Prob. 1	Prob. 2

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Problem 1.

$INT_{TM} := \{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) \neq \emptyset\}$
INT_{TM} is the language recognizing every couple of Turing Machine such that there existe at least one word let same w that are reconise by both two Machine M_1 and M_2

The language INT_{TM} is not Turing-recongnizable to proof that let's first proof that the language $\{\langle M_1, M_2 \rangle \mid L(M_1) \cap L(M_2) = \emptyset\}$ is undecidable but Turing-recognizable.
We can compute a TM M_3 that simply is the intersection of M_1 and M_2.
Now we define a language similar to E_{DFA} that we saw in class namely $E_{TM} := \{\langle M_3 \rangle \mid L(M_3) = \emptyset\}$. By a similar argument we know that this language is undecidable but Turing-recognizable. Because in opposite of DFA which their langugage is always recognizable, it is possible that our input M_3 is undecidable. So by theorem 6 we can say that INT_{TM} is not Turing-recongnizable

Problem 2.

We have that $R := \{w \mid \text{either } w = 0x \text{ for some } x \in A_{TM} \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$, then we can find $\overline{R}$ such that $\overline{R} := \{w \mid \forall x \in A_{TM}, \forall y \in \overline{A_{TM}} : w \neq 0x \wedge w \neq 1y\}$. Since R and $\overline{R}$ contain $\overline{A_{TM}}$ We can use Theorem 11 : $\overline{A_{TM}} \leq_m R$ and $\overline{A_{TM}} \leq_m R$
We know from Theorem 5 that $\overline{A_{TM}}$ is unrecognizable. So it implies that R and $\overline{R}$ are unrecognizable.