EARTH AND OCEAN SCIENCES 453

MATLAB homework assignment # 3:
VOLCANIC ERUPTIONS AND CLIMATE CHANGE

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Due 20 November, 2020

Preliminary Notes

This assignment involves developing a simple radiative energy balance model for the Earth and then investigating the effect of volcanism on this energy balance. Computationally, it is only slightly more involved than the carbon cycle model you did in project 1. However, the end product is far more ambitious: I want to see you do some interesting and solid science. This assignment asks explicitly for some creativity and for you to "follow your nose". I am looking forward to the results of your curiosity. This project is based very loosely on the *Robock's* [2000] review paper "Volcanic eruptions and climate"– the results in Figure 4. You might explore the effect of volcanism on winter climate variability through its effect on the North Atlantic Oscillation (see: Schleussner, C.F. and G. Feulner, A volcanically triggered regime shift in the subpolar North Atlantic Ocean as a possible origin of the Little Ice Age, Clim. Past, 9, 1321-1330, 2013). Alternatively, you could investigate how the effect of volcanism on global climate might change under the "Faint Young Sun" conditions of Early Earth (see the review paper by *Feulner* you read first) or if the volcanic forcing occurs during a so-called "freshening" event, where freshwater from Lake Aggasiz, say, or from melting icebergs associated with Heinrik events reduces the strength of Atlantic deep-water formation and overturning (see review papers by *Barreiro et al.* and *Hemming* from the reading list as well as recent papers by Ramstorff (e.g., S. Rahmstorf: Thermohaline Ocean Circulation. In: Encyclopedia of Quaternary Sciences, Edited by S. A. Elias. Elsevier, Amsterdam 2006)).

The class of model that we will work with is sometimes referred to as a "Budyko model" in recognition of the work of the Russian scientist Mikhail I. Budyko who was among the first to develop such models. Budyko is a bit of a legend. Among other things he used this modeling approach to do the first theoretical investigation of the ice-albedo feedback mechanism for climate change, which he discovered by accident. That is, the more snow and ice, the more the solar radiation is reflected back into space, leading, in turn, to more snow and ice. You will have an opportunity to play with this feedback as well.

A usual starting point for modeling the radiative energy balance of a planet is the Stefan-Boltzmann radiation law

$$e = \sigma_{\mathrm{B}} T^4 \tag{1}$$

where e is the flux of energy emitted ($\mathrm{W\,m^{-2}}$), $\sigma_{\mathrm{B}} = 5.6696 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan-Boltzmann constant and T is absolute temperature. From the concepts summarized in Figure 1 an expression for the radiation balance at the surface of a planet is

$$\rho c [Z] \frac{\mathrm{d}T}{\mathrm{d}t} = (1 - \alpha_{\mathrm{S}}) S(t) - \varepsilon \sigma_{\mathrm{B}} T^4 \tag{2}$$

where ρ is the density ($\mathrm{kg\,m^{-3}}$) of material comprising the surface, c is the specific heat capacity ($\mathrm{J\,kg^{-1}\,K^{-1}}$), $[Z]$ is a length scale that represents the effective thickness or "skin depth" of the irradiated surface, α_{S} is the albedo, $S(t)$ is the incident energy flux ($\mathrm{W\,m^{-2}}$) and ε is the total emissivity of the surface (for an ideal black body $\varepsilon = 1$). Note that the units for equation 1 are ($\mathrm{W\,m^{-2}}$). In words, the LHS of equation 1 is the rate of change of energy stored within the irradiated surface layer and the LHS gives the net flux of energy from this layer by radiative processes (i.e. [short-wavelength UV energy in from the sun] - [outgoing long-wavelength energy radiated from the top of the atmosphere]).

