OPTIMUMG OPTIMUM TIRE - FORMULA STUDENT

Wheel configuration choice for the vehicle Invictus (2020 season)

Direction and management
JACQUET Mathieu (Team Leader) mathieu.jacquet@ecl18.ec-lyon.fr
RODRIGUEZ Arthur (Team Manager) arthur.rodriguez@ecl18.ec-lyon.fr

Suspension system
KAWCZYNSKI Martin (Department leader) martin.kawczynski@ecl18.ec-lyon.fr
SCHIO Michele michele.schio@ecl18.ec-lyon.fr

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1 Introduction

1.1 Stakeholder analysis

For the 2020 vehiclea budjet limitation had to be applied in order to couterfit for the 2019 season. Rim choice was therefore limited to already bought rims from the past seasons.

The team needs a mathematical model that allows a comparison between different tire models and rims in a dynamic fashion. Experience from the last competitions had shows us how non realistis is the statc hypothesis so far given fro granted in the team's past models.

The structure of the present work had to follow the engineering V-model as standard practice within the team. The structure of the document is therefore divided in 3 different sections: concept generation, concept selection and result analysis.

1.2 Wheel requirements specification

is therer something to say with respect to the rules ??

2 Concept generation

Table 1 presents different tires and rims easily available in the FS competition. Overall mass , rolling inertia and yaw inertia are calculated for each tyre and rim with the hypothesis of a finite cylindrical shell with the equivalent mass of the tire (rim). The same variables were added in order to calculate the properties of each possible configuration. A number code was given to each tire whereas a letter was chosen for each rim. A given configuration was therefore identified by a letter-number code.

Tx	Ty	p
mm	mm	mm
1650	1250	1035

usd2€	0,9
in2mm	25,4
lbs2g	453,592

tyre thickness [mm]	5

tyre		model	outer radiu	width	mass	Iy	Iz	price	TTC round	
code		str	mm	mm	g	kg m^2	kg m^2	€ HT sans sped.	cornering	drive/brake
1		Hoosier 20,5/7 - 13	260	178	4990	0,33	0,18	205	5	5
2		Hoosier 16/7.5 -10	203	191	3402	0,14	0,08	180	8	x
3		Hoosier 18/7.5 - 10	229	191	4536	0,23	0,13	180	5, 6	5, 6
4		C19 205/470 r13	235	205	3924	0,21	0,12	183	5, 8	5, 8
5		Avon 16/7-10	203	178	3266	0,13	0,07	104	7	x

rim thickness [mm]	5

rim		model	radius	width	mass	Iy	Iz	price	notes
code		str	mm	mm	g	kg m^2	kg m^2	€ HT sans sped.	
a		OZ 13 Mg	165	178	2450	0,06	0,04	250	not center lo
b		OZ 10 Mg	127	178	1660	0,03	0,02	250	center lock
c		keizer 10i Al	127	178	2041	0,03	0,02	248	$265 8 piece
d		keizer 13 Al	165	178	2835	0,07	0,04	338	
e		Oz 13 Al	165	178	3400	0,09	0,05	260	sa coute au n

config	outer radius	mass	Iyy	Izz	Iz_G	price
rim tyre	m	g	kg m^2	kg m^2	kg m^2	€ HT
a1*	0,260	7440	0,40	0,22	8,19	205
a4 *	0,235	6374	0,28	0,16	6,99	183
d1	0,260	7824	0,41	0,22	8,61	543
d4	0,165	6759	0,29	0,16	7,40	521
b2	0,203	5062	0,16	0,10	5,52	430
b3	0,229	6196	0,26	0,15	6,78	430
b5	0,203	4926	0,16	0,09	5,37	354
c2 *	0,203	5443	0,17	0,10	5,93	180
c3 *	0,229	6577	0,26	0,15	7,20	180
c5	0,203	5307	0,16	0,10	5,78	351
c5 *	0,203	5307	0,16	0,10	5,78	104
e1	0,260	8390	0,42	0,23	9,22	465
e4	0,235	7324	0,30	0,17	8,02	443

* use old rims — configuration to analyze

Table 1: Concept generation table: chosen configuration are highlighted in green

3 Concept selection with a dynamic simulation

3.1 A nonlinear single track model

I this work a nonlinear single track dynamic model was implemented in MATLAB Simulink following the structure proposed in section 10.2 of [1] whose schema is presented in fig.1. This model presented no suspension system and an infinitely rigid chassis. To allow the wheel configuration choice, rim (and tire) mass and inertia were included as a parameter.

