

Prob. 1	Prob. 2

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Problem 1.

$$INT_{TM} := \{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) \neq \emptyset\}$$

INT_{TM} is the language recognizing every couple of Turing Machine such that there exists at least one word w that are recognised by both two machines M_1 and M_2 .

The language INT_{TM} is not Turing-recognizable to prove that let's first prove that the language $\{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$ is undecidable but Turing-recognizable.

We can compute a TM M_3 that simply is the intersection of M_1 and M_2 .

Now we define a language similar to E_{DFA} that we saw in class namely $E_{TM} := \{\langle M_3 \rangle \mid L(M_3) = \emptyset\}$. By a similar argument we know that this language is undecidable but Turing-recognizable. Because in opposite of DFA which their language is always recognizable, it is possible that our input M_3 is undecidable. So by theorem 6 we can say that INT_{TM} is not Turing-recognizable.

Problem 2.

We have that $R := \{w \mid \text{either } w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$, then we can find \bar{R} such that $\bar{R} := \{w \mid \forall x \in A_{TM}, \forall y \in \overline{A_{TM}} : w \neq 0x \wedge w \neq 1y\}$. Since R and \bar{R} can be reduced to $\overline{A_{TM}}$, we have $\overline{A_{TM}} \leq_m R$ and $\overline{A_{TM}} \leq_m \bar{R}$. As we know from Theorem 5 that $\overline{A_{TM}}$ is unrecognizable. We can use Theorem 11 to conclude that R and \bar{R} are unrecognizable.