Wentworth Institute of Technology

Department of Sciences

Deflection of an Electric Beam

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Contents

1 Theory & Introduction

This investigation aims to measure the deflection of an electron beam as it passes between a set of parallel charged plates. Using this measurement, we will be able to determine the electric field strength between the plates.

Using Newton's 2nd Law, which states that the net force on an object is equal to the product of the object's mass and its acceleration, we can express the force exerted on an object in terms of the electric field strength:

$$[\mathbf{F} = m\mathbf{a}] \text{ and } [\mathbf{F} = -e\mathbf{E}]$$

$$\therefore -e\mathbf{E} = m\mathbf{a}$$

$$\mathbf{a} = -\frac{e}{m}\mathbf{E} \tag{1}$$

Using this equation, if the electric field is uniform, then the electrons will have a constant acceleration given by:

$$\mathbf{a}_y = -\frac{e}{m}\mathbf{E}_y \tag{2}$$

Using the component kinematic equations, we can model the deflection of the electron beam from its initial path:

$$x = x_0 + \mathbf{v}_{0x}t + \frac{1}{2}\mathbf{a}_x t^2 \text{ and } y = y_0 + \mathbf{v}_{0y}t + \frac{1}{2}\mathbf{a}_y t^2$$

In this case, $\mathbf{a}_x = 0$ as the beam is only being accelerated in the y-direction, and $\mathbf{v}_{0y} = 0$ as the initial velocity of the beam is only in the x-direction.

$$\begin{aligned} x &= x_0 + \mathbf{v}_{0x}t + \frac{1}{2}\mathbf{a}_x t^2 \\ x - x_0 &= \mathbf{v}_{0x}t + \frac{1}{2}(0)t^2 \\ \Delta x &= \mathbf{v}_{0x}t \\ \therefore t &= \frac{\Delta x}{\mathbf{v}_{0x}} \end{aligned}$$

$$\begin{aligned} y &= y_0 + \mathbf{v}_{0y}t + \frac{1}{2}\mathbf{a}_y t^2 \\ y - y_0 &= (0)t + \frac{1}{2}\mathbf{a}_y t^2 \\ \therefore \Delta y &= \frac{1}{2}\mathbf{a}_y t^2 \end{aligned}$$

$$t = \frac{\Delta x}{\mathbf{v}_{0x}} \tag{3}$$

$$\Delta y = \frac{1}{2}\mathbf{a}_y t^2 \tag{4}$$

As the movement of the beam follows a parabolic path in the xy plane, we can substitute equation (3) for t in equation (4), which allows us to find a direct relationship with the parabolic path of motion given by $y = Ax^2 + Bx + C$.

$$t = \frac{\Delta x}{\mathbf{v}_{0x}} \text{ and } \Delta y = \frac{1}{2}\mathbf{a}_y t^2$$

$$\Delta y = \frac{1}{2}\mathbf{a}_y \left(\frac{\Delta x}{\mathbf{v}_0 x}\right)^2$$

$$\therefore \Delta y = \frac{1}{2}\frac{\mathbf{a}_y}{\mathbf{v}_x^2}x^2 \tag{5}$$

$$\left[y = Ax^2 + Bx + C\right]$$

$$\Delta y = \frac{1}{2}\frac{\mathbf{a}_y}{\mathbf{v}_x^2}x^2 + 0x + 0$$

$$\therefore A = \frac{1}{2}\frac{\mathbf{a}_y}{\mathbf{v}_x^2} \tag{6}$$

$$A = \frac{1}{2}\frac{\mathbf{a}_y}{\mathbf{v}_x^2}$$
$$2A\mathbf{v}_x^2 = \mathbf{a}_y$$

$$\therefore \mathbf{a}_y = 2A\mathbf{v}_x^2 \tag{7}$$

Since we now know that the vertical acceleration and horizontal velocity are directly related with the quadratic coefficient A, all we need is an accurate value of the initial velocity $\mathbf{v}_x$ in order to compute the approximate vertical acceleration $\mathbf{a}_y$ for each deflecting voltage. We can use the Work-Energy Theorem to compute the initial velocity as shown below:

$$\left[eV_A = \frac{1}{2}m_e\mathbf{v}_x^2\right]$$

$$\therefore \mathbf{v}_x = \sqrt{2\left(\frac{e}{m_e}\right)V_A} \tag{8}$$

Finally, in order to express the separation of plates d we can use the definition of a uniform electric field, which states that the electric field strength E is equal to the amount of potential difference (voltage) V per unit length between the plates d. Moreover, as only the deflection voltage influences the electric field, we can rewrite the equation to be as follows:

$$\left[E = \frac{V}{d}\right]$$

$$E = \frac{V_D}{d} \tag{9}$$

Using equations (2), (7), (8), and (9), we can determine the initial velocity of the electron beam $\mathbf{v}_x$, an approximate equation for the path of the beam of the form $y = Ax^2 + Bx + c$, the vertical acceleration produced by the electric field $\mathbf{a}_y$, the electric field strength E_y, and the separation between the charged plates d.

