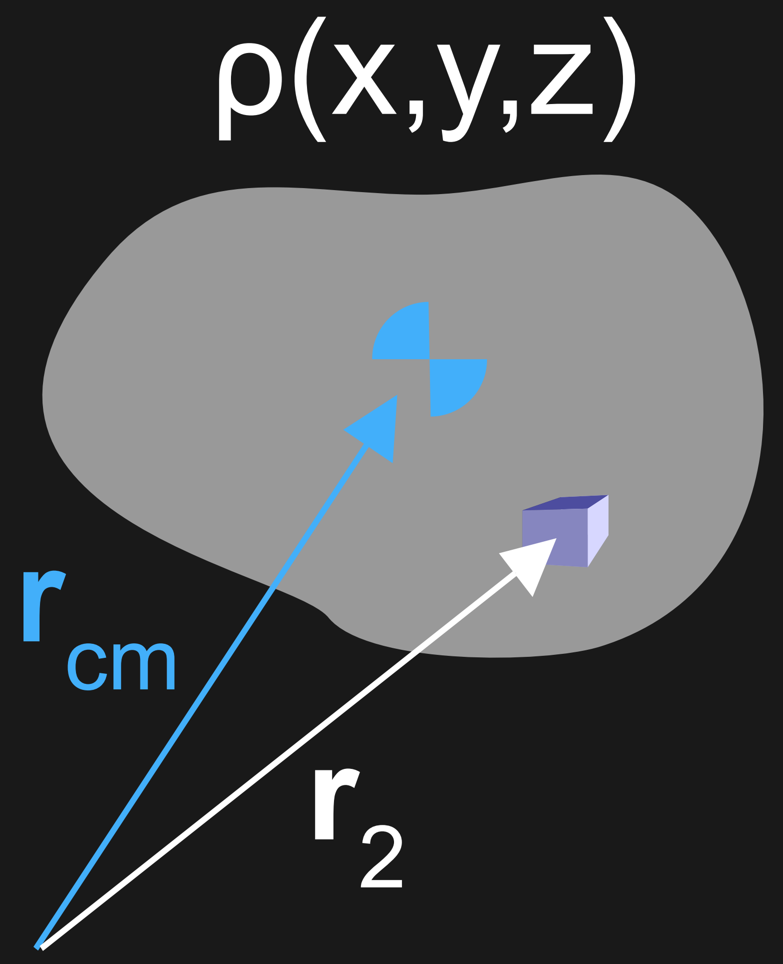

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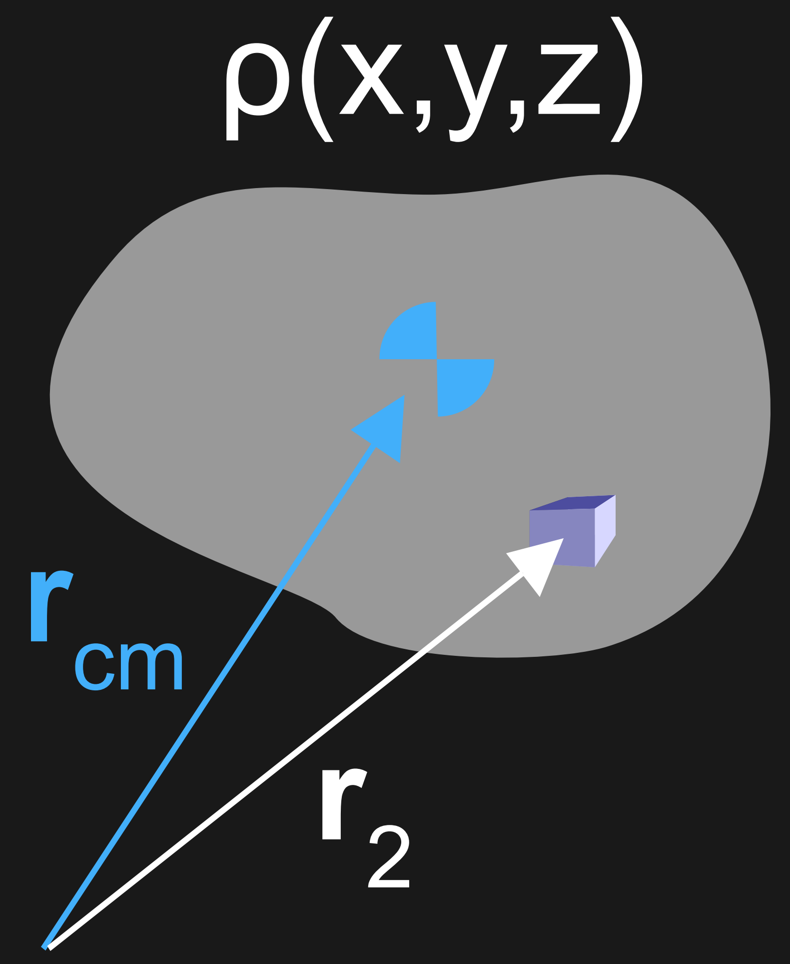

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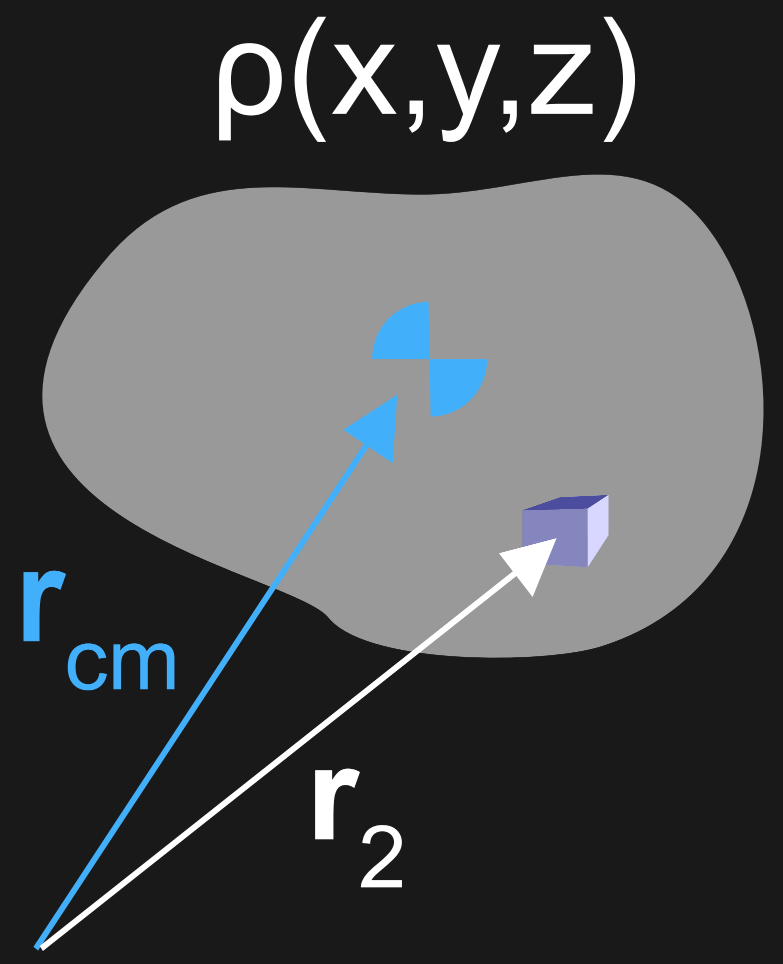
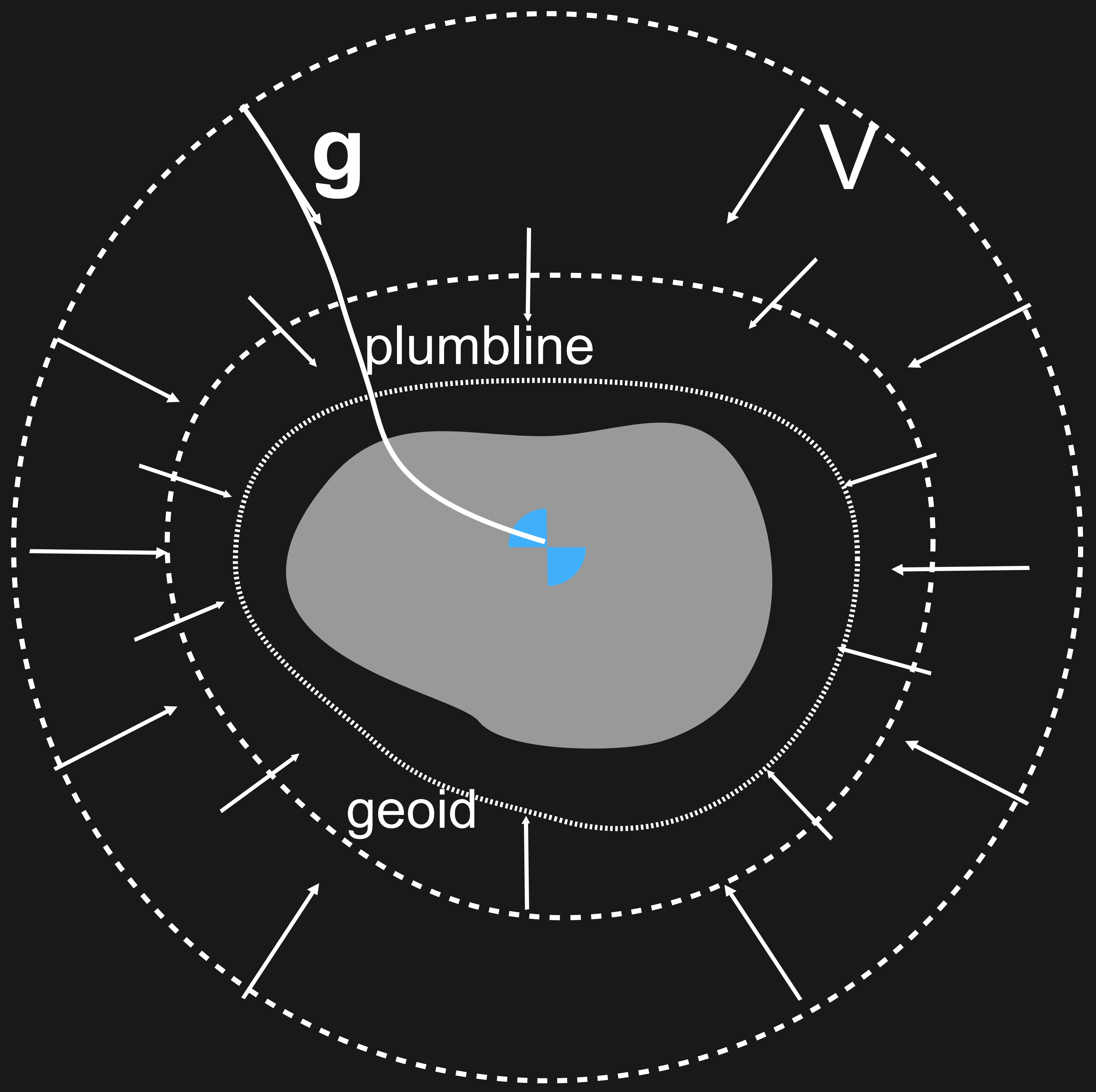
  - Objects far away can be considered point masses e.g. for Keplerorbits, three-body problem etc.



Properties of g?



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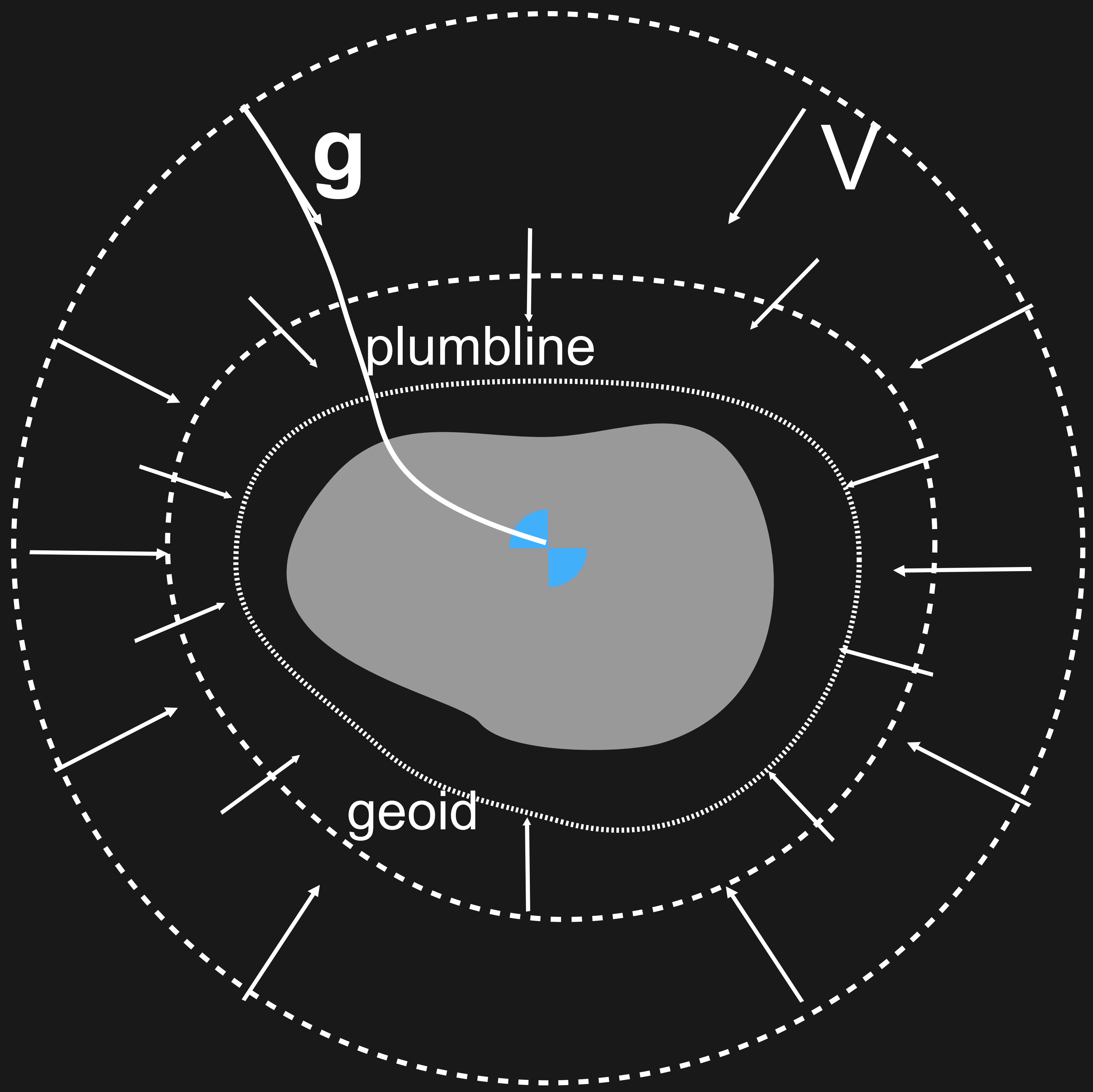

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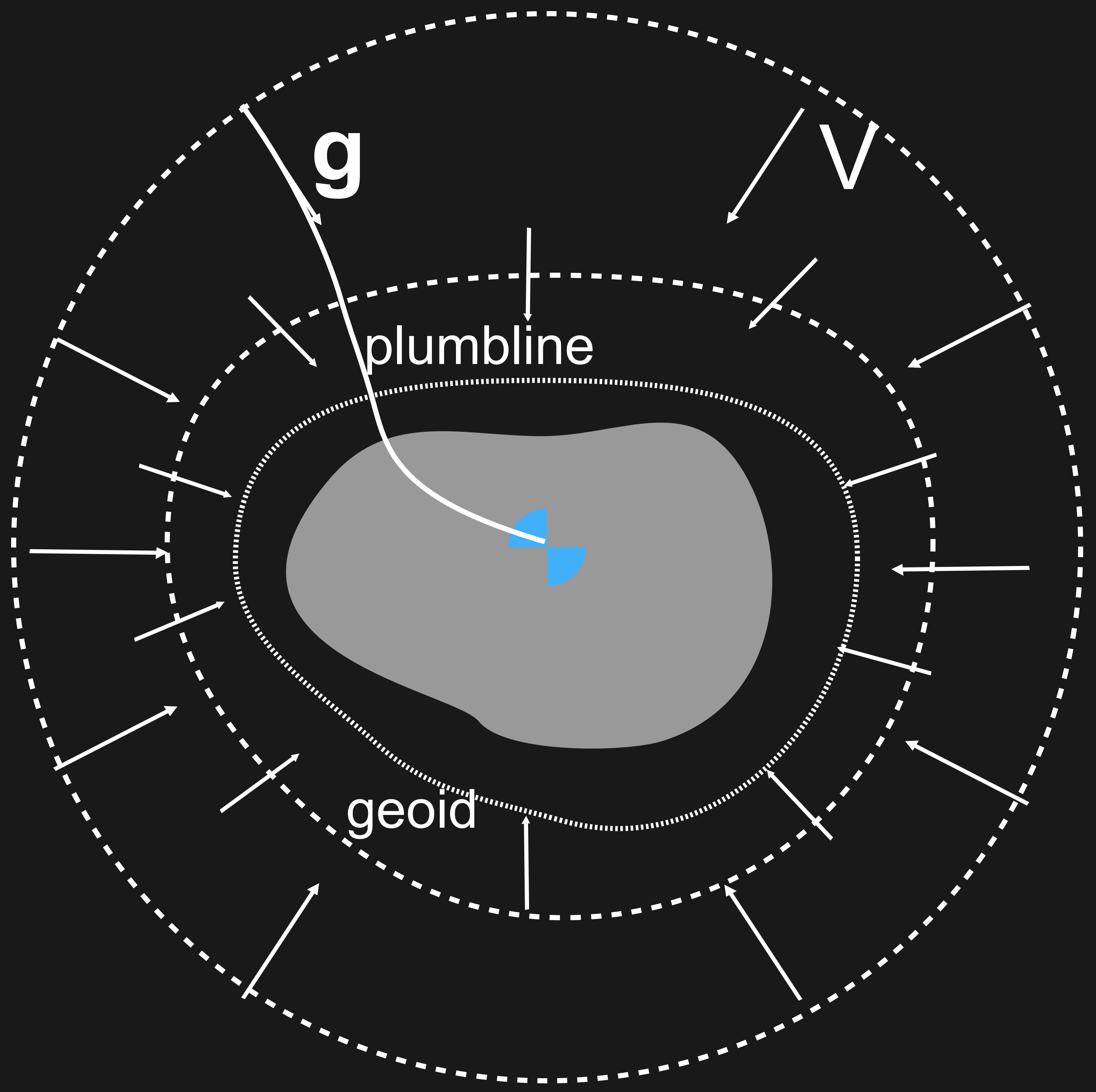

