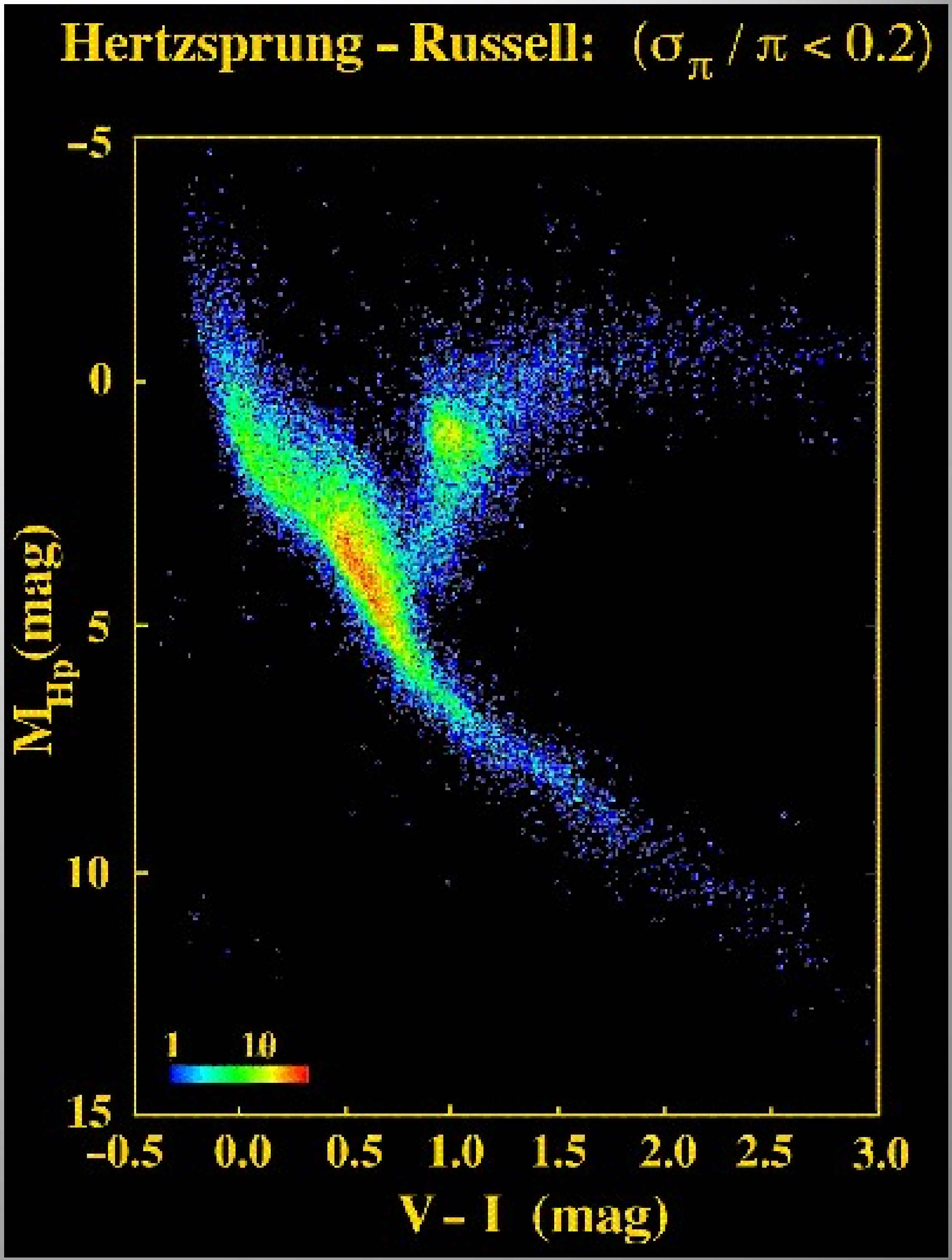
The Main Sequence



Main Sequence Star:

Powered by fusion of $4H \longrightarrow {}^4He$ in core

Main features:

- High abundance of H in core
- Temp high enough to fuse H
- Temp not high enough to fuse heavier elements

Generally the longest-lived phase of a star

Energy from nuclear fusion

Mass of a nucleus is < sum of masses of constituent particles due to **binding energy** of nucleus.

Since binding energy differs for different nuclei, can release or absorb energy when nuclei either fuse or fission.