Some additional problems associated with describing a planetary radiation balance are that spinning planets have a day-side and a night-side and that the angle of incidence of light on the planetary surface depends on latitude. These considerations lead to

$$\rho c[Z]\,\frac{\mathrm{d}T}{\mathrm{d}t} = \gamma\,(1-\alpha_{\mathrm{S}})S(t) - \varepsilon\sigma_{\mathrm{B}}T^4 \tag{3}$$

where γ is a geometric factor accounting for the fact that, for a spinning spherical planet, radiation is intercepted by the disc area πR^2 but reradiated to space by the spherical surface area $4\pi R^2$. It is easy to show that for this case $\gamma = \frac{1}{4}$, the ratio of the two areas. Finally, for planets with atmospheres including clouds there is an albedo associated with the top of the atmosphere α_{sky} as well as an atmospheric transmissivity τ for infrared radiation emitted back to space from the planetary surface. The combined results give

$$\rho c[Z]\,\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{4}\,(1-\alpha_{\mathrm{sky}})(1-\alpha_{\mathrm{S}})S(t) - \varepsilon\tau\sigma_{\mathrm{B}}T^4, \tag{4}$$

where the temperature T represents some kind of average surface temperature for the entire planet. Setting $\mathrm{d}T/\mathrm{d}t = 0$ yields the steady-state condition (i.e., heat in = heat out)

$$\frac{1}{4}\,(1-\alpha_{\mathrm{sky}})(1-\alpha_{\mathrm{S}})S_0 = \varepsilon\tau\sigma_{\mathrm{B}}T^4 \tag{5}$$

where S_0 is the solar constant (the radiative flux of solar energy at the Earth–Sun distance). Figure 1 summarizes the concepts leading to (5).

Note that Equation (5) provides no information about the distribution of temperature over the planetary surface and does not take into account such details as the distribution of land, sea and ice. We can introduce these possibilities by considering a *zonally-averaged model* in which the planet is divided into latitude-dependent zones or bands (Figure 2). Such a model can distinguish between equatorial and polar regions but has no longitudinal variation. Let us consider an Earth-like planet that is divided into six zones with the zone boundaries at 60°S, 30°S, 0°, 30°N and 60°N and suppose that each zone has its own temperature. Taking $\varepsilon = 1$ and following the spirit of (4), one might write

$$A_k\rho_k c_k[Z_k]\,\frac{\mathrm{d}T_k}{\mathrm{d}t} = A_k\left\{\gamma_k\,(1-\alpha_k^{\mathrm{sky}})(1-\alpha_k)S(t) - \tau_k\sigma_{\mathrm{B}}T_k^4\right\} \tag{6}$$

where the k index identifies an individual zone so that, for our case, $k = 1\ldots 6$; A_k is the surface area of zone k, γ_k represent the geometric factor for the zone (no longer equal to 0.25 as for the whole sphere) and the values of ρ_k, c_k, $[Z_k]$, etc. remain to be assigned. I will take τ to be constant for the entire atmosphere rather than zonally dependent. Note that, as written, the system of equations (6) would result in a complete thermal decoupling of the zones. This is obviously wrong: there are substantial heat fluxes between zones that are associated with atmospheric and oceanic circulation. The Gulf Stream, for example, transfers something like 10^{15}W from Zone 4 to Zone 5. To take account of these exchanges we follow the layout of Figure 2 and modify (6) as follows:

$$A_1\overline{\rho_1 c_1[Z_1]}\,\frac{\mathrm{d}T_1}{\mathrm{d}t} \qquad = A_1\left\{\gamma_1\,(1-\alpha_1^{\mathrm{sky}})(1-\overline{\alpha_1})S_0 - \tau\sigma_{\mathrm{B}}T_1^4\right\}$$

$$+L_{12}k_{12}(T_2 - T_1) \quad (7a)$$

$$A_2\overline{\rho_2 c_2[Z_2]}\,\frac{\mathrm{d}T_2}{\mathrm{d}t} = A_2\left\{\gamma_2\,(1-\alpha_2^{\mathrm{sky}})(1-\overline{\alpha_2})S_0 - \tau\sigma_{\mathrm{B}}T_2^4\right\} - L_{12}k_{12}(T_2 - T_1) + L_{32}k_{32}(T_3 - T_2) \quad (7b)$$

