An open-loop bio-economic differential game of diagnostic test adoption for sheep-scab

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Diseases, too, their causes and tokens, I will teach you. Foul scab attacks sheep, when chilly rain and winter, bristling with hoar frost, have sunk deep into the quick, or when the sweat, unwashed, clings to the shorn flock, and prickly briars tear the flesh. Therefore the keepers bathe the whole flock in fresh streams; the ram is plunged in the pool with his dripping fleece, and let loose to float down the current. Or, after shearing, they smear the body with bitter oil lees, blending sliver scum and native sulphur with pitch from Ida and richly oiled wax, squill, strong hellebore, and black bitumen. Yet no help for their ills is of more avail than when one has dared to cut open with steel the ulcer's head; the mischief thrives and lives by concealment, while the shepherd refuses to lay healing hands on the wounds, and sits idle, calling upon the gods for happier omens.

-Virgil, Third Georgic.

Background

- Farmer behaviour is an important factor in determining rates of disease transmission.
- How farmers manage disease through monitoring for its presence and controlling its spread has the potential to change the dynamics of disease spread.
- Sheep-scab is a parasitic mite disease which may give rise to clinical signs but also may be present in sheep which show no clinical signs. In order to determine the presence of scab in sub-clinical cases current tests employ a skin scraping test which has relatively low sensitivity.
- More recently Moredun Research Institute have developed an ELISA (blood) test to detect the presence of sheep-scab in sub-clinical cases.

Background

Background

Figure 2: sheep-scab signs

Background

- We develop a game theoretic model to investigate the circumstances under which farmer's would voluntarily adopt such a test taking into account both within and between flock transmission of the disease. In order to do this we first developed a two-farm meta-population model of within and between flock transmission of the disease without treatment or testing.
- We reformulated this in terms of a differential game model of diagnostic test adoption in which two farmers make strategic decisions as to whether or not to adopt the test.
- We extend previous work by Bicknell et al. 1999 AJARE and Ceddia 2012 ERE to study diagnostic test adoption for sheep-scab in a competitive situation to farmer behaviour amongst sheep farmers. Previous bio-economic analyses of diagnostic tests include Bicknell et al. 1999 and Posner and Philipson 1993.

Time series

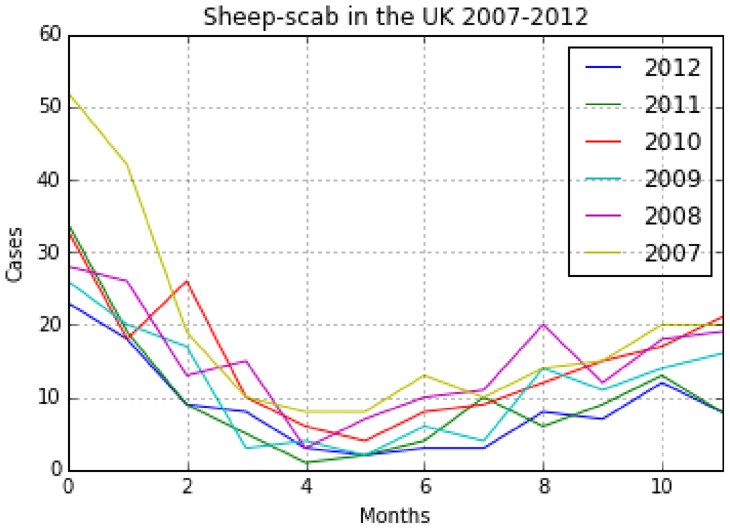


Figure 3: Sheep-scab time series, UK 2007-2012

Economic epidemiology

- Seminal work: Posner, R. A. (1993). Private choices and public health: The AIDS epidemic in an economic perspective. Harvard University Press.
- Geoffard, P. Y., & Philipson, T. (1997). Disease eradication: private versus public vaccination. The American Economic Review, 222-230.
- Ceddia, M. G. (2012). Optimal disease eradication in sympatric metapopulations. Environmental and Resource Economics, 52(4), 499-530.

