Learning how structures form in drift-wave turbulence

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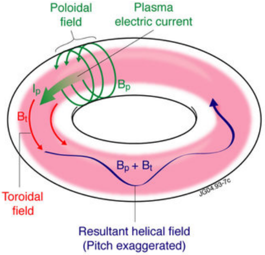
Introduction

Brief outline

- This talk is on some old stuff (ca. 2020) that I recently started revisiting (with O. Gurcan and G. Dif-Pradalier)
- Mostly will discuss methods and results from Heinonen and Diamond, *PRE* 101, 061201(R) (2020) and *PPCF* 62, 105017 (2020)
- Unlike most of the ML techniques discussed here, this work is strictly offline/a priori—BOUT++ used as data generation tool
- Goal is interpretable reduced modeling, beyond what is analytically tractable
- Final couple slides will outline how I'm trying to extend and improve on the earlier work (no results yet, just general ideas)

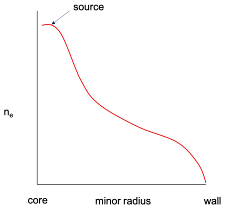
Tokamak physics basics

- Toroidal fusion device that uses strong helical magnetic field to confine plasma
- Key challenge: $\langle n \rangle \langle T \rangle \tau_E > 10^{21}\ \mathrm{keV\ s/m^3}$ (Lawson criterion) $\rightarrow$ maximize confinement time τ_E $\rightarrow$ minimize losses due to transport
- But: n, T gradients $\rightarrow$ instabilities $\rightarrow$ turbulence $\rightarrow$ anomalous transport. How to understand?
- In this work: develop simple, interpretable model for turb. transport informed by machine learning



Drift-wave turbulence I

- Drift wave turbulence is useful paradigm for turbulence due to gradient instabilities (universal)
- Drift wave: collective oscillations associated with ion/electron diamagnetic drifts, which form in response to temperature/density gradients $v_d = 1/(qnB^2)\nabla p \times \mathbf{B}$
- Driven unstable by resonances, collisions. Turbulence results when many drift modes are driven unstable, interaction becomes important



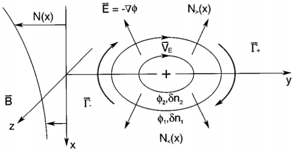


FIG. 1. Drift-wave mechanism showing $\mathbf{E}\times\mathbf{B}$ convection in a nonuniform, magnetized plasma.

Drift-wave turbulence II

- In certain regime, **zonal flows** spontaneously build up via secondary instability. Special drift modes with $m = n = 0$, $\omega \simeq 0$.
- ZFs regulate turbulence via shearing; extremely important to confinement problem.
- DWT features complex interaction between mean profile, ZF, and turbulence
- Of special interest to this talk: how does the flow feed back on the profile?

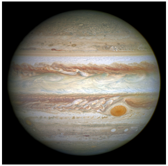


Figure Obligatory image of Jupiter's bands to illustrate ZF

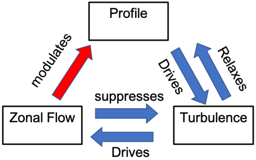


Figure Feedback loop illustrating interaction of mean fields in DW turbulence

Hasegawa-Wakatani

- Simplest realistic framework for understanding collisional drift wave turbulence

$$\frac{dn}{dt} = \alpha(\tilde{\phi} - \tilde{n}) + D\nabla^2 n$$
$$\frac{d\nabla^2\phi}{dt} = \alpha(\tilde{\phi} - \tilde{n}) + \mu\nabla^4\phi,$$
$$\text{where } \frac{d}{dt} \equiv \frac{\partial}{\partial t} + (\hat{z} \times \nabla\phi) \cdot \nabla$$

- Potential serves as stream function for the flow $\mathbf{v} = (\partial_y\phi, -\partial_x\phi)$
- $\alpha \equiv k_\parallel^2 T_e/(n_0\eta\Omega_i e^2)$ “adiabaticity parameter,” measures parallel electron response

$\alpha \to 0$ yields 2-D Navier-Stokes, $\alpha \to \infty$ yields quasigeostrophic equation from atmospheric dynamics

