1- Probability is a measure of the likelihood of an event occurring, defined mathematically on a probability space. A probability space consists of three components: a sample space Ω (the set of all possible outcomes), a set of events (subsets of Ω) and a probability measure P that assigns to each event a number P(A) such that.

1. $0 \leq P(A) \leq 1$
2. $P(\Omega) = 1$
3. $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=i}^{\infty} P(A_i)$

Equiprable Sample Space: In an equiproble (or equally likely) sample space, all outcomes are assumed to have the same probability. If Ω has n outcomes, the probability of an event A is:

$$P(A) = \frac{\text{Number of total outcomes of } A}{\text{Total number of outcomes of } \Omega} = \frac{|A|}{n}$$

Non-Equiproble Sample Space = In a non-equiprable sample space, outcomes have different probabilities, and the probability measure P is based on a given distribution. For a discrete sample space, $P(A) = \sum_{s \in A} P(s)$, where P(s) is the probability of outcome s and $\sum_{s \in \Omega} P(s) = 1$.

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2- If direction proof: Suppose $X_1, X_2 \dots X_n$ are mutually independent. By definition, the probability of any joint event $P(X_1 \in A_1, X_2 \in A_2, \dots, X_n \in A_n) = P(X_1 \in A_1) P(X_2 \in A_2) \dots P(X_n \in A_n)$. For continuous variables, joint density $f(x_1, x_2 \dots x_n)$ must satisfy this for all infinitesimal regions, implying:

$$f(x_1, x_2, \dots x_n) = f_1(x_1) f_2(x_2) \dots f_n(x_n)$$

Only if Direction Proof: Suppose $f(x_1, x_2 \dots x_n) = f_1(x_1) f_2(x_2) \dots f_n(x_n)$. To show independence, compute the probability of a joint event:

- Continuous case: $P(X_1 \in A_1, \dots, X_n \in A_n) =$

$$\int_{A_1} \dots \int_{A_n} f(x_1, \dots, x_n) \, dx_1 \dots dx_n =$$

$$\int_{A_1} f_1(x_1) \, dx_1 \dots \int_{A_n} f_n(x_n) \, dx_n = P(X_1 \in A_1) \dots P(X_n \in A_n)$$

- Discrete case: similar factorization holds for summation over probabilities. This satisfies the definition of mutual independence.

Thus, the condition is both necessary and sufficient.

3-

a) $P(\text{Bribe} \mid \text{Stay}) = \dfrac{P(\text{Bribe}) \cdot P(\text{Stay} \mid \text{Bribe})}{P(\text{Stay})}$

$P(\text{Bribe}) = \dfrac{1}{5}$

$P(\text{Stay} \mid \text{Bribe}) = 1$

$P(\text{Stay}) = \dfrac{1}{5} \cdot 1 + \dfrac{4}{5} \cdot \dfrac{1}{3} = \dfrac{7}{15}$

$\Rightarrow \dfrac{\frac{1}{5} \cdot 1}{\frac{7}{15}} = \boxed{\dfrac{1}{\cancel{5}} \cdot \dfrac{\cancel{15}^{3}}{7} = \dfrac{3}{7}}$

b) $P(\text{Stay Both}) = P(\text{Stay both} \mid \text{Bribe}) \cdot P(\text{Bribe}) + P(\text{Stay both} \mid \text{No Bribe}) \cdot P(\text{No Bribe})$

$\boxed{\Rightarrow 1 \cdot 1 \cdot \dfrac{1}{5} + \dfrac{1}{3} \cdot \dfrac{1}{3} \cdot \dfrac{4}{5} = \dfrac{13}{45}}$

c) $P(\text{No 2nd. round} \mid \text{Stay in 1st. round}) =$

$1 - P(\text{Stay in 2nd round} \mid \text{Stay in 1st round})$

$\Rightarrow P(\text{Stay in 2nd round} \mid \text{Stay in 1st round}) = \dfrac{P(\text{Stay Both})}{P(\text{Stay 1st round})}$

$= \dfrac{\frac{13}{45}}{\frac{7}{15}} = \dfrac{13}{21} \Rightarrow \boxed{1 - \dfrac{13}{21} = \dfrac{8}{21}}$

4- P (Affectuee | Positive) = P(A | Poz)

$$P(A \mid Poz) = \frac{P(A) \cdot P(Poz \mid A)}{P(Poz)}$$

$P(A) = \frac{1}{500}$. $P(Poz) = \frac{1}{500} \cdot \frac{95}{100} + \frac{499}{500} \cdot \frac{1}{100}$

$= 0,01188$

$P(Poz \mid A) = \frac{95}{100}$.

$$\boxed{P(A \mid Poz) = \frac{\frac{1}{500} \cdot \frac{95}{100}}{0,01188} = 0,1599}$$

5-

$\Omega = \{HHH, HHT, HTT, TTT, TTH, THH, HTH, THT)$

HHH → # H = 3 , # T = 0 w = 3

HHT, THH, HTH → # H = 2 , # T = 1 w = 1

TTH, THT, HTT → # H = 1 , # T = 2 w = -1

TTT → # H = 0 , # T = 3 w = -3

$W = \{3, 1, -1, -3\}$

6-

a) $\int_{-\infty}^{\infty} k y^4(-y^3+3y^2-3y+1) = 1$

$\Rightarrow k\int -y^7+3y^6-3y^5+y^4) = 1$

$\Rightarrow k\left(-\frac{y^8}{8}+\frac{3y^7}{7}-\frac{3y^6}{6}+\frac{y^5}{5}\right)\Big|_0^1 = 1$

$\Rightarrow k\left(-\frac{1}{8}+\frac{3}{7}-\frac{3}{6}+\frac{1}{5}\right) = 1 \Rightarrow k \cdot \frac{176-175}{280}$

$\frac{k}{280} = 1$ $\boxed{k = 280}$

b) $P(0 < y \leq 0{,}5) = \int_0^{0{,}5} 280\, y^4 (1-y)^3\, dy$

$\Rightarrow 280\left(-\frac{y^8}{8}+\frac{3y^7}{7}-\frac{3y^6}{6}+\frac{y^5}{5}\right)\Big|_0^1 = \boxed{0{,}3632}$

c) $280\left(-\frac{y^8}{8}+\frac{3y^7}{7}-\frac{3y^6}{6}+\frac{y^5}{5}\right)\Big|_0^{0{,}8} = \boxed{0{,}0563}$