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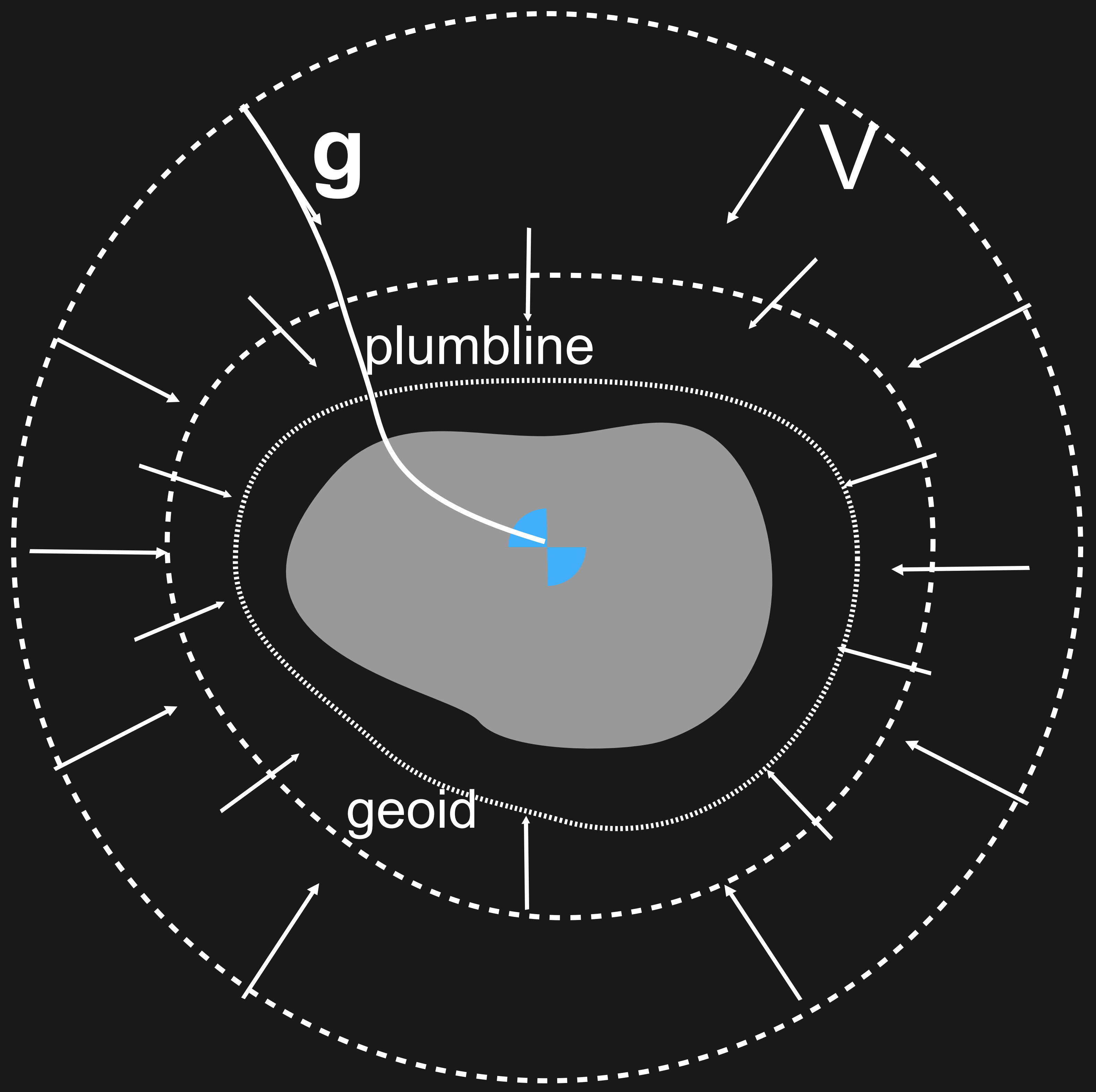

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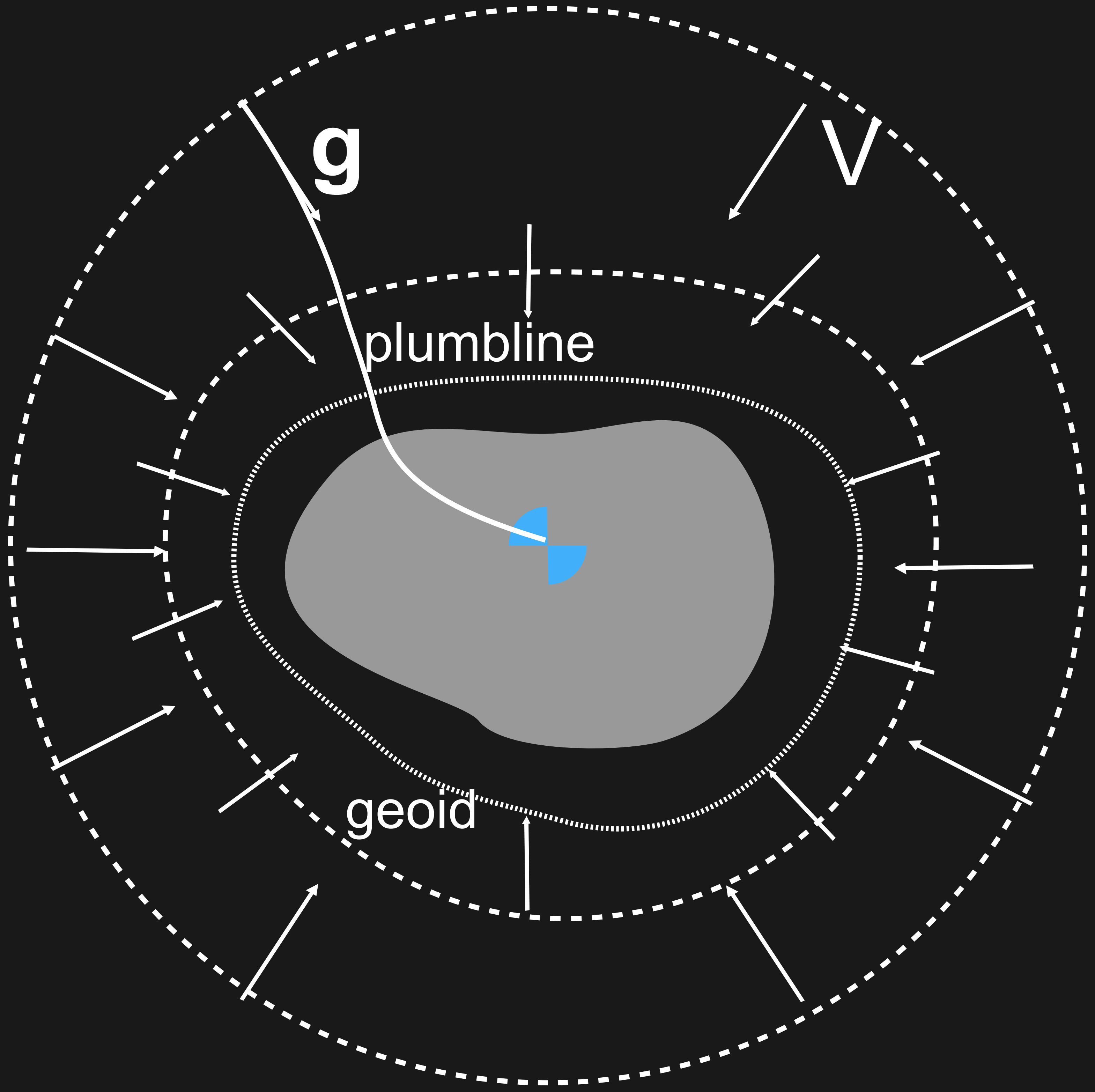

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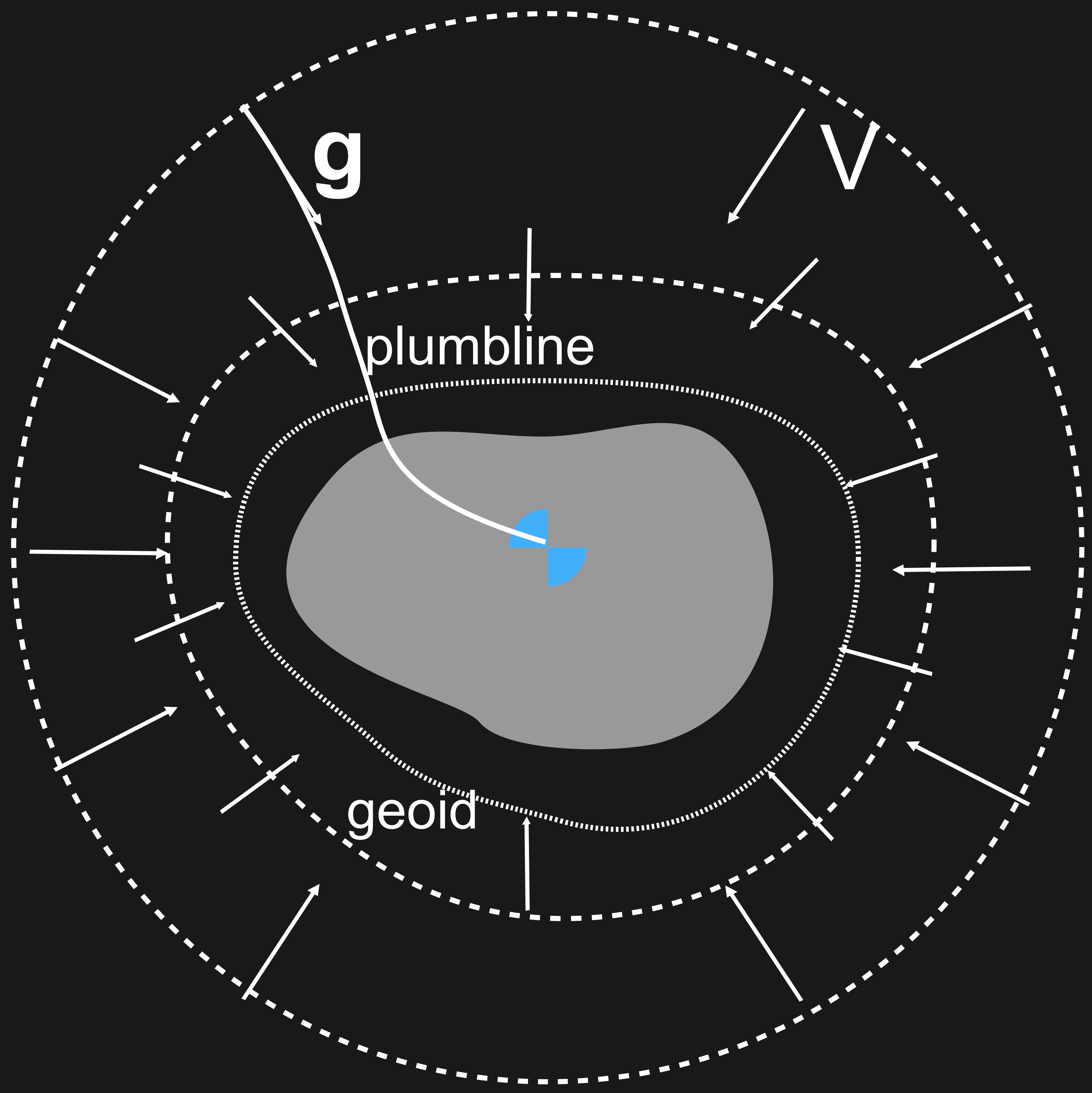

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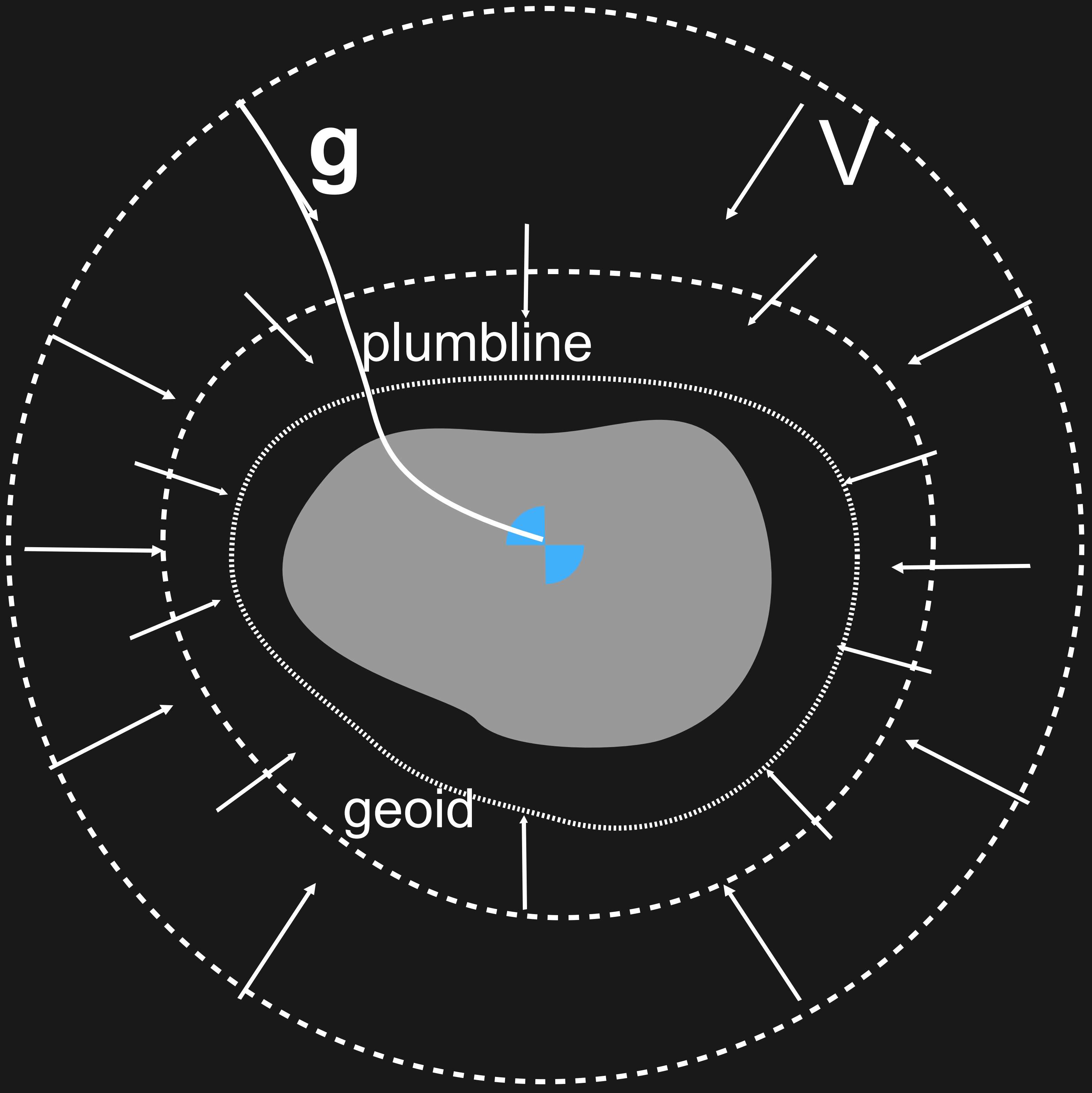

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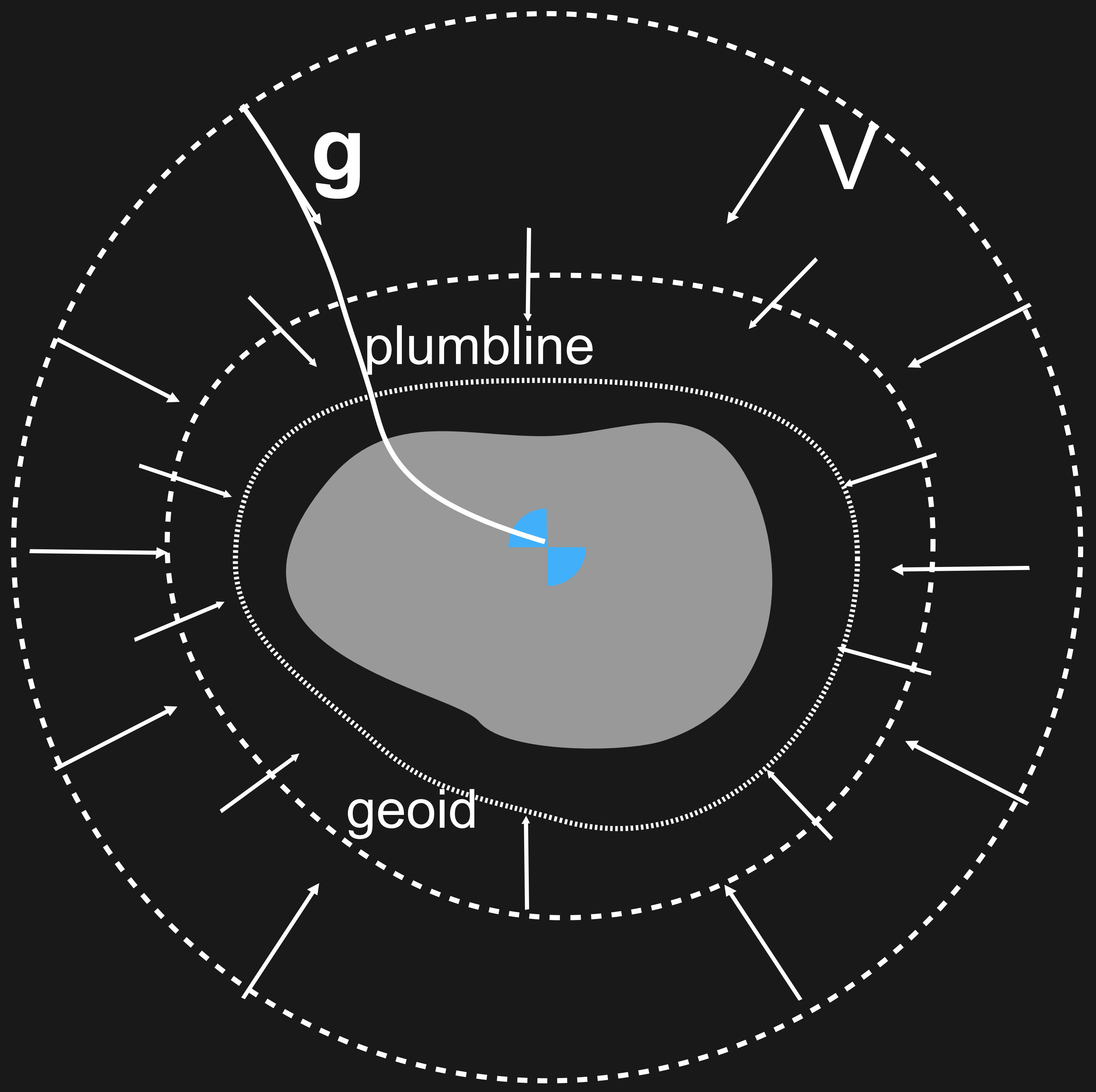

