Q1

a)

$$\lim_{n\to\infty} \frac{(n^2-3n)^2}{5n^3+n} = \frac{\infty}{\infty}$$

$$\xrightarrow{L'hospital} \lim_{n\to\infty} \frac{2(n^2-3n).(2n-3)}{15n^2+1} = \frac{4n^3-18n^2-18n}{15n^2+1} = \frac{\infty}{\infty}$$

$$\xrightarrow{L'hospital} \frac{12n^2-36n-18}{30n} = \frac{\infty}{\infty}$$

$$\xrightarrow{L'hospital} \frac{24n-36}{30} = \infty$$

Therefore $f(n) \in \Omega(g(n))$

b)

$$\lim_{n\to\infty} \frac{n^3}{\log_2 n^4} = \frac{n^3}{4\log_2 n} = \frac{\infty}{\infty}$$

$$\xrightarrow{L'hospital} \lim_{n\to\infty} \frac{3n^2}{4.\frac{1}{n.\ln 2}} = \frac{3}{4}\lim_{n\to\infty} n^3 \ln_2 = \infty$$

Therefore $f(n) \in \Omega(g(n))$

c)

$$\lim_{n\to\infty} \frac{5n\log_2(4n)}{n\log_2(5^n)} = \frac{5\log_2(4n)}{\log(5^n)} = \frac{5\log_2(4n)}{n\log(5)} =$$

$$\frac{5}{\log(5)} \lim_{n\to\infty} \frac{\log_2(4n)}{n} = \frac{\log_2 4 + \log_2 n}{n} = \frac{2+\log_2 n}{n}$$

$$= \frac{\infty}{\infty}$$

$$\xrightarrow{L'hospital} \frac{5}{\log(5)} \lim_{n\to\infty} \frac{\frac{1}{n.\ln 2}}{1} = \frac{1}{n.\ln 2} = \frac{1}{\infty} = 0$$

Therefore $f(n) = O(g(n))$

d) $\lim_{n\to\infty} \frac{n^n}{10^n} = \lim_{n\to\infty} \left(\frac{n}{10}\right)^n = \lim_{n\to\infty} \infty^\infty = \infty$

Therefore $f(n) \in \Omega(g(n))$

e)

$\lim_{n\to\infty} \frac{8n\sqrt[5]{2n}}{n\sqrt[3]{n}} = \lim_{n\to\infty} \frac{8(2n)^{1/5}}{n^{1/3}} = 8 \cdot 2^{1/5} \cdot \lim_{n\to\infty} \frac{n^{1/5}}{n^{1/3}} =$

$8 \cdot 2^{1/5} \cdot \lim_{n\to\infty} \frac{1}{n^{2/15}} = 8 \cdot 2^{1/5} \cdot \frac{1}{\infty} = 0$

Therefore $f(n) \in O(g(n))$

)

a) Q 2

a) This method replaces all elements of array named str_array with an empty string (""). Since there is a single for loop we assume it runs once for each element, Therefore for an array with n element there will be n operations.

b) Therefore time complexity is $\boxed{O(n)}$ in method A.

b) This method first calls method A, then prints each element of the array. Since complexity of method A is $O(n)$ and it will be called n times in for loop so complexity is $O(n^2)$. Second foor loop's complexity is $O(n)$. $O(n) + O(n^2) = O(n^2)$. Therefore complexity is $O(n^2)$ for this method.

c) In this method there are two nested for loops, and the inner loop calls method B, which has $O(n^2)$ complexity. In this case, the complexity for the inner for loop will be $O(n^2 \cdot n)$. Since the outer for loop will run up to n, which is the size of the array, a factor n is added from there and the total complexity becomes $O(n^4)$.

d) This method is incorrect because while integer i is increased by one by the for loop, it is decreased by one within the loop. In this case, the loop will continue forever, complexity cannot be accounted.

e) In this method, in the worst case scenario, the empty string is at the end of the array or there is no empty string in the array. In this case the loop runs up to size of the array and complexity is $O(n)$.

Q3

a) ALGORITHM max_difference_sorted (A)

```
BEGIN
    n = length (A)
    max_diff = A[n-1] - A[0]
    RETURN max_diff
END
```

b) ALGORITHM max_difference_unsorted (A)

```
BEGIN
    n = length (A)
    min_val = A[0]
    max_val = A[0]
    FOR i FROM 1 TO n-1
        IF A[i] < min_val THEN
            min_val = A[i]
        ELSE
            max_val = A[i]
        ENDIF
    ENDFOR
    max_diff = max_val - min_val
    RETURN max_diff
```

Algorithm A only accesses two elements and substracts one, so complexity is $O(1)$

Algorithm B checks all the elements of array one by one so complexity is $O(n)$.