Optique

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Optique géométrique

Snell-Descartes: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

Réflexion totale: $\theta \geq \theta_c = \arcsin\frac{n_2}{n_1}\ (n_1 > n_2)$

Lentilles

$$\frac{1}{f} = (n_l + 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{s_o} + \frac{1}{s_i}$$

$f > 0\ (R > 0) \rightarrow$ convexe

$f < 0\ (R < 0) \rightarrow$ concave

Méthode matricielle

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = M \begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix}$$

Milieu homogène: $M = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$

Interface droit: $M = \begin{pmatrix} 1 & 0 \\ 0 & \frac{n_1}{n_2} \end{pmatrix}$

Interface courbe: $M = \begin{pmatrix} 1 & 0 \\ -\frac{n_2 - n_1}{n_2 R} & \frac{n_1}{n_2} \end{pmatrix}$

Lentille: $M = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$

Réflexion droite: $M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (changement de $\vec{z}$)

Réflexion courbe: $M = \begin{pmatrix} 1 & 0 \\ \frac{2}{R} & 1 \end{pmatrix}$ (changement de $\vec{z}$)

Optique ondulatoire

Équation fondamentale:

$$\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0$$

$$c = \frac{c_0}{n}$$

Fonction d'onde monochromatique harmonique:

$$u(\vec{r}, t) = a(\vec{r}) \cos\left(\omega t + \phi(\vec{r})\right)$$

$$\omega = 2\pi\nu$$

Fonction d'onde complexe:

$$U(\vec{r}, t) = a(\vec{r}) e^{j\phi(\vec{r})} e^{j\omega t} \Rightarrow \nabla^2 U + k^2 U = 0$$

$$k = \frac{\omega}{c} = \frac{\omega n}{c_0} = \frac{2\pi\nu}{c} = \frac{2\pi\nu n}{c_0} = \frac{2\pi}{\lambda}$$

Onde plane:

$$U(\vec{r}) = A e^{-j\vec{k}\cdot\vec{r}} = A e^{-j\left(k_x x + k_y y + k_z z\right)}$$

$$k_x^2 + k_y^2 + k_z^2 = k^2$$

Onde progressive: $U(\vec{r}, t) = A e^{-j\vec{k}\cdot\vec{r} + j\omega t}$

Onde rétrograde: $U(\vec{r}, t) = A e^{j\vec{k}\cdot\vec{r} + j\omega t}$

Onde dans un milieu:

$$c = \frac{c_0}{n}$$

$$\lambda = \frac{\lambda_0}{n}$$

$$k = n k_0$$

Changement de milieu:

$$c_2 = \frac{c_0}{n_2} = c_1 \frac{n_1}{n_2}$$

$$\lambda_2 = \lambda_1 \frac{n_1}{n_2}$$

$$k_2 = n_2 k_0 = k_1 \frac{n_2}{n_1}$$

La fréquence ν et la pulsation ω (temporelles) dépendent de l'énergie $\rightarrow$ constantes

Avec $\tilde{n} = n + jp$:

$$k = n k_0 = k' + jk''$$

$$\Rightarrow U(\vec{r}, t) = A e^{k'' z} e^{-jk' z} e^{j\omega t}$$

$k'' < 0 \rightarrow$ convergent

$k'' > 0 \rightarrow$ divergent

Accumulation de la phase: $\phi(z) = k_1 z_1 + k_2(z_2 - z_1) + k_1(z - z_2) = \frac{2\pi n_1}{\lambda_0} z_1 + \frac{2\pi n_2}{\lambda_0}(z_2 - z_1) + \frac{2\pi n_1}{\lambda_0}(z - z_2)$

Interférences

$U(\vec{r}) = U_1(\vec{r}) + U_2(\vec{r}) = \sqrt{I_1} e^{j\phi_1(\vec{r})} + \sqrt{I_2} e^{j\phi_2(\vec{r})}$

$I = |U|^2 = \left|U_1 + U_2\right|^2 = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\phi_2 - \phi_1)$