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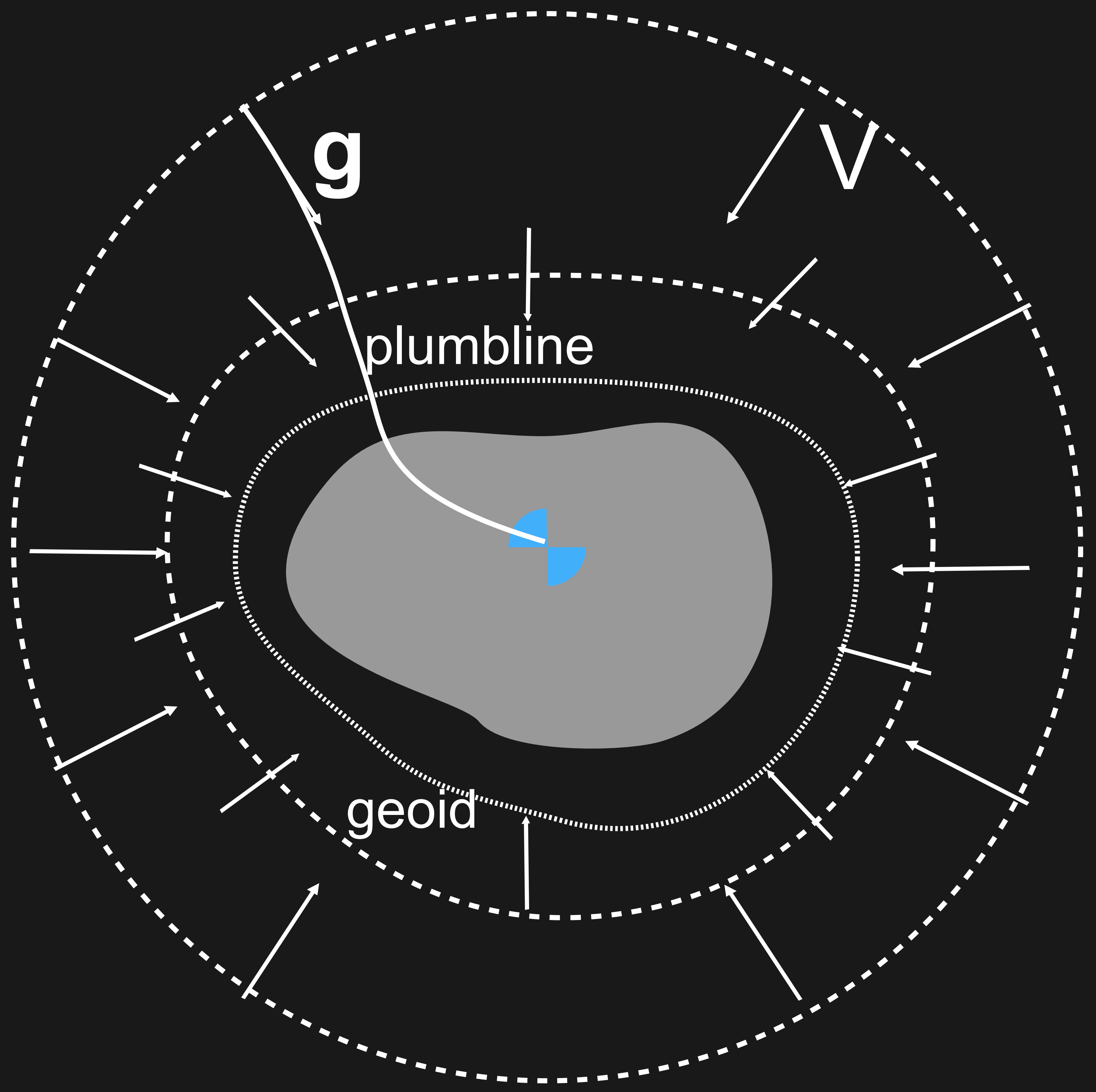

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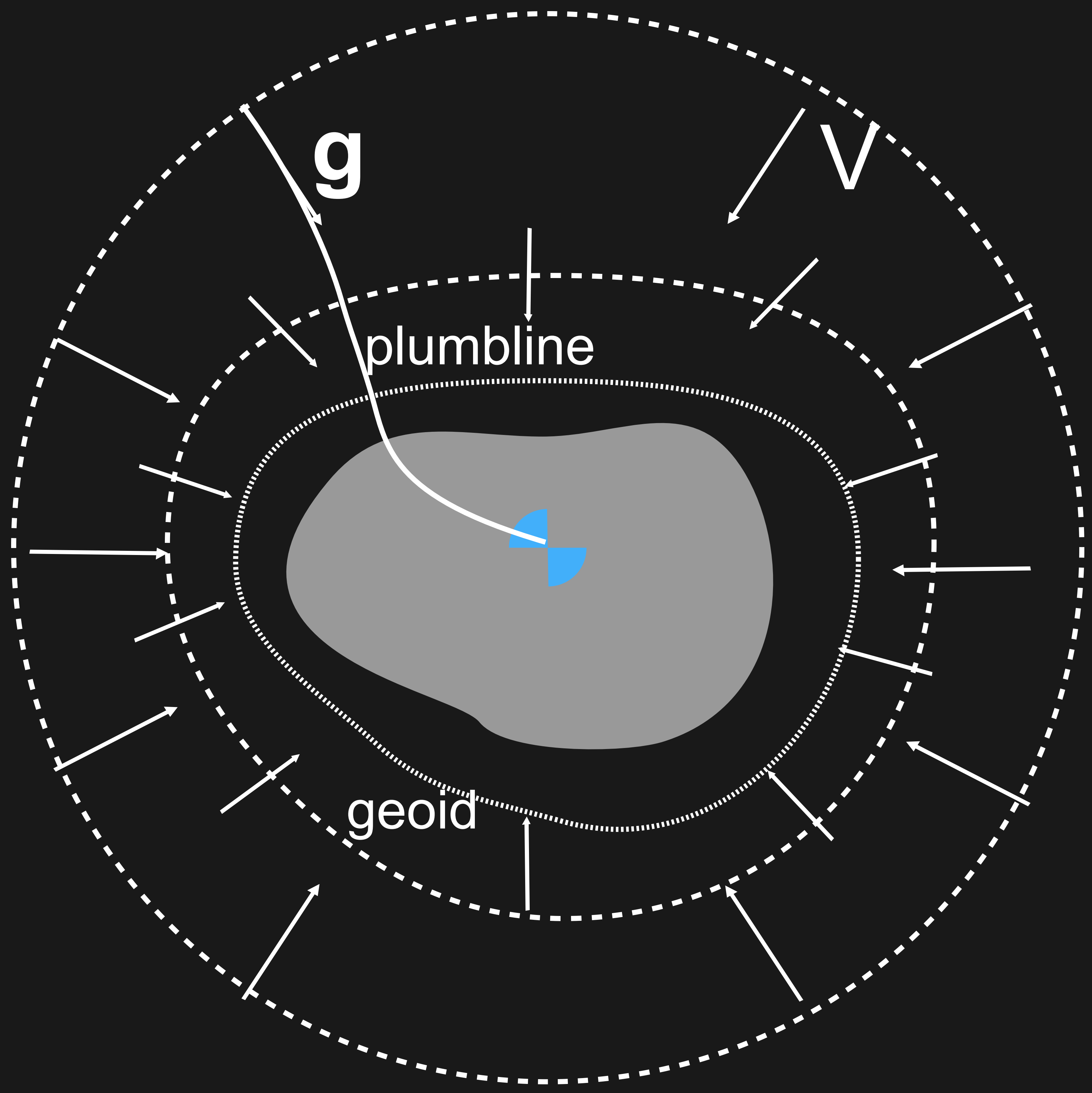

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  - gravity follows plumblines and is orthogonal to equipotential surfaces
  - the geoid is special (coincides with an ocean surface at rest)

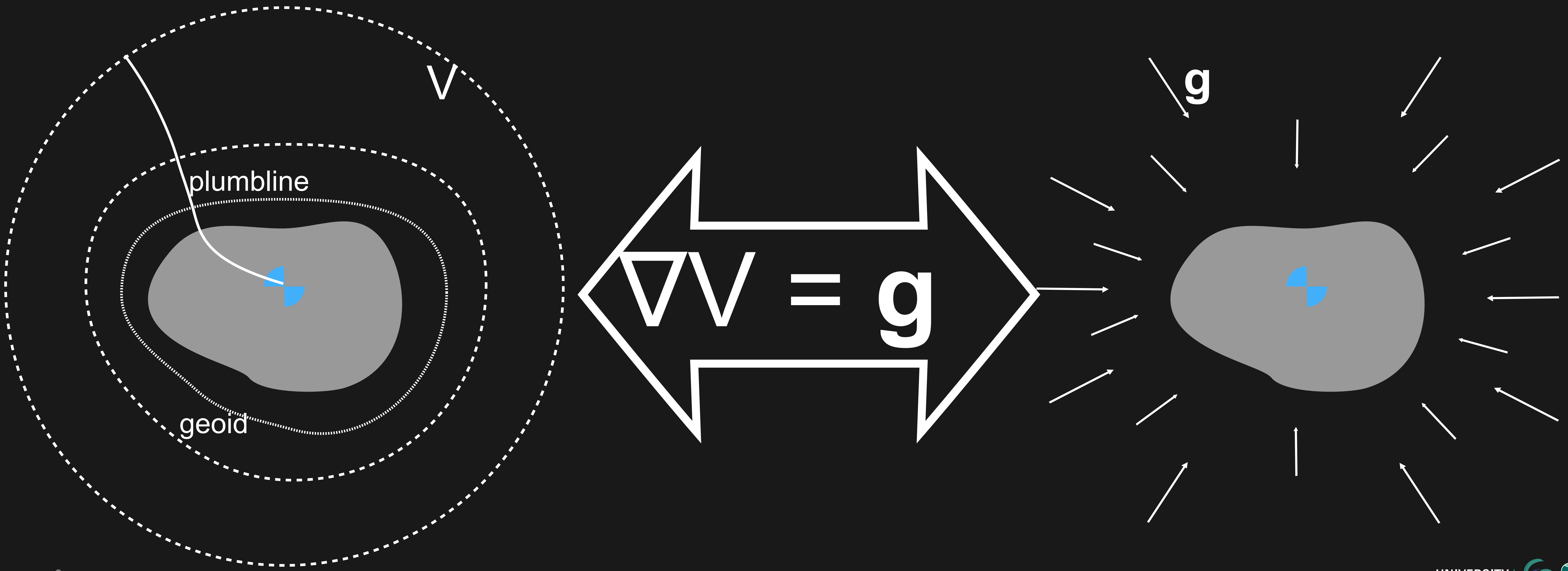
Gravity versus Potential

Potential V

- Scalar field
- unit m^2/s^2
- not directly observable
- Mathematical aid

Gravity **g**

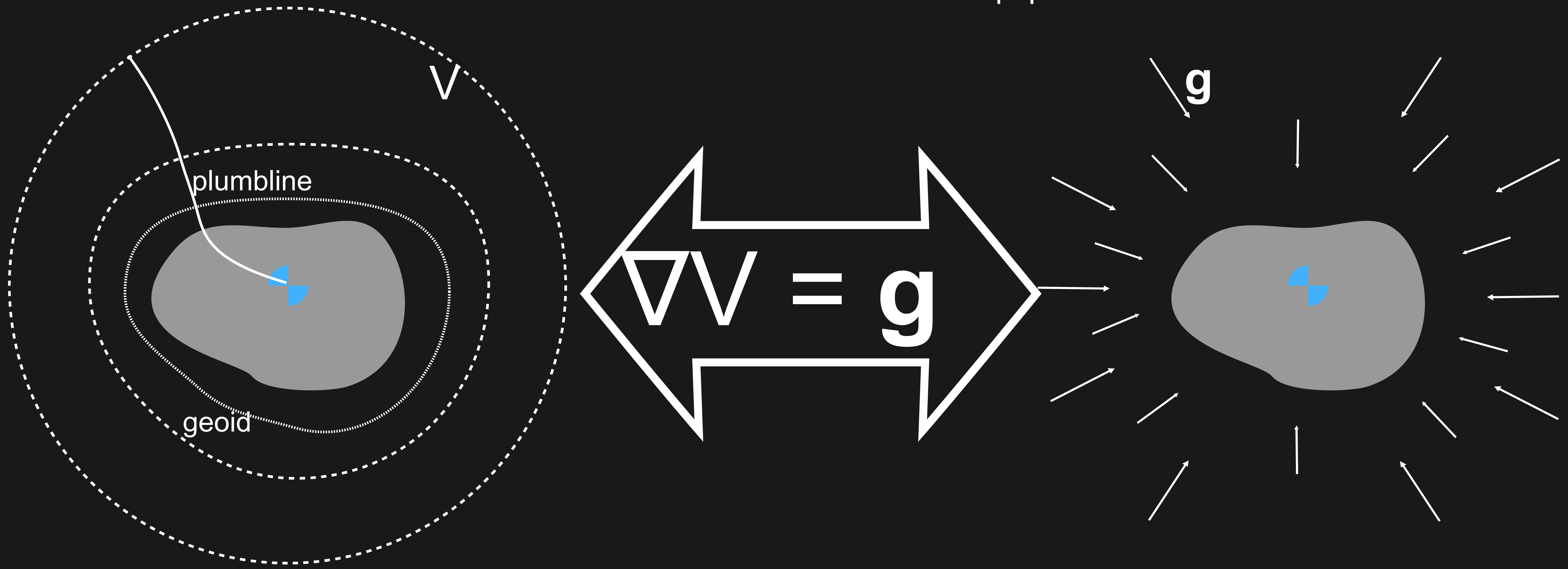
- Vector field
- unit m/s^2
- Observable
- Gradient of the Potential



Gravity versus Potential from infinite distance

No matter how complex the body is: from a distance it looks like a point mass

$$\lim_{r \to \infty} V = \frac{GM}{|\mathbf{r}|} \qquad \lim_{r \to \infty} \mathbf{g} = -\frac{GM\mathbf{r}}{|\mathbf{r}|^3}$$



The gravity field obeys the Laplace or Poisson equation

How does the potential behave inside and outside of a body?

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- $\mathbf{g} = \nabla V$

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Outside:

$$\nabla^2 V = 0$$

(Laplace Equation)

The gravity field obeys the Laplace or Poisson equation

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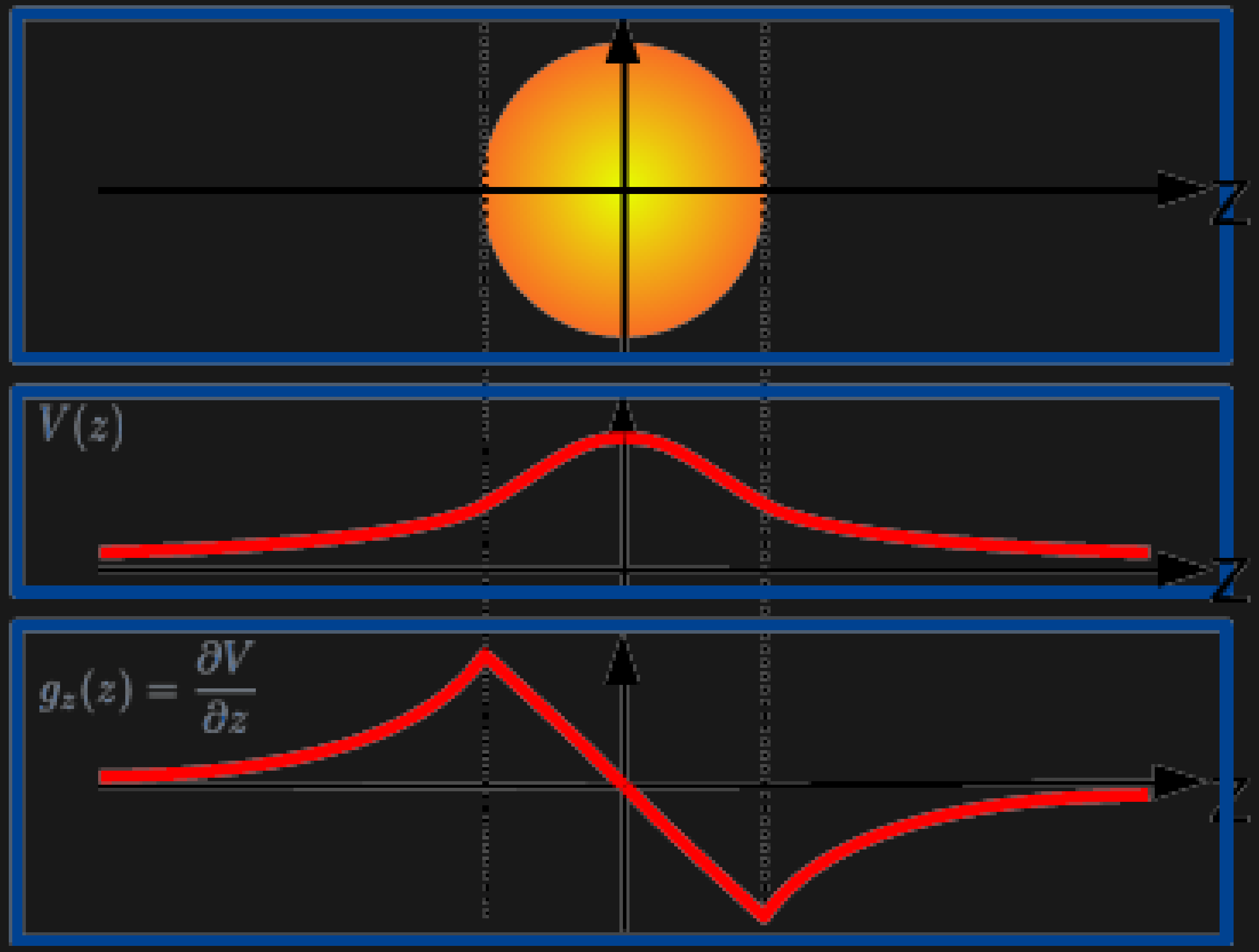
Outside:
$\nabla^2 V = 0$
(Laplace Equation)

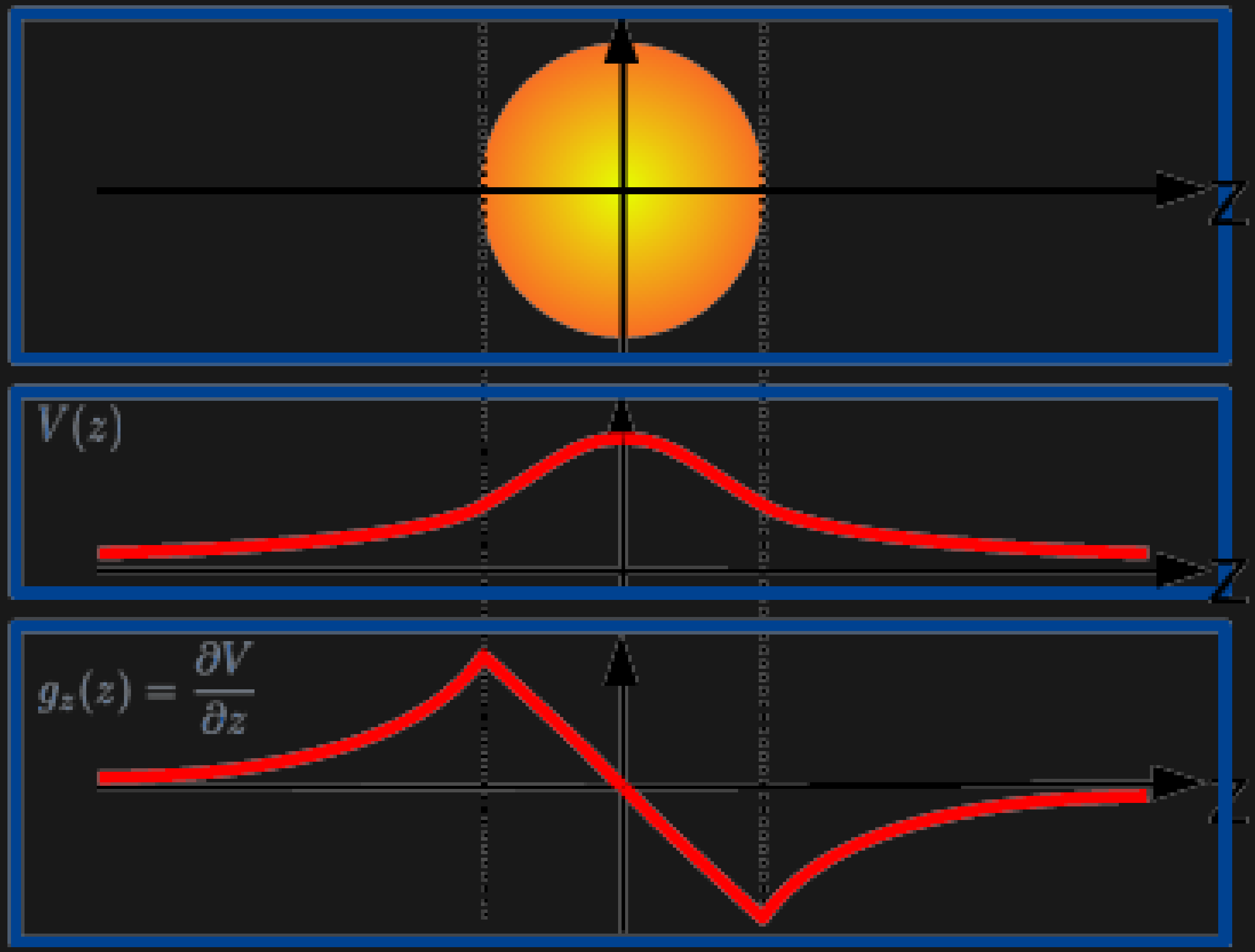
Inside:
$\nabla^2 V = -4\pi G\rho$
(Poisson Equation)

Example: How does gravity change inside the Earth?