$$A_3\overline{\rho_3 c_3[Z_3]}\,\frac{\mathrm{d}T_3}{\mathrm{d}t} = A_3\left\{\gamma_3\,(1-\alpha_3^{\mathrm{sky}})(1-\overline{\alpha_3})S_0 - \tau\sigma_{\mathrm{B}}T_3^4\right\} - L_{23}k_{32}(T_2 - T_3) + L_{43}k_{43}(T_4 - T_3) \quad (7c)$$

$$A_4\overline{\rho_4 c_4[Z_4]}\,\frac{\mathrm{d}T_4}{\mathrm{d}t} = A_4\left\{\gamma_4\,(1-\alpha_4^{\mathrm{sky}})(1-\overline{\alpha_4})S_0 - \tau\sigma_{\mathrm{B}}T_4^4\right\} - L_{43}k_{43}(T_4 - T_3) - L_{45}k_{45}(T_4 - T_5) \quad (7d)$$

$$A_5\overline{\rho_5 c_5[Z_5]}\,\frac{\mathrm{d}T_5}{\mathrm{d}t} = A_5\left\{\gamma_5\,(1-\alpha_5^{\mathrm{sky}})(1-\overline{\alpha_5})S_0 - \tau\sigma_{\mathrm{B}}T_5^4\right\} + L_{45}k_{45}(T_4 - T_5) - L_{56}k_{56}(T_5 - T_6) \quad (7e)$$

$$A_6\overline{\rho_6 c_6[Z_6]}\,\frac{\mathrm{d}T_6}{\mathrm{d}t} = A_6\left\{\gamma_6\,(1-\alpha_6^{\mathrm{sky}})(1-\overline{\alpha_6})S_0 - \tau_6\sigma_{\mathrm{B}}T_6^4\right\} + L_{56}k_{56}(T_5 - T_6) \quad (7f)$$

(1)

where the overlined quantities represent zonally-averaged properties (see Appendix B). This tactic allows us to think of zones as containing a mix of land, water and ice surfaces. Dimensional analysis of the above equations confirms that the LHS and RHS have dimensions of power (W). Given that L_{ij} is a length, the transfer coefficients k_{ij} must have dimensions of $\mathrm{W\,m^{-1}\,K^{-1}}$. That is, they are "effective" thermal conductivities that provide a parametric means to include the influence of longitudinal (i.e. meridonal) heat transfer across the zones without adding a fully dynamic model.

For computational purposes we reorganize (7), noting that $k_{ij} = k_{ji}$ and $L_{ij} = L_{ji}$, to obtain

$$\frac{\mathrm{d}T_1}{\mathrm{d}t} = \frac{1}{\overline{\rho_1 c_1[Z_1]}}\left\{\gamma_1\,(1-\alpha_1^{\mathrm{sky}})(1-\overline{\alpha_1})S_0 - \tau\sigma_{\mathrm{B}}T_1^4\right\} + \frac{L_{12}k_{12}}{A_1\overline{\rho_1 c_1[Z_1]}}(T_2 - T_1) \quad (8a)$$

$$\frac{\mathrm{d}T_2}{\mathrm{d}t} = \frac{1}{\overline{\rho_2 c_2[Z_2]}}\left\{\gamma_2\,(1-\alpha_2^{\mathrm{sky}})(1-\overline{\alpha_2})S_0 - \tau\sigma_{\mathrm{B}}T_2^4\right\} + \frac{1}{A_2\overline{\rho_2 c_2[Z_2]}}\left\{-L_{12}k_{12}(T_2 - T_1) + L_{23}k_{23}(T_3 - T_2)\right\} \quad (8b)$$