Si $I_1 = I_2 = I_0$:

$$I = 2I_0 + 2I_0 \cos\phi = 4I_0 \cos^2\frac{\phi}{2}$$

Avec $U_1 = \sqrt{I_0} e^{-jkz}$ et $U_2 = \sqrt{I_0} e^{-jk(z-d)}$:

$$I = 2I_0\left(1 + \cos 2\pi \frac{d}{\lambda}\right)$$

Avec $U_1 = e^{-jk_0(x\sin\theta + y\cos\theta)}$ et $U_2 = e^{-jk_0(-x\sin\theta + y\cos\theta)}$:

$$U = U_1 + U_2 = 2e^{-jk_0 y\cos\theta}\cos(k_0 x\sin\theta) = 2e^{-jk_y y}\cos k_x x$$

$$T = \frac{\lambda}{2n\sin\theta}$$

Superposition de deux ondes de fréquences différentes:

$$U_1(z,t) = U_0 e^{j\omega_1 t - jk_1 z}, \qquad U_2(z,t) = U_0 e^{j\omega_2 t - jk_2 z}$$

$$U(z,t) = U_0 e^{j\omega_0 t - jk_0 z}\left(e^{j\Delta\omega t - j\Delta k z} + e^{-(j\Delta\omega t - j\Delta k z)}\right) = 2U_0 e^{j\omega_0 t - jk_0 z}\cos(\Delta\omega t - \Delta k z)$$

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} \qquad \Delta\omega = \frac{\omega_2 - \omega_1}{2} \qquad k_0 = \frac{k_1 + k_2}{2} \qquad \Delta k = \frac{k_2 - k_1}{2}$$

<u>Milieux dispersifs</u>

$$c = \frac{\omega}{k} \Rightarrow n = \frac{c_0}{c} = \frac{c_0 k}{\omega}$$

$$\Rightarrow \frac{\partial n}{\partial \omega} = \frac{c_0}{\omega}\left(\frac{\partial k}{\partial \omega} - \frac{k}{\omega}\right) = \frac{c_0}{\omega}\left(\frac{1}{v} - \frac{1}{c}\right) \Rightarrow v = \frac{c_0}{n + \omega \frac{\partial n}{\partial \omega}}$$

<u>Réseau de diffraction</u>

Surface de fréquence spatiale $v_x = \frac{k_x}{2\pi}$ et période Λ:

$$U(x,z) = Ae^{-j(k_x x + k_z z)}$$

$$\sqrt{k_x^2 + k_z^2} = k = \frac{2\pi}{\lambda} = \frac{\omega}{c}$$

$$\sin\theta_q = \sin\theta_i + q\frac{\lambda}{\Lambda}$$

Si $k_x > k = \frac{2\pi}{\lambda}$: $k_z = \pm\sqrt{\frac{4\pi^2}{\lambda^2} - k_x^2} = \pm\sqrt{\frac{\omega^2}{c^2} - k_x^2} = -jk_z'' \Rightarrow U(x,z) = Ae^{-jk_x x}e^{-k_z'' z}$ (onde évanescente)

Redirection par une lentille avec $\sin\theta_x \approx \theta_x, \sin\theta_y \approx \theta_y$:

$$\left(f\theta_x, f\theta_y\right) = \left(\lambda f v_x, \lambda f v_y\right)$$

$$\Rightarrow v_x = \frac{x}{\lambda f}, v_y = \frac{y}{\lambda f} \text{ (à la distance focale)}$$

$$\Rightarrow I = \left(\frac{1}{\lambda f}\right)^2 \left|F\left(\frac{x}{\lambda f}, \frac{y}{\lambda f}\right)\right|^2 \text{ (transformée de Fourier spatiale)}$$

<u>Diffraction</u>

Cas général:

$$U(x,y,z) = \frac{1}{j\lambda}\int_{S'} U(x',y')\frac{e^{-jkr}}{r}\cos\theta\, dx'dy'$$