Deep learning approach

Motivation: mean-field Hasegawa-Wakatani

- Want theory for radial transport
- Averaging over symmetry directions ($\langle \cdots \rangle$) yields

$$\partial_t \langle n \rangle + \partial_x \Gamma = \mathrm{dissipation}$$

$$\partial_t \langle \nabla^2 \phi \rangle - \partial_x^2 \Pi = \mathrm{dissipation}$$

$$\partial_t \varepsilon + 2\varepsilon(\Gamma - \partial_x \Pi)(\partial_x \langle n \rangle - \partial_x^3 \langle \phi \rangle) = -\gamma \varepsilon - \gamma_{NL} \varepsilon^2$$

where $\Gamma = \langle \tilde{n} \tilde{v}_x \rangle$ (particle flux) and $\Pi = \langle \tilde{v}_x \tilde{v}_y \rangle$ (poloidal momentum flux or "Reynolds stress").

- $\varepsilon = \langle (\tilde{n} - \nabla^2 \tilde{\phi})^2 \rangle$ is turbulent potential enstrophy. Proxy for turbulence intensity
- Seek 1D mean-field closure: Γ, Π as function of $\langle n \rangle$, $\langle \phi \rangle$, ε, radial derivatives. Idea: use supervised learning. What can we learn?

Feature selection

- Assume a **local** model: local mean fields (in space and time) suffice to specify the local fluxes
- HW invariant under uniform shifts $n \to n + n_0$ and $\phi \to \phi + \phi_0 \implies$ eliminate dependence on $\langle n \rangle, \langle \phi \rangle$
- Invariant under poloidal boosts

$$\begin{cases} \phi & \to \phi + v_0 x \\ y & \to y - v_0 t \end{cases}$$

  $\to$ eliminates dependence on ZF speed $V_y = -\partial_x \langle \phi \rangle$.
- Confine ourselves to adiabatic regime ($\alpha > 1$) so $\tilde{n} \sim \tilde{\phi} \implies \varepsilon$ reasonably suffices to specify intensity
- Anticipate that hyperviscosity necessary to regularize ZF, so need derivatives up to V_y'''

Methods

- Thus choose minimal set of inputs $N', U, U', U'', \varepsilon$ $(N = \langle n \rangle, U = V_y')$
- 32 simulations of 2D HW, with $\alpha = 2$, various initial conditions for mean density, flow
- Smooth and compute inputs, Γ, Π. Locality means each point in space, time treated on equal footing $\rightarrow$ lots of data per simulation
- Fit MLP to output fluxes as functions of inputs. Use logcosh loss instead of MSE due to large fluctuations
- Exploit/enforce 3 reflection symmetries via data augmentation

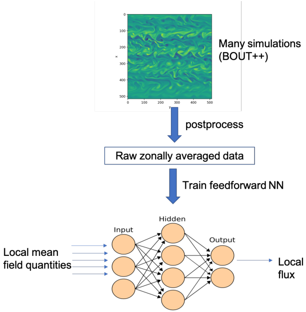


Figure Schematic of deep learning method

Results

Particle flux

DNN learns a model roughly of the form (for small gradients)

$$\Gamma \simeq -D_n \varepsilon N' + D_U \varepsilon U'.$$

Large gradients: fluxes saturate. Diffusive term $\propto N'$ well-known, tends relax driving gradient. Second (non-diffusive) term is not so well-known, driven by vorticity gradient!

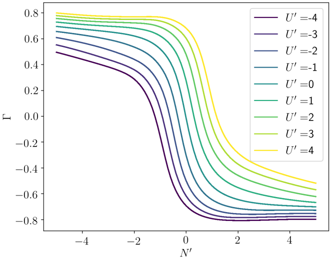


Figure Particle flux at constant ε as function of density and vorticity gradients