$$\frac{\mathrm{d}T_3}{\mathrm{d}t} = \frac{1}{\overline{\rho_3 c_3[Z_3]}}\left\{\gamma_3\,(1-\alpha_3^{\mathrm{sky}})(1-\overline{\alpha_3})S_0 - \tau\sigma_{\mathrm{B}}T_3^4\right\} + \frac{1}{A_3\overline{\rho_3 c_3[Z_3]}}\left\{-L_{23}k_{23}(T_3 - T_2) + L_{34}k_{34}(T_4 - T_3)\right\} \quad (8c)$$

$$\frac{\mathrm{d}T_4}{\mathrm{d}t} = \frac{1}{\overline{\rho_4 c_4[Z_4]}}\left\{\gamma_4\,(1-\alpha_4^{\mathrm{sky}})(1-\overline{\alpha_4})S_0 - \tau\sigma_{\mathrm{B}}T_4^4\right\} + \frac{1}{A_4\overline{\rho_4 c_4[Z_4]}}\left\{-L_{34}k_{34}(T_4 - T_3) + L_{45}k_{45}(T_5 - T_4)\right\} \quad (8d)$$

$$\frac{\mathrm{d}T_5}{\mathrm{d}t} = \frac{1}{\overline{\rho_5 c_5[Z_5]}}\left\{\gamma_5\,(1-\alpha_5^{\mathrm{sky}})(1-\overline{\alpha_5})S_0 - \tau\sigma_{\mathrm{B}}T_5^4\right\} + \frac{1}{A_4\overline{\rho_5 c_5[Z_5]}}\left\{-L_{45}k_{45}(T_5 - T_4) + L_{56}k_{56}(T_6 - T_5)\right\} \quad (8e)$$

$$\frac{\mathrm{d}T_6}{\mathrm{d}t} = \frac{1}{\overline{\rho_6 c_6[Z_6]}}\left\{\gamma_6\,(1-\alpha_6^{\mathrm{sky}})(1-\overline{\alpha_6})S_0 - \tau_6\sigma_{\mathrm{B}}T_6^4\right\} - \frac{L_{56}k_{56}}{A_6\overline{\rho_6 c_6[Z_6]}}(T_6 - T_5) \quad (8f)$$

(2)

APPENDIX A. SOME TRIGONOMETRY

Area fraction of zones

For a sphere of radius R, the surface area of a zone in the latitude range $[\theta_{k+1}, \theta_k]$ is given by

$$A_{k:k+1} = 2\pi R^2 \int_{\theta_k}^{\theta_{k+1}} \mathrm{d}\theta = 2\pi R^2 [\sin\theta_{k+1} - \sin\theta_k] \tag{A.1}$$

where θ is in radians. The total surface area of the sphere is $A = 4\pi R^2$ so that the area fraction of the zone is

$$a_{k:k+1} = \frac{1}{2}[\sin\theta_{k+1} - \sin\theta_k]. \tag{A.2}$$

The length of the boundary separating zone k and zone $k+1$ at latitude $\theta_{k:k+1}$ (radians) is

$$L_{k:k+1} = 2\pi R \cos\theta_{k:k+1}. \tag{A.3}$$

Taking $R = R_{\mathrm{E}}$ these values are applied in Tables 2 and 3.

APPENDIX B. AVERAGING

To calculate area-averaged properties such as $\overline{\rho c[Z]}$ and $\overline{\alpha}$ for each zone, use the area fractions of land, water and ice for each zone:

$$\begin{aligned} \overline{\rho c[Z]}_k &= f_k^{\mathrm{s}} \rho_{\mathrm{s}} c_{\mathrm{s}} [Z_{\mathrm{s}}] + f_k^{\mathrm{w}} \rho_{\mathrm{w}} c_{\mathrm{w}} [Z_{\mathrm{w}}] + f_k^{\mathrm{i}} \rho_{\mathrm{i}} c_{\mathrm{i}} [Z_{\mathrm{i}}] && (B.1a) \\ \overline{\alpha}_k &= f_k^{\mathrm{s}} \alpha_{\mathrm{s}} + f_k^{\mathrm{w}} \alpha_{\mathrm{w}} + f_k^{\mathrm{i}} \alpha_{\mathrm{i}} && (B.1b) \end{aligned}$$