Example: How does gravity change inside the Earth?

- Does gravity decrease when descending in a mine?



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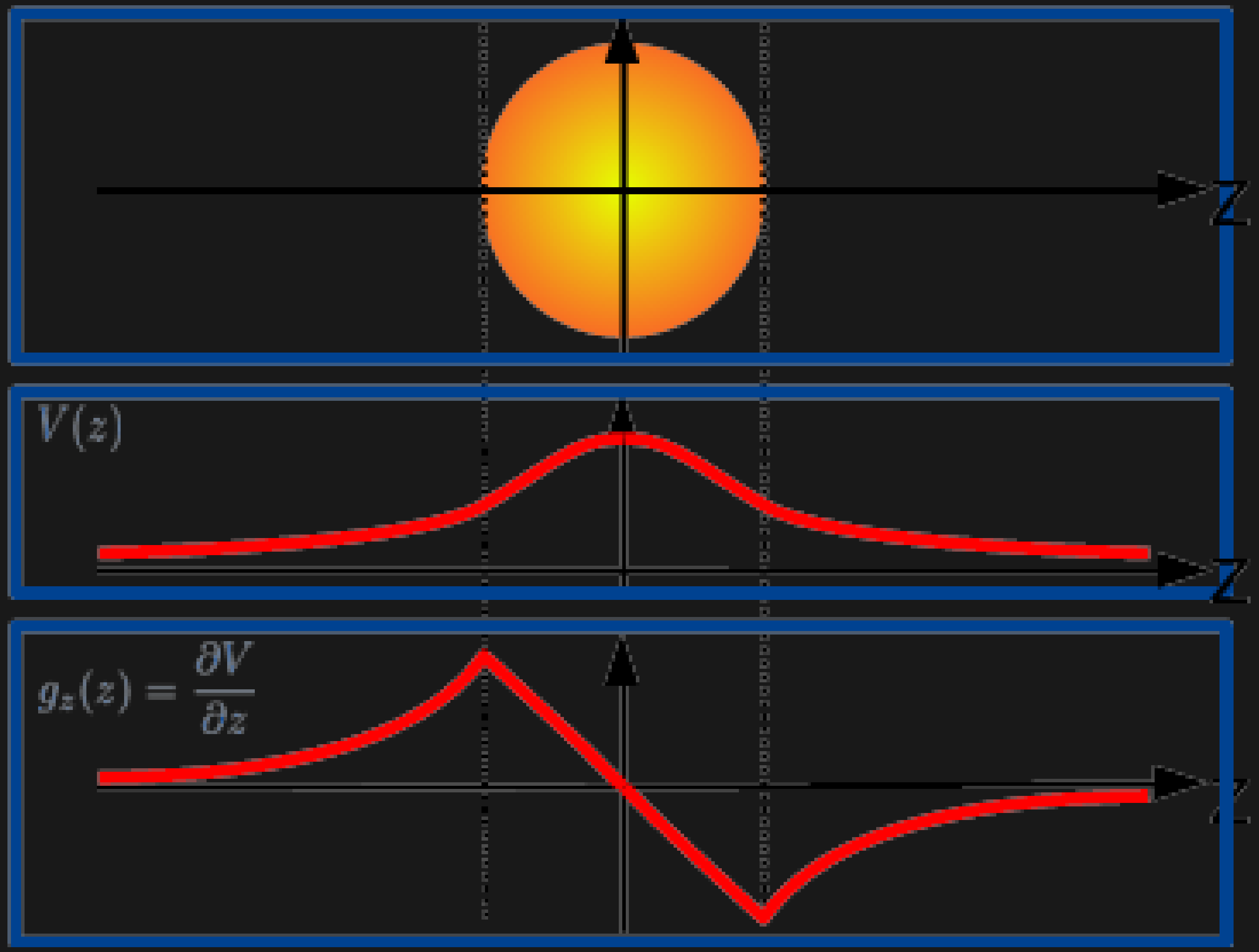
EO AFRICA R&D Facility

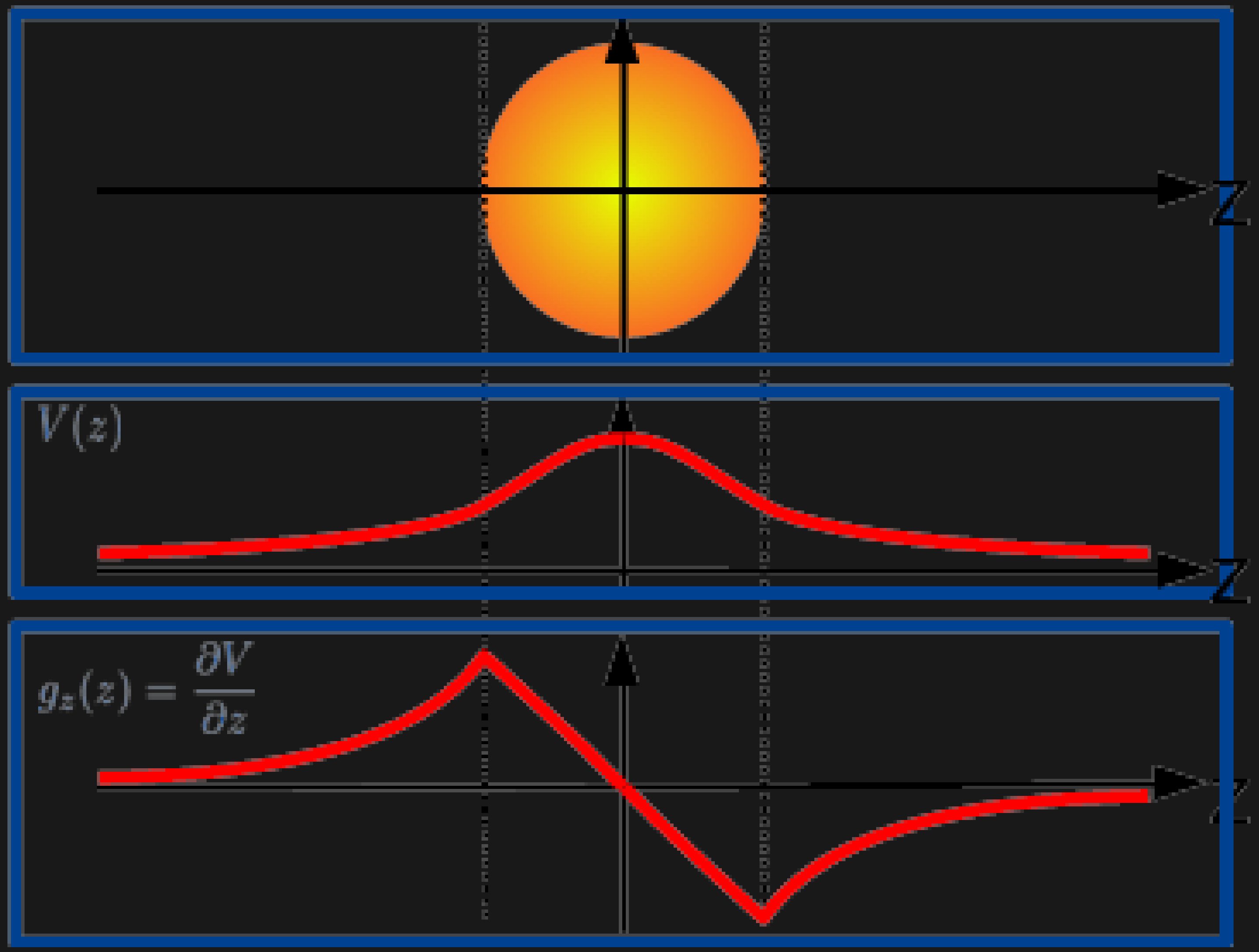
Example: How does gravity change inside the Earth?

- Does gravity decrease when descending in a mine?
- But gravity actually INCREASES when you go into a mine!

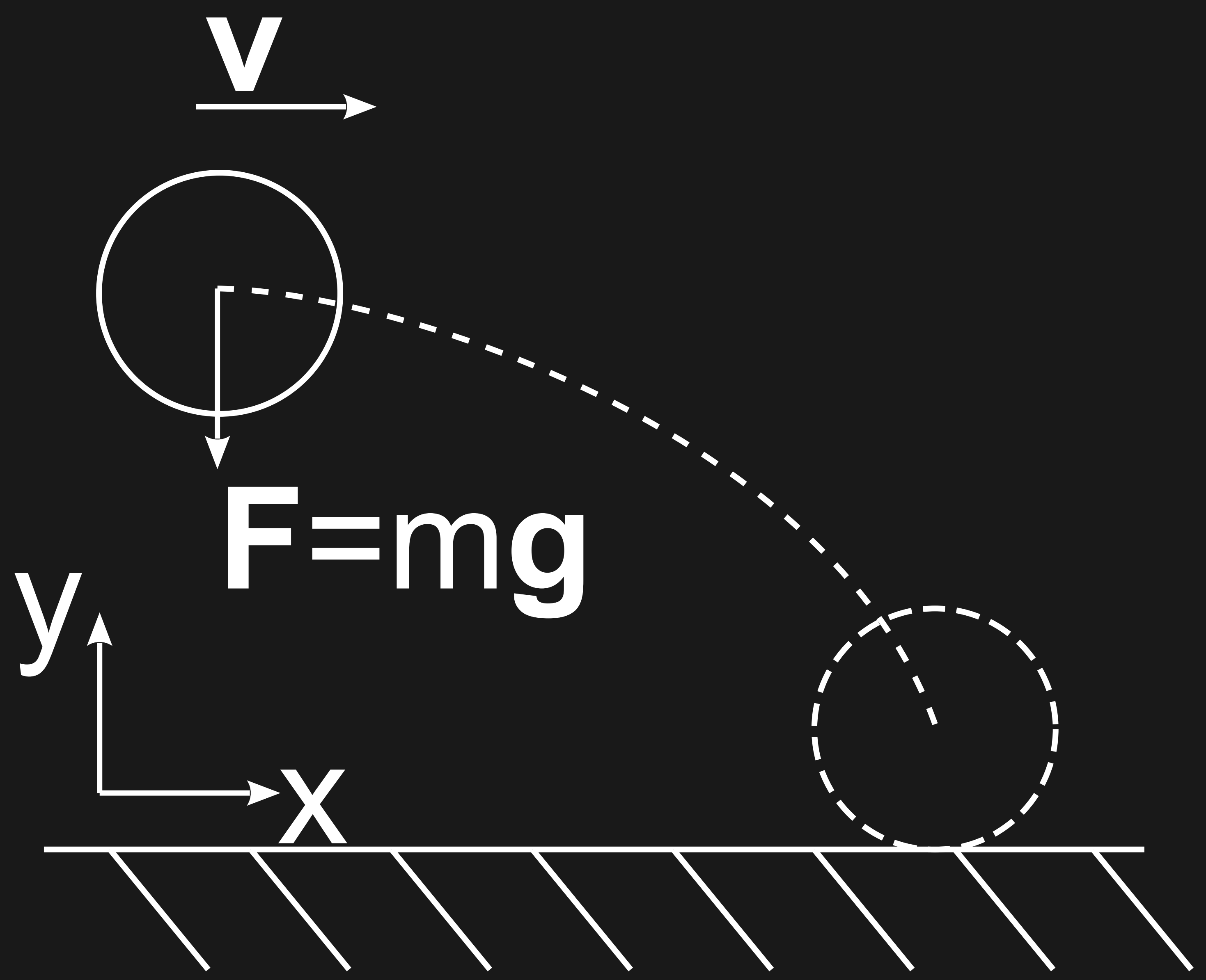


Example: How does gravity change inside the Earth?

- Does gravity decrease when descending in a mine?
- But gravity actually INCREASES when you go into a mine!
- Due to nonhomogenous density distribution (mantle is much denser than crust)



Throwing a stone



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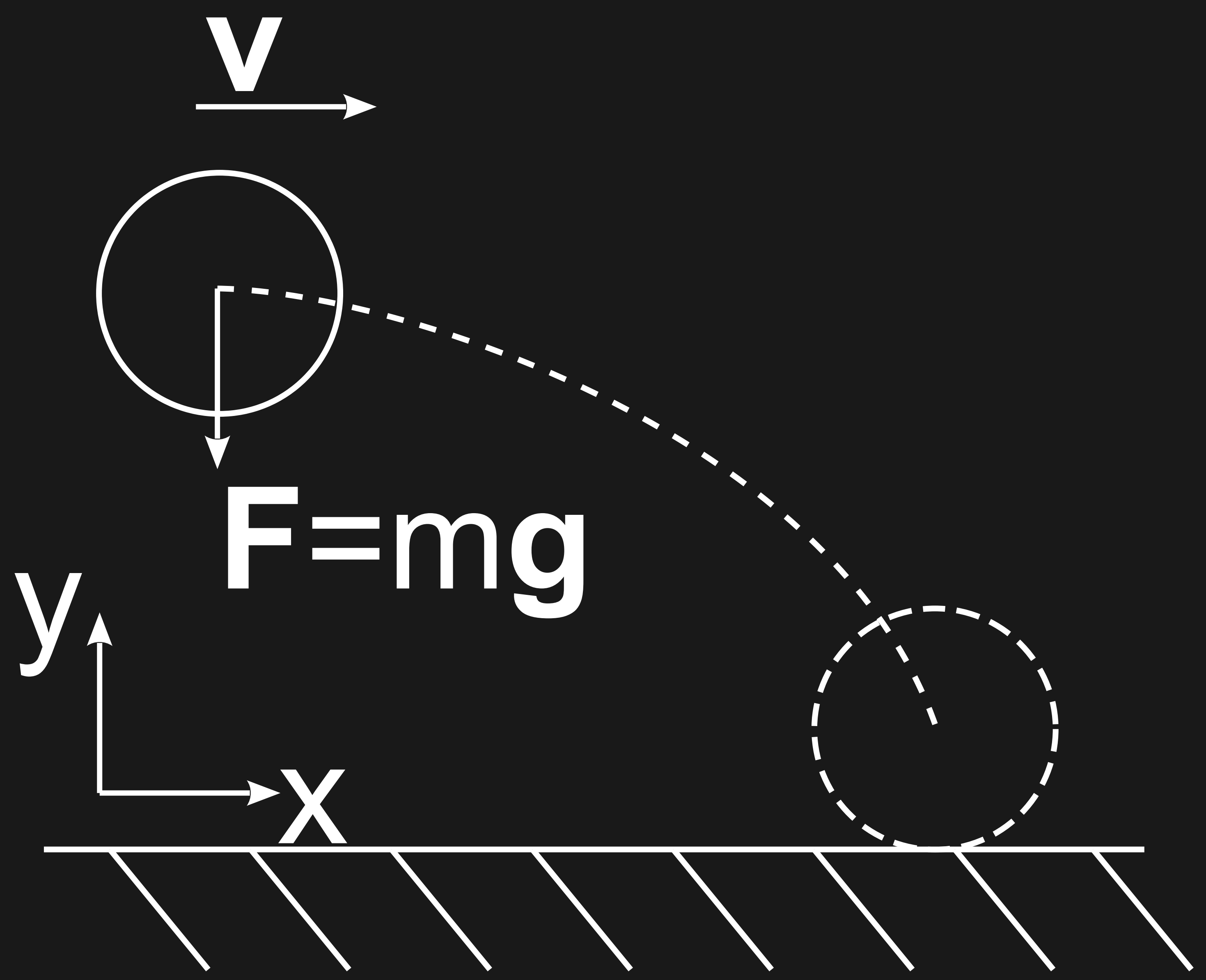
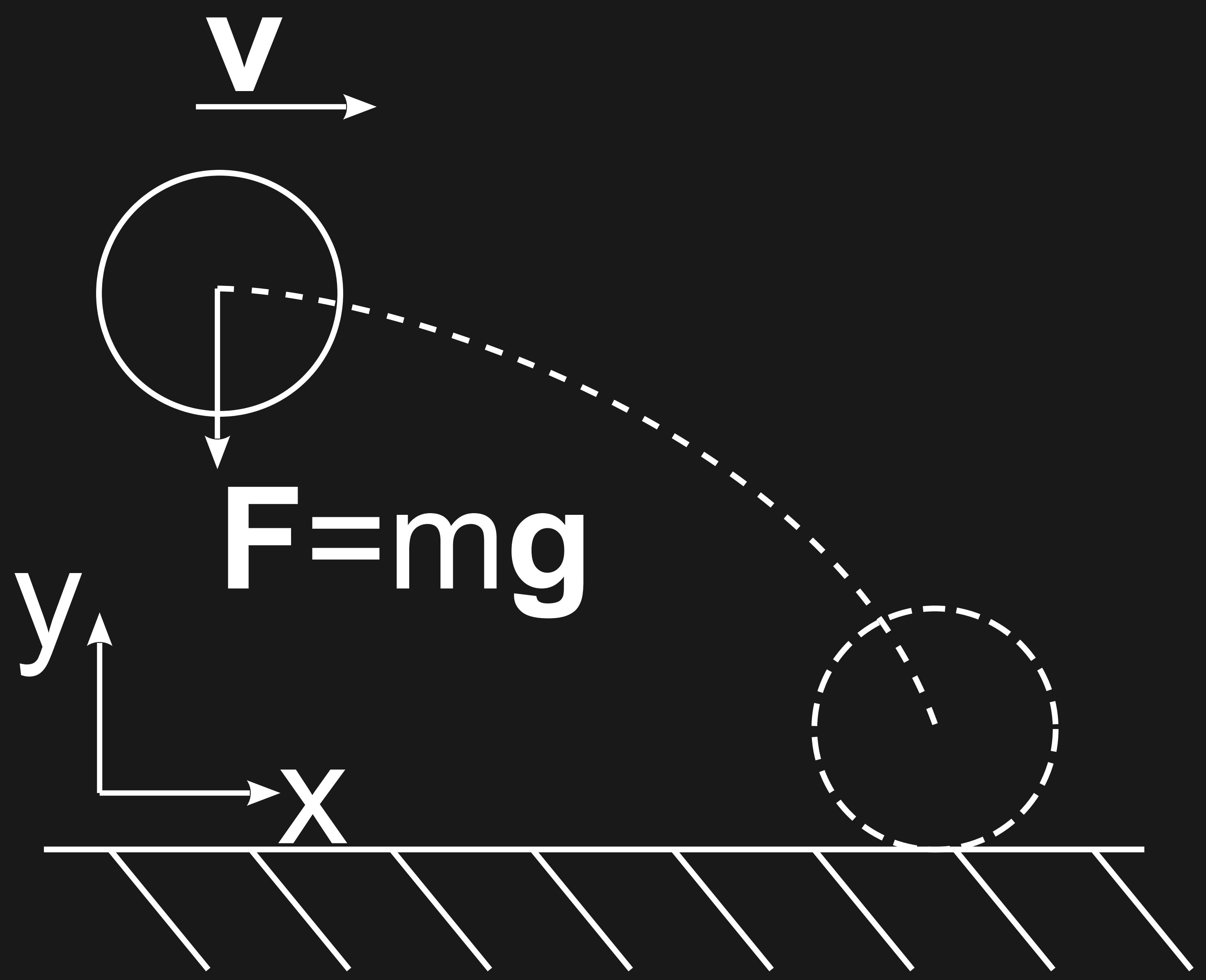


- What is the shape of the path?

