IPX Reference Guide

July 6, 2019

Contents

1	Functionality	1
	1.1 Problem Formulation	1
	1.2 Interior Point Method	
	1.3 Crossover	2
2	Usage	2
	2.1 C++ Interface	2
	2.2 C Interface	4
3	Parameters	5
4	Info	9

Functionality 1

Problem Formulation 1.1

IPX solves linear programming (LP) problems in the form

subject to
$$Ax\{\geq, \leq, =\}$$
rhs, (1b)

$$1b \le x \le ub.$$
 (1c)

The matrix A has num_constr rows and num_var columns. Associated with (1b) are dual variables y with the sign convention that

$$y[i] \ge 0$$
 if constraint is of type \ge , (2a)

$$y[i] \le 0$$
 if constraint is of type \le , (2b)

$$y[i]$$
 free if constraint is of type =. (2c)

Associated with $1b \le x$ and $x \le ub$ are dual variable $z1 \ge 0$ and $zu \ge 0$ respectively. Entries of -1b and ub can be infinity, in which case the dual is fixed at zero.

1.2 Interior Point Method

The interior point method (IPM) computes a primal-dual point (x, slack, xl, xu, y, zl, zu) that approximately satisfies

$$Ax + slack = rhs, \quad x - xl = lb, \quad x + xu = ub,$$
 (3a)

$$A^T y + z1 - zu = obj, (3b)$$

and that is guaranteed to satisfy $xl \ge 0$, $xu \ge 0$, (2) and

$$slack[i] \le 0$$
 if constraint is of type \ge , (4a)

$$\operatorname{slack}[i] \ge 0$$
 if constraint is of type \le , (4b)

$$slack[i] = 0$$
 if constraint is of type =. (4c)

In theory, the IPM iterates will in the limit