Example: $4\times{}^{1}\mathrm{H}\rightarrow{}^{4}\mathrm{He}$

4 protons, each of mass
1.0081 atomic mass units:
4.0324 amu

mass of helium nucleus:
4.0039 amu

Mass difference: 0.0285 amu = 4.7 x 10^{-26} g

$$\Delta E = \Delta M c^{2} = 4.3\times10^{-5}\ \mathrm{erg} = 27\ \mathrm{MeV}/{}^{4}\mathrm{He}$$

$$\mathrm{Efficiency} = \Delta E / M_{He} c^{2} = 0.7\%$$

- Binding energy of a nucleus = energy required to break it up into constituent protons and neutrons
- A given reaction can be exothermic or endothermic depending on difference of binding energy

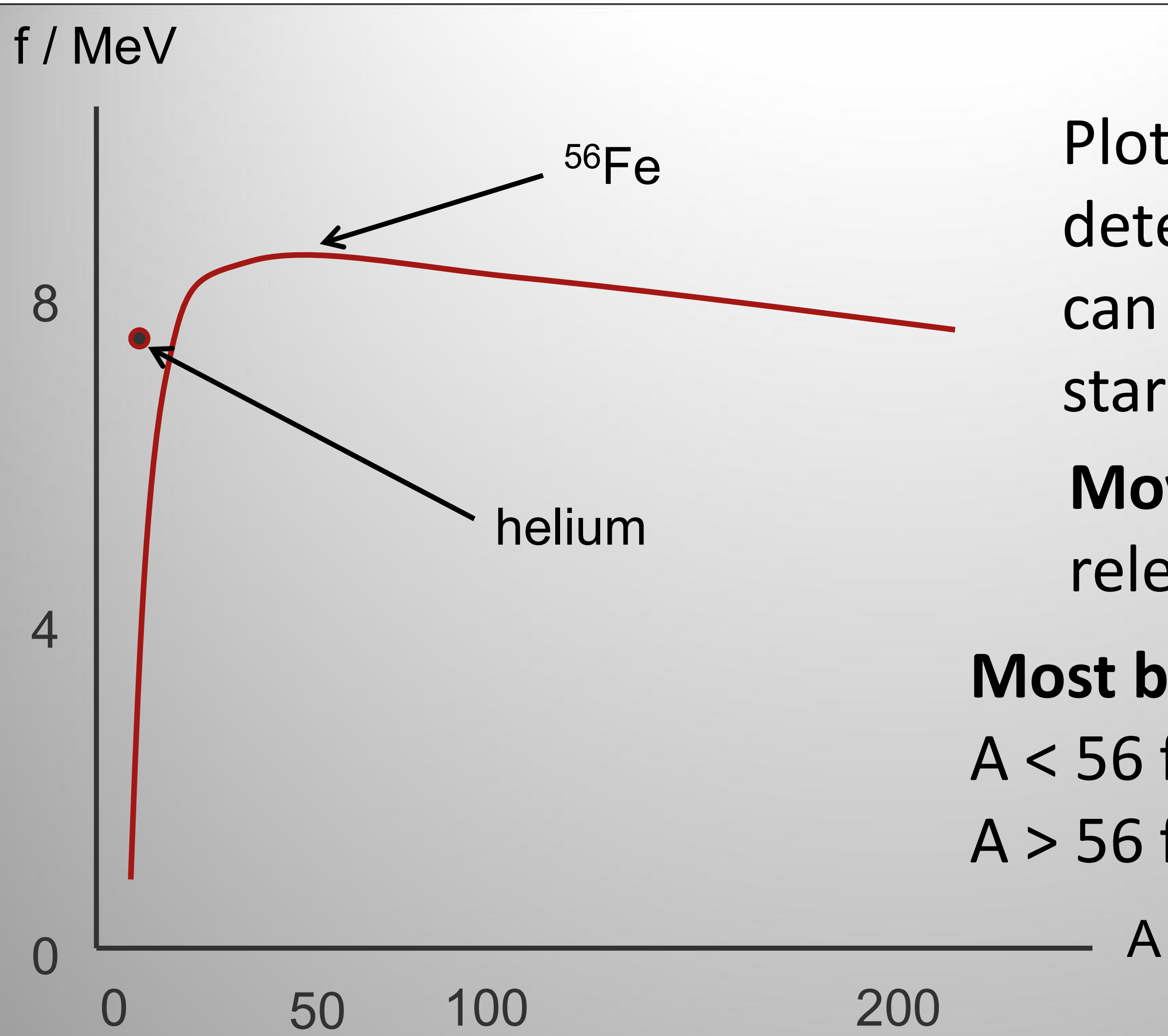
Suppose nucleus has:

- Proton number Z
- Atomic mass number A (number of protons + neutrons)
- Mass M_{nuc}

Binding energy: $$E_B = \left[(A-Z)m_n + Zm_p - M_{nuc}\right]c^2$$

Useful quantity for considering which nuclear reactions yield energy is the binding energy *per nucleon* - defined as:

$$f = \frac{E_B}{A}$$



Plot of f vs A largely determines which elements can be formed in different stars

Moving up along the curve releases energy

Most bound nucleus: ^{56}Fe

A < 56 fusion releases energy

A > 56 fusion requires energy

Yield for fusion of hydrogen to ^{56}Fe: ~8.5 MeV per nucleon

Most of this is already obtained in forming helium (6.6 MeV)

(Curve drawn as smooth – actually fluctuates for small A.

He is more tightly bound than ‘expected’)

How to trigger a fusion reaction

- Nuclei are positively charged - repel each other.
- If charges on the nuclei are Z_1e and Z_2e, then at distance d the electrostatic energy is:

$$E = \frac{Z_1 Z_2 e^2}{d}$$

If the nuclei approach sufficiently closely, short range nuclear forces (attractive) dominate and allow fusion to take place.