A simplified magic formula tire model was used as suggested in [1]. Coefficients equations to link between this simplified model and the magic formula 5.2 structure were derived from [2] as follows:

pure longitudinal (slip angle = 0)

$$\begin{aligned} F_{x,0} &= D_x \sin[C_x \arctan(B_x \kappa)] \\ C_x &= p_{Cx1} \\ D_x &= \mu_x F_z \\ \mu_x &= (p_{Dx1} + p_{Dx2}\, df_z) \\ df_z &= \frac{F_z - F_{z,0}}{F_{z,0}} \\ K_{x\kappa} &= F_z(p_{Kx1} + p_{Kx2}\, df_z)\exp(p_{Kx3} df_z) \\ B_x &= \frac{K_{x\kappa}}{C_x D_x + \varepsilon} \end{aligned}$$

pure lateral (slip ratio = 0)

$$\begin{aligned} F_{y,0} &= D_y \sin[C_y \arctan(B_y \alpha)] \\ C_y &= p_{Cy1} \\ D_y &= \mu_y F_z \\ \mu_y &= p_{Dy1} + p_{Dy2}\, df_z \\ K_y &= p_{Ky1} F_{z,0} \\ B_y &= \frac{K_y}{C_y D_y + \varepsilon} \end{aligned}$$

The model was given the steering angle δ and the rear axle thrusting torque $M_{A,h}$ as inputs. All of the required parameters are listed in tab.2. Output variables were chosen with respect to the test cases defined in sec.3.3 among the following variables:

s_v, α_v, $s_{v,a}$ front axle slip ration, slip angle and normalized slip

s_h, α_h, $s_{h,a}$ rear axle slip ration, slip angle and normalized slip

ψ_V, $\dot{\psi}_V$ overall vehicle angle (and its time derivative) with respect to a fixed reference system

$r\rho_v$, $r\rho_h$ front and rear axle wheel speed at contact point

$\dot{x}_V$, $\dot{y}_V$ longitudinal an lateral vehicle speed

An overall view of the MATLAB Simulink schema is presented below with a focus on the tire block implementation.