2 Materials, Apparatus, & Procedure

In order to conduct this experiment, we will be using a Tel-Atomic Deflection Tube which is constructed to have a *"highly evacuated electron tube with focusing electron gun and fluorescent screen inclined relative to the beam axis"*[1]. Moreover, there will be two sources of high voltage: one connected between the filament and the electron gun in order to accelerate the electrons (accelerating voltage V_A), and the other connected to the deflection plates in order to create an approximately uniform electric field (deflecting voltage V_D). An illustration of the apparatus is provided below.[2]

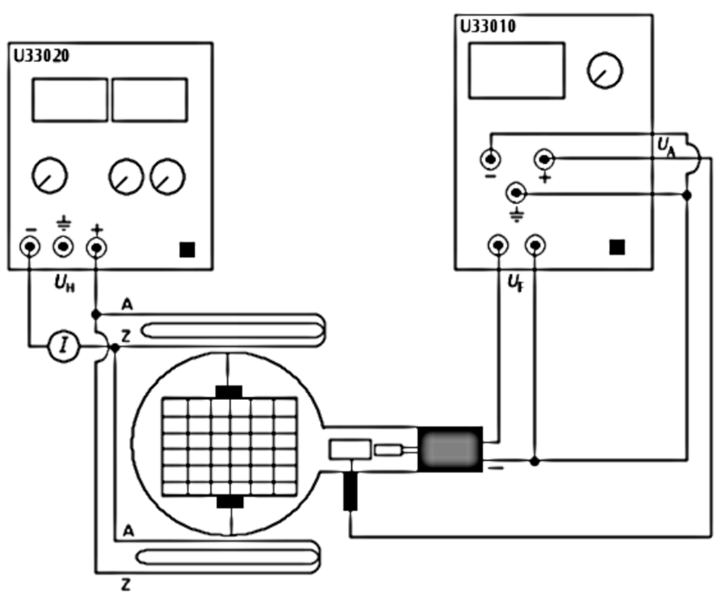

Figure 1: *Illustration of apparatus*

[1] "Deflection e/m Tube." TEL-Atomic Incorporated.
[2] Diamond, Josh, and John Cummings. Modern Physics Laboratory e/m with Teltron Deflection Tube. 2010.

Materials Used

- Tel-Atomic Deflection Tube
- Three sources of voltage (one low and two high)
- Wires
- Camera

Procedure

1. The filament must be supplied with a potential difference of 6.3 volts AC. Electrical connections to the filament are made at the plastic cap at the end of the neck of the tube where there are two banana plug sockets. This is also connected to the negative terminal of the accelerating voltage power supply.
2. A source of high voltage must be connected between the filament and the anode of the electron gun to accelerate the electrons. The terminal for connection to the anode of the electron gun is located on the side of the neck of the tube. This point is connected to ground and also to the Center Tap of the power supply, not the positive terminal, so we are only using half the possible range for the accelerating voltage.
3. A source of high voltage must be connected to the deflection plates. This is connected to the +/- terminals of the deflection voltage power supply to use the full range.
4. Set the adjustable control on each of the power supplies to its minimum setting and plug each of the supplies into an AC outlet.
5. Turn on the accelerating voltage power supply only. You should be able to see light coming from the filament.
6. Adjust the accelerating voltage to 1000 volts on the 2500-volt scale. You should now see a blue beam.
7. Turn on the deflection voltage power supply and set the deflecting voltage to 1000 volts on the 5000-volt scale.
8. Measure the y-coordinates of the deflected beam.
9. Repeat your measurement for four more values of the deflecting voltage (1250 V, 1500 V, 1750 V, and 2000 V), keeping the accelerating voltage at 1000 volts the whole time.