Throwing a stone



- What is the shape of the path?
  - Parabola is a good approximation

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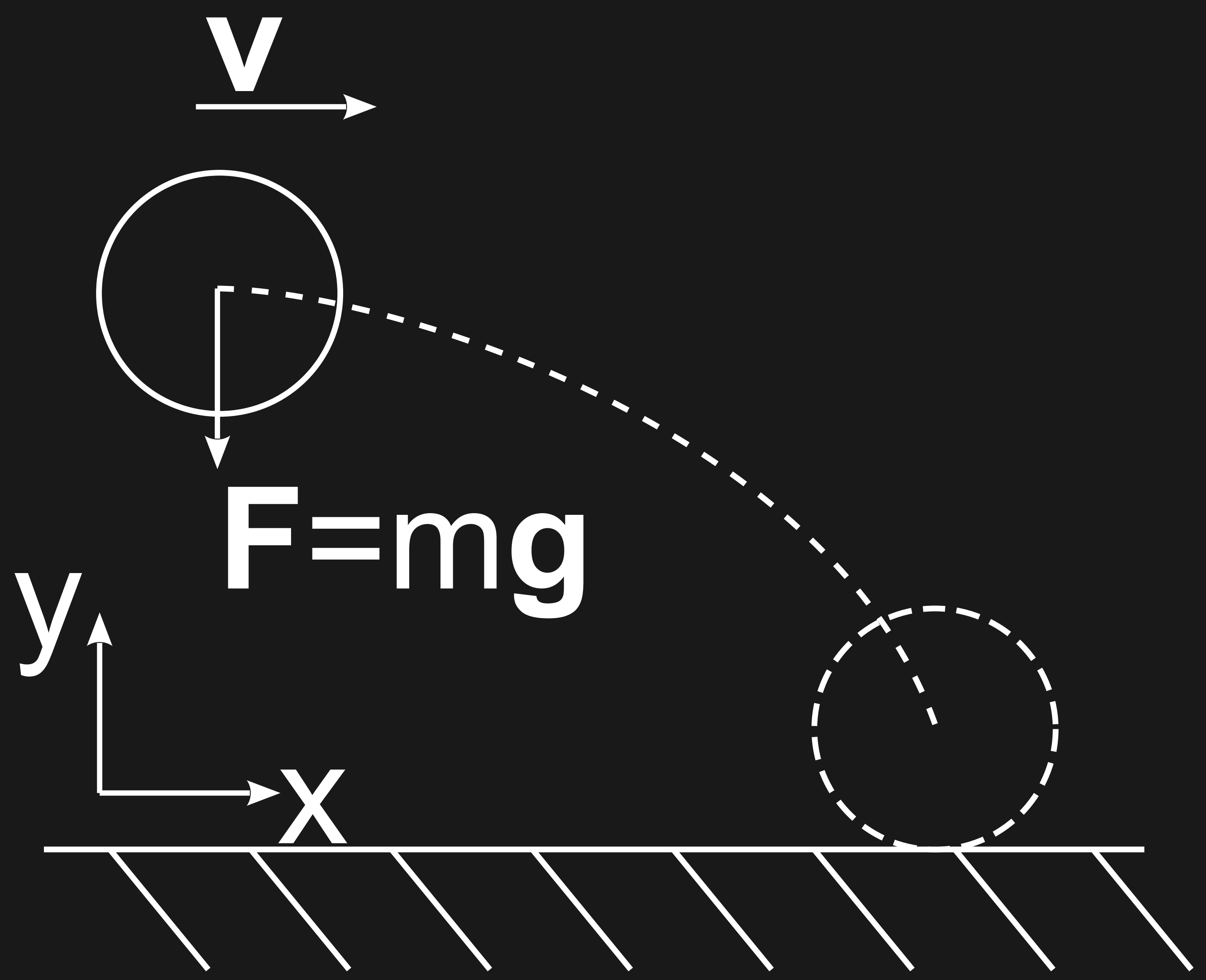
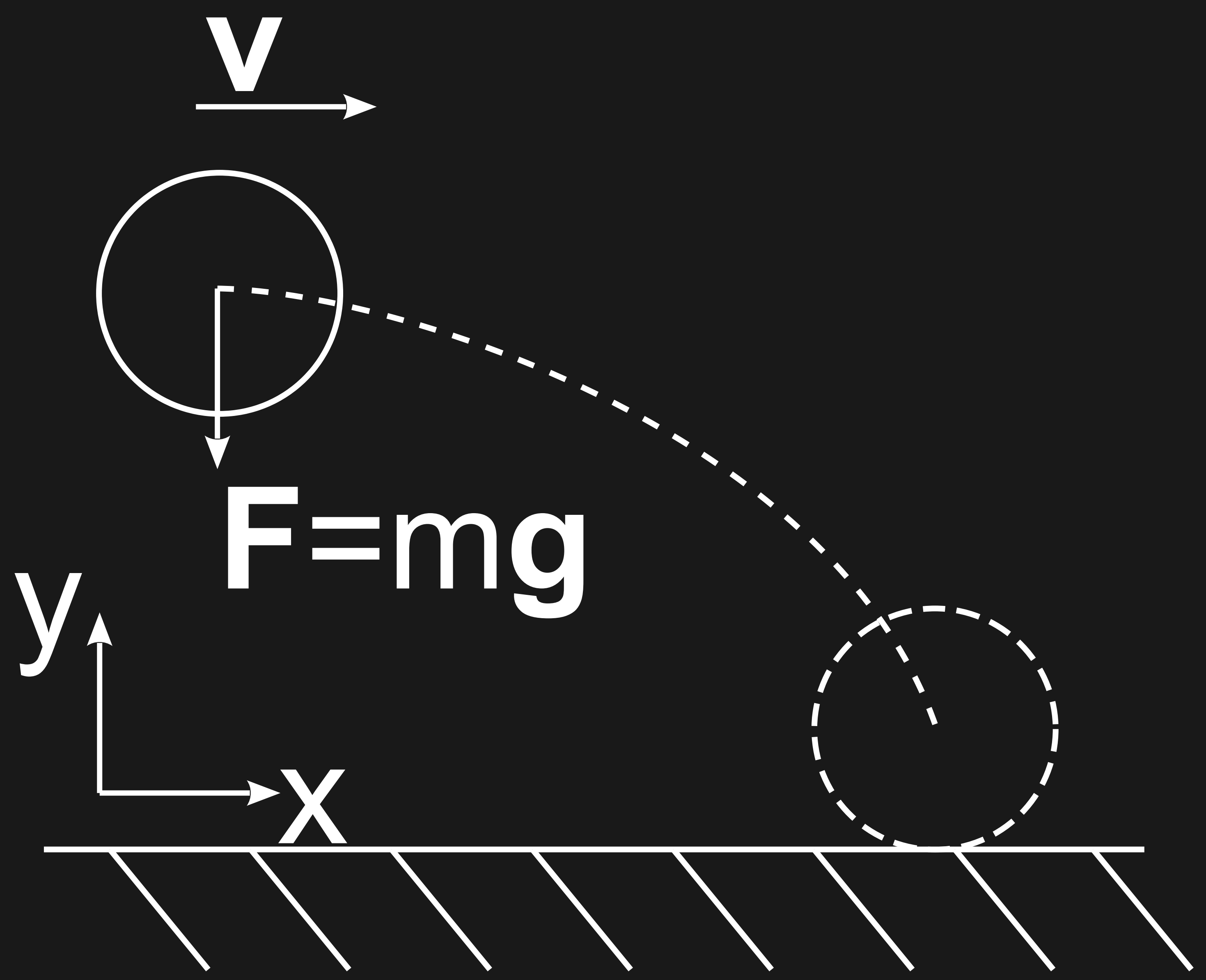


- What is the shape of the path?
    - Parabola is a good approximation
        - $x(t) = x_0 + Vt$

Throwing a stone



- What is the shape of the path?
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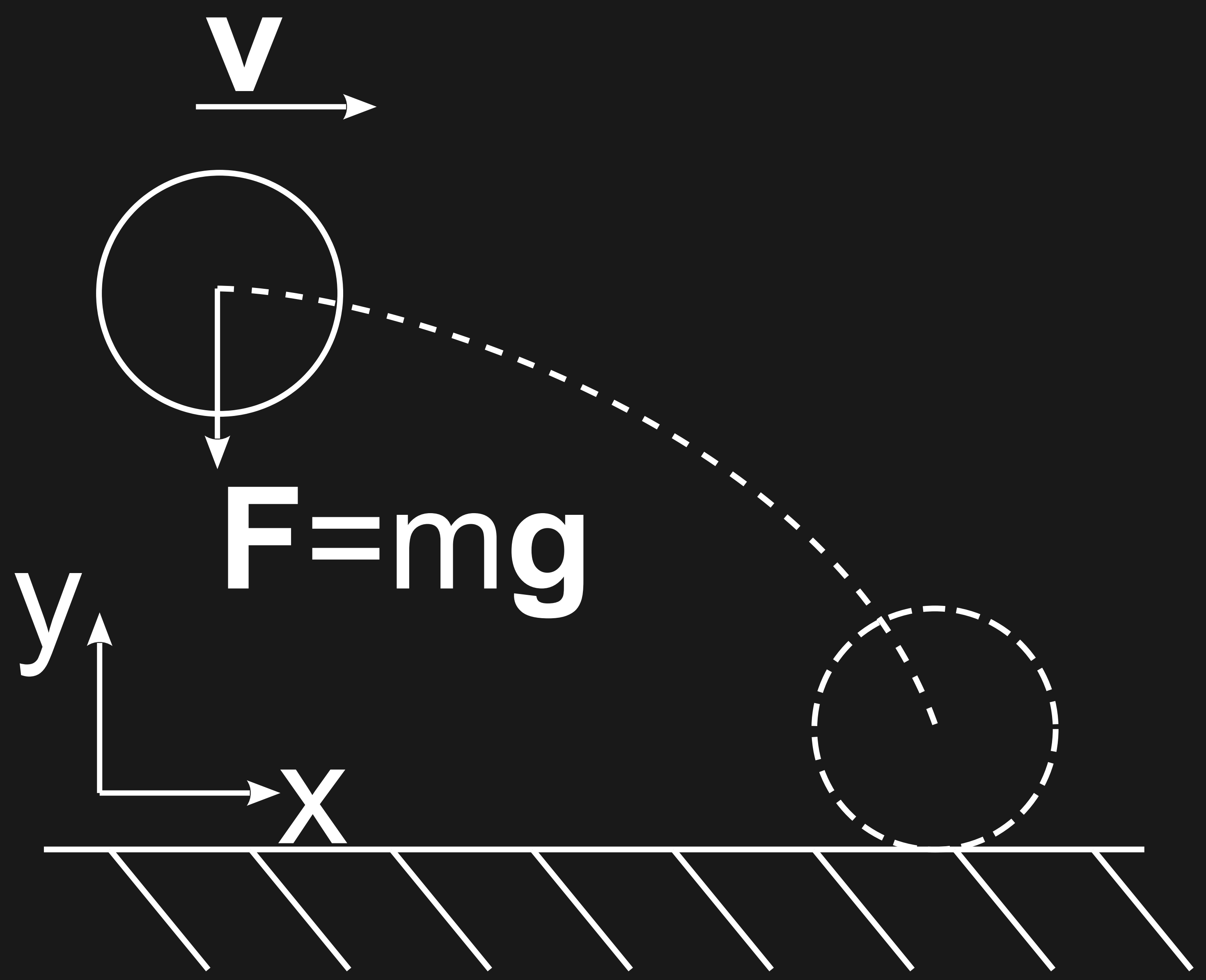
Throwing a stone



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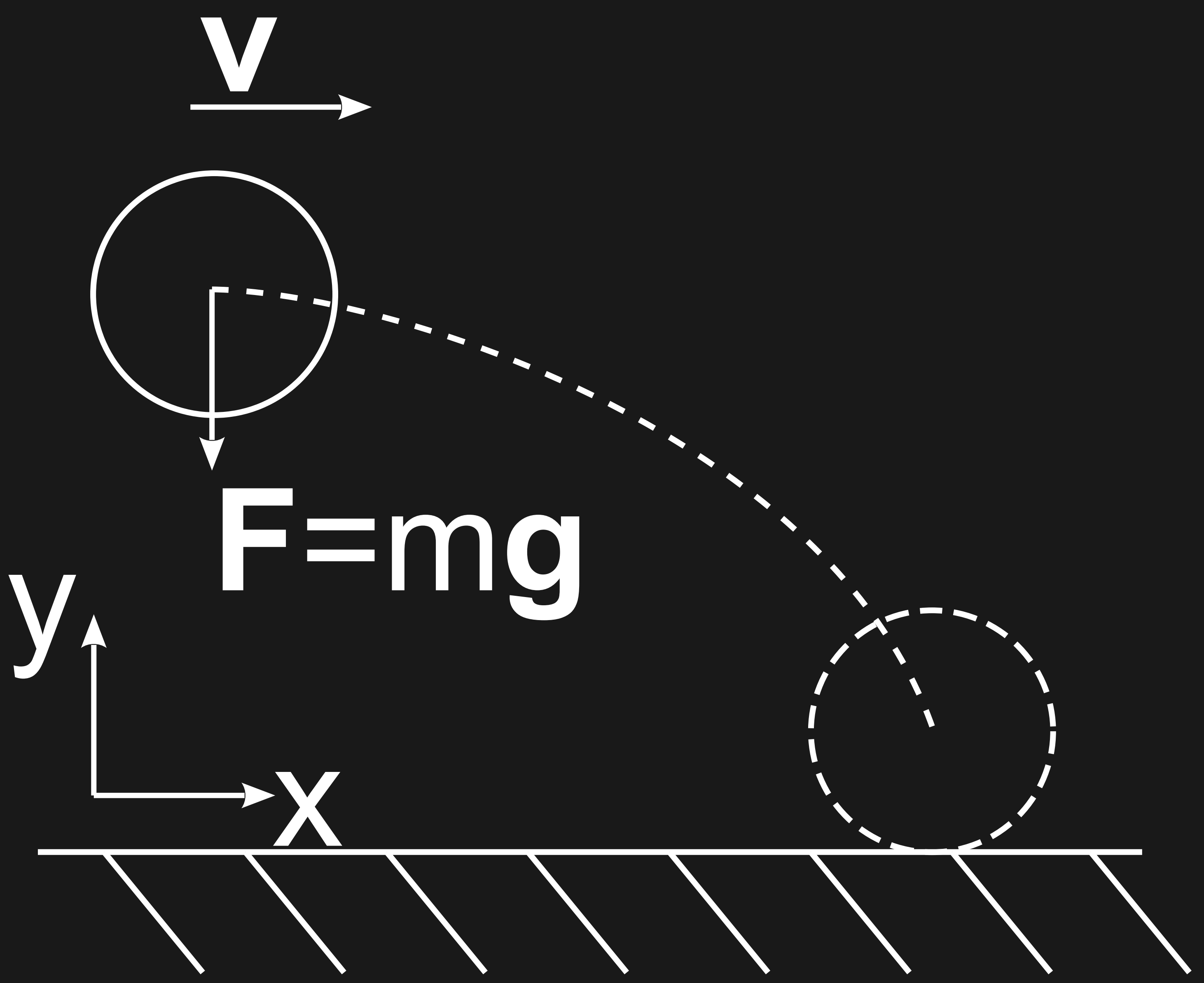

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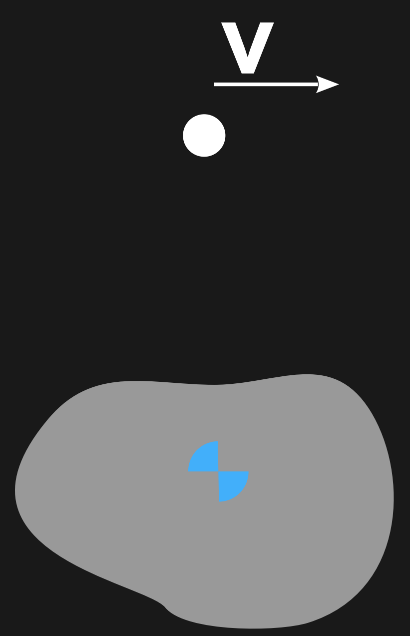
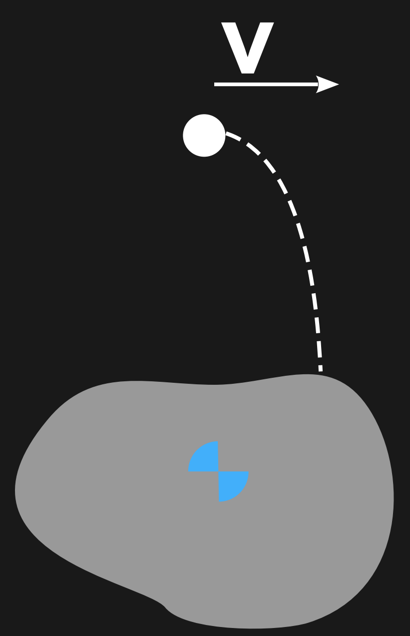
- Now use 💪 and increase horizontal velocity, **V**.

Orbital paths



- With increasing **V**
  - Orbit
  - Escape Orbit
- Earth as a point mass
  - Kepler orbit
  - Conical sections

Orbital paths



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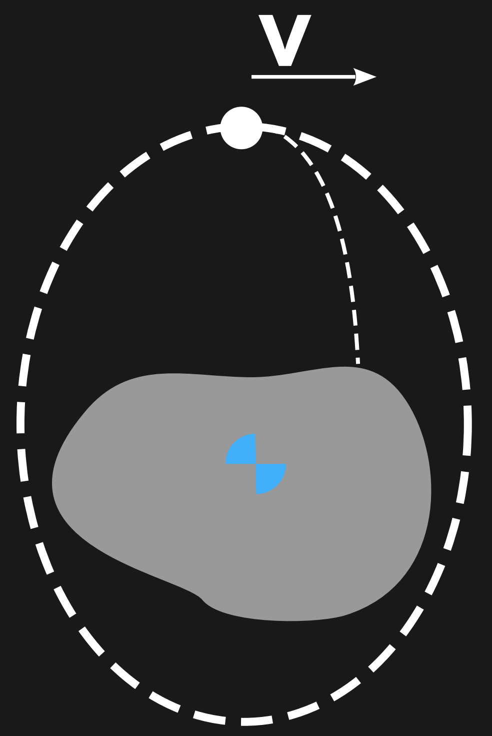

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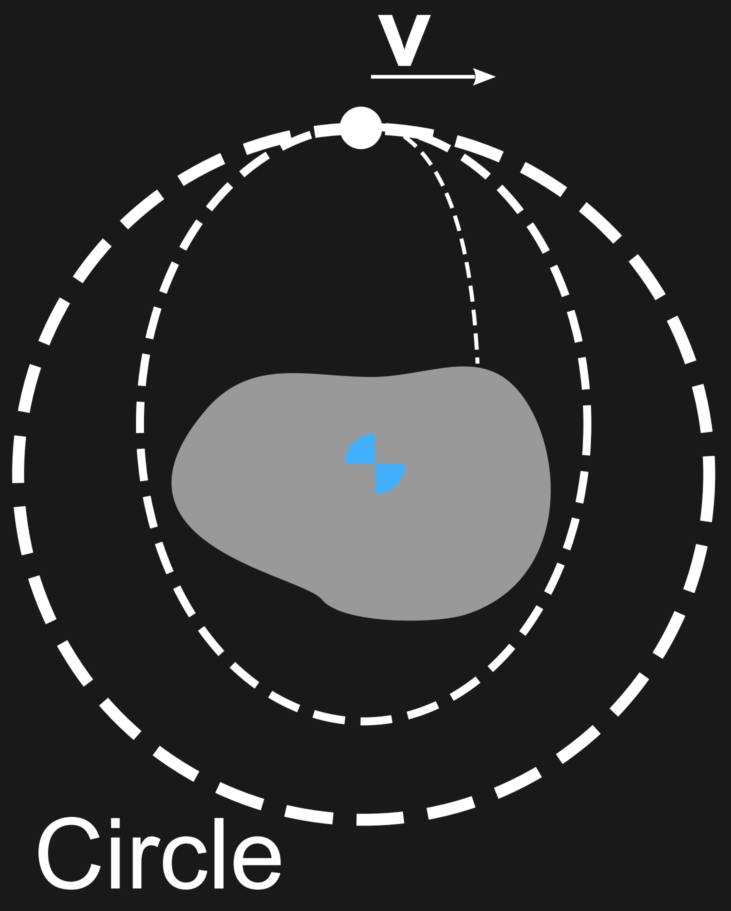

