NLP: Maximum Entropy Models

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1 Features

Why do we like feature?

- Give us additional, useful information.
- Parts of speech: prefixes, suffixes, capitalization, word shape, is a number, ...

Why do we like sequence models?

- Nouns and Adjectives more likely to follow Determiners
- "I enjoy walks". Typically, "walks" is a verb, but not here since "enjoy" is definitely a verb.

2 Linear Regression

- Given a set of data points, find a line approximating the function that produced the points.
- Equation for a 2D line: y = b + mx
- Each feature f_i has an associated weight w_i
- So: $y = w_0 + w_1 f_1$, where $y \in (-\infty, \infty)$ is a predicted value
 - $-f_1$ is the observation
- With multiple features: $y = w_0 + w_1 f_1 + w_2 f_2 + w_3 f_3 + \dots = \sum_{i=0}^{N} w_i \times f_i = \vec{\mathbf{w}} \cdot \vec{\mathbf{f}}$ (dot product)
 - $-\vec{\mathbf{f}}$ is the observation (aka x)
- \bullet For a particular instance $j,\,y_{pred}^{(j)} = \sum_{i=0}^{N} w_i \times f_i^{(j)}$
- \bullet Learning: choose weights W that minimize the sum-squared error:

$$\hat{w} = \operatorname{argmin}_{x} \sum_{j=0}^{M} (y_{pred}^{(j)} - y_{obs}^{(j)})^{2}$$

- can be solved in closed form

3 Logistic Regression

- For many NLP applications, we don't want a real value output, we want a classification.
- Moreover, we want to assign a probability to each class
- Want to be able to use weighted features
- But, can't simply apply linear regression: it gives us a real value, not a probability

Binary Classification

- Need $p(y = \text{true} \mid x)$
 - For observation x (which is $\vec{\mathbf{f}}$), we want to make use of $\sum_{i=0}^{N} w_i \times f_i$
 - * but can't directly because it's a real value
 - We can use a ratio of probabilities, $\frac{p(y=\text{true}|x)}{p(y=\text{false}|x)} = \frac{p(y=\text{true}|x)}{1-p(y=\text{true}|x)}$, but this yields a value between 0 (definitely false) and ∞ (definitely true)
 - Logarithm gets us a value between $-\infty$ and ∞ : $\ln(\frac{p(y=\text{true}|x)}{1-p(y=\text{true}|x)})$
 - * so we can say this equals $\vec{\mathbf{w}} \cdot \vec{\mathbf{f}}$, since it is also in $(-\infty, \infty)$
 - Exponentiating both sides gives us: $\frac{p(y=\text{true}|x)}{1-p(y=\text{true}|x)} = e^{\vec{\mathbf{w}} \cdot \vec{\mathbf{f}}}$

- So,
$$\frac{p(y = \text{true } | x)}{1 - p(y = \text{true } | x)} = e^{\vec{\mathbf{w}} \cdot \vec{\mathbf{f}}}$$
$$p(y = \text{true } | x) = \frac{e^{\vec{\mathbf{w}} \cdot \vec{\mathbf{f}}}}{1 + e^{\vec{\mathbf{w}} \cdot \vec{\mathbf{f}}}} = \frac{1}{1 + e^{-\vec{\mathbf{w}} \cdot \vec{\mathbf{f}}}} \quad \text{(logistic function)}$$

• To classify, just use $\sum_{i=0}^{N} w_i f_i > 0$

$$p(y = \text{true} \mid x) > p(y = \text{false} \mid x)$$

$$\frac{p(y = \text{true} \mid x)}{p(y = \text{false} \mid x)} > 1$$

$$\frac{p(y = \text{true} \mid x)}{1 - p(y = \text{true} \mid x)} > 1$$

$$e^{\vec{\mathbf{w}} \cdot \vec{\mathbf{f}}} > 1 \quad \text{from above}$$

$$\vec{\mathbf{w}} \cdot \vec{\mathbf{f}} > 0$$

Learning

$$\begin{split} \hat{w} &= \text{argmax}_w \ \prod_i \ p(y^{(i)} \mid x^{(i)}) \\ &= \text{argmax}_w \ \prod_i \ \left\{ \begin{array}{l} p(y^{(i)} = 1 \mid x^{(i)}) & \text{for } y^{(i)} = 1 \\ p(y^{(i)} = 0 \mid x^{(i)}) & \text{for } y^{(i)} = 0 \end{array} \right. \end{split}$$

- convex optimization
- gradient ascent or L-BFGS

4 Maximum Entropy

- Multi-class logistic regression
- Generalizing the above: we want the probability of a class c for some observation x, $p(c \mid x)$

$$p(c \mid x) = \frac{\exp\left(\sum_{i=0}^{N} w_{ci} f_i\right)}{Z} = \frac{\exp\left(\sum_{i=0}^{N} w_{ci} f_i\right)}{\sum_{c' \in C} p(c' \mid x)} = \frac{\exp\left(\sum_{i=0}^{N} w_{ci} f_i\right)}{\sum_{c' \in C} \exp\left(\sum_{i=0}^{N} w_{c'i} f_i\right)}$$

- Z is the normalization factor
- \bullet So, we need to assign a per-class, per-feature weight w_{ci} for each feature f_i and class c

Binary Features

- In NLP, we typically use binary features: "ends in -ed", "starts with capital", etc
- Indicator function: feature that takes only values 0 or 1
- $f_i(c, x)$: feature i for class c for observation x:

$$p(c \mid x) = \frac{\exp\left(\sum_{i=0}^{N} w_{ci} f_i(c, x)\right)}{\sum_{c' \in C} \exp\left(\sum_{i=0}^{N} w_{c'i} f_i(c', x)\right)}$$

Example: POS prediction based on word (not sequence tagging!)

