1-

a) For a Poisson distribution $\lambda = 4$ requests per second, the probability of 6 requests in a second is:

$$P(X=6) = \frac{e^{-4} \cdot 4^6}{6!} = \frac{0{,}0183 \cdot 4096}{720} = 0.1042$$

b) For a 0.5 - second interval, adjusts $\lambda = 4 \times 0.5 = 2$

The probability of at least one requests is:

$$P(X \geq 1) = 1 - P(X=0) = 1 - e^{-2} \cong 1 - 0{,}1353 \cong 0{,}8647$$

2- For negative binomial distribution with $p = 0{,}7$ and $r = 4$ success, and $k = 6$ trials, the probability that the 4th success occurs on 6th trial is

$$p = \binom{5}{3} \cdot 0{,}7^4 \cdot 0{,}3^2 = 0{,}2161$$

3- For $X \sim$ Geometric $(p = 0{,}25)$:

a) The probability that $X > 3$ is:

$$P(X>3) = (1-p)^3 = 0{,}75^3 = 0{,}4219$$

b) The expected value and variance are:

$$E(X) = \frac{1}{p} = 4 \qquad Var(X) = \frac{1-p}{p^2} = 12$$

4- For an exponential distribution with mean 10 hours ($\lambda = 1/10$)

a) The probability that a component lasts more than 12 seconds is:

$$P(X > 12) = e^{-\lambda \cdot 12} = e^{-1,2} \simeq 0,3012$$

b) The median time to failure, where $P(X < m) = 0.5$ is:

$$m = \frac{-\ln(0,5)}{\lambda} = 6,93 \text{ hours.}$$

5- For $X \sim \text{Uniform}(2,8)$:

a) The PDF is $f(x) = \frac{1}{6}$ for $2 < x < 8$ and CDF is $F(x) = \frac{x-2}{6}$ for $2 < x < 8$

b) The probability that $3 < X < 6$ is:

$$P(3 < X < 6) = \frac{6-3}{8-2} = 0,5$$

c) The expected value and variance are:

$$E(X) = \frac{2+8}{2} = 5 \quad Var(X) = \frac{(8-2)^2}{12} = 3$$

6- For $X \sim N(50, 5^2)$:

a) The probability that $45 < X < 55$ is:

$$P\left(\frac{45-50}{5} < Z < \frac{55-50}{5}\right)$$

$$= P(-1 < Z < 1) \approx 0,6826$$

b) The value x such that $P(X < x) = 0,90$ is:

$Z = 1,2816$

so,

$x = 50 + 5 \cdot 1,2816 = 56,41$

7- a) Mean $= \frac{118}{7} \approx 17$ Median $= 17$

Mode = All of them is unique

b) Variance $= s^2 = \frac{\Sigma(x_i - 17)^2}{6} = \frac{56}{6} = 9.33$

Standard deviation $= 3,06$

c) $IQR = 20 - 15 = 5$ Fences $= [7,5 , 27,5]$

No outliers.

8- a) Method of moments $= E(x) = \frac{1}{\lambda} = \bar{x}$ so $\hat{\lambda} = \frac{1}{\bar{x}}$.

b) MLE = Maximize $L(\lambda) = \lambda^{n} e^{-\lambda \sum x_i}$ yielding $\hat{\lambda} = \frac{n}{\sum x_i} = \frac{1}{\bar{x}}$

c) $\bar{x} = 2$ so $\hat{\lambda} = 0{,}5$

$\hookrightarrow \frac{8}{2} = \frac{\text{Sum}}{n}$

9- $n = 16$ $\bar{x} = 13{,}4$ $s = 2{,}1$ %95 CI

$t_{15,0{,}025} \approx 2{,}131$

$13{,}4 \pm 2{,}131 \cdot \frac{2{,}1}{\sqrt{16}} = (12{,}28,\ 14{,}52)$

10- $n = 120$, 86 prefer online, $\hat{p} = \frac{86}{120} \approx 0.717$;

a) 95% CI $\pm 1{,}96 \cdot \sqrt{\frac{0{,}717 \cdot 0{,}283}{120}} = 0{,}717 \pm 0{,}081$

$= (0{,}636,\ 0{,}797)$

b) We are %95 confident the true proportion preferring online learning is between %63,6 and %79,7.

CamScanner ile tarandı

11- Since X and Y are independent:

$f_X(y) f_Y(x)$:

$E[XY] = \sum \sum xy f_X(y) f_Y(x) = \left(\sum x f_X(x)\right)\left(\sum y f_Y(y)\right)$

$= E[X] E[Y]$

12- $X \sim$ Exponential (λ):

$P(X > s+t \mid X > s) = \frac{P(X > s+t)}{P(X > s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}}$

$= e^{-\lambda t} = P(X > t)$

13-
14- Both of the codes are in:
Codes > HW2_13_14.py