Derivation of nondiffusive term

Analytic treatment in $\alpha \to \infty$ limit reproduces nondiffusive term.
Need include frequency shift due to convection of mean vorticity.
In QLT:

$$\omega_{\mathbf{k}} = \frac{k_y}{1+k^2}(N' + U') + O(\alpha^{-2})$$

$$\gamma_{\mathbf{k}} = \frac{k_y^2}{\alpha(1+k^2)^3}(N' + U')(k^2 N' - U') + O(\alpha^{-2})$$

$$\begin{aligned}\Gamma &= \mathrm{Re} \sum_{\mathbf{k}} -ik_y \tilde{n}_{\mathbf{k}} \tilde{\phi}^*_{\mathbf{k}} \\ &= \sum_{\mathbf{k}} \frac{-k_y^2 \partial_x n(\gamma_{\mathbf{k}} + \alpha) + \alpha k_y \omega_{r,\mathbf{k}}}{\omega_{r,\mathbf{k}}^2 + (\gamma_{\mathbf{k}} + \alpha)^2} |\tilde{\phi}_{\mathbf{k}}|^2 \\ &= \frac{1}{\alpha} \sum_{\mathbf{k}} -\frac{k_y^2}{1+k^2}\left(k^2 N' - U'\right) |\tilde{\phi}_{\mathbf{k}}|^2 + O(\alpha^{-2})\end{aligned}$$

Inserting a reasonable ansatz spectrum yields good agreement with DNN result for small gradients

Implications of nondiffusive term

- Neglected in literature, but coupling same order of magnitude (~ 0.5) that of usual N' term. DNN picks it out very clearly!
- Consequence: ZF can induce "staircase" pattern on profile. If $V_y = V_0 \sin(qx)$, U' term will contribute

$$\partial_t \langle n \rangle \sim -\frac{k_y^2 q^3 V_0 \langle \varepsilon \rangle}{\alpha(1+k^2)^3} \cos(qx)$$

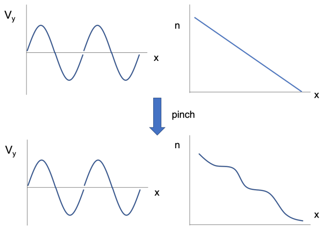


Figure Cartoon indicating how ZF may induce profile staircase via nondiffusive flux/pinch

Reynolds stress

- Learns model of Cahn-Hilliard form (leading order)

$$\Pi = \varepsilon(-\chi_1 U + \chi_3 U^3 - \chi_4 U'')$$

  with $\chi_1, \chi_3, \chi_4 > 0$
- $\partial_t U = \partial_x^2 \Pi \sim \chi_1 \varepsilon k^2 U$. Zonal flow generation by *negative viscosity* $\varepsilon\chi_1$
- Large U stabilized by nonlinearity $\propto U^3$, small scales by hyperviscosity χ_4
- Power law decay of Π with U at large U agrees with wave kinetic calculation

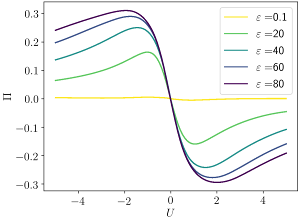


Figure Reynolds stress as function of U, at fixed U', U''

Reynolds stress: gradient corrections

- How does Reynolds stress depend on N', U'? Not easy to calculate
- Learned dependence well-described by overall suppression factor $f \simeq 1/(1 + 0.04(N' + 4U')^2)$, i.e. gradients generally reduce Reynolds stress
- Found to be crucial for stability of learned model. Kinks tend to form in flow in its absence

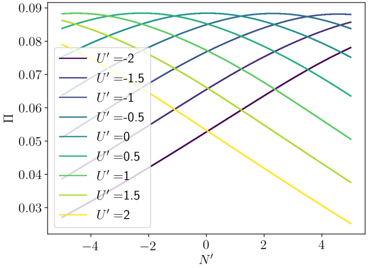
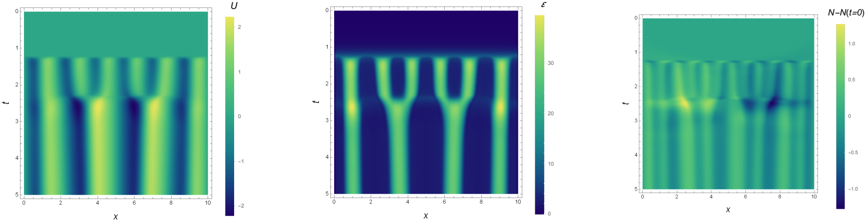