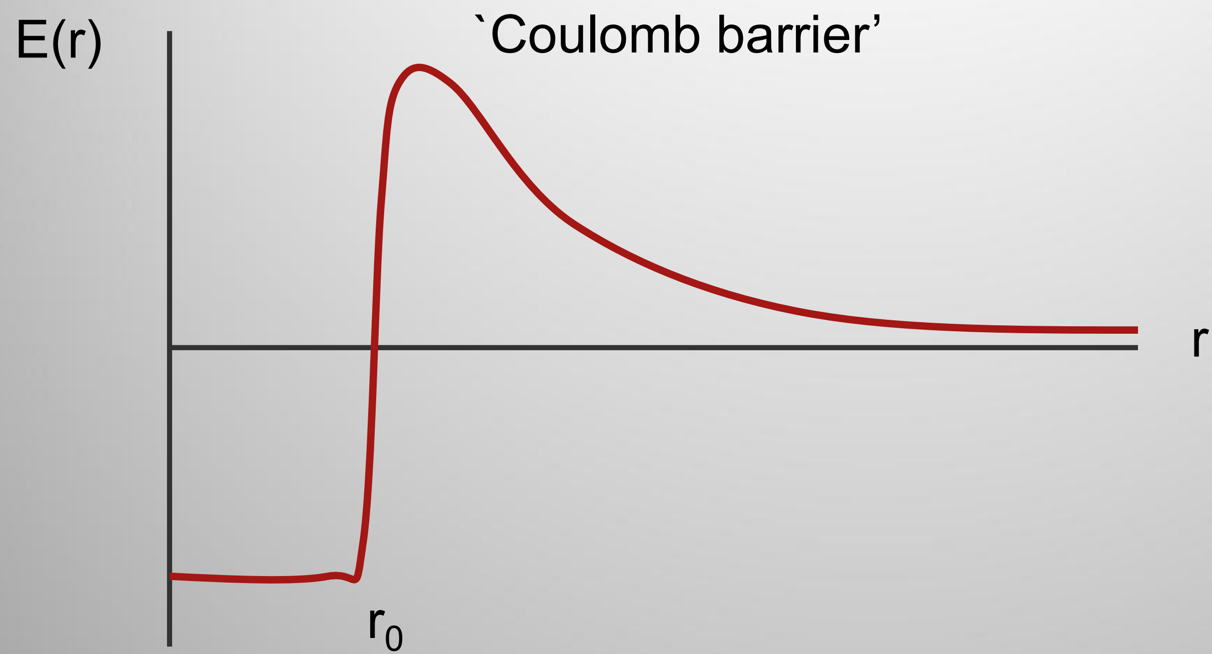
Nuclear material has ~ constant density ⇒ 'close enough' means:

$$r_0 \approx 1.44 \times 10^{-13} A^{1/3} \text{ cm}$$

↑

atomic mass number

Schematically:

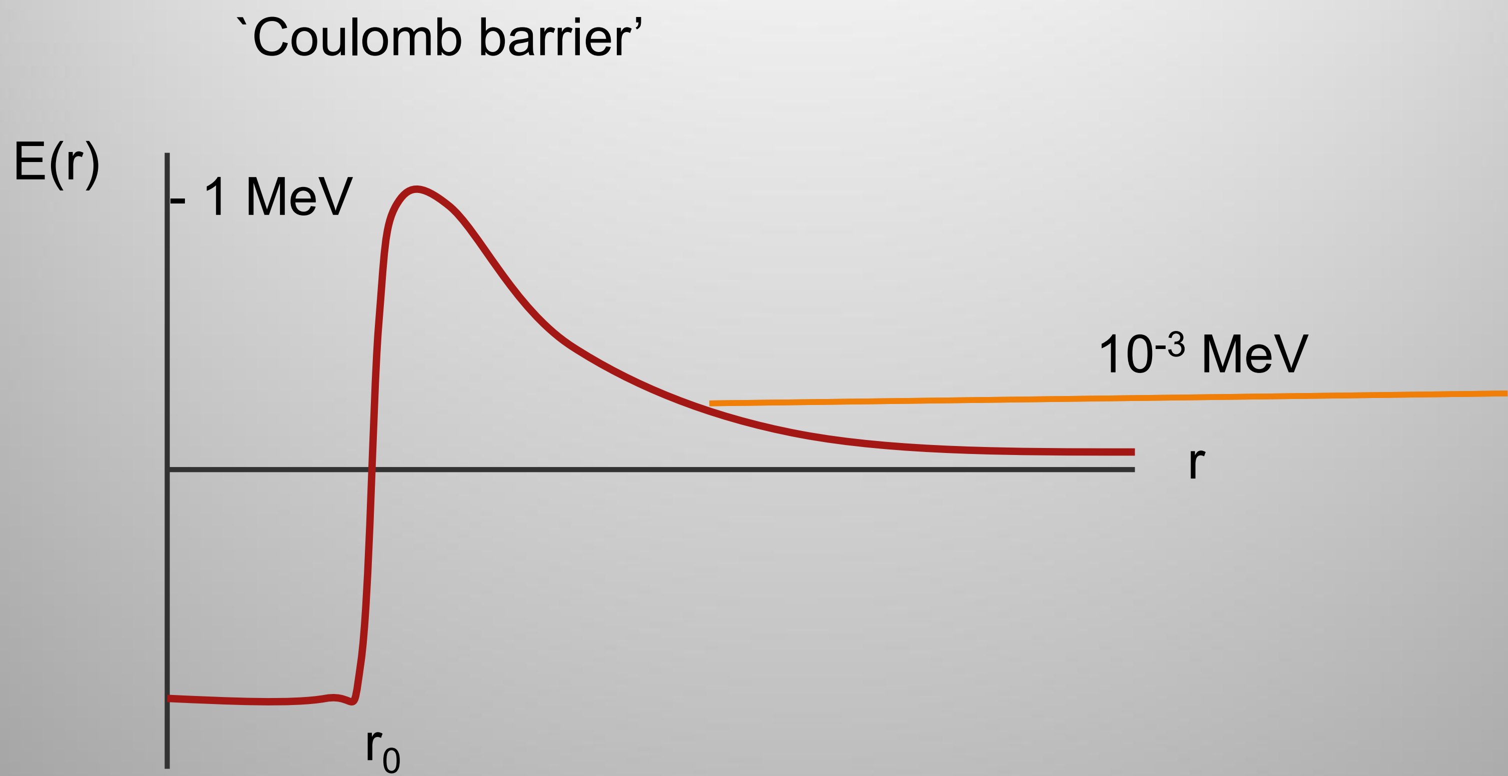


At r = r_0, height of the Coulomb barrier is:

$$E = \frac{Z_1 Z_2 e^2}{r_0} \sim Z_1 Z_2 \text{ MeV}$$

i.e. of the order of 1 MeV for two protons…

Schematically:



For Solar core conditions T = 1.5 x 10^7 K
Thermal energy of particles = kT = 1300 eV = 10^{-3} MeV

Classically, *no* particles in the Sun would have enough energy to surmount the Coulomb barrier and fuse

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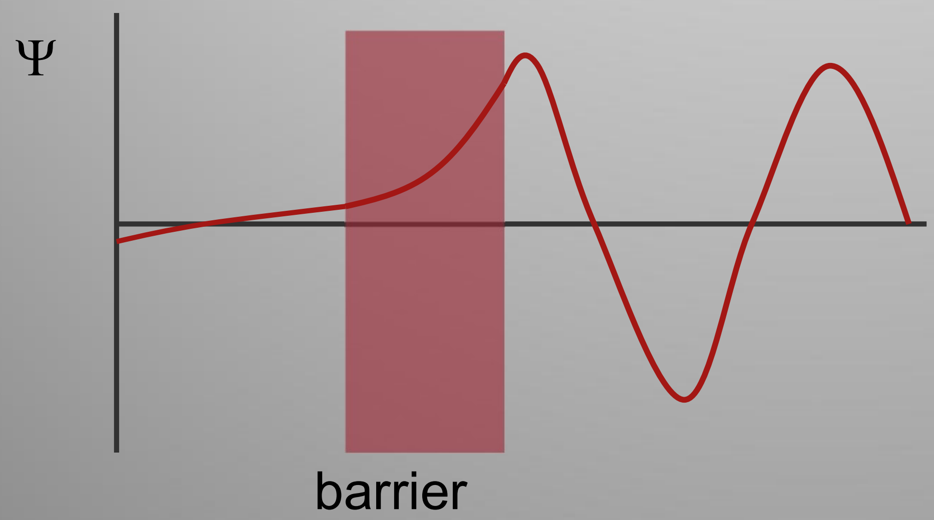
Quantum mechanically, lower energy particles have a very small but non-zero probability of tunneling through the barrier:



Probability of finding particle ~ $|\Psi|^2$ - if barrier is not too wide then non-zero wavefunction allows some probability of tunneling…

Tunneling allows fusion in stars

Qualitative argument: tunneling requires particles to approach within ~ de Broglie wavelength – does not depend on details of nuclear force (r_0)

Uncertainty principle ⇒ particles smeared out over size

$$\lambda_{\text{de Broglie}} \sim \frac{h}{\underset{\text{momentum}}{p}} = \frac{h}{mv}$$