(3)

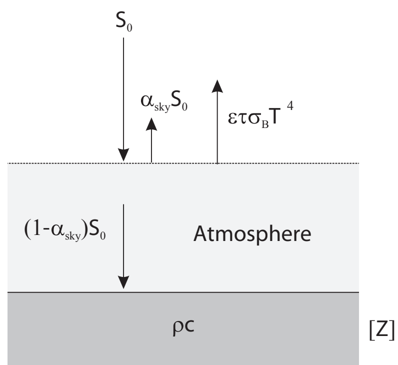

Figure 1. Basic radiative energy balance.

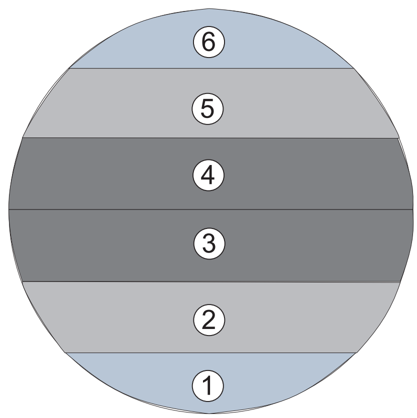

Figure 2. Zonally-averaged model.

MATLAB ASSIGNMENT

In this assignment you will construct a radiative energy balance model for the Earth and then examine perturbations to this model related to volcanism. One goal is to learn about the effects of volcanism on climate change. Another is to learn how analyze this problem quantitatively.

Write MATLAB code to perform energy balance modelling as summarized in Equation 8. As a starting point use the parameter values summarized in Tables 1–3. Note that data for the fractional area of land, water and ice for the zones are not given. Estimate these for yourself using Google Earth. I strongly suggest that you write your program using consistent SI units. Thus you should be integrating in seconds even though you are probably interested in the month-to-year scale response. Following this advice I also strongly suggest that you use a Kelvin temperature scale and only after you have finished solving the system and are interested in graphic the results should you consider introducing Celsius temperatures. Note that you require initial conditions $T_k(0)$ for each zone in order to launch the Runge–Kutta integration. You could, for example, assume that all zones were at 0°K at time $t = 0$. The only consequence of such a poor start is that it would take longer for the start-up transient to disappear and the solution to reach a steady state.

1. Develop an algorithm for solving this problem on a computer. Please do it graphically and then add some text as needed. Please do this *before* you start writing your code.

2. Write a code to examine the present, undisturbed (i.e. steady-state), energy balance of Earth. Start by assuming that all intra-zonal transfer is suppressed (i.e. set k_{ij} to zero). Next, change to the non-zero values suggested in Table 3. Note that k_{45} has been set to a larger value than other transfer coefficients, which emulates the effect of the Gulf Stream on Northern Hemisphere climate. You might wish to tinker with these coefficients (e.g. setting them to higher or lower values) to see what happens. That is, please play around with these values and see what you learn. Be aware of the questions you are asking when you do this. *Keep track of what you are trying to learn.*

3. Once you are satisfied that you have the program running and consider the results to be realistic, you can introduce the effect of volcanism. Refer to Figures 2 and Plate 5 of *Robock* [2000]. Note that Plate 5 is concerned with the latitudinal (i.e. zonal) spreading of volcanic aerosols. Note that Figure 2 is concerned with the change in solar radiation associated with the 1982 El Chichón and Pinatubo eruptions as observed at the Mauna Loa observatory. Examine the graph for "Direct Radiation" and use this to guide your choice of a magnitude, shape and timescale for a volcanic effect on incoming radiation. Use your examination of these graphs to propose a simple function $\Phi_k(t)$ (where k is zone number and t is time (in seconds!!)) to represent the decrease in incoming radiation related to the outgassing of aerosols. This is another place where you might wish to play with the mathematical form of this forcing to the model.
   To implement this perturbation define a function $\Phi_k(t)$ such that $\Phi_k(t) = 1$ for the undisturbed case and, for example, $\Phi_k(t) = 0.6$ for a 40% reduction in incoming radiation. Referring to Equation 8, modify the incoming radiation terms