Approximation $r = R\left(1 - \frac{xx'+yy'}{R^2}\right)$ (Fraunhofer):

$$U(x,y,R) = \frac{e^{-jkR}}{j\lambda R}\cos\theta\int_{S'} U(x',y')e^{jk\frac{xx'+yy'}{R}}\,dx'dy' = \frac{e^{-jkR}}{j\lambda R}\int_{S'} U(x',y')e^{j2\pi\left(p_x x' + p_y y'\right)}dx'dy'$$

Approximation $p_x = \frac{x}{\lambda R} = \frac{\sin\theta_x}{\lambda} \approx \frac{\theta_x}{\lambda}, p_y \approx \frac{\theta_y}{\lambda}$:

$$U(x,y,R) = \frac{e^{-jkR}}{j\lambda R}\mathcal{F}\{U(x',y')\} = \frac{e^{-jkR}}{j\lambda R}U\left(p_x, p_y\right)$$

Diffraction par un réseau de fentes:

$$U(x) = \sum_{q=-\frac{N-1}{2}}^{\frac{N-1}{2}} \mathrm{rect}\frac{x + qb}{a}$$

$$U(p_x) = ae^{-\frac{j\pi b p_x}{2}}\,\mathrm{sinc}\, a\pi p_x\,\mathrm{sinc}\, bN\pi p_x$$

$$I(p_x) = \frac{a^2}{\lambda^2 R^2}\frac{\sin^2 a\pi p_x}{\left(a\pi p_x\right)^2}\frac{\sin^2 bN\pi p_x}{\sin^2 b\pi p_x} \text{ } (N-2 \text{ pics secondaires})$$

Diffraction par une ouverture circulaire:

$$U(\rho) = \mathrm{rect}\frac{\rho}{D}$$

$$U\left(p_\rho\right) = \frac{\pi D^2}{2}\frac{J_1\left(\pi D p_\rho\right)}{\pi D p_\rho}$$

$$I\left(p_\rho\right) = \left(\frac{\pi D^2}{4\lambda R}\right)^2\left(\frac{2J_1\left(\pi D p_\rho\right)}{\pi D p_\rho}\right)^2$$

Premier minimum: $\pi D p_\rho = 3.83 \Rightarrow \sin\theta_\rho = \frac{3.83}{\pi}\frac{\lambda}{D} = 1.22\frac{\lambda}{D}$

Optique de Maxwell

$$\vec{\nabla}^2\vec{E} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \qquad c = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c_0}{\sqrt{\varepsilon_r\mu_r}} \qquad n = \frac{c_0}{c} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0\varepsilon_r\vec{E} = \varepsilon_0(1+\chi)\vec{E}$$

Modèle de Lorentz:

$$\vec{P} = \frac{\omega_p^2}{\omega_0^2 - \omega^2 - j\gamma\omega}\varepsilon_0\vec{E}$$

$$\varepsilon_r = 1 + \chi = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 - j\gamma\omega}$$

Onde plane monochromatique:

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{-j\vec{k}\cdot\vec{r} + j\omega t} \qquad -j\vec{k}\times\vec{E}(\vec{r}) = -j\omega\vec{B}(\vec{r}) \qquad -j\vec{k}\cdot\vec{D}(\vec{r}) = 0 \qquad \vec{k}\times\vec{E}_0 = \omega\mu\vec{H}_0$$

$$\vec{H}(\vec{r},t) = \vec{H}_0 e^{-j\vec{k}\cdot\vec{r} + j\omega t} \qquad -j\vec{k}\times\vec{H}(\vec{r}) = j\omega\vec{D}(\vec{r}) \qquad -j\vec{k}\cdot\vec{B}(\vec{r}) = 0 \qquad \vec{k}\times\vec{H}_0 = -\omega\varepsilon\vec{E}_0$$

Impédance: $\eta = \frac{E}{H} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} \Rightarrow \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.6\ \Omega$

Conditions d'interface

Composantes continues (inchangées): $\vec{E}, \vec{H}$ parallèles à l'interface, et $\vec{D}, \vec{B}$ perpendiculaires à l'interface

Composantes changées: $D_{1\perp} = D_{2\perp} = \varepsilon_1 E_{1\perp} = \varepsilon_2 E_{2\perp}$