⇒ just need $r \lesssim \lambda_{\text{de Broglie}}$ to get nuclear reaction

For typical nucleus: $\frac{1}{2}mv^2 \sim kT \gtrsim \frac{e^2}{\lambda_{deB}} \sim \frac{e^2 mv}{h}$

$$\Longrightarrow \quad v > \frac{2e^2}{h} \sim 2.3\times10^{-3}\, c$$

$$\boxed{T \sim \frac{mv^2}{3k} \gtrsim \frac{4}{3}\frac{me^4}{kh^2} \sim 2\times10^{7}\ \text{K}}$$

close to Temp. inside Sun

Typically fusion occurs “out on the tail” at a few times higher energy

Nuclei with higher masses/charges require higher temperatures to fuse

$$T \sim \frac{mv^2}{3k} \gtrsim \frac{4}{3}\frac{me^4}{k\hbar^2}$$

$$T_{nuc} \propto (\text{nuclear charge})^4 \times (\text{mass/nucleon})$$

$\Rightarrow$ fusion of heavier nuclei require much hotter temps.

e.g. $3 \times {}^4He \rightarrow {}^{12}C$ (triple-α process)

needs ~ few $\times$ 10 $\times$ $T(4H \rightarrow He) \sim 4 \times 10^8$ K

since charge ~ $2e$, mass ~ $4m_H$

$$\Rightarrow T \sim (2^4)\cdot(4) \sim 64\, T_{4H \rightarrow He}$$

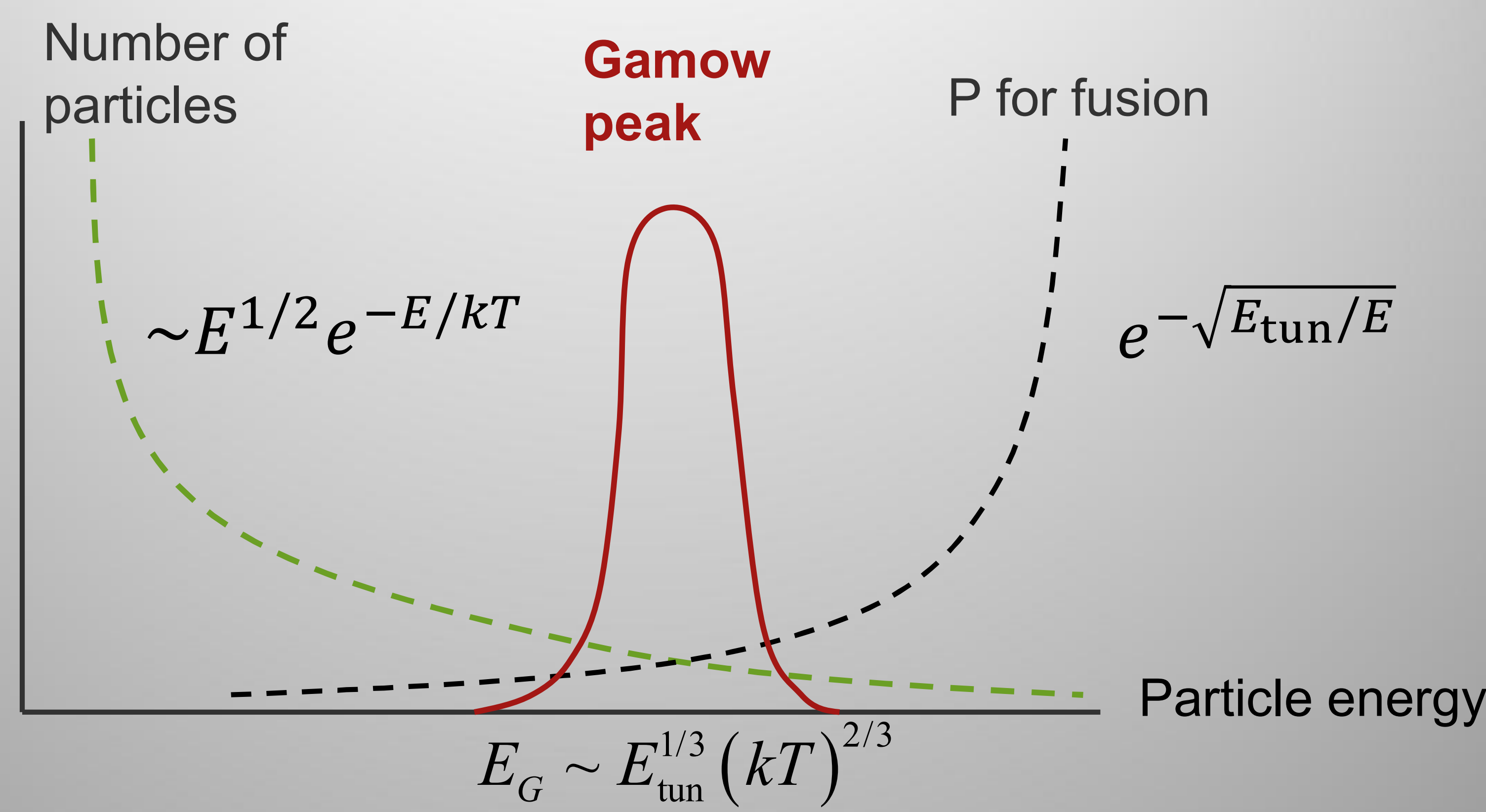
Probability of tunneling depends upon the energy of the particles, their mass, and the charge:

$$P \propto E^{-1/2} e^{-2\pi\eta} \qquad \eta = \left(\frac{m}{2}\right)^{1/2} \frac{Z_1 Z_2 e^2}{\hbar E^{1/2}} = \frac{1}{2\pi}\left(\frac{E_{\text{tun}}}{E}\right)^{1/2}$$

$$E_{\text{tun}} = \frac{8\pi^4 m e^4}{h^2} \sim 10 \text{ keV}$$

- P increases rapidly with E
- P decreases with $Z_1 Z_2$ - lightest nuclei can fuse more easily than heavy ones
- Higher energies/temperatures needed to fuse heavier nuclei, so different nuclei burn in well-separated phases during stellar evolution.