   $$\gamma_k\,(1 - \alpha_k^{\text{sky}})(1 - \overline{\alpha_k})S_0$$

   to read

   $$\gamma_k\,(1 - \alpha_k^{\text{sky}})(1 - \overline{\alpha_k})\Phi_k(t)S_0$$

   and rerun your model.

4. Now torture the planet with volcanism in a way that you define. Here are some ideas (this is NOT a "to-do" list, just some ideas– please take as much latitude as you like):

- What happens if you include a CO_2 greenhouse effect in the problem in a way that is a little more realistic than the ε value in the table? The simplest way to do this is to rewrite the outgoing radiative flux in equation 2 using a fit for the outgoing long-wavelength radiation from a state-of-the-art model. That is: $\epsilon\sigma_B T^4 \approx OLR(H_2O_v, CO_2)$. For example the OLR heat flux (Pierrehumbert, 2011):

$$OLR(T, CO2) = A(CO_2) + B(CO_2) * (T - T_{reference}),$$

  with $B = 2W/m^2$ K (independent of CO2) and $A = A(CO2(0)) - 4. * log2(CO2/CO2(0))$, where you choose $A(CO2(0))$ to be where $T = T_{reference}$. This form gives a climate sensitivity of 2 K per doubling of $p\text{CO}_2$. You will note that because changes in OLR are proportional to the log of the change in CO2 that it takes a really big change in CO2 to have a large effect on the surface temperature.
- What happens if your atmosphere is a "gray body" or, alternatively, has an emissivity that depends on optical wavelength because of clouds (that may vary in structure or altitude or be distributed only in certain latitudes) and greenhouse gases? (Please find some data on this to justify your approach. Google sky might be a place to start for estimating cloud cover).
- Do your results change if you add more zones with appropriate meridional (i.e., longitudinal) transport coefficients?
- What if the residence time of volcanic aerosols is a few times longer than might be estimated from the Robock measurements or varies according to eruption intensity (i.e., the more energetic the eruption, the higher the eruption column and the longer the residence time– look, for example, in the *Encyclopedia of Volcanoes* to learn about eruption intensity and column height (the measure of energy release is the "Volcano Explosivity Index " or VEI index with 8 as the highest value).
- What happens if the "diffuse radiation" in Figure 2 of the Robock paper is also considered?
- What if the continents were distributed differently– how might the effect of volcanism on climate change during different epochs in Earth's history? Be specific about which epoch and justify your distribution of continental landmass.
- What is the effect of the monotonic increase in the solar constant over Earth's history?
- What if you include effects related to the differing albedos of fresh snow, sea ice, glacier ice, dirty ice, forest, grassland, desert, tundra, shallow water, deep water, ... etc?
- (This is a little more ambitious and interesting). The review paper by *Barreiro et al* [2008, sections 3 and 4] in the course reading list show that a critically large freshening of the North Atlantic will drastically reduce the strength of the thermohaline circulation (Figure 8) and change the meridional (longitudinal) heat transfer in the Atlantic. To parameterize this effect in terms of the rate of freshening, you might use Figure 9 in: S. Rahmstorf: Thermohaline Ocean Circulation. In: Encyclopedia of Quaternary Sciences, Edited by S. A. Elias. Elsevier, Amsterdam 2006. Yes... for a start, just use the stability diagram in Figure 9 to define approximately bounds for when the gulf stream is on or off and for the effect of freshening rate. See the last bullet on the ice-albedo feedback for a way in which such a bifurcation diagram might be implemented computationally.