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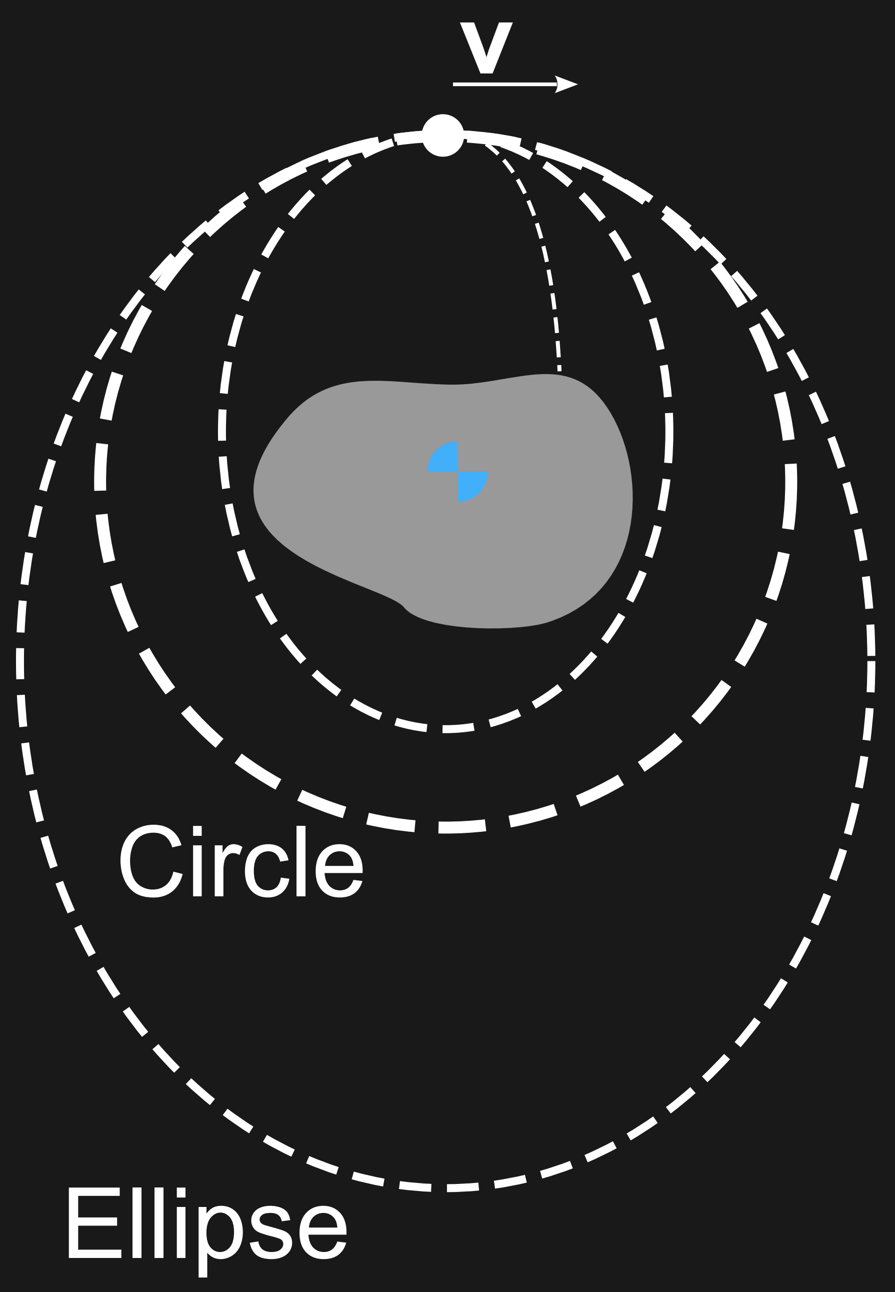

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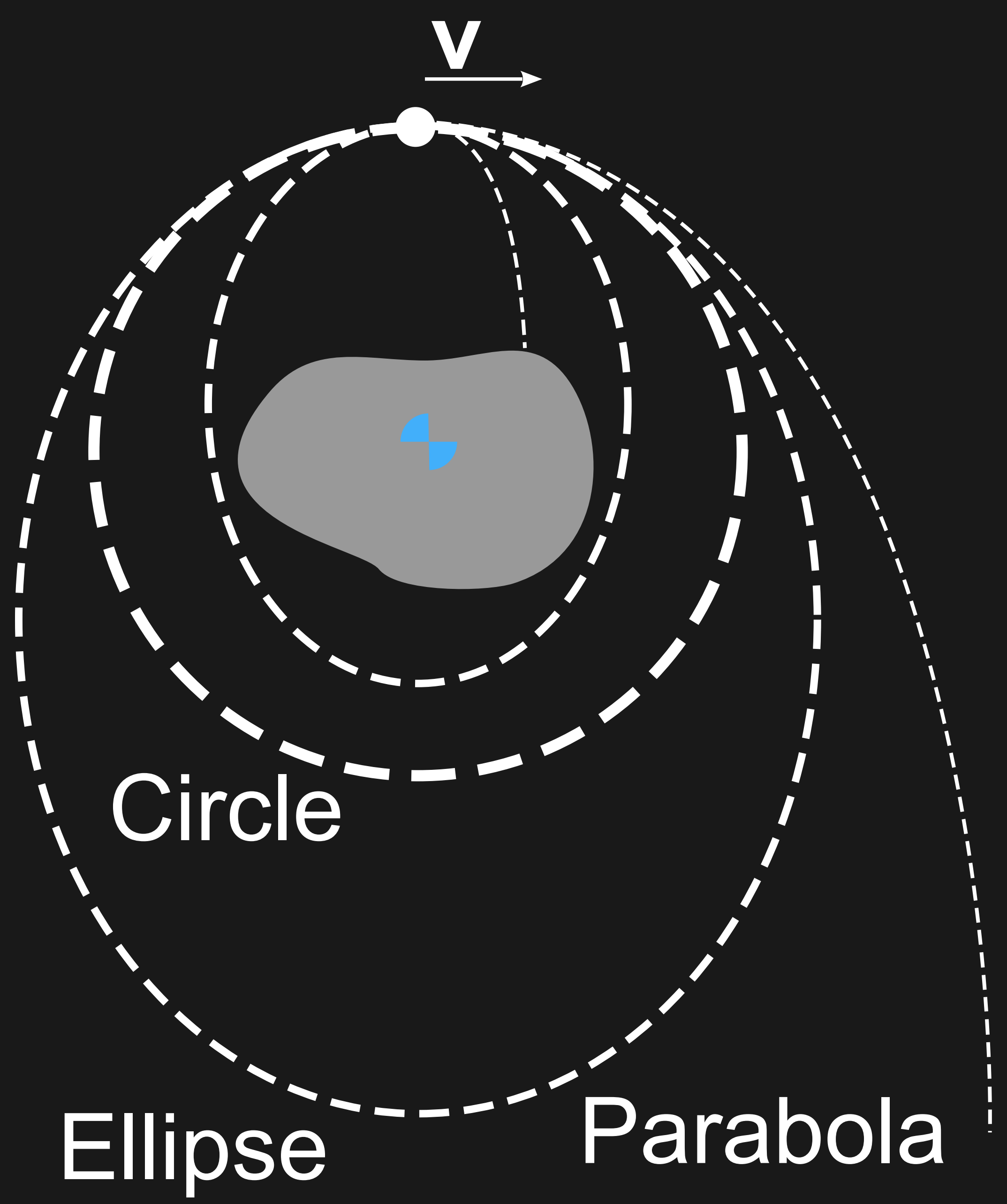

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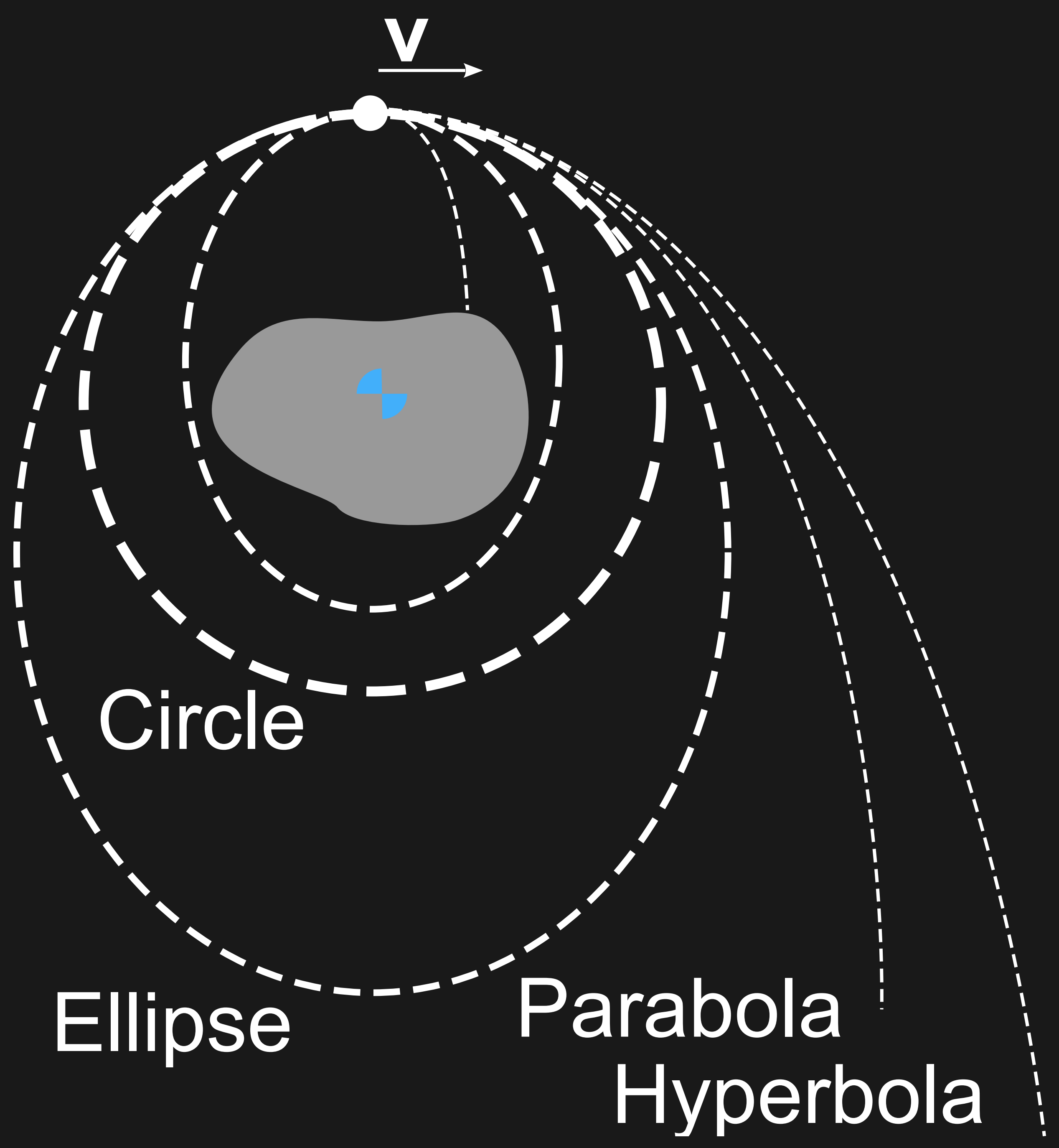

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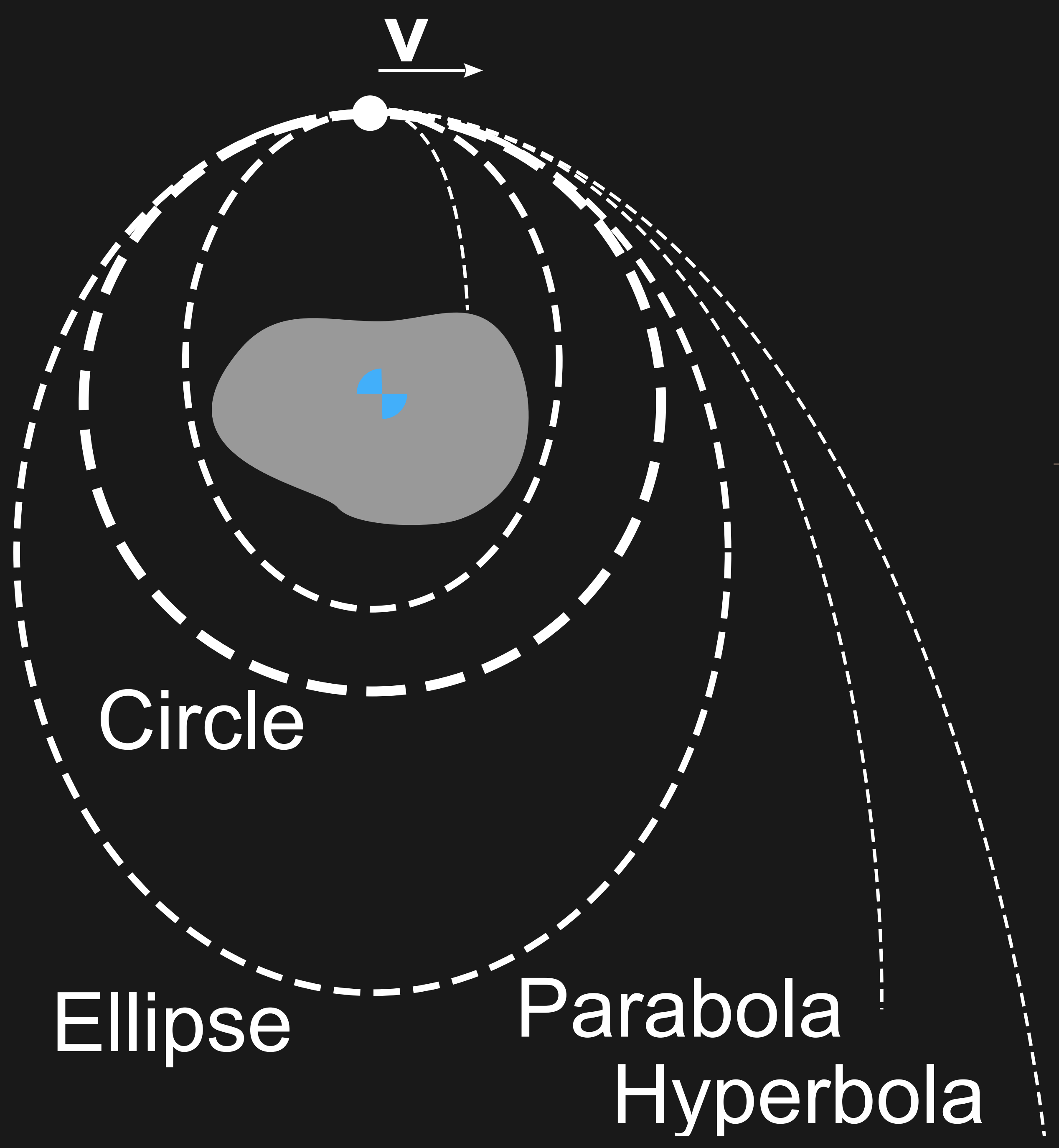

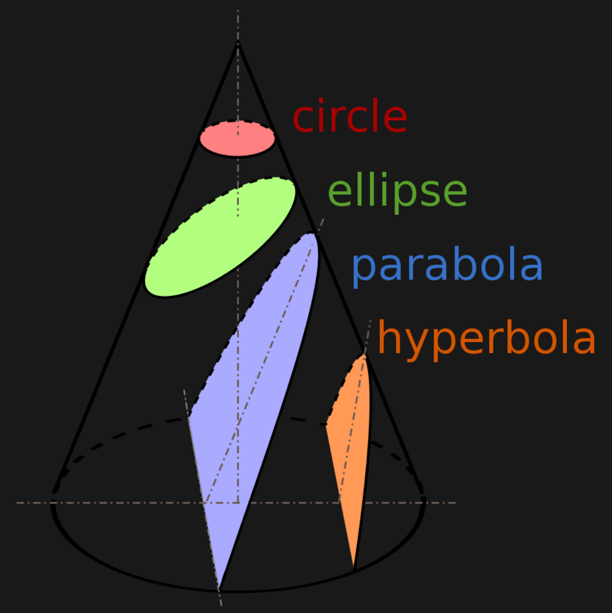

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How do we know? Painstaking work from Tycho Brahe and Johannes Kepler

Tycho Ottesen Brahe

Brahe wearing the Order of the Elephant

Born	14 December 1546 Knutstorp Castle, Scania, Denmark, Denmark–Norway (now Sweden)
Died	24 October 1601 (aged 54) Prague, Habsburg Bohemia, Holy Roman Empire (now Czech Republic)
Nationality	Danish
Alma mater	University of Copenhagen Leipzig University University of Rostock
Occupation	Nobleman, astronomer, writer
Known for	Tychonic system *Rudolphine Tables*
Spouse(s)	Kirsten Barbara Jørgensdatter
Children	8
Parent(s)	Otte Brahe Beate Clausdatter Bille

Signature

Wikipedia

Johannes Kepler

Portrait of Kepler by an unknown artist, 1610

Born	December 27, 1571 Free Imperial City of Weil der Stadt, Holy Roman Empire
Died	November 15, 1630 (aged 58) Regensburg, Electorate of Bavaria, Holy Roman Empire
Residence	Württemberg; Styria; Bohemia; Upper Austria
Nationality	German
Alma mater	Tübinger Stift, University of Tübingen
Known for	Kepler's laws of planetary motion Kepler conjecture *Rudolphine Tables*
Scientific career	
Fields	Astronomy, astrology, mathematics and natural philosophy
Institutions	University of Linz
Doctoral advisor	Michael Maestlin
Influences	Nicolaus Copernicus Tycho Brahe
Influenced	Sir Isaac Newton

Signature

Wikipedia

- Tycho Brahe→ compiled a comprehensive database of astronomical observations
- Johannes Kepler → explained these astronomical observations with his 3 laws of planetary motion
- Kepler was the assistant and follow up of Brahe during his time in Prague