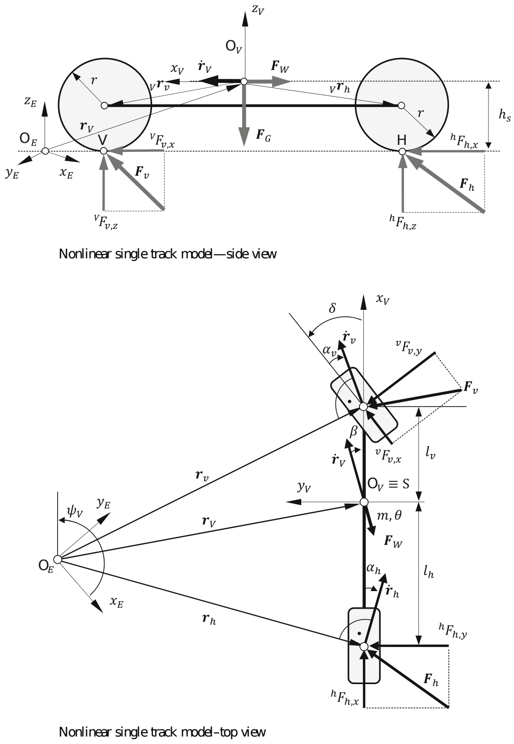
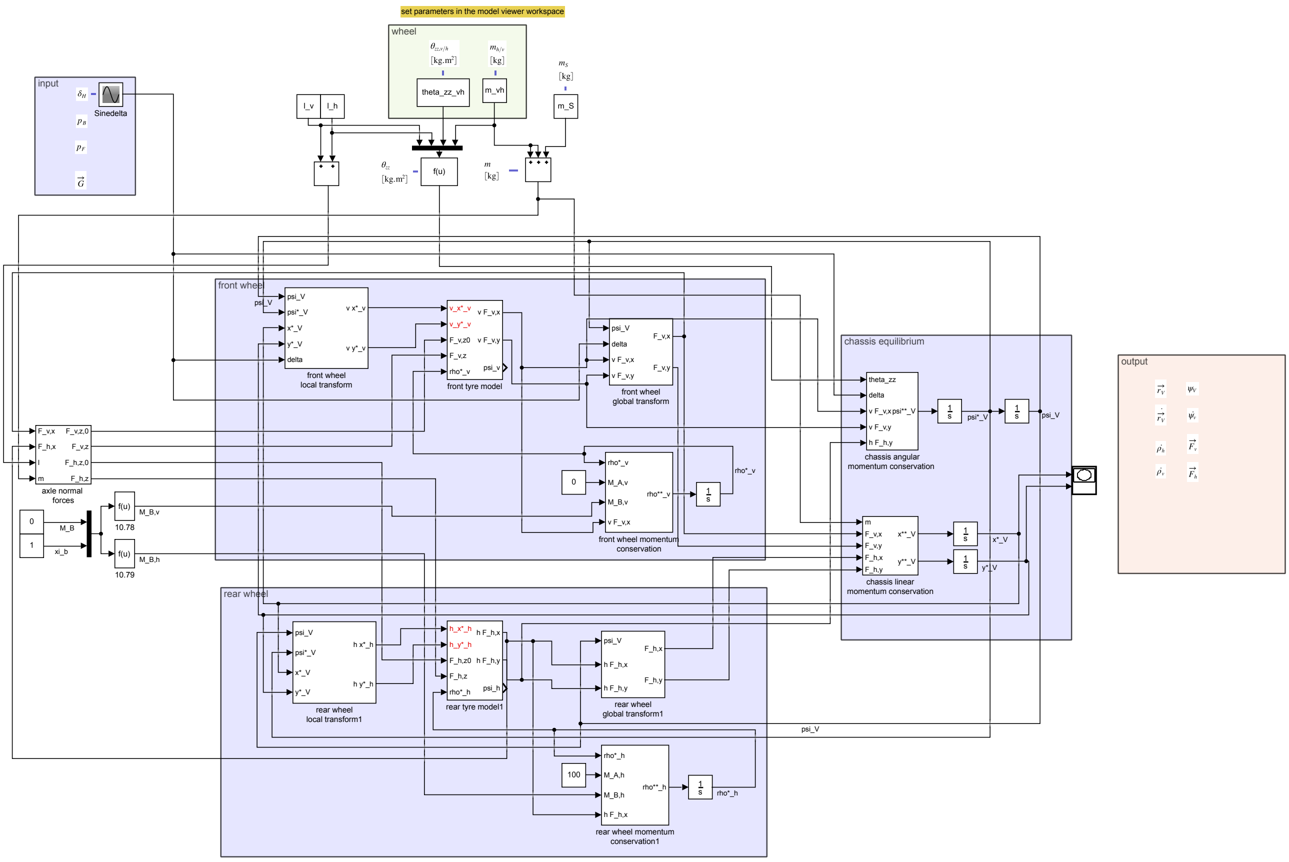
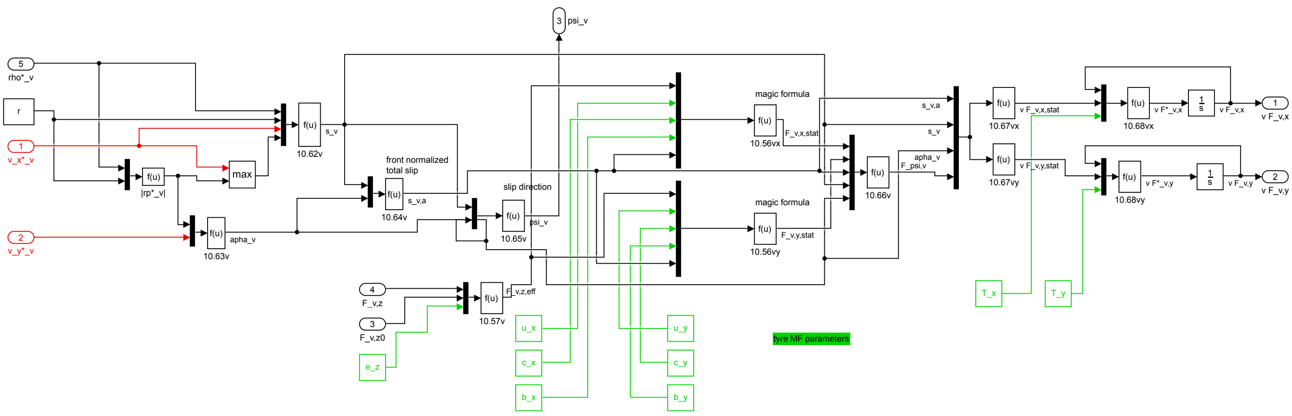