- (This is also pretty interesting and builds on the Atlantic freshening idea above) An emerging potential cause of the widespread famines that defined the Medieval Little Ice Age (LIA) is the collective effect of multiple explosive eruptions on the dynamics of the North Atlantic Oscillation. The North Atlantic Oscillation (NAO) is the main driver of winter climate variability in the northern hemisphere (NH) and is related to a seesaw driven by variations in the atmospheric masses above the Azores (a permanent high-pressure region) and Iceland (a permanent low-pressure region). A positive mode corresponds to a large pressure difference, strong westerly winds, warm and wet winters in Europe and an enhanced production of sea ice near Labrador and Greenland. Reduced Atlantic deep-water formation and a weakened meridional (equator to pole) heat transfer related to this enhanced sea ice production can drive NH cooling. In contrast, a negative mode corresponds to weak winds and relatively cold, dry winters in northern Europe. A provocative correlation between explosive volcanism within or near the tropics and the persistence of a positive mode suggests a link between stratospheric eruptions and this class of multidecadal climate variability. Climate models suggest that a temporal correlation between the LIA with a number of explosive eruptions is explained by a volcanically-stabilized positive mode. Using figure 1 of the paper by Schleussner et al. as a guide for the forcing (Schleussner, C.F. and G. Feulner, A volcanically triggered regime shift in the subpolar North Atlantic Ocean as a possible origin of the Little Ice Age, Clim. Past, 9, 1321-1330, 2013), together with Figure 9 from the Ramstorff paper above, see if you can parameterize effects of explosive volcanism on the NAO and build some understanding of the sensitivity to, say, where volcanism occurs, how intense are the eruptions in terms of their radiative forcing, etc..
- From Archer's 2000 review paper in the reading list, it is clear that glacial and interglacial oceans are different beasts. In particular, the longitudinal temperature difference driving ocean stirring is weaker during glacial times because the whole planet is colder (there are smaller equator - pole temperature gradients). This leads to to proportionally less "meridional" heat transfer. How does your model change if you implement a weaker gulf stream? How about a stronger one? Qualitatively, varying the meridional temperature gradient has an effect similar to the freshening discussed in the Barriero paper but for different reasons. Which effect is larger?
- (This is also a little more ambitious and interesting). What if the reduction in solar radiation is particularly extreme? One way to explore this is to investigate what happens if the zonally-averaged albedo varies nonlinearly with temperature– this is a proxy for the so-called "ice-albedo feedback" that may have given rise to so-called "snowball Earth" events (see the display on the first floor of ESB and the Hoffmann and Schrag paper at: http://www-eps.harvard.edu/people/faculty/hoffman/snowball_paper.html). Should you choose to investigate this effect note that the temperature-dependent albedo in each zone will vary between the zonally-averaged initial value α_o, say, used for part 2 above and the value for ice α_i. In addition, it will obviously take more cooling (a greater temperature change) in an equatorial zone to change its average albedo than a zone in the high latitudes.

  One parameterization for the variation of albedo with surface temperature that is not inconsistent with results from fully-dynamic climate models is:
  $$\alpha(T) = \begin{cases} \alpha_i & T \leq T_i \\ \alpha_o + (\alpha_i - \alpha_o)\frac{(T-T_o)^2}{(T_i-T_o)^2} & T_i < T < T_o \\ \alpha_o & T \geq T_o \end{cases},$$
  where I take $T_i = 260$ K and $T_o = 290$ K is a threshold temperature below which α_o will

increase quadratically with declining temperature (to a minimum value of α_i. Can you find a snowball solution related to volcanism? Can you recover from a snowball solution (we are not living on a frozen planet currently)?

5. Submit a **concise** and **precisely-written** report on your modelling efforts, including the MATLAB script and any plots that you generate.