3 Data Analysis Results

In order to find the electric field strength and the separation of plates, I first needed to obtain a quadratic regression for the parabolic path of the electron beam. I did this by taking a picture of what the path appeared like for each value of the deflection voltage V_D and uploading each image into Logger Pro, where the program can analyze the image and generate a regression. After doing the same process for each deflection voltage, I was able to obtain the following data with accompanying graphs:

Raw Data

Trial	V_D	Retrieved Expression of Path of Beam	Correlation Coefficient
1	1000 V	$3.066x^2 - 0.02055x - 0.0001524$	0.9999
2	1250 V	$4.318x^2 - 0.03735x + 0.0004948$	0.9999
3	1500 V	$6.439x^2 + 0.007229x + 0.001338$	0.9999
4	1750 V	$7.528x^2 - 0.02255x + 0.001280$	0.9997
5	2000 V	$7.958x^2 - 0.02284 + 0.0007083$	0.9998

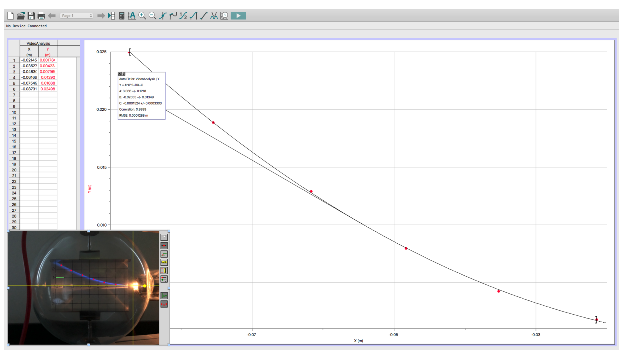

Figure 2: *Graph of the deflection (y) versus the distance (x) for a deflection voltage of 1000 V.*

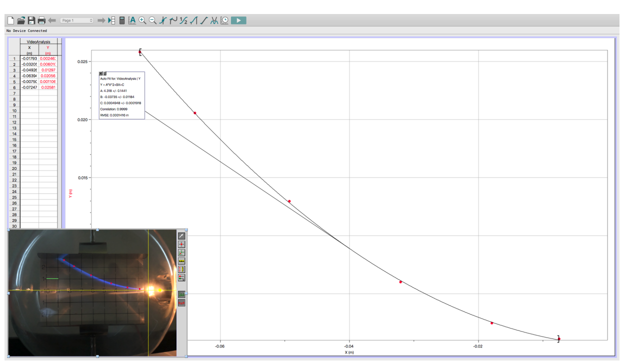

Figure 3: *Graph of the deflection (y) versus the distance (x) for a deflection voltage of 1250 V.*

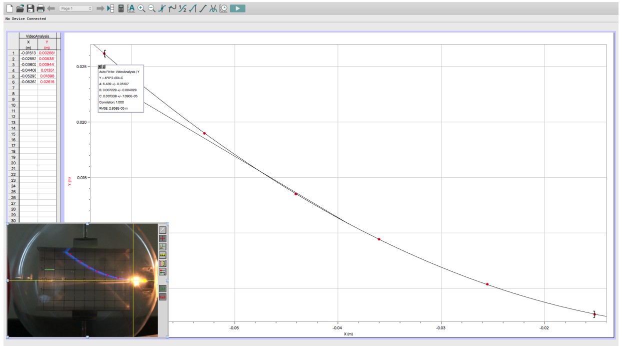

Figure 4: *Graph of the deflection (y) versus the distance (x) for a deflection voltage of 1500 V.*

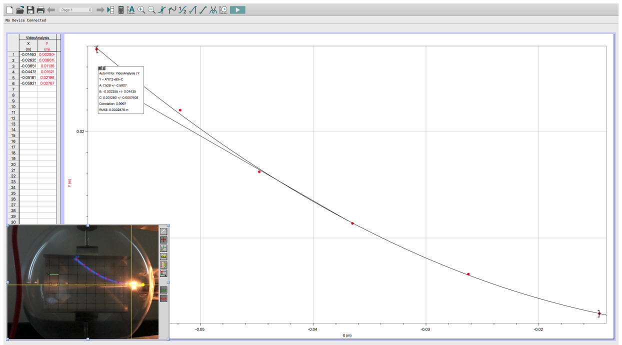

Figure 5: *Graph of the deflection (y) versus the distance (x) for a deflection voltage of 1750 V.*