Figure 1: nonlinear single track model from [1]

			constant	unit	description	configuration	a1	a4	c2	c3	c5
wheel (tire+rim)	nonlinear single track model	rim + tyre	m_vh	kg	wheel mass		7.44	6.374	5.443	6.577	5.307
			theta_vh	kg.m^2	tyre rolling inertia		0.40	0.28	0.17	0.26	0.16
			theta_zz_vh	kg.m^2	tyre z inertia		0.22	0.16	0.10	0.15	0.10
			r	m	tyre outer radius		0.260	0.235	0.203	0.229	0.203
		eff. load	e_z		effective load degressive parameter			0			
			T_x	s	settling time of the tires during fast changes of course or velocity			0.02			
			T_y	s				0.02			
	tyre magic formula	pure longitudinal (SA=0)	PCX1		Shape factor Cfx for longitudinal force		1.278793	0.509153		1.441082	
			PDX1		Longitudinal friction Mux at Fznom		2.824687	5.447613		3.097078	
			PDX2		Variation of friction Mux with load		-0.723840	-0.441576		-0.758640	
			PKX1		Longitudinal slip stiffness Kfx/Fz at Fznom		-60.730470	-54.143220		-43.330700	
			PKX2		Variation of slip stiffness Kfx/Fz with load		30.189940	-7.051341		2.939219	
			PKX3		Exponent in slip stiffness Kfx/Fz with load		0.005985	0.171439		-0.253608	
		pure lateral (SR=0)	PCY1		Shape factor Cfy for lateral forces		1.389410	1.459571	0.735002	0.829714	1.598327
			PDY1		Lateral friction Muy		-2.509494	-2.470600	-2.985970	-3.151897	-2.294378
			PDY2		Variation of friction Muy with load		0.283008	0.628891	0.243913	0.129540	0.334261
			PKY1		Maximum value of stiffness Kfy/Fznom		73.211130	37.798840	49.328500	55.598290	54.820040
single track	geom.		l_v	m	distance between S and the front axle (vehicle reference frame)				0.78		
			l_h	m	distance between S and the rear axle (vehicle reference frame)				0.77		
			h_S	m	height of S (ground reference frame)				0.3		
			m_S	kg	suspended mass				90		
			g	m/s^2	acceleration of gravity				9.81		
			eps		avoid singularity				0.001		