Intermediate Lessons learned

- Geodesy is build around three pillars: Shape,earth rotation and gravity
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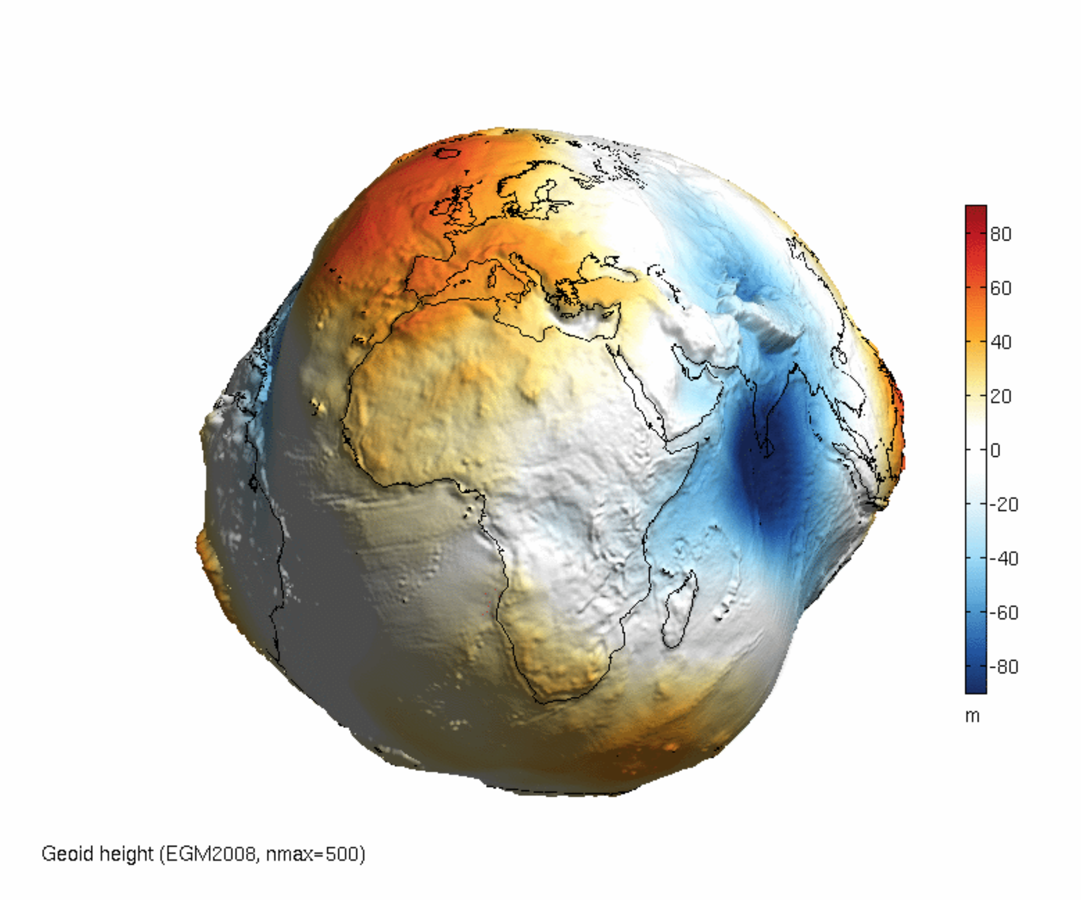
The Earth as a potato (aka Geoid)



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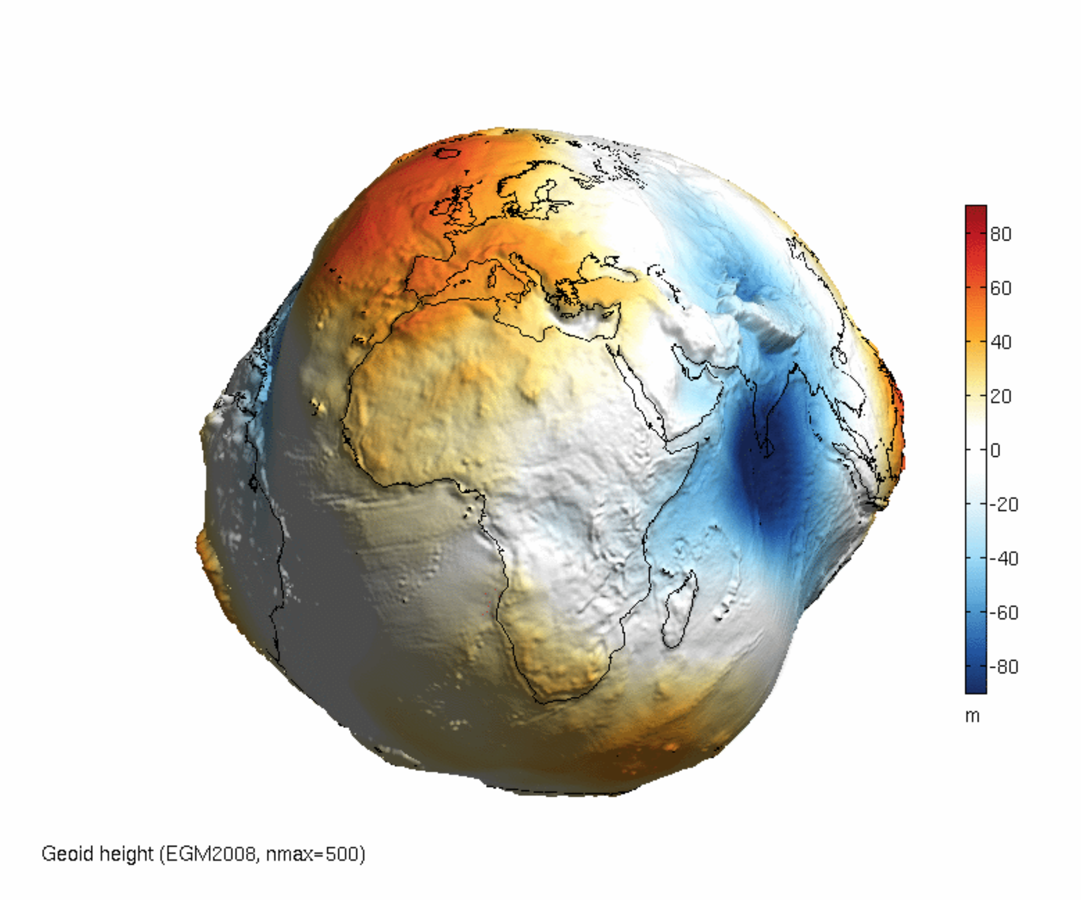
The Earth as a potato (aka Geoid)

- Geoid (surface of the ocean in rest sea without motion)



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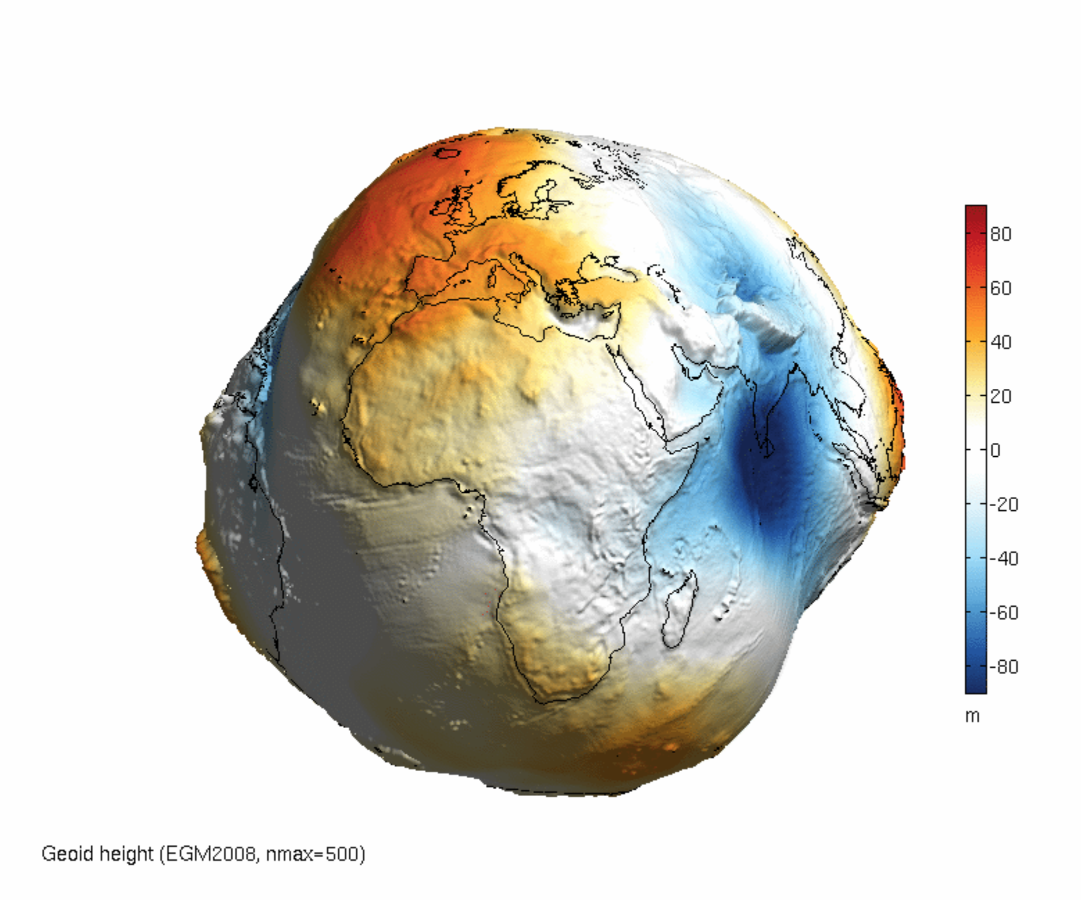
The Earth as a potato (aka Geoid)

- Geoid (surface of the ocean in rest sea without motion)
- Natural choice as a reference surface (where is 0m height?)



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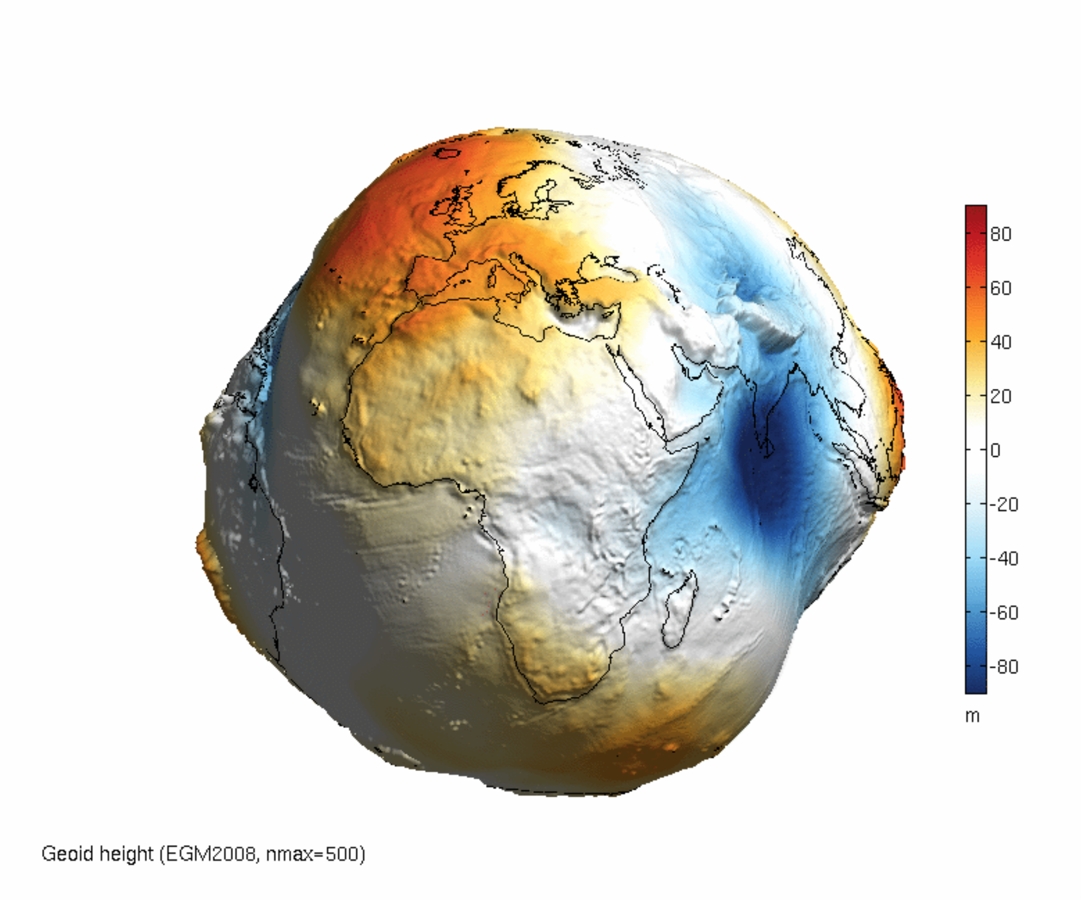
- Natural choice as a reference surface (where is 0m height?)
- max -100 meter difference from Ellipsoid



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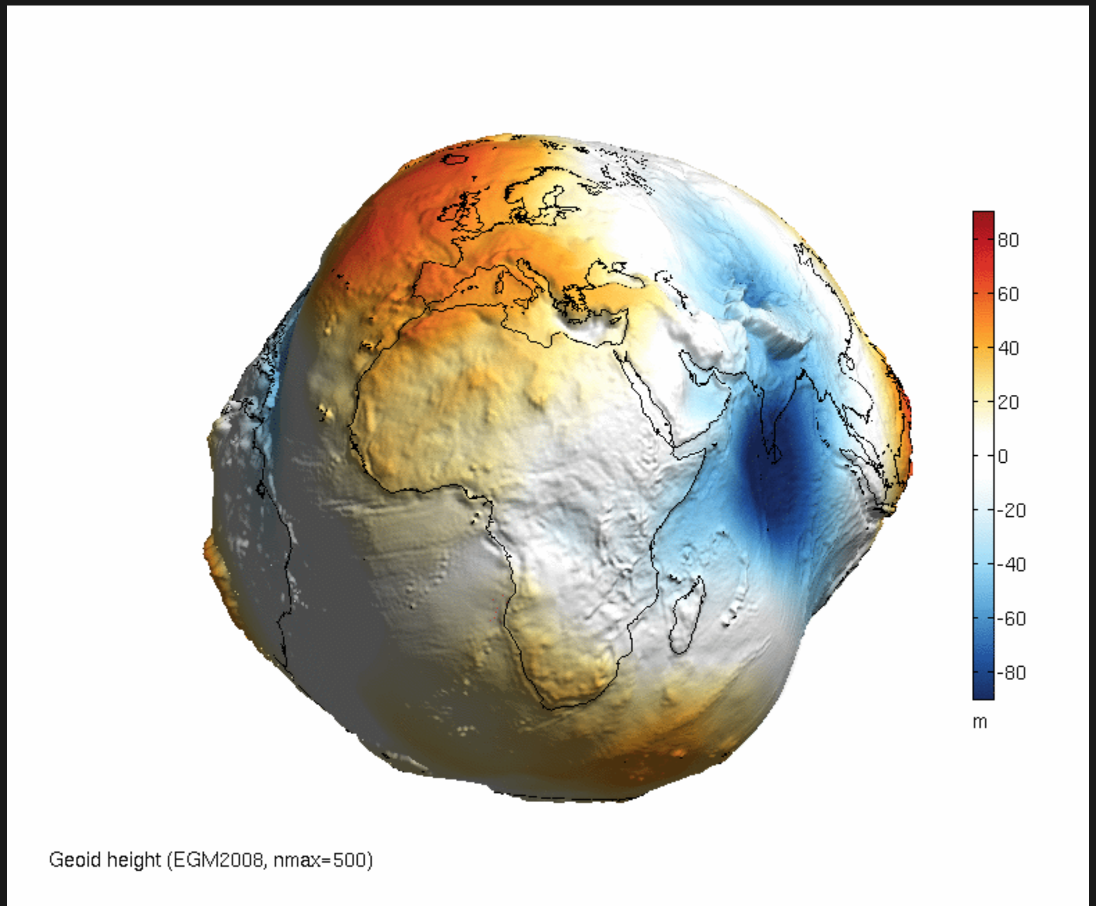
- Geoid (surface of the ocean in rest sea without motion)
- Natural choice as a reference surface (where is 0m height?)
- max -100 meter difference from Ellipsoid
- Equipotential surface: Paths along the geoid do not involve work



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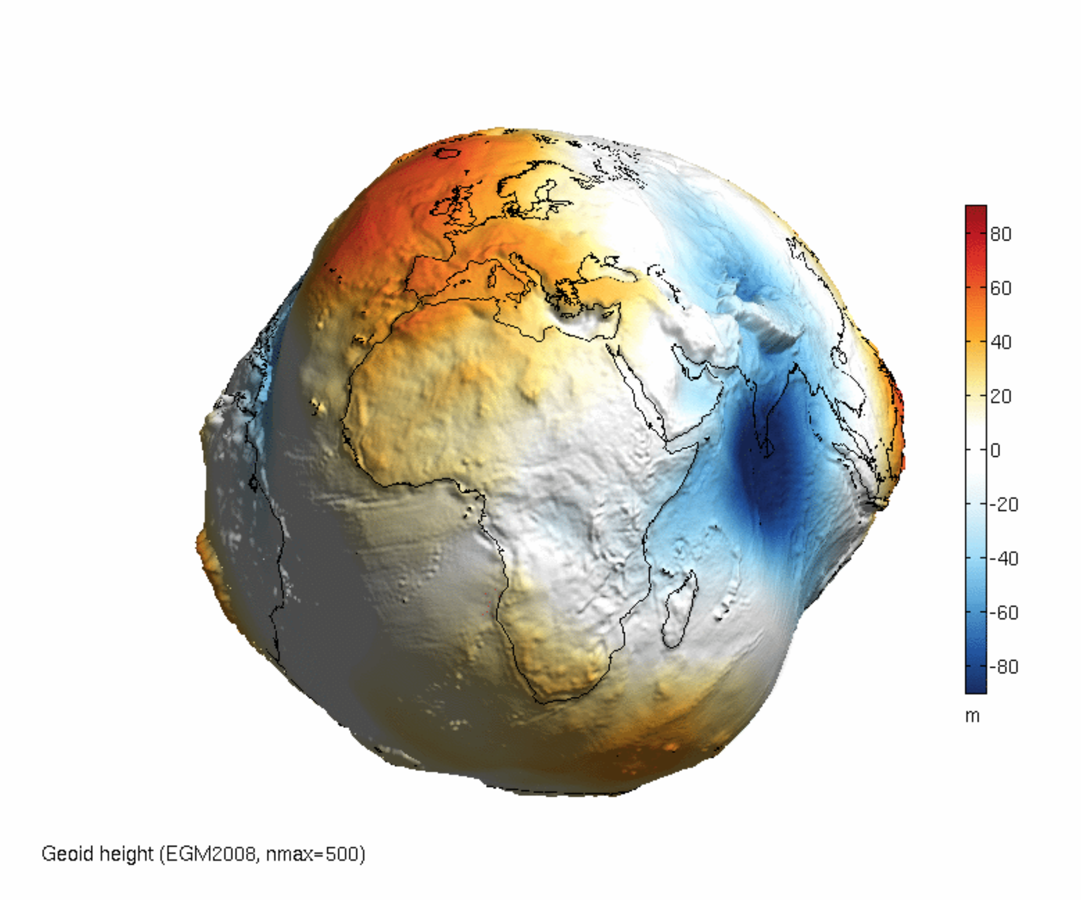
The Earth as a potato (aka Geoid)

- Geoid (surface of the ocean in rest sea without motion)
- Natural choice as a reference surface (where is 0m height?)
- max -100 meter difference from Ellipsoid
- Equipotential surface: Paths along the geoid do not involve work
- Visible: Tectonics, mantle convection



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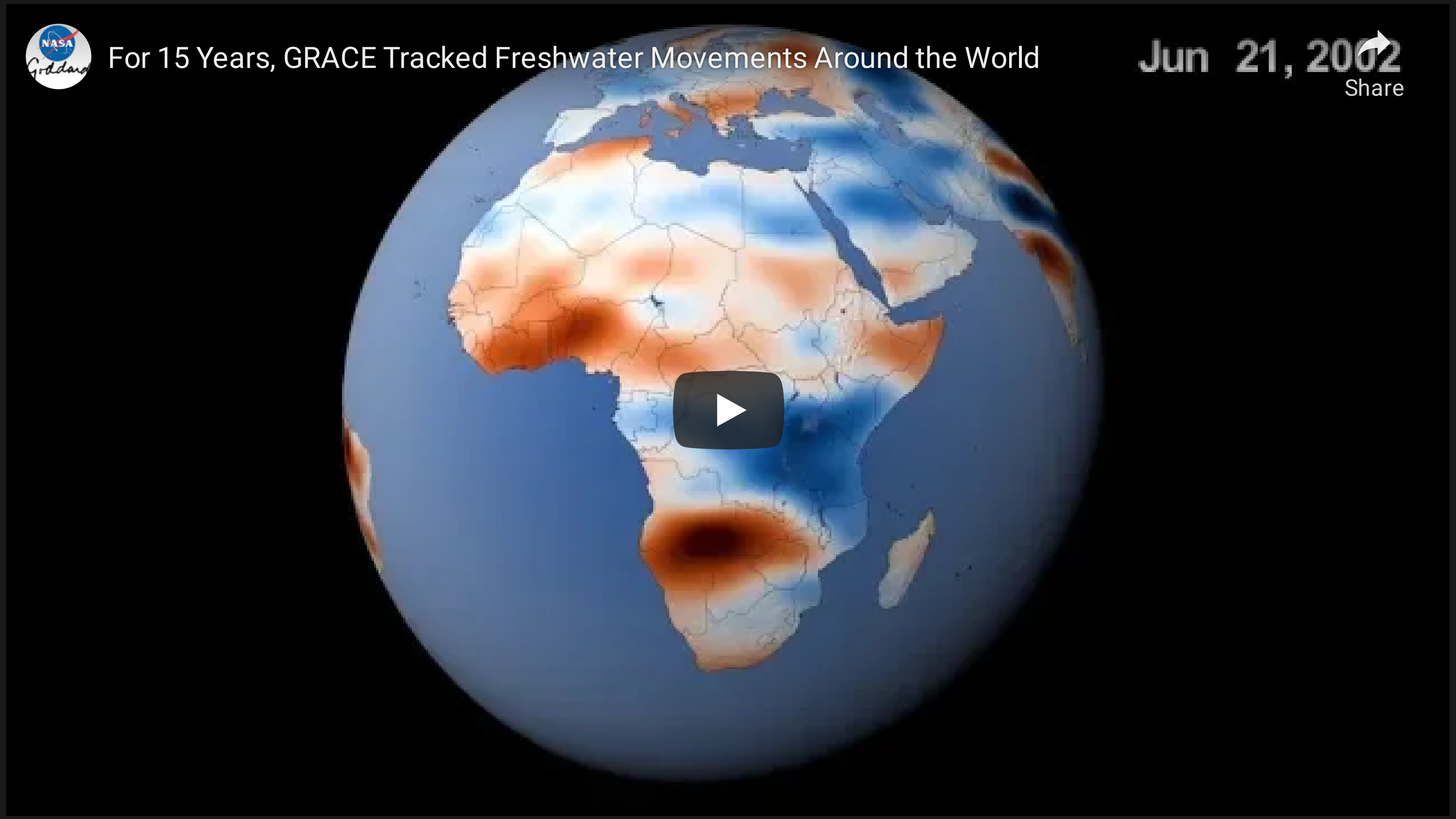
But there is a time variable component!