Economics of diagnostic testing

- Model of diagnostic test adoption Posner, R. A. (1993). Private choices and public health: The AIDS epidemic in an economic perspective. Harvard University Press.
- Bicknell, K. B., Wilen, J. E., & Howitt, R. E. (1999). Public policy and private incentives for livestock disease control. Australian Journal of Agricultural and Resource Economics, 43(4), 501-521.
- Hockstra, D. J. (1973). Partially Observable Markov Decision Processes with Applications (No. TR-156). STANFORD UNIV CALIF DEPT OF OPERATIONS RESEARCH.
- Hockstra, D. J., & Miller, S. D. (1976). Sequential games and medical diagnosis. Computers and Biomedical Research, 9(3), 205-215.
- Laking, G., Lord, J., & Fischer, A. (2006). The economics of diagnosis. Health economics, 15(10), 1109-1120.

Bang-bang differential games

- Many game-theoretic epidemiological models are of bang-bang type as decisions are dichotomous
- Examples include: Rowthorn, B. R., & Toxvaerd, F. (2012). The optimal control of infectious diseases via prevention and treatment. CEPR Discussion Paper No. DP8925.
- Olsder, G. J. (2002). On open-and closed-loop bang-bang control in nonzero-sum differential games. SIAM journal on control and optimization, 40(4), 1087-1106.

Diagnostic testing

Assuming adoption sensitivity and specificity are calculated as follows:

	disease	No disease	
+ve	a	b (Type I)	a/(a+b)=PPV
-ve	c (Type II)	d	d/(c+d)=NPV
	a/(a+c) =Se	d/(b+d)=Sp	

perfect test corresponds to c=0 and b=0 no false negatives and no false positives.

In general not all sheep will be tested.

The model: set-up

- There are two farms, farm 1 and farm 2, sheep are assumed to homogeneously mix on each farm but mixing between farms is not necessarily homogenous.
- So the contact rates β_{12}, β_{21} between farms may differ.
- each farm consists of a sheep flock that is divided into five compartments:
  - susceptibles S_i
  - sub-clinically infective I_{sci}
  - clinically infective I_{ci}
  - protected due to treatment P_i and
  - removed R_i due to death.
  - Infected animals that have reached the clinical stage are assumed to either die or recover at rate ρ.

Instantaneous farm profit

$$GM_i(N_i - I_{ci} - R_i - P_i) - GM_i D(I_{sci} + I_{ci})$$
$$-c_T T_i(N_i - R_i - P_i) - c_D(S_e + 1 - S_P)(N_i - I_{ci} - R_i - P_i)T_i$$

$$-c_D 0.5(N_i - I_{ci} - R_i - P_i)(1 - T_i)$$

- GM_i gross-margin for the i-th farm
- N_i flock size
- D damage parameter
- c_T marginal test cost
- c_D treatment (dipping) cost
- $T_i \in [0, \tau]$ treatment decision

The model

Farmers maximize the following

$$\max_{T_i \in [0,\tau]} \int_0^{\infty} \{ GM_i(N_i - I_{ci} - R_i - P_i) - GM_i D(I_{sci} + I_{ci})$$
$$-c_T T_i(N_i - R_i - P_i) - c_D(S_e + 1 - S_P)(N_i - I_{ci} - R_i - P_i)T_i$$
$$-c_D 0.5(N_i - I_{ci} - R_i - P_i)(1 - T_i))\} \, e^{-rt} dt, \, i = 1, 2$$

subject to the disease dynamics depicted on the following slides

Disease dynamics

Farm 1

$$\frac{dS_1}{dt} = -\frac{\tilde{\beta}_{12}}{N_1 - R_1} S_1(I_{sc1} + \tilde{\phi} I_{c1} + \tilde{\gamma}_{12}(I_{sc2} + \tilde{\phi} I_{c2}))$$

$$-(1 - S_p) T_1 S_1 - 0.5 S_1 (1 - T_1) + \delta P_1, S_1(0) = S_0$$

$$\frac{dI_{sc1}}{dt} = \frac{\tilde{\beta}_{12}}{N_1 - R_1} S_1(I_{sc1} + \tilde{\phi} I_{c1} + \tilde{\gamma}_{12}(I_{sc2} + \tilde{\phi} I_{c2}))$$

$$-\tilde{\psi} I_{sc1} + \rho I_{c1} - S_e T_1 I_{sc1} - 0.5 I_{sc1}(1 - T_1), I_{sc1}(0) = 0$$

$$\frac{dI_{c1}}{dt} = \tilde{\psi} I_{sc1} - \rho I_{c1} - \mu I_{c1}, I_{c1}(0) = 0$$

