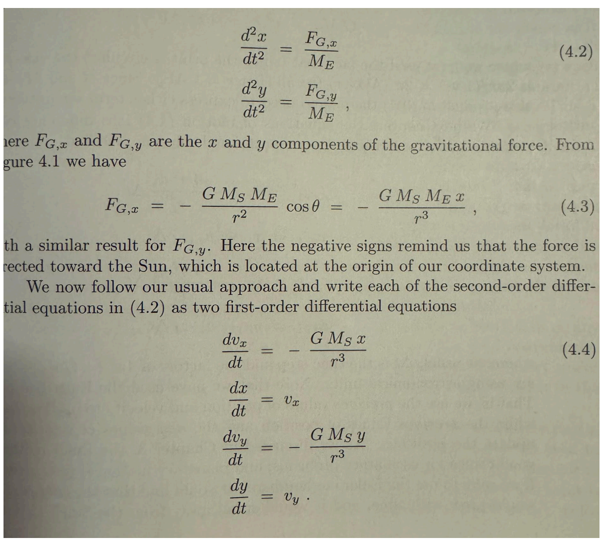
Chapter 4 Reading Notes

The Solar System

4.1 Kepler's Law

-Newton's Law of gravitation, Fg= G*Ms*Me/r^2 (Ms is mass of sun, Me is mass of earth, r is distance between them, G is gravitational constant)

-(Derivation that would be too hard to type out on google docs)



$$\frac{d^2x}{dt^2} = \frac{F_{G,x}}{M_E} \qquad (4.2)$$
$$\frac{d^2y}{dt^2} = \frac{F_{G,y}}{M_E} ,$$

ere $F_{G,x}$ and $F_{G,y}$ are the x and y components of the gravitational force. From gure 4.1 we have

$$F_{G,x} = -\frac{G\,M_S\,M_E}{r^2}\cos\theta = -\frac{G\,M_S\,M_E\,x}{r^3} , \qquad (4.3)$$

th a similar result for $F_{G,y}$. Here the negative signs remind us that the force is ected toward the Sun, which is located at the origin of our coordinate system.

We now follow our usual approach and write each of the second-order differtial equations in (4.2) as two first-order differential equations

$$\frac{dv_x}{dt} = -\frac{G\,M_S\,x}{r^3} \qquad (4.4)$$
$$\frac{dx}{dt} = v_x$$
$$\frac{dv_y}{dt} = -\frac{G\,M_S\,y}{r^3}$$
$$\frac{dy}{dt} = v_y .$$

- 1 AU is equivalent to 1.5e^11m which is the average distance between the earth and the sun.

-I was under the impression that the earth orbits the sun in an oval shape and based on how massive the earth is wouldn't the average distance change dramatically?

- G ms = v^2 r = 4pi^2 AU^3/yr^2

-All planets move in elliptical orbits with the sun at focus

- The line joining a planet to the sun sweeps out equal areas in equal times

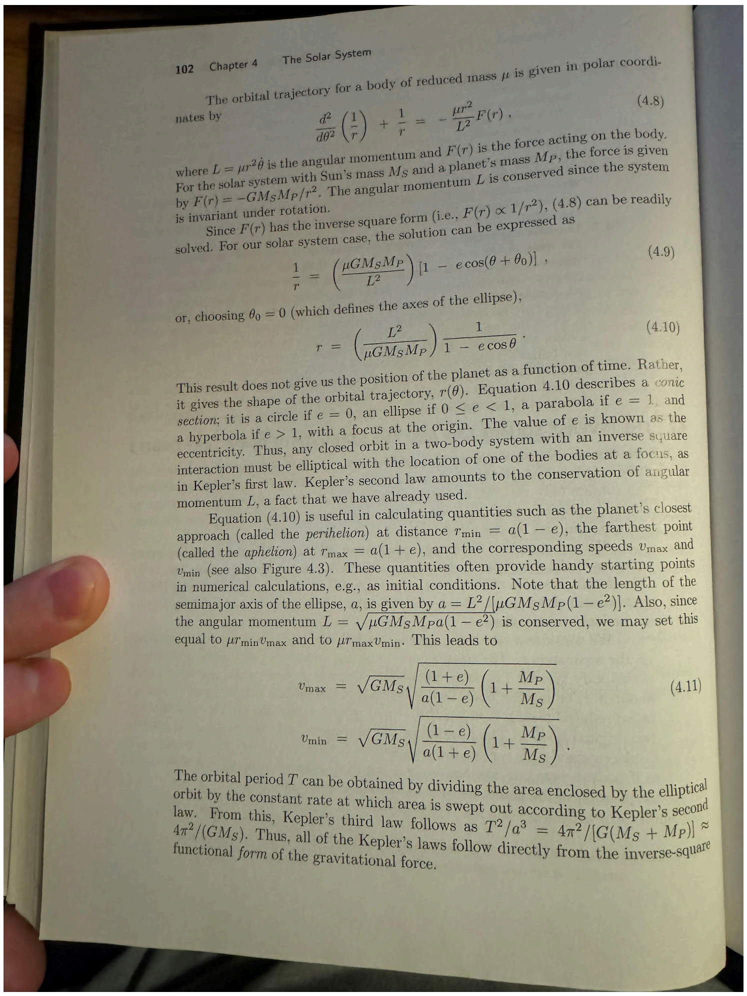
- If T is the period and a the semimajor axis of the orbit then T^2/a^3 is a constant