Coefficients de Fresnel:

$r_x = \dfrac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}, t_x = 1 + r_x$ (polarisation transverse électrique)

$r_y = \dfrac{n_1 \sec\theta_1 - n_2 \sec\theta_2}{n_1 \sec\theta_1 + n_2 \sec\theta_2}, t_y = \left(1 + r_y\right)\dfrac{\cos\theta_1}{\cos\theta_2}$ (polarisation transverse magnétique)

Réflectance: $R = |r|^2$

Transmittance: $T = 1 - R$

Polarisation

$E_x = a_x \cos\left(\omega\left(t - \frac{z}{c}\right) + \phi_x\right), E_y = a_y \cos\left(\omega\left(t - \frac{z}{c}\right) + \phi_y\right)$

Polarisation linéaire: $a_x = 0$ ou $a_y = 0$ ou $\phi = 0$ ou $\phi = \pi$

Polarisation circulaire: $\phi = \pm\frac{\pi}{2}, a_x = a_y = a_0$

$\vec{J} = \begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} a_x e^{j\phi_x} \\ a_y e^{j\phi_y} \end{pmatrix}$

Polarisation linéaire selon x: $\vec{J} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Polarisation linéaire: $\vec{J} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$

Polarisation circulaire à droite: $\vec{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ j \end{pmatrix}$

Polarisation circulaire à gauche: $\vec{J} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -j \end{pmatrix}$

$\vec{J}_2 = T\vec{J}_1$

Polarisateur linéaire: $T = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Polarisation retardateur: $T = \begin{pmatrix} 1 & 0 \\ 0 & e^{j\Gamma} \end{pmatrix}$

Polarisateur rotateur: $T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

Matériaux anisotropiques

$\vec{D} = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$

Diagonalisation sur les axes d'un cristal:

$\begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{pmatrix}\begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}, n_1 = \sqrt{\dfrac{\varepsilon_1}{\varepsilon_0}}$

Propagation de modes

$k^2 = \dfrac{\omega^2}{c^2} = \beta_m^2 + k_{ym}^2$

$\sin\theta_m = m\dfrac{\lambda}{2d}$

$k_{ym} = nk_0 \sin\theta_m = nk_0 m\dfrac{\lambda}{2d}$

$\beta_m = \sqrt{k^2 - \dfrac{m^2\pi^2}{d^2}}$

$\Rightarrow E_x(y, z) = a_m u_m(y) e^{j\beta_m z}$

m impair: $u_m(y) = \sqrt{\frac{2}{d}}\cos\frac{m\pi y}{d}, a_m = \sqrt{2d}A_m$

m pair: $u_m(y) = \sqrt{\frac{2}{d}}\sin\frac{m\pi y}{d}, a_m = j\sqrt{2d}A_m$

Propriétés:

Normalisation: $\int_{-\frac{d}{2}}^{\frac{d}{2}} u_m^2(y)dy = 1$

Orthogonalité: $\int_{-\frac{d}{2}}^{\frac{d}{2}} u_l(y)u_m(y)dy = 0 \ (l \neq m)$

Nombre de modes: $M = \left\lfloor\frac{2d}{\lambda}\right\rfloor \left(0 \text{ si } d = \frac{\lambda}{2}\right)$

Fréquence de coupure: $k = \frac{2\pi\nu}{c} = \frac{\pi}{d} \Rightarrow \nu_c = \frac{c}{2d}$ (pas de modes pour $\nu < \nu_c$)

Vitesse de groupe: $v_m = \frac{\partial\omega}{\partial k} = \frac{\partial\omega}{\partial\beta_m} = \frac{c^2\beta_m}{\omega} = c\sqrt{1 - m^2\frac{\omega_c^2}{\omega^2}} \left(\beta_m^2 = k^2 - \frac{m^2\pi^2}{d^2} = \frac{\omega^2}{c^2} - \frac{m^2\pi^2}{d^2} \Rightarrow \beta_m = \frac{\omega}{c}\sqrt{1 - m^2\frac{\omega_c^2}{\omega^2}}\right)$