Protection

$$\frac{dP_1}{dt} = (1 - S_p)T_1 S_1 + S_e T_1 I_{sc1}$$

$$+0.5(S_1 + I_{sc1})(1 - T_1) - \delta P_1, P_1(0) = 0$$

$$\frac{dR_1}{dt} = \mu I_{c1}, R_1(0) = 0$$

Farm 2

$$\frac{dS_2}{dt} = -\frac{\tilde{\beta}_{21}}{N_2 - R_2} S_2(I_{sc2} + \tilde{\phi} I_{c2} + \tilde{\gamma}_{21}(I_{sc1} + \tilde{\phi} I_{c1})) -$$

$$(1 - S_p) T_2 S_2 - 0.5 S_2 (1 - T_2) + \delta P_2, S_2(0) = S_0$$

$$\frac{dI_{sc2}}{dt} = \frac{\tilde{\beta}_{21}}{N_2 - R_2} S_2(I_{sc2} + \tilde{\phi} I_{c2}) + \tilde{\gamma}_{21}(I_{sc1} + \tilde{\phi} I_{c1})) - \tilde{\psi} I_{sc2} + \rho I_{c2} -$$

$$S_e T_2 I_{sc2} - 0.5 I_{sc2}(1 - T_2), I_{sc2}(0) = 1$$

$$\frac{dI_{c2}}{dt} = \tilde{\psi} I_{sc2} - \rho I_{c2} - \mu I_{c2}, I_{c2}(0) = 0$$

Protection

$$\frac{dP_2}{dt} = (1 - S_p) T_2 S_2 + S_e T_2 I_{sc2}$$

$$+0.5(S_2 + I_{sc2})(1 - T_2) - \delta P_2, P_2(0) = 0$$

$$\frac{dR_2}{dt} = \mu I_{c2}, R_2(0) = 0 \text{ and } N_i = S_i + I_{sci} + I_{ci} + R_i + P_i, i = 1, 2$$

and

$$N_i = S_i + I_{sci} + I_{ci} + R_i + P_i, i = 1, 2$$

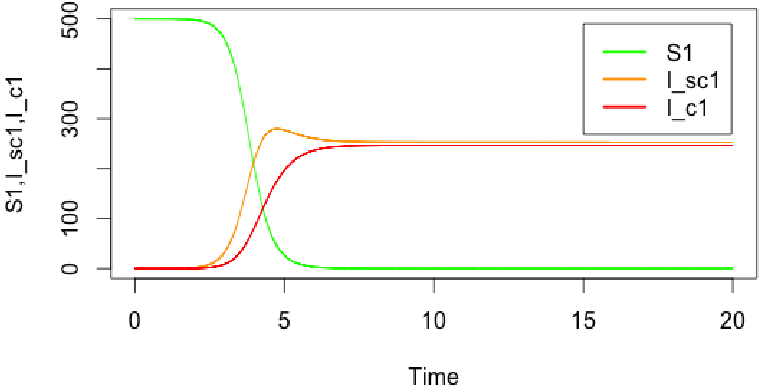


Figure 4: Sheep-scab dynamics

Bang-bang solution

$$\frac{\partial \tilde{H}_i}{\partial T_i} = -c_T(N_i - R_i - P_i) - c_D(S_e + 1 - S_P)(N_i - I_{ci} - R_i - P_i) +$$

$$c_D 0.5(N_i - I_{ci} - R_i - P_i)$$

$$-\lambda_{i1}(1 - S_p)S_i - \lambda_{i2} S_e I_{sci} + \lambda_{i4}((1 - S_p)S_i + S_e I_{sci}) > 0, i = 1, 2$$

then farmers will choose to test.

- Farmer's either test to the maximum or don't test.
- Test decisions vary with prevalence
- solution determines optimal timing of test adoption

Numerical challenges

- Two point-boundary value problem
- The problem is a coupled-system of high-dimensional control problems
- High-dimensionality (typical of compartmental models) creates additional challenges
- Solution procedure employs the shooting algorithm bvpshoot in the R package bvpSolve
- Bang-bang solution presents some additional challenges but these are easily resolvable

Shooting method

- Idea is to pick an initial guess and solve forward to get as close to known terminal conditions as possible.
- Then iterate by choosing another value and minimizing distance between known and estimated terminal condition
- standard method in control theory
- not much used in dynamic games, but needed for numerical problems