Table 2: simulation parameters summary table





3.2 Magic formula tire models from TTC data with Optimum Tire

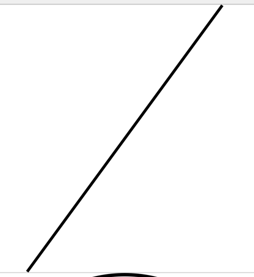
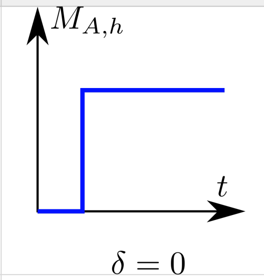
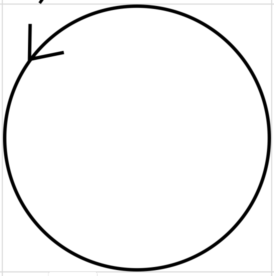
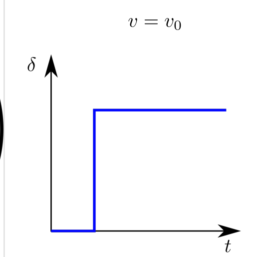
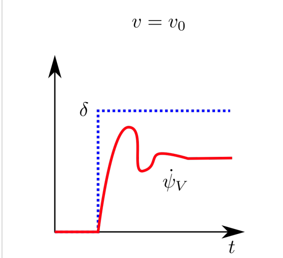
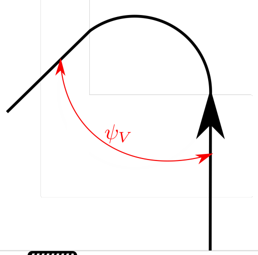
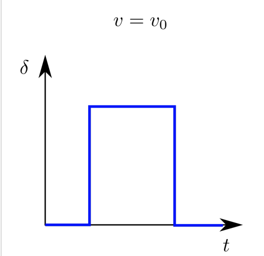
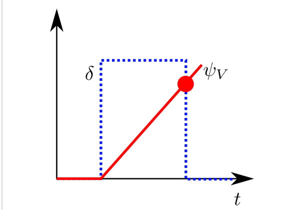
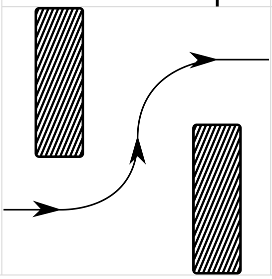
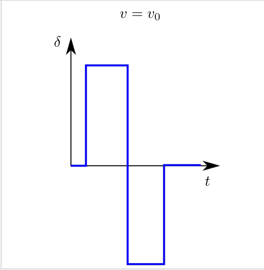
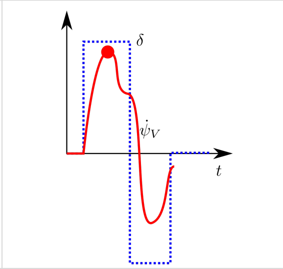
In order to obtain the magic formula parameters required by the single track model, TTC data was analyzed and fitted through the help of Optimum Tire. Only tires chosen in tab.1 where analyzed. Optimum Tire greatly facilitated the fitting process and provided a great tool for validating the calculated magic formula model against raw data points. In tab.**??** validation plots are presented for each fitted tire. The calculated models were exported to `.TIR` format to easily save magic formula coefficients.

tyre	Fy -SA	Fx - SR	Mz - SA
1			
2			
3			
4			
5			

Table 3: Tire magic formula validation. Notation of sec-**??** is used.

3.3 Test cases list

Following the implementation of the nonlinear single track model and the calculation of the magic formula tire models, a list of test cases was created (tab.**??**) in order to define different simulation scenarios.

test	event	schema	input	output
T1	acceleration			max speed
T2	skidpad			
T3	autocross / endurance			
T4				

3.4 Results

The chosen

test	variable	unit	a1		a4		c2		c3		c5	
MF temperature		°C	55		60		70		60		60	
rear axle torque		Nm	50	60	50	60	50	60	50	60	50	60
T1	SR		0.016	0.018	0.037	0.043	0.032	0.038	0.027	0.033	0.032	0.038
	x*_V	km/h	27	32.3	29.8	35.6	35.6	42.5	30.7	36.7	35.8	42.7
speed		km/h	30	40	30	40	30	40	30	38	30	40
T2	psi*_V (stabilized)	deg/s	173	175	174	173	175	177	174	176	181	176
T3	psi_V	deg	109	136	118	141	107	142	113	140	114	140
T4	psi*_V (max)	deg/s	200	227	250	262	250	320	274	330	215	267

Table 4: Caption

4 Conclusion

4.1 Wheel configuration choice

In order to choose a final wheel configuration, a selection criteria was needed. The idea of summing all of the speed results from the different simulation was followed but a different coefficient was applied to distinguish between 30km/h and 40 km/h runs. Since the TTC data fitted in the magic formula was taken at an average speed of 11m/s, a 0.7 coefficient was applied to the 40Km7h runs of test T2, T3 and T4.

The table below summarises such a selection criteria and displays 3 different performance levels:

1.

test	a1	a4	c2	c3	c5
T1	59.3	65.4	78.1	67.4	78.5
T2	174.4	173.3	176.4	175.4	177.5
T3	127.9	134.1	131.5	131.9	132.2
T4	218.9	258.4	299	313.2	251.4
	580.5	631.2	685	687.9	639.6

4.2 Future work

A Optimum Tire software feedback

References

[1] Bardini Schramm, Hiller: *Vehicle Dynamics: Modeling and Simulation*. 2018, ISBN 9783662544822.

[2] Hans B. Pacejka and Igo Besselink: *Tire and Vehicle Dynamics*. Elsevier Ltd., third edition, 2012.