Section carrée: $\vec{k} = \left(k_x, k_y, k_z\right) = \left(k_x, k_y, \beta\right), k_x^2 + k_y^2 + \beta^2 = k^2 = n^2 k_0^2 = \frac{n^2\omega^2}{c_0^2} \Rightarrow \begin{cases} 2k_x d = 2\pi m_x \\ 2k_y d = 2\pi m_y \end{cases}$

Fibres optiques

Réflexion interne: $\theta < \theta_a$

$\sin\theta_a = n_1 \cos\theta_c = n_1\sqrt{1 - \sin^2\theta_c} = \sqrt{n_1^2 - n_2^2} = NA$

$\Delta = \dfrac{n_1^2 - n_2^2}{2n_1^2} \approx \dfrac{n_1 - n_2}{n_1} \ll 1 \Rightarrow NA \approx n_1\sqrt{2\Delta}$

Fréquence normalisée: $V = \frac{2\pi a}{\lambda_0}NA = k_0 a NA$ (a: rayon du coeur)

Atténuation:

$P_2 = P_1 e^{-\alpha(z_2 - z_1)}$

$$\alpha = \frac{\ln \frac{P_1}{P_2}}{z_2 - z_1}$$

$$\alpha_{dB} = \frac{10 \log \frac{P_1}{P_2}}{z_2 - z_1} \Rightarrow P_2 = P_1 10^{-\frac{\alpha_{dB}(z_2 - z_1)}{10}}$$

Diffusion Rayleigh: $\alpha = \frac{A_0}{\lambda_0^4}$

Photons

Énergie: $E = h\nu = \hbar\omega, h = 6.63 \cdot 10^{-34}\, Js, \hbar = \frac{h}{2\pi}$

$$E(eV) = \frac{1.24}{\lambda_0(\mu m)}$$

Quantité de mouvement: $\vec{p} = \hbar\vec{k} \Rightarrow p = \hbar k = \hbar\frac{\omega}{c} = \frac{h}{\lambda} = \frac{E}{c}$

Piégeage optique: $\vec{F}(x, y, z) = 2\pi n_0^2 \varepsilon_0 a^3 \frac{m^2 - 1}{m^2 + 2} \nabla E^2(x, y, z), m = \frac{n_1}{n_0} = \frac{n_{sphère}}{n_{milieu}}$

Lasers et photodétecteurs

Transitions

Émission spontanée: $p_{sp} = \frac{c}{V}\sigma(\nu) \Rightarrow N(t) = N(0)e^{-p_{sp}t}$

Absorption: $p_{ab} = \frac{c}{V}\sigma(\nu), P_{ab} = n\frac{c}{V}\sigma(\nu) = W_i$

Émission stimulée: $p_{st} = \frac{c}{V}\sigma(\nu), P_{st} = n\frac{c}{V}\sigma(\nu) = W_i$

Probabilité d'émission totale: $p_{sp} + P_{st} = (n+1)\frac{c}{V}\sigma(\nu)$

Force d'oscillateur: $S = \int_0^{+\infty} \sigma(\nu) d\nu, \Delta\nu = \frac{1}{g(\nu_0)} = \frac{S}{\sigma(\nu_0)}$

Coefficients d'Einstein

Source monochromatique: $\phi = \frac{I}{h\nu} \Rightarrow W_i = \phi\sigma(\nu)$

Source polychromatique $\rho(\nu)$: $\bar{n} = \frac{c^3}{8\pi h \nu_0^3}\rho(\nu_0) = \frac{\lambda^3}{8\pi h}\rho(\nu_0) \Rightarrow W_i = \frac{\bar{n}}{t_{sp}}$

Colorimétrie

Couleur décrite par X, Y, Z: $X = \int_0^\infty I(\lambda)\bar{x}(\lambda) d\lambda$

Réduction à x, y: $x = \frac{X}{X + Y * Z}, y = \frac{Y}{X + Y + Z}, z = 1 - x - y$