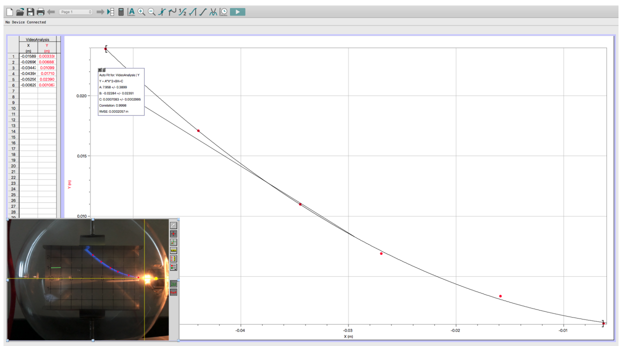

Figure 6: *Graph of the deflection (y) versus the distance (x) for a deflection voltage of 2000 V.*

As shown in equation (7), we can use the quadratic coefficient A in each of the above expressions to compute an approximate value of the vertical acceleration $\mathbf{a}_y$ experienced by the electron beam. However, before we can calculate this value, we first need to know the approximate initial velocity of the beam, which can be found by using equation (8), as shown below:

$$\left[\mathbf{v}_x = \sqrt{2\left(\frac{e}{m}\right)V_A}\right]$$

$$\mathbf{v}_x = \sqrt{2\left(1.759 \times 10^{11}\text{C kg}^{-1}\right)(1000\text{V})}$$
$$\mathbf{v}_x \approx 18,756,332.2640648\text{m s}^{-1}$$

$$\boxed{\mathbf{v}_x \approx 1.88 \times 10^{7}\text{m s}^{-1}}$$

Now that we have a value for the initial velocity of the beam, we can utilize equation (7) to obtain an approximate value for the vertical acceleration due to the electric field $\mathbf{a}_y$ for each value of the deflection voltage V_D, as follows:

Values of Vertical Acceleration $\mathbf{a}_y = 2A\mathbf{v}_x^2$

Deflection Voltage V_D	Value of A	$\mathbf{a}_y = 2A\mathbf{v}_x^2$	$\mathbf{a}_y$
1000 V	3.066	$\mathbf{a}_y = 2(3.066)(1.88 \times 10^7)^2$	$2.17 \times 10^{15}\text{m s}^{-2}$
1250 V	4.318	$\mathbf{a}_y = 2(4.318)(1.88 \times 10^7)^2$	$3.05 \times 10^{15}\text{m s}^{-2}$
1500 V	6.439	$\mathbf{a}_y = 2(6.439)(1.88 \times 10^7)^2$	$4.55 \times 10^{15}\text{m s}^{-2}$
1750 V	7.528	$\mathbf{a}_y = 2(7.528)(1.88 \times 10^7)^2$	$5.32 \times 10^{15}\text{m s}^{-2}$
2000 V	7.958	$\mathbf{a}_y = 2(7.958)(1.88 \times 10^7)^2$	$5.63 \times 10^{15}\text{m s}^{-2}$

Now that we have the precise values of the vertical acceleration of the beam, we can calculate the electric field strength using equation (2), as shown below:

$$\left[\mathbf{a}_y = -\frac{e}{m}\mathbf{E}\right]$$

$$\therefore \mathbf{E} = -\frac{m}{e}\mathbf{a}_y \tag{10}$$

Electric Field Strength for Varying Values of Acceleration

V_D	$\mathbf{a}_y$	$\mathbf{E} = -\frac{m}{e}\mathbf{a}_y$	$\mathbf{E}$
1000 V	2.17×10^{15}m s^{-2}	$\mathbf{E} = -(5.69 \times 10^{-12}$kg s^{-1} A^{-1})($2.17 \times 10^{15}$m s^{-2})	-1.25×10^{4}N C^{-1}
1250 V	3.05×10^{15}m s^{-2}	$\mathbf{E} = -(5.69 \times 10^{-12}$kg s^{-1} A^{-1})($3.05 \times 10^{15}$m s^{-2})	-1.74×10^{4}N C^{-1}
1500 V	4.55×10^{15}m s^{-2}	$\mathbf{E} = -(5.69 \times 10^{-12}$kg s^{-1} A^{-1})($4.55 \times 10^{15}$m s^{-2})	-2.59×10^{4}N C^{-1}
1750 V	5.32×10^{15}m s^{-2}	$\mathbf{E} = -(5.69 \times 10^{-12}$kg s^{-1} A^{-1})($5.32 \times 10^{15}$m s^{-2})	-3.03×10^{4}N C^{-1}
2000 V	5.63×10^{15}m s^{-2}	$\mathbf{E} = -(5.69 \times 10^{-12}$kg s^{-1} A^{-1})($5.63 \times 10^{15}$m s^{-2})	-3.20×10^{4}N C^{-1}