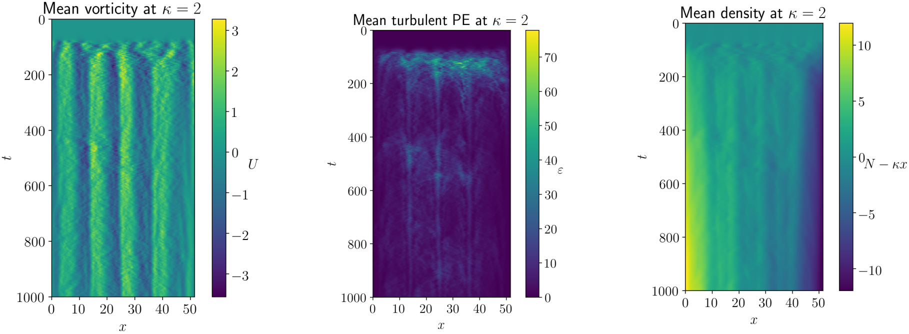
Figure Reynolds stress dependence on gradients at fixed ε, U, U''

Reduced 1-D model

Now have 3 coupled, **one-dimensional** mean field equations describing nonlinear turbulent dynamics. Construct expressions for Γ, Π capturing NN behavior, and numerically solve



Compare to zonally averaged 2D DNS



1D resembles simplified version of DNS. One key difference: 3-field model equivalent to taking stationary “best-fit” spectrum. Some system memory lost

Towards bigger and better things

Weaknesses of previous work

- Constrained ourself to fixed $\alpha = 2$: "easy" regime with robust ZFs. Much more interesting to ask what happens as $\alpha \to 1$
- Extracted "mean" fluxes for given parameters, but signal-to-noise ratio seemed quite low esp. for Reynolds stress. "Aleatoric uncertainty"
- Symbolic regression, feature extraction all done by hand. Should automate in principle
- Couldn't make sense of turbulent spreading: failed to find a good model for triplet term $\langle \tilde{v}_x \tilde{\varepsilon} \rangle$

Ideas for present and future I

- α is now free parameter. Need a lot more data (active learning?)
- For $\alpha \lesssim 1$, $\tilde{n}$ and $\tilde{\phi}$ begin to decouple $\implies$ need more variables for fluctuations. Leads us to a model of the form

$$\partial_t N + \partial_x \Gamma = 0$$
$$\partial_t U + \partial_x^2 \Pi = 0$$
$$\partial_t \xi_i = f_i(N', N'', \ldots, U, U', \ldots, \xi_1, \ldots, \xi_n, {\xi_1}', \ldots, {\xi_n}', \ldots)$$

for some choice of fluctuation variables $\{\xi_i\}_{i=1}^{n}$ (turb. kinetic energy, turb. potential enstrophy, $\langle \tilde{n}^2 \rangle$, etc.). Need to learn Γ, Π, f_i

- The choice of ξ_i and other variables should be at least partially automated to maximize information about dynamics

Ideas for present and future II

- Can we learn probability distributions for the fluxes and f_i instead of deterministic functions? I.e. learn a Langevin equation. “Bayesian deep learning” (assume Gaussian, choose loss in such a way to learn mean and std dev) is one option, but fat tails are an issue
- Is SiNDy the answer? Advances in the last couple years give us more hope for turbulent systems with lots of noise

Extra slides

Comparison to theory (diffusive term)

Compare DNN result to theory result using spectrum centered at most unstable **k** for $U' = 0$

$$\varepsilon_{\mathbf{k}} = \frac{\langle \varepsilon \rangle}{2\pi^2 \Delta k_x \Delta k_y} \frac{1}{1 + k_x^2/\Delta k_x^2} \left(\frac{1}{1 + (k_y - \sqrt{2})^2/\Delta k_y^2} + \frac{1}{1 + (k_y + \sqrt{2})^2/\Delta k_y^2} \right)$$

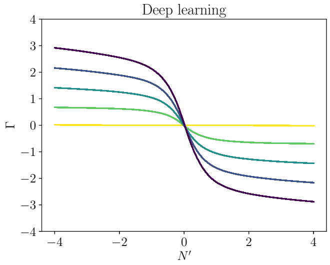


Figure Curves (at fixed $U = U' = U'' = 0$, and various ε) of Γ vs density gradient from DNN