Rate of fusion: most energetic nuclei most likely to fuse, but very few of them in a thermal distribution of speeds:



Narrow range of energies around the Gamow peak where significant numbers of particles in the plasma are able to fuse. Energy is >> typical thermal energy, so fusion is slow.

Nuclear reactions in the Sun

Almost all reactions involve collisions of only two nuclei. So making helium from four protons involves a sequence of steps. In the Sun, this sequence is called the **proton-proton chain**:

Step 1

$$^{1}H + {}^{1}H \longrightarrow {}^{2}H + e^{+} + \nu_{e}$$

deuteron – one proton + one neutron

positron

electron neutrino

This is the critical reaction in the proton-proton chain. It is slow because forming a deuteron from two protons requires transforming a proton into a neutron – this involves the weak nuclear force so it is slow (rate-limiting step) …

Beyond this point, several possibilities. Simplest:

Step 2 $^{2}H + {}^{1}H \longrightarrow {}^{3}He + \gamma$

Step 3 $^{3}He + {}^{3}He \longrightarrow {}^{4}He + {}^{1}H + {}^{1}H$

Results of this chain of reactions:

- Form one ^{4}He nucleus from 4 protons
- Inject energy into the gas via energetic particles:
  2 positrons, 2 photons, 2 protons
- Produce 2 electron neutrinos, which carry little
  energy and escape the star without being absorbed.

Energy yield is ~10^{-5} erg per proton, so ~4×10^{38} reactions per second needed to yield L_{sun}. About 0.65 billion tons of hydrogen fusing per second.

Nuclear Reaction Network in Sun

Basic reaction $4H \rightarrow {}^4He$ + energy

but several routes to this. – same net energy release in each.

Proton-proton chain – add protons in sequence

PP I: ${}^1H + {}^1H \rightarrow {}^2D$ ← deuterium = ${}^2H$ $+ e^+$ ← positron $+ \nu$ ← neutrino

${}^2D + {}^1H \rightarrow {}^3He + \gamma$ ← photon

${}^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$

also ${}^1H + e^- + {}^1H \rightarrow {}^2D + \nu$ "pep" reaction somewhat higher ν energy

2 important variants – 1st 2 steps same, 3rd step different

PP II: ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^7Be + e^- \rightarrow {}^7Li + \gamma$

${}^7Li + {}^1H \rightarrow {}^4He + {}^4He$

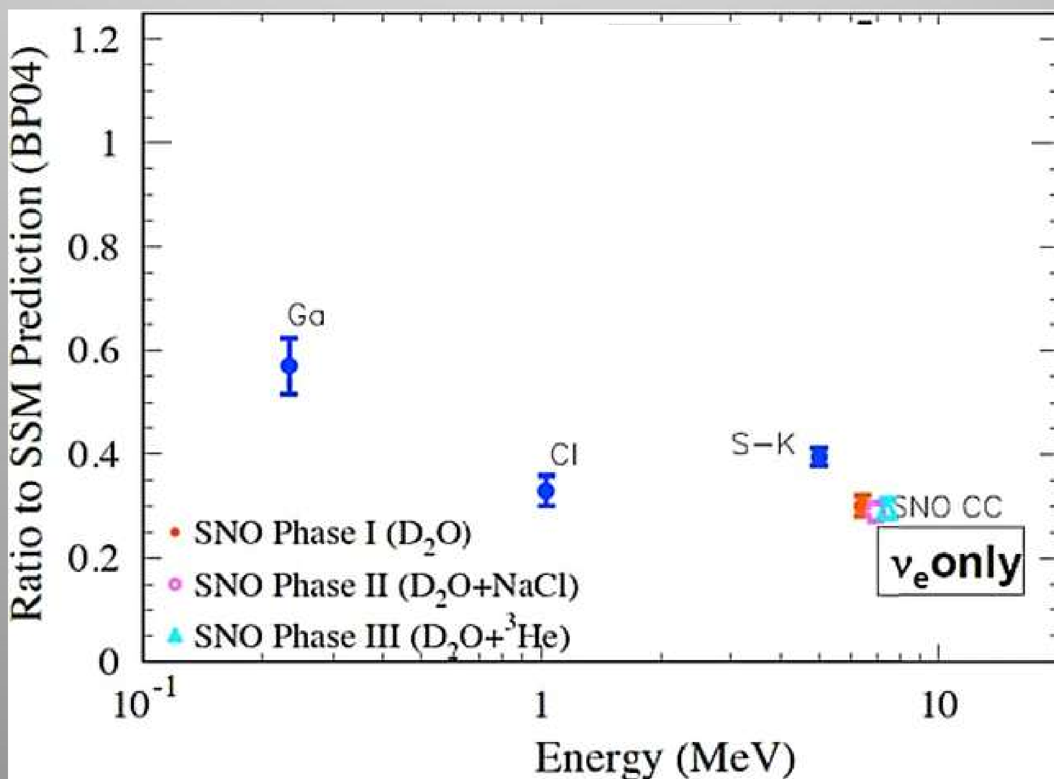
PP III: ${}^3He + {}^4He \rightarrow {}^7Be + \gamma$

