Interlocking Test

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Interlocking Test

The original interlocking test can be formulated as:

$$\texttt{Interlocking} \iff \{x | Ax \geq 0\} = \{\mathbf{0}\}$$

Linear Programming

Thanks to Samara, we reformulate the feasibility problem into a linear programming.

$$\begin{aligned} \min_{t,x} & -\sum_{i=1}^{m} t_i \\ \texttt{s.t.} \quad & Ax = t \\ & t \geq 0 \end{aligned} \tag{1}$$

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The challenge here is how to solve this optimization.

Interior Point Method

A simple idea would be using the **interior point method**. However, the interior point method needs a strictly feasible point of Eq 5, which leads to the "chicken and egg" problem. If this strictly feasible point is found in advance, we already can confirm the structure to be non-interlocking without starting the optimization.

Big-M method

When M tends to be infinity, the solution of right optimization is a strictly feasible point of the left optimization. Meanwhile, the right optimization has a nature strictly feasible point:

$$\lambda = 1, x = 0, t = t_0.$$

$$\begin{aligned} \min_{t,x} \; & -\sum_{i=1}^{m} t_i \\ \texttt{s.t.} \quad & Ax = t \\ & t \geq 0 \end{aligned}$$

$$\begin{aligned} \min_{t,x,\lambda} \; & -\sum_{i=1}^{m} t_i + \lambda M \\ \texttt{s.t.} \quad & Ax + \lambda t_0 = t \\ & t, \lambda \geq 0 \\ & t_0 = (1, \cdots, 1)^T \end{aligned}$$

A New Interlocking Test

Further, we find a link between interlocking property and the big-M method. If let $t_{\texttt{sol}}(M)$ be the optimal point of the big-M method with respect to a given M, we have:

$$\texttt{Interlocking} \iff \lim_{M\to\infty} t_{\texttt{sol}}(M) = 0$$

Affine Scaling

For a general linear programming:

$$\begin{aligned} &\min_x \; c^T x \\ \texttt{s.t.} \quad & Ax = 0 \\ & x \geq 0 \end{aligned}$$

The affine scaling iteration is:

(I) Suppose x^0 is a strictly feasible point.

(II) Let X_k be the **diagonal matrix** with x^k on its diagonal.

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(VI) Otherwise, Update $x^{k+1} = x^k - \beta\frac{X_k^2r^k}{\|X_kr^k\|}$