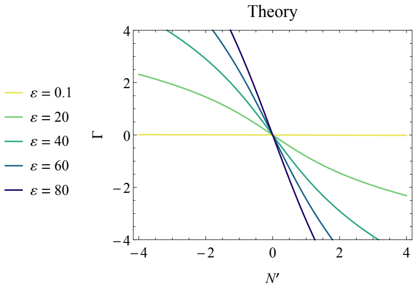


Figure Corresponding curves from QLT+ansatz with $\Delta k_x = \Delta k_y = 0.8$

Comparison to theory (nondiffusive term)

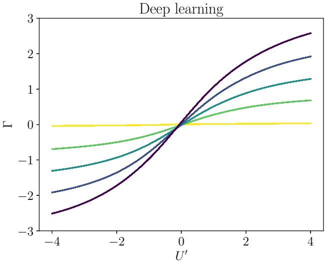


Figure Curves (at fixed $N' = U = U'' = 0$, and various ε) of Γ vs U' from DNN

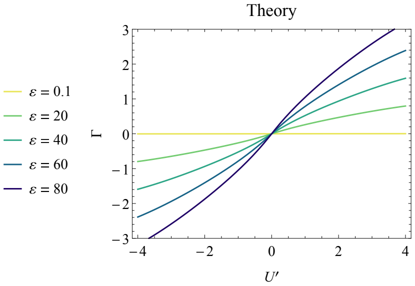


Figure Corresponding curves from QLT+ansatz with $\Delta k_x = \Delta k_y = 0.8$

Good agreement when N', U' are small!

Reynolds stress: intensity scaling

- Whereas learned Γ is essentially $\propto \varepsilon$, Π scaling with ε is nontrivial
- Learned exponent is 1 for small intensity, close to zero for large intensity
- Jibes with intuition from strong turbulence theory

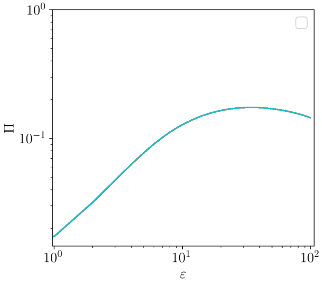


Figure Reynolds stress dependence on gradients at fixed ε, U, U''

Reynolds stress: hyperviscosity

Hyperviscous term, crucial for stability, has small coefficient.
Sensitive test of method

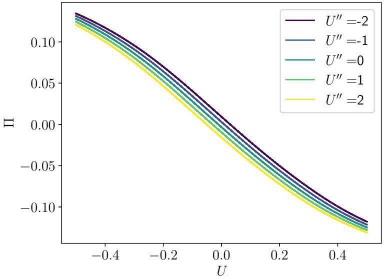


Figure U'' level curves of Reynolds stress as function of U, at fixed ε, U', N'

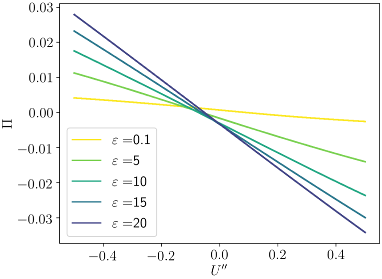


Figure ε level curves of Reynolds stress as function of U'', at fixed U, U', N''

Error quantification

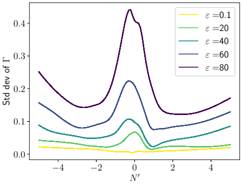


Figure Plot of the standard deviation among the ensemble of DNN models for the diffusive/diagonal part of the particle flux ($U = U' = U'' = 0$).

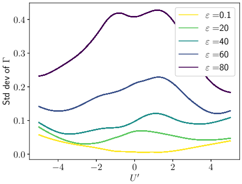


Figure Plot of the standard deviation among the ensemble of DNN models for the nondiffusive/off-diagonal part of the particle flux ($U = U'' = N' = 0$).

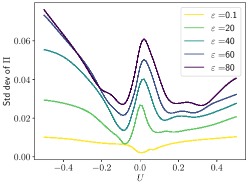


Figure Plot of the standard deviation among the ensemble of DNN models for the Reynolds stress, when $N' = 2$ and $U' = U'' = 0$.

$$L^{-1}(\text{mean validation loss for } \Pi) \simeq 0.09$$

$$L^{-1}(\text{mean validation loss for } \Gamma) \simeq 0.07$$

$$L(x) = \log\cosh x$$

Compare: typically $|\Pi| \leq 0.3$ and $|\Gamma| \leq 3$