${}^1H + {}^7Be \rightarrow {}^8Be + e^+ + \nu$

${}^8Be \rightarrow 2\ {}^4He$

Solar neutrino problem

- Neutrinos produced in the Sun are "electron" neutrinos, one of 3 types
- Early experiments detected ~1/3 of the expected number!
- Later experiments showed that the **total** number of neutrinos matches theory to good accuracy
- Accepted explanation is that neutrions **oscillate** between types, due to being massive particles

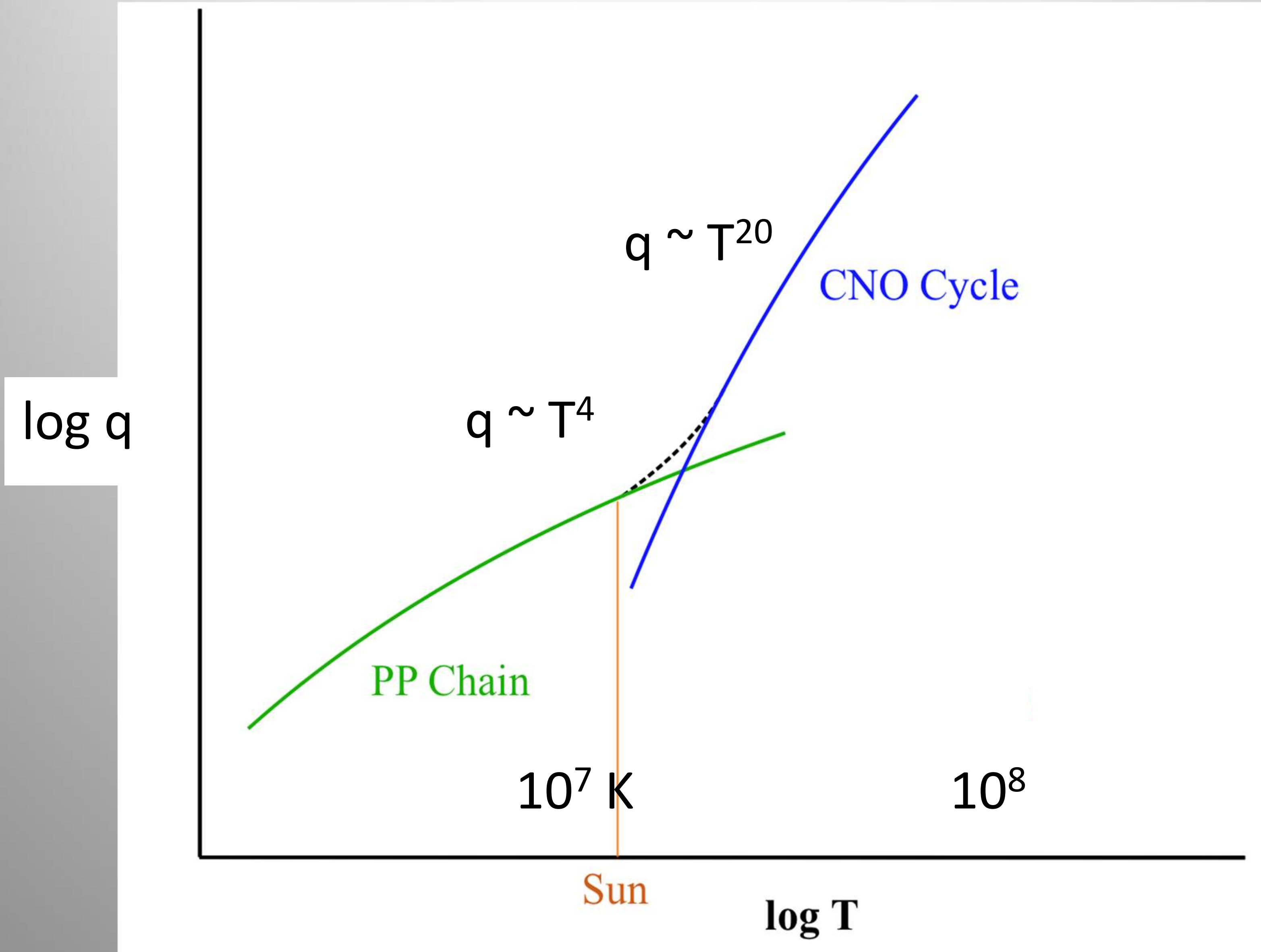


CNO cycle main process at higher mass

$$^{1}\mathrm{H} + {}^{12}\mathrm{C} \rightarrow {}^{13}\mathrm{N} + \gamma$$
$$^{13}\mathrm{N} \rightarrow {}^{13}\mathrm{C} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$$^{1}\mathrm{H} + {}^{13}\mathrm{C} \rightarrow {}^{14}\mathrm{N} + \gamma$$
$$^{1}\mathrm{H} + {}^{14}\mathrm{N} \rightarrow {}^{15}\mathrm{O} + \gamma$$
$$^{15}\mathrm{O} \rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu_{\mathrm{e}}$$
$$^{1}\mathrm{H} + {}^{15}\mathrm{N} \rightarrow {}^{12}\mathrm{C} + {}^{4}\mathrm{He}$$

Requires higher temperatures than pp chain
Does not actually use up CNO (“catalytic”)

Temperature sensitivity of fusion rates



Central temperature in stars is set by nuclear physics

How does a star maintain equilibrium if its core temperature suddenly increases by a small amount?

In turn, what happens to the:

1. Nuclear fusion energy generation rate q
2. Gas pressure (ideal gas law)
3. Stellar radius (hydrostatic equilibrium)
4. Temperature (virial theorem)

Temperature of main sequence stars is set by what is needed to fuse enough 4 $^{1}H \rightarrow {}^{4}He$ to support the star!

Temperature Sensitivity of Nuclear Reactions

$$\frac{d\dot{E}_{nuc}}{dt} = L_\star$$ very sensitive to temp

at center of star (T_c)

$T \sim \frac{m}{k}\frac{GM}{R} \Rightarrow$ $\boxed{\text{R adjusts so that } T \sim \text{about right}}$

T too high → too much energy → star expands

→ T decreases

T too low → too little energy → star contracts

→ T increases

⇒ stable, T is regulated

Main sequence scaling laws