Plot of Switching Functions

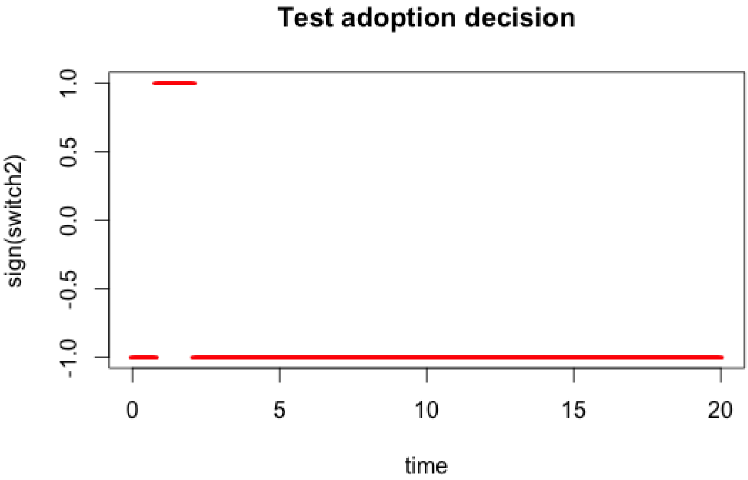


Figure 5: Discount rate 3%

Myopic Behaviour and diagnostic test adoption

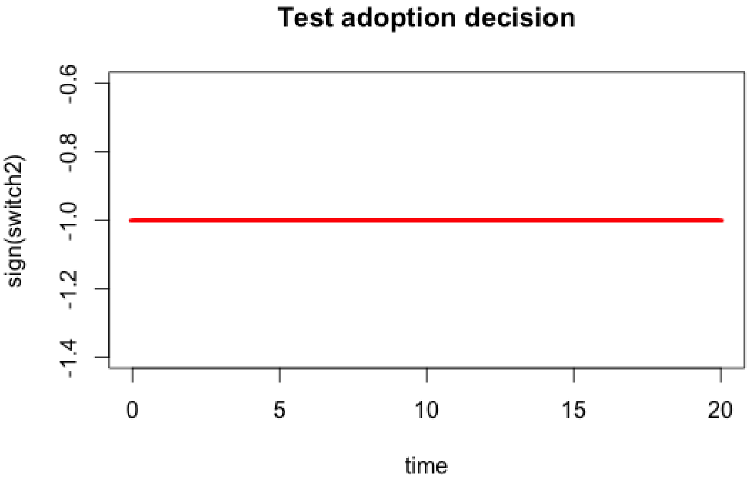
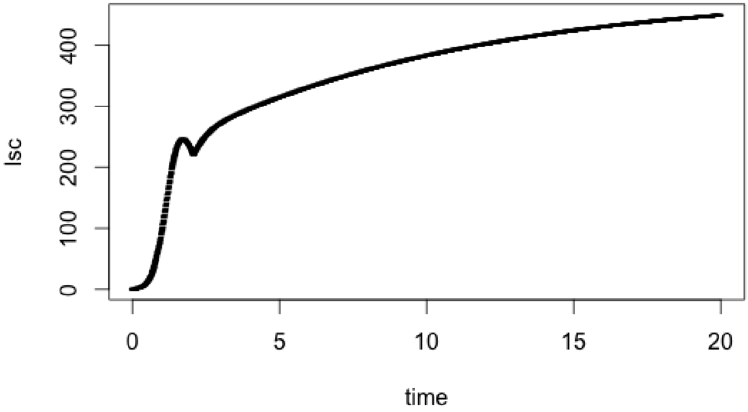


Figure 6: Discount rate 7 %

Sub-clinical infection over time



Conclusion and further research

- Farmer's switch behaviour up to two or three times over the observed period
- Infected farm's switch and adopt early
- Uninfected farm's tend to wait and see
- Test adoption has a short-term impact on sub-clinical cases but doesn't bring disease under control on one farm
- Test adoption does seem to stop spread to neighbouring farms (compare to plain vanilla meta-population model)

Conclusion and further research

- Sheep-scab is an important economic disease in the UK and particularly Scotland's livestock sector
- Evaluation of diagnostic tests is important as it is unclear as to whether diagnostic test adoption reduces or exacerbates disease incidence (Posner and Philippson show it can exacerbate an epidemic)
- Dynamic game theoretic models using differential game theory are a feasible means of evaluating such tests

Thanks for listening!

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