Lasers

Cavité laser: $I = \frac{I_{\max}}{1 + \left(\frac{2F}{\pi}\right)^2 \sin^2\frac{\pi\nu}{\nu_F}}, I_{\max} = \frac{I_0}{(1 - |r|)^2}, F = \frac{\pi\sqrt{|r|}}{1 - |r|}$ (r: réflectance des miroirs)

Facteur de qualité: $Q = 2\pi \frac{\text{énergie enmagasinée}}{\text{énergie perdue}} = \frac{\nu_0}{\nu_F}F$

Amplification: $\phi(\nu) = \frac{I(z)}{h\nu} = \frac{|E(z)|^2}{2\eta h\nu} = \frac{|E(z)|^2}{2h\nu\sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}}}, \frac{\partial\phi}{\partial z} = \gamma(\nu)\phi(z)$

Courbe de gain: $g(\nu) = \frac{\text{section efficace de transition}}{\text{force d'oscillateur}} = \frac{\sigma(\nu)}{S} \Rightarrow \sigma(\nu) = \frac{\lambda}{8\pi t_{sp}} g(\nu) = \frac{\gamma(\nu)}{N}$

Probabilité d'absorption ou émission stimulée: $W_i = N\frac{c}{V}\sigma(\nu)$

$N_1 W_i$ photons absorbés, $N_2 W_i$ photons émis → $NW_i = (N_2 - N_1)W_i$ photons produits ($N > 0$: pompage, $N = 0$: transparence, $N < 0$: absorption)

Flux de photons: $\phi(z) = \phi(0)e^{\gamma(\nu)z} = \phi(0)e^{\frac{N\lambda^2}{8\pi t_{sp}}g(\nu)z}$

$$\text{Courbe de gain: } g(\nu) = \frac{\frac{\Delta\nu}{2\pi}}{(\nu - \nu_0)^2 + \left(\frac{\Delta\nu}{2\pi}\right)^2} \text{ (courbe Lorentzienne)}$$

Intensité: $I(z) = I(0)e^{\gamma(\nu)z}$

$$\text{Gain: } \gamma(\nu) = \gamma(\nu_0)\frac{\left(\frac{\Delta\nu}{2}\right)^2}{(\nu - \nu_0)^2 + \left(\frac{\Delta\nu}{2}\right)^2}, \gamma(\nu_0) = \frac{N\lambda^2}{4\pi^2 t_{sp}\Delta\nu}$$

$$\text{Phase: } \varphi(\nu) = \frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu)$$

Rate equations:

$$\frac{\partial N_2}{\partial t} = R_2 - \frac{N_2}{\tau_2} - N_2 W_i + N_1 W_i$$

$$\frac{\partial N_1}{\partial t} = -R_1 - \frac{N_1}{\tau_1} + \frac{N_2}{\tau_{21}} + N_2 W_i - N_1 W_i$$

$$N_{eq} = \frac{N_0}{1 + \tau_s W_i} < N_0 \left(N_0 = R_2\tau_2\left(1 - \frac{\tau_1}{\tau_{21}}\right) + R_1\tau_1\right)$$

Pompage: $N_0 \approx \frac{t_{sp}N_a W}{1 + t_{sp}W}, \tau_s = \frac{t_{sp}}{1 + t_{sp}W}$

Système laser:

Gain: $\gamma(\nu) = N_0\sigma(\nu) = \frac{\lambda^2}{8\pi t_{sp}}g(\nu) = \frac{\gamma_0(\nu)}{1 + \frac{\phi}{\phi_s(\nu)}}, \varphi(\nu) = \frac{\nu - \nu_0}{\Delta\nu}\gamma(\nu), \phi_s(\nu) = \frac{1}{\tau_s\sigma(\nu)}$

Pertes: $R_2 R_1 e^{-2\alpha_s d} = e^{-2\alpha_r d}, \tau_p = \frac{1}{\alpha_r c}$

Lasage: $\gamma_0(\nu) > \alpha_r \Rightarrow M \approx \frac{B}{\nu_F}, N_t = \frac{8\pi}{\lambda^2 c}\frac{t_{sp}}{\tau_p}\frac{1}{g(\nu)}$