Nuclear sensitivity: T ~ const in core (only slight incr. w/ M)

Radiative energy transport $L = -\dfrac{16\pi r^2 ac T^3}{3\kappa\rho}\dfrac{dT}{dr} \sim \dfrac{T^4 R^4}{\kappa M}$

VERY ROUGHLY:

Gas pressure dominates:

Lower mass range

$M < M_{\mathrm{Sun}}$

$\kappa \propto \rho T^{-7/2}$

$T \propto \dfrac{M}{R} \rightarrow R \propto M$

$L \propto T^{15/2} R^7 M^{-2}$
$\propto R^5 \propto M^5$

$M = 1 - 30\ M_{\mathrm{Sun}}$

$\kappa \propto \mathrm{constant}$

$T \propto \dfrac{M}{R} \rightarrow R \propto M$

$L \propto R^3 \propto M^3$

Rad pressure important:

Upper mass range

$M > 30\ M_{\mathrm{Sun}}$

$\kappa \propto \mathrm{constant}$

$T^4 \propto \dfrac{M^2}{R^4} \rightarrow R^2 \propto M$

$L \propto \dfrac{R^4}{M} \propto R^2 \propto M$

Effective temperature of photosphere

$$4\pi\sigma T_{eff}^4 R^2 = L(M)$$

$$\Rightarrow \quad T_{eff} \propto L^{1/4}/R^{1/2}$$

$M \lesssim 1 M_\odot \quad L \propto M^5 \quad R \propto M \qquad T_{eff} \propto \frac{M^{5/4}}{M^{1/2}} \propto M^{3/4}$

$1 M_\odot \lesssim M \lesssim 10 M_\odot \quad L \propto M^3 \quad R \propto M \qquad T_{eff} \propto \frac{M^{3/4}}{M^{1/2}} \propto M^{1/4}$

$M \gtrsim 10 M_\odot \quad L \propto M \quad R \propto M^{1/2} \qquad T_{eff} \propto \frac{M^{1/4}}{M^{1/4}} \sim$ const

(actually continues to incr. slowly with M since interior temp incr. slowly)

$\Rightarrow$ Effective temp. $\Leftrightarrow$ color determined by mass

Plot of L vs. T_{eff} called .

Hertzsprung - Russell Diagram (HR diagram)

Main sequence stars are powered by fusion of $4H \rightarrow {}^4He$ in their cores

- $M <\sim 2.5\ M_{Sun}$: mostly pp-chain
- $M >\sim 2.5\ M_{Sun}$: mostly CNO cycle

Main features:

- High abundance of H in core
- T high enough to fuse H ($\sim 10^7$ K)
- T not high enough to fuse He ($\sim 10^8$ K)

Longest-lived phase of a star

The main sequence is a sequence in initial stellar mass

Take average value for < 1 M_{Sun} and 1-30 M_{Sun} stars, $L \sim R^4 \sim M^4$

$L \propto T_{\mathrm{eff}}^4 R^2$