- Tides
- Hydrological water cycle
- Atmosphere
- Ocean
- Solid Earth (e.g. Earthquakes,..)
- Highly variable sub-daily to inter-annual
- 3 orders of magnitude smaller than the static field
- Most changes occur in a thin layer at the surface of the Earth (~15 km thick)

video link

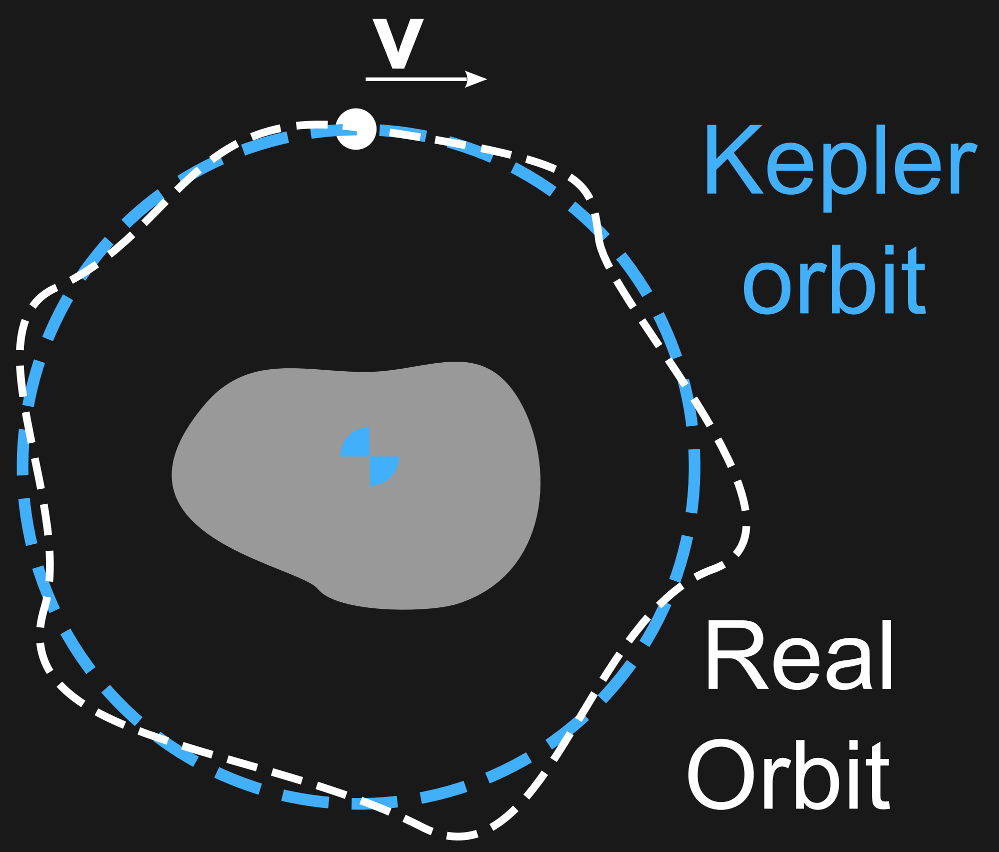


Video opens up more questions

- How GRACE can measure water height changes?
    - after all, there is an infinite number of density distributions which generate the same external gravity field
- The principle measurement has to do with the Earth
- How does one separate all those different contributions (soil moisture, groundwater, atmosphere)?
- Why does the gravity field from GRACE look so smoothed?

Principle of Satellite Gravimetry

- Anomalies from a perfect Kepler orbit contain gravity field information
- Anomalies decreases for higher orbital altitudes
- Largest gravity anomaly?
  - The Earth's flattening



Satellite orbit(Position, Velocity)

← Precise orbit determination ←

→ Gravity field estimation →

Gravity field(Potential, gravity)

GRACE Mission (2002 - 2017)

- Launch March 2002, Plesetsk (Rockot, reused ICBM)
- End Nov/Dec 2017
- Orbit height: 500km (350 km during last phase)
- Satellite separation:~250km
- Orbital period: 94 min
- Inclination: 89 degs

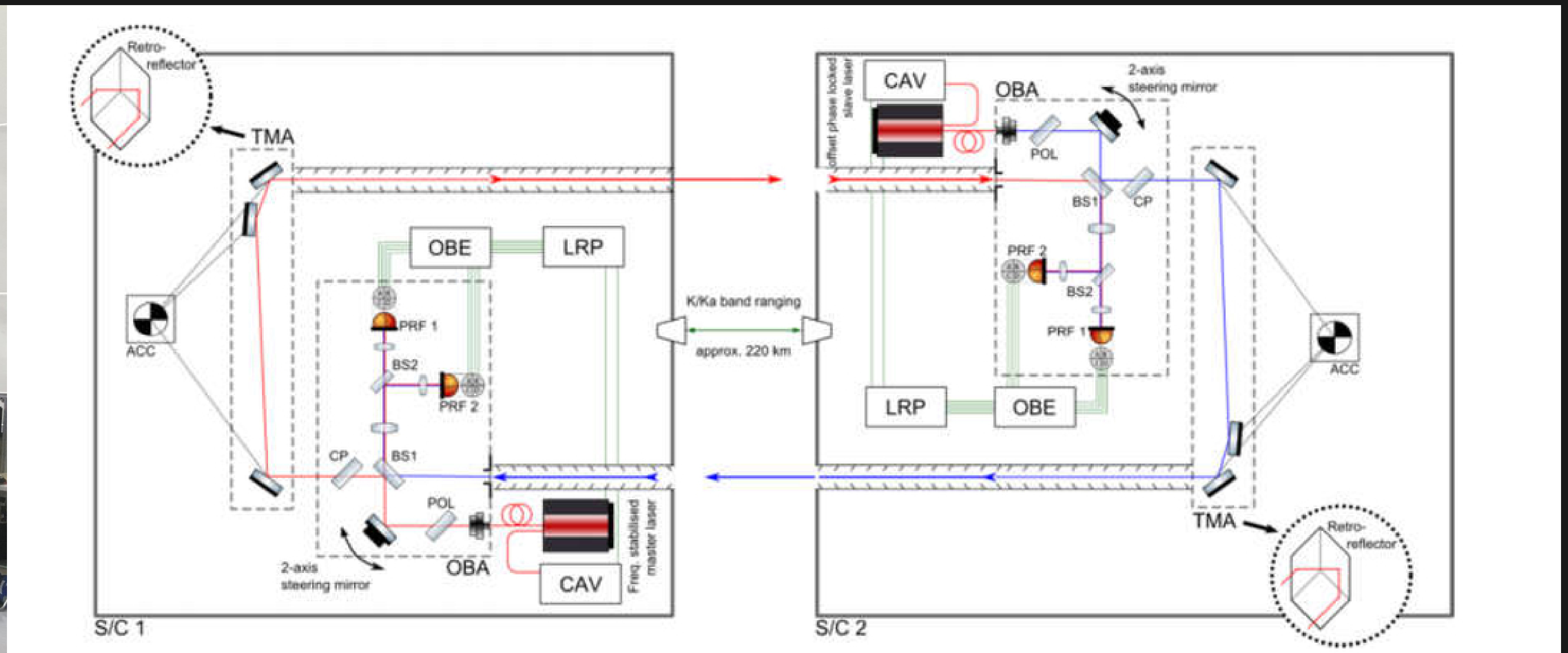
GRACE Mission cont'd

- Mission life time: 5 yrs!
- Satellite weight: 460kg+34kg fuel
- Power 160W
- Microwave ranging instrument (σ < 1 µm/s)
- Accelerometer (σ = 3e-10 m/s^2)
- Global positioning System (σ = 2 - 3 cm)
- NASA: 97 million USD, DLR: 30 million USD

GRACE Follow on Mission

- Launched: May 22, 2018
- Piggy-back with Iridium satellites on Falcon launcher
- Demonstrator: Laser interferometer for intersatellite ranging (~10x more accurate)

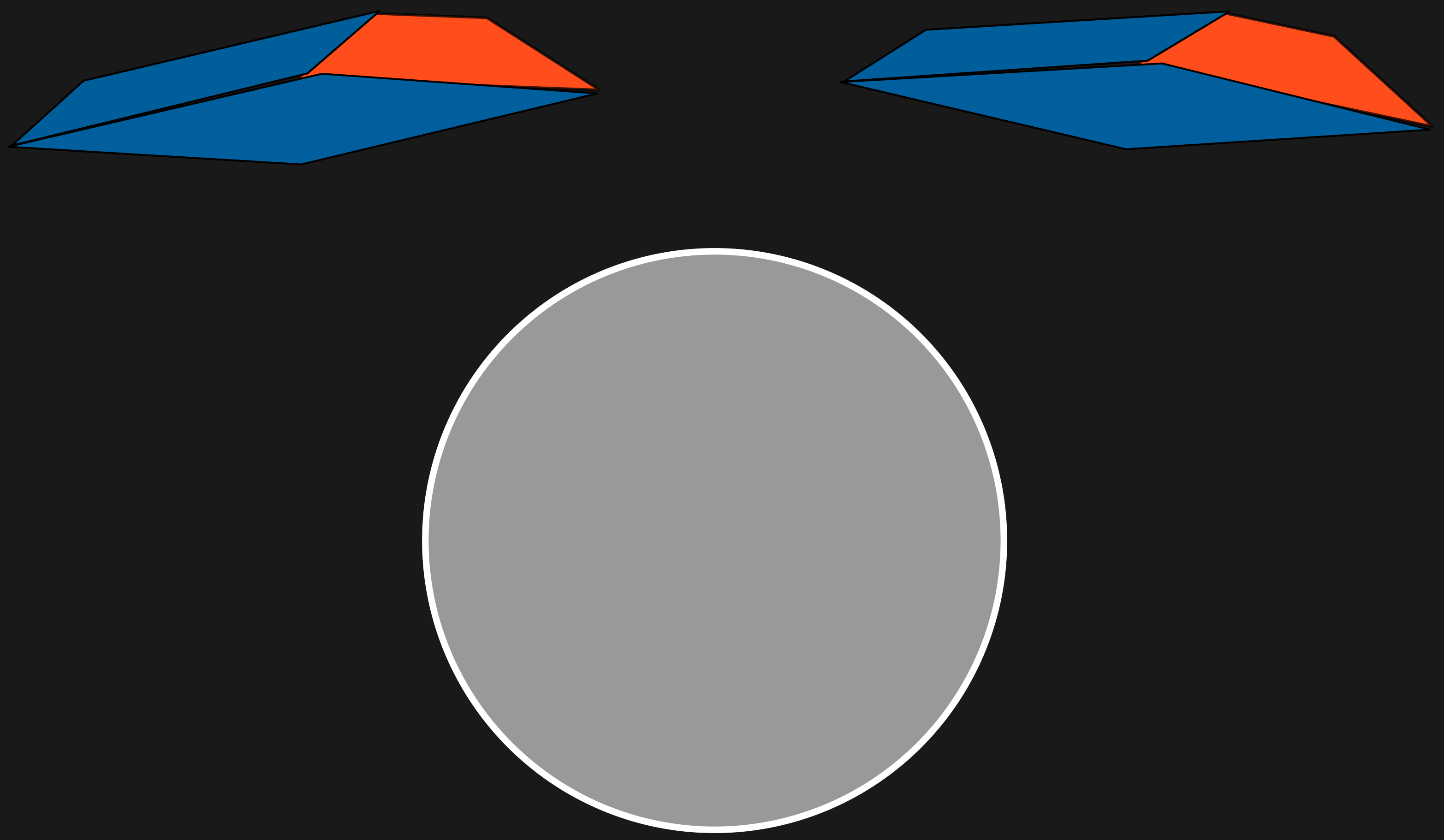
Frank Flechtner (DLR/GFZ) with the GRACE-FO satellites



Surface loading and satellite gravimetry

- Surface load causes
  - A **direct** change in gravity
  - an **indirect** response due to deformation
- Total Water Storage can be retrieved from satellite orbits when:
  - We assume mass change occurs in a thin layer
  - We have a dense coverage of accurate orbit measurements (e.g. from GNSS or range instruments)
  - We know the mechanical properties of the solid Earth

Surface loading and satellite gravimetry



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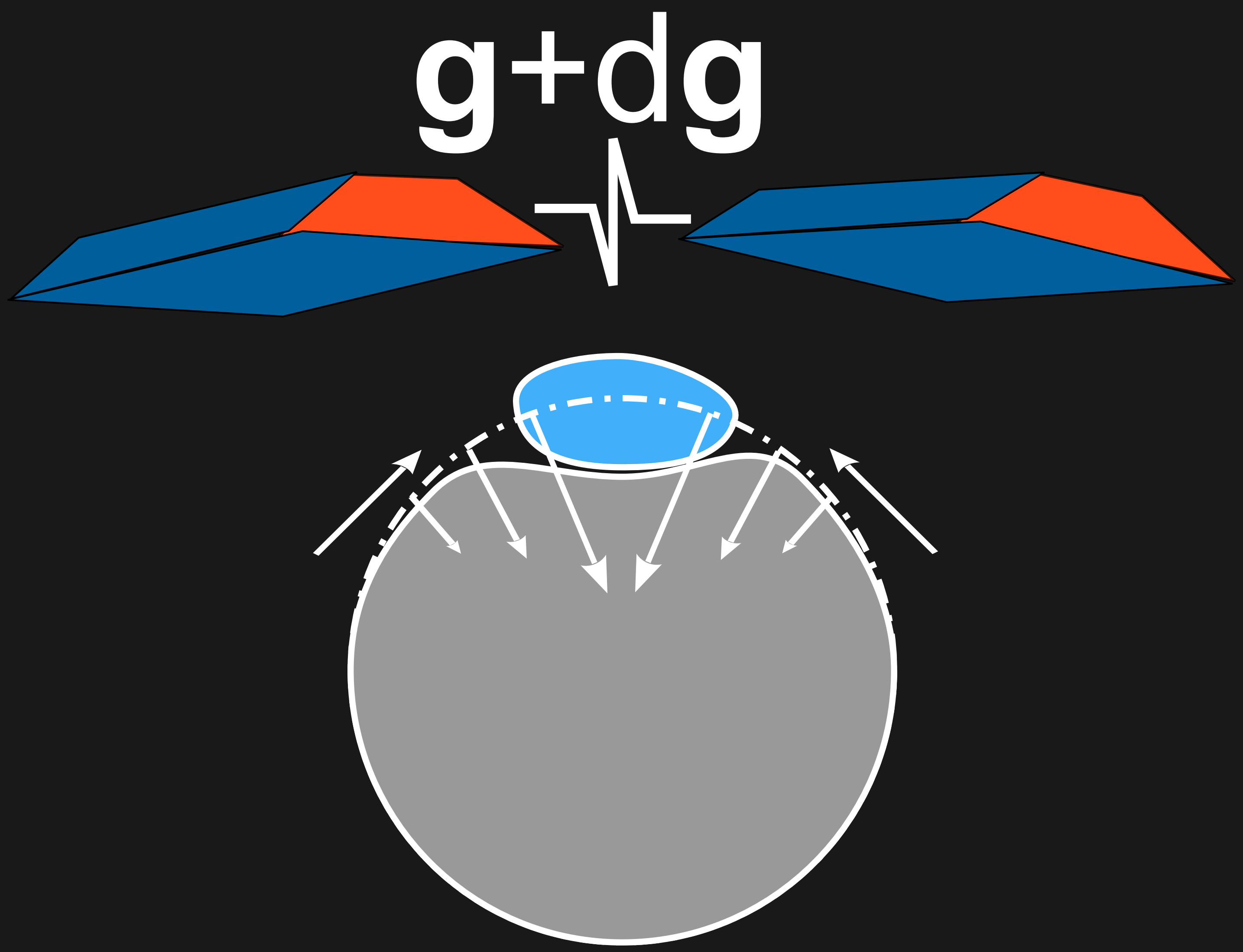

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Working with monthly Level 2 GRACE/GRACE-FO products...

$$V(\theta, \lambda, r) = \frac{GM}{a} \sum_{n=0}^{\infty} \sum_{m=0}^{m=n} \left(\frac{a}{r}\right)^{n+1} P_{nm}(\cos\theta)\,(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)$$

- Spherical harmonics are solutions of the Laplace equations!
- Where do we find the information of the gravity field?
- Infinitely many terms with combinations of degree, n, and order m, how many do we need for an ellipsoid?

$$TWS(\theta, \lambda) = a \sum_{n=0}^{\infty} \sum_{m=0}^{m=n} \frac{2n+1}{1+k_n} \frac{\rho_e}{\rho_w} P_{nm}(\cos\theta)\,(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)$$

Working with Level 2 GRACE/GRACE-FO products cont'd...

- Other corrections needed
  - Degree 1 Coefficients are missing
  - Coefficients with large n are too noisy and spatial filters are needed
  - Replace flattening coefficient (n=2,m=0) with that from Satellite Laser Ranging (better)
  - Static gravity field needs to be subtracted
  - For Ocean applications the a priori background model needs to be restored
- Many options available, but may be overwhelming for new users
- Alternative: Level 3 data (e.g. gridded with a variety of corrections applied)

Alternatively, let's look at Level 3/4 data (e.g. gridded, specific applications)

You can find level 3/4 products here:

- Gravis service (Germany)
- Tellus website (US)

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- GRACE and GRACE-FO carry inter-satellite ranging instruments which provide more information than just the orbits from GNSS tracking
- Using GRACE data does not appear straightforward but higher level producst are available