- He/PRP is/VBZ expected/VBN to/TO race/?? tomorrow/
- Indicator function: feature that takes only values 0 or 1
- $f_i(c,x)$: feature i for class c for observation x:

$$\begin{split} f_1(c,x) &= 1 \text{ iff } word_i = \text{``race''} & \& c = \text{NN} \\ f_2(c,x) &= 1 \text{ iff } t_{i-1} = \text{TO} & \& c = \text{VB} \\ f_3(c,x) &= 1 \text{ iff suffix}(word_i) = \text{``ing''} & \& c = \text{VBG} \\ f_4(c,x) &= 1 \text{ iff isLowerCase}(word_i) & \& c = \text{VB} \\ f_5(c,x) &= 1 \text{ iff } word_i = \text{``race''} & \& c = \text{VB} \\ f_6(c,x) &= 1 \text{ iff } t_{i-1} = \text{TO} & \& c = \text{NN} \end{split}$$

• Now we can compute the relative probabilities (can leave out $f_i(c,x) = 0$ entries):

$$p(VB \mid x) = \frac{e^{0.8} e^{0.01} e^{0.1}}{e^{0.8} e^{-1.3} + e^{0.8} e^{0.01} e^{0.1}} = 0.80$$
$$p(NN \mid x) = \frac{e^{0.8} e^{-1.3}}{e^{0.8} e^{-1.3} + e^{0.8} e^{0.01} e^{0.1}} = 0.20$$

- Features can be arbitrarily complex
 - For example, this feature can be used to recognize that words at the start of the sentence may be capitalized even though they are common nouns.

$$f_{125}(c,x) = 1 \text{ iff } word_{i-1} = \langle S \rangle \& \text{ startsUpper}(word_i) \& c = NN$$

Learning

- $\hat{w} = \operatorname{argmax}_{w} \sum_{i} \log p(y^{(i)} \mid x^{(i)})$
- Regularization is like smoothing: $\hat{w} = \operatorname{argmax}_w \sum_i \log p(y^{(i)} \mid x^{(i)}) \alpha \sum_{i=1}^n w_i^2$
 - Reduce the score if the weights are high
 - Penalizes overly-specific weights
 - $-\alpha$ controls the amount of regularization (analogous to λ in smoothing)
 - Assume a Gaussian prior on the weights, saying they prefer to have a value 0

5 MaxEnt Markov Model

- Descriminative Model: discriminate between various possible tag sequences
- use o for "observation" (word) since w means "weight"
- Single probabilistic model to estimate $p(t_i \mid o_i, t_{i-1})$ instead of two separate models like HMM

Recall for HMM:

- Likelihood of sequence: $p(t_1...t_n \mid o_1...o_n) = \prod_{i=1}^n p(o_i \mid t_i) \cdot p(t_i \mid t_{i-1})$
- $\hat{t}_i^n = \operatorname{argmax}_{t_1^n} p(t_1...t_n \mid o_1...o_n) \approx \operatorname{argmax}_{t_1^n} \prod_{i=1}^n p(o_i \mid t_i) \cdot p(t_i \mid t_{i-1})$
- $v_i(k) = p(w_i \mid k) \cdot \max_{k' \in T} v_{i-1}(k') \cdot p(k \mid k')$

For an MEMM:

- $p(t_i \mid o_i, t_{i-1}) = \frac{1}{Z(t_{i-1}, o_i)} \exp(\sum_i w_i f_i(t_i, o_i))$
- $\hat{t}_i^n = \operatorname{argmax}_{t_1^n} \ p(t_1...t_n \mid o_1...o_n) \approx \operatorname{argmax}_{t_1^n} \ \prod_{i=1}^n p(t_i \mid o_i, t_{i-1})$
- $v_i(k) = \max_{k' \in T} v_{i-1}(k') \cdot p(k \mid o_i, k')$

6 Information Extraction

"United States President Barack Obama discussed changes in the U.S. economy. Obama presented his ideas to Congress."

- Named Entity Recognition: Identify all the named entities (sequences of proper nouns)
 - United States, President Barack Obama, U.S., Obama, Congress
- Detect all entities: all nouns or pronouns
 - {Named entities} + changes, economy, his, ideas
- Co-reference: Determine whether two mentions refer to the same entity
 - United States = U.S.
 - President Barack Obama = Obama = his
- Relation Extraction: Find all words relating two entities

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\{\text{Obama}\} \leftarrow \text{discuss} \rightarrow \text{changes} \{\text{Obama}\} \leftarrow \text{present} \rightarrow \text{ideas} \{\text{Obama}\} \leftarrow \text{present\_to} \rightarrow \text{Congress}
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• Semantic Role Labeling (VerbNet, PropBank)

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{Obama} discussant_of discuss {Obama} giver_of present changes topic_of discuss ideas theme_of present Congress recipient_of present
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Identifying sequence spans

- B-TYPE (beginning), I-TYPE (inside), O (outside)
- United States President Barack Obama discussed changes in the U.S. economy B-ORG I-ORG B-PER I-PER I-PER O O O B-ORG O
- Predict with an MEMM based on words, previous tag sequence, and any other features (word, wordshape, POS)
- Can use a BIO model of POS as a features:
 - if bt_{i-1} =B-NNP, bt_i =I-NNP, and ne_{i-1} =B-PER, then probably ne_i =I-PER
- Or tree-path features: NP \uparrow S \downarrow NP

Pipelines

- POS useful in NER (NNP \rightarrow named entity)
- NER useful in Co-Reference (a is a person, b is an organization $\rightarrow a \neq b$)
- NER and Co-Reference useful in SRL ("giver" likely to be a PER, or maybe an ORG)