Table 1. Model parameters

Property	Value	Units
Stefan-Boltzmann constant, σ_B	5.6696×10^{-8}	$\mathrm{W\,m^{-2}\,K^{-4}}$
Solar constant, S_0	1368	$\mathrm{W\,m^{-2}}$
Radius of Earth, R_E	6371×10^3	m
Surface area of Earth, A_E	πR_E^2	$\mathrm{m^2}$
Total emissivity of Earth, ε	1	
Atmospheric transmissivity, τ	0.63	
Atmospheric albedo, α_{sky}	0.2	
Albedo of land surface, α_s	0.4	
Albedo of ocean surface, α_w	0.1	
Albedo of ice, α_i	0.6	
Density of land surface, ρ_s	2500	$\mathrm{kg\,m^{-3}}$
Density of ocean water, ρ_w	1028	$\mathrm{kg\,m^{-3}}$
Density of ice, ρ_i	900	$\mathrm{kg\,m^{-3}}$
Thermal scale depth for land, $[Z_s]$	1.0	m
Thermal scale depth for ocean, $[Z_w]$	70.0	m
Thermal scale depth for ice, $[Z_i]$	1.0	m
Specific heat capacity for land, c_s	790	$\mathrm{J\,kg\,K^{-1}}$
Specific heat capacity for water, c_w	4187	$\mathrm{J\,kg\,K^{-1}}$
Specific heat capacity for ice, c_i	2060	$\mathrm{J\,kg\,K^{-1}}$

Table 2. Zone properties

Property	Value	Units
Zone 1 (90°S–60°S)		
Area fraction, a_1	0.0670	
Geometric factor, γ_1	0.1076	
Land fraction, f_1^{s}	–	
Ocean fraction, f_1^{w}	–	
Ice fraction, f_1^{i}	–	
Zone 2 (60°S–30°S)		
Area fraction, a_2	0.1830	
Geometric factor, γ_2	0.2277	
Land fraction, f_2^{s}	–	
Water fraction, f_2^{w}	–	
Ice fraction, f_2^{i}	–	
Zone 3 (30°S–0°)		
Area fraction, a_3	0.2500	
Geometric factor, γ_3	0.3045	
Land fraction, f_3^{s}	–	
Water fraction, f_3^{w}	–	
Ice fraction, f_3^{i}	–	
Zone 4 (0°–30°N)		
Area fraction, a_4	0.2500	
Geometric factor, γ_4	0.3045	
Land fraction, f_4^{s}	–	
Water fraction, f_4^{w}	–	
Ice fraction, f_4^{i}	–	
Zone 5 (30°N–60°N)		
Area fraction, a_5	0.1830	
Geometric factor, γ_5	0.2277	
Land fraction, f_5^{s}	–	
Water fraction, f_5^{w}	–	
Ice fraction, f_5^{i}	–	
Zone 6 (60°N–90°N)		
Area fraction, a_6	0.0670	
Geometric factor, γ_6	0.1076	
Land fraction, f_6^{s}	–	
Water fraction, f_6^{w}	–	
Ice fraction, f_6^{i}	–	

Table 3. Intra-zonal exchanges

Property	Value	Units
Boundary 12 (60°S)		
Boundary length, L_{12}	2.0015×10^7	m
Thermal exchange coefficient, k_{12}	1.0×10^7	$\mathrm{W\,m^{-1}\,K^{-1}}$
Boundary 23 (30°S)		
Boundary length, L_{23}	3.4667×10^7	m
Thermal exchange coefficient, k_{23}	1.0×10^7	$\mathrm{W\,m^{-1}\,K^{-1}}$
Boundary 34 (0°)		
Boundary length, L_{34}	4.0030×10^7	m
Thermal exchange coefficient, k_{34}	1.0×10^7	$\mathrm{W\,m^{-1}\,K^{-1}}$
Boundary 45 (30°N)		
Boundary length, L_{45}	3.4667×10^7	m
Thermal exchange coefficient, k_{45}	5.0×10^7	$\mathrm{W\,m^{-1}\,K^{-1}}$
Boundary 56 (60°N)		
Boundary length, L_{56}	2.0015×10^7	m
Thermal exchange coefficient, k_{56}	1.0×10^7	$\mathrm{W\,m^{-1}\,K^{-1}}$