Using these values of the acceleration of the beam $\mathbf{a}_y$ and electric field strength $\mathbf{E}$, we can graphically compare each to the initial value of the deflection voltage V_D for each trial:

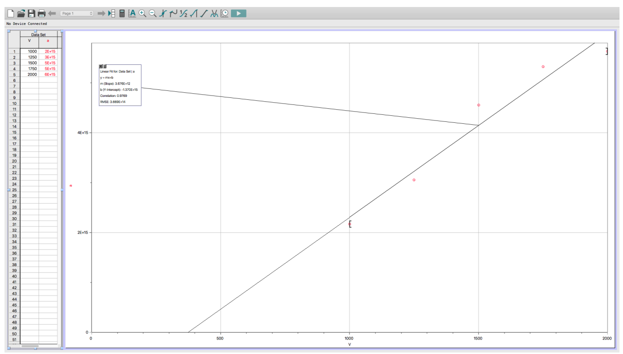

Figure 7: *Graph of deflection voltage V_D versus vertical acceleration $\mathbf{a}_y$*

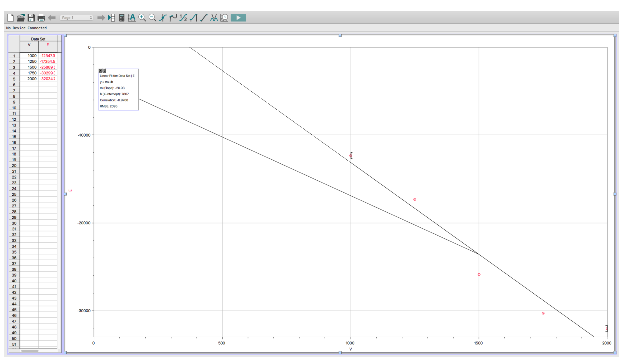

Figure 8: *Graph of deflection voltage V_D versus electric field strength $\mathbf{E}$*

Based on Figure 8, we can utilize the slope of the regression line to approximately figure out what the plate separation d is, using equation (9), as follows:

$$\left[E = \frac{V_D}{d}\right]$$

$$\therefore d = \frac{V_D}{E} = \frac{1}{m}, \text{ where } m \text{ is the slope of the regression line.} \tag{11}$$

$$d = \left|\frac{1}{-20.93}\right|$$

$$\boxed{d = 0.0478 \text{ m} = 4.78 \text{ cm}}$$

Based on these calculations, it is shown that by using the acceleration of the beam $\mathbf{a}_y$, the initial velocity of the beam $\mathbf{v}_x$, the electric field strength E, and the acceleration and deflection voltages V_A and V_D, the approximate value for the separation of charged plates is $d \approx 4.78$cm.

4 Conclusion/Discussion & Error Analysis

In an ideal environment, this experiment would have worked perfectly without any error nor mistake during the process. However, the real world prohibits us from conducting an absolutely perfect experiment as many factors and parts of the experiment can falter. For example, in order to calculate the acceleration due to the electric field $\mathbf{a}_y$, we needed to find a quadratic regression for the parabolic path of the electron beam. For this, the best method we had of finding the curve of best fit was to take a picture of the curved, blue light emitted from the beam and import it into Logger Pro, where the computation would take place. In this scenario, there is an abundance of systematic error, or errors and miscalculations by the scientist and/or the equipment. In addition to this, we needed to plot points on each deflection path to tell Logger Pro where to reference the regression. Both of these acts can be inaccurate as we are using both the camera lens and our eyes to accurately measure something. Due to these errors, the approximation error of the real separation and the acquired separation is:

$$\text{Error} = 100 \times \frac{d_{true} - d}{d_{true}} = 100 \times \frac{0.05 \text{ m} - 0.0478 \text{ m}}{0.05 \text{ m}}$$

$$\text{Error} \approx 4.44\% \tag{12}$$

If I were to execute this experiment again I would do more trials with different values of the deflection voltage, each with smaller increments between them. This, when plotted, would generate regressions and graphs that are as close of an approximation as humanly possible.

References

"Deflection e/m Tube." TEL-Atomic Incorporated,
www.telatomic.com/all-produts/deflection-em-tube.

Diamond, Josh, and John Cummings. Modern Physics Laboratory e/m with Teltron Deflection Tube. 2010, www.sos.siena.edu/ jcummings/teaching/modernlab/instr/eom.pdf.