Metropolis - Hastings for observables

$$\langle \hat{O} \rangle = \frac{\mathrm{Trace}(\hat{O} \cdot \hat{\rho})}{\mathrm{Trace}(\hat{\rho})} = \frac{\langle O^T | \rho \rangle}{\mathrm{Trace}(\hat{\rho})}$$

$$\langle O^T | \rho \rangle = \sum_x \langle O^T | x \rangle \langle x | \rho \rangle = \sum_x \frac{\langle O^T | x \rangle}{\langle \rho | x \rangle} \cdot |\langle x | \rho \rangle|^2$$

↑ not c.c.
just transposed $\hat{O}$
is row vector form

where $|x\rangle = |s, s'\rangle$

$$\langle O^T | x \rangle = \langle s | O^T | s' \rangle = \langle s' | O | s \rangle$$

$$\mathrm{Trace}(\hat{\rho}) = \sum_x \langle x | \rho \rangle \, \delta(s - s') = \sum_x |\langle x | \rho \rangle|^2 \cdot \frac{1}{\langle \rho | x \rangle} \cdot \delta(s - s')$$

$$\langle s' | \hat{\sigma}^z | s \rangle = \frac{1}{N} \cdot \sum_i s[i]$$

$$\langle s' | \hat{\sigma}^x | s \rangle = \frac{1}{N} \langle s' | \sum_i \hat{\sigma}_i^+ + \hat{\sigma}_i^- | s \rangle =$$

= if count_nonzero(s'^s) == 1
  return 1/N
  else 0

$\langle s' | \hat{\sigma}^y | s \rangle = \frac{1}{N} \cdot \langle s' | \sum_i (-i)(\sigma_i^+ - \sigma_i^-) | s \rangle$ if count_nonzero(s'^s) — s'[] == −1 → $-\frac{i}{N}$; == +1 → $+\frac{i}{N}$
else 0