4.2 The Inverse-Square Law and the Stability of Planetary Orbits

-Mu= m1m2/(m1+m2)

-Position of equivalent body →r= →r2- →r1

- equations I can't type out but are important to the movement of planets (L is the angular momentum and Fr is the force acting on the body)



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The orbital trajectory for a body of reduced mass μ is given in polar coordinates by

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{\mu r^2}{L^2}F(r)\,, \tag{4.8}$$

where $L = \mu r^2\dot{\theta}$ is the angular momentum and $F(r)$ is the force acting on the body. For the solar system with Sun's mass M_S and a planet's mass M_P, the force is given by $F(r) = -GM_SM_P/r^2$. The angular momentum L is conserved since the system is invariant under rotation.

Since $F(r)$ has the inverse square form (i.e., $F(r) \propto 1/r^2$), (4.8) can be readily solved. For our solar system case, the solution can be expressed as

$$\frac{1}{r} = \left(\frac{\mu GM_SM_P}{L^2}\right)[1 - e\cos(\theta + \theta_0)]\,, \tag{4.9}$$

or, choosing $\theta_0 = 0$ (which defines the axes of the ellipse),

$$r = \left(\frac{L^2}{\mu GM_SM_P}\right)\frac{1}{1 - e\cos\theta}\,. \tag{4.10}$$

This result does not give us the position of the planet as a function of time. Rather, it gives the shape of the orbital trajectory, $r(\theta)$. Equation 4.10 describes a *conic section*; it is a circle if $e = 0$, an ellipse if $0 \le e < 1$, a parabola if $e = 1$, and a hyperbola if $e > 1$, with a focus at the origin. The value of e is known as the eccentricity. Thus, any closed orbit in a two-body system with an inverse square interaction must be elliptical with the location of one of the bodies at a focus, as in Kepler's first law. Kepler's second law amounts to the conservation of angular momentum L, a fact that we have already used.

Equation (4.10) is useful in calculating quantities such as the planet's closest approach (called the *perihelion*) at distance $r_{\min} = a(1 - e)$, the farthest point (called the *aphelion*) at $r_{\max} = a(1 + e)$, and the corresponding speeds $v_{\max}$ and $v_{\min}$ (see also Figure 4.3). These quantities often provide handy starting points in numerical calculations, e.g., as initial conditions. Note that the length of the semimajor axis of the ellipse, a, is given by $a = L^2/[\mu GM_SM_P(1 - e^2)]$. Also, since the angular momentum $L = \sqrt{\mu GM_SM_Pa(1 - e^2)}$ is conserved, we may set this equal to $\mu r_{\min}v_{\max}$ and to $\mu r_{\max}v_{\min}$. This leads to

$$v_{\max} = \sqrt{GM_S}\sqrt{\frac{(1+e)}{a(1-e)}\left(1 + \frac{M_P}{M_S}\right)} \tag{4.11}$$

$$v_{\min} = \sqrt{GM_S}\sqrt{\frac{(1-e)}{a(1+e)}\left(1 + \frac{M_P}{M_S}\right)}\,.$$

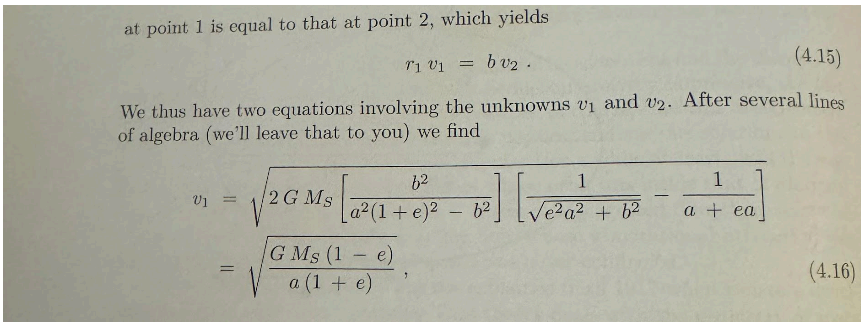
The orbital period T can be obtained by dividing the area enclosed by the elliptical orbit by the constant rate at which area is swept out according to Kepler's second law. From this, Kepler's third law follows as $T^2/a^3 = 4\pi^2/[G(M_S + M_P)] \approx 4\pi^2/(GM_S)$. Thus, all of the Kepler's laws follow directly from the inverse-square functional *form* of the gravitational force.

- Beta= 2 is the inverse square law and will create an elliptical orbit
- When beta = 3 it does not follow a stable orbit
- When beta is closer to 2 the the increasingly becomes more stable as shown in the pictures

4.3 Precession of the Perihelion of Mercury

-Find it impressive that they are able to track the orbit of mercury's perihelion when it only completes a full rotation every 230,000 years but then proven wrong

-Important to find the initial conditions



at point 1 is equal to that at point 2, which yields

$$r_1 v_1 = b v_2 . \tag{4.15}$$

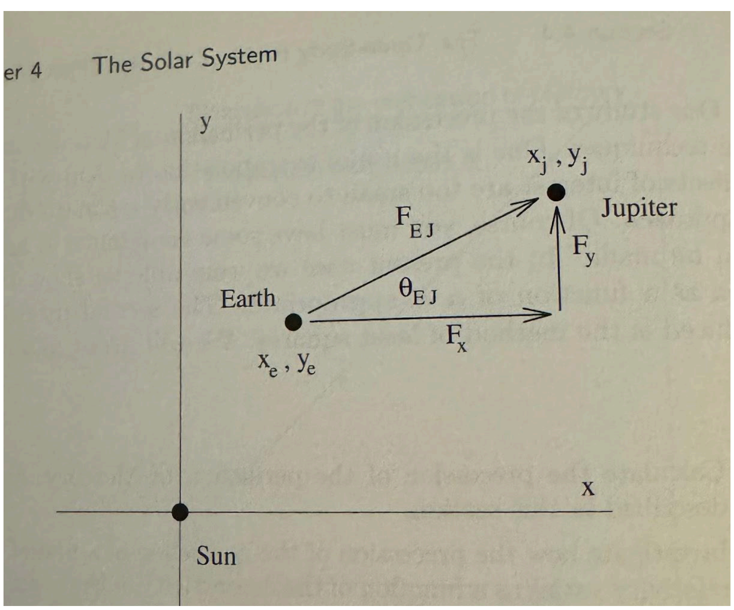
We thus have two equations involving the unknowns v_1 and v_2. After several lines of algebra (we'll leave that to you) we find

$$\begin{aligned} v_1 &= \sqrt{2\,G\,M_S \left[\frac{b^2}{a^2(1+e)^2 - b^2}\right]\left[\frac{1}{\sqrt{e^2a^2 + b^2}} - \frac{1}{a + ea}\right]} \\ &= \sqrt{\frac{G\,M_S\,(1-e)}{a\,(1+e)}} , \end{aligned} \tag{4.16}$$

4.4 The Three-Body Problem and the Effect of Jupiter on Earth

-Jupiter has a big effect on the other plants because it is by far the biggest planet in the solar system

- Mag force of Jupiter and Earth Fe,j = G*Mj*Me/r^2ej (rej is the distance between earth and jupiter)

- 

-Helps understand the relation between the planets.

-Although Jupiter is large is doesn't have an effect on earth which is proven by earth orbiting around the sun for billions of years

-In lab we showed that when we increased the mass by 1000 times it had a severe impact

-Jupiter is the largest but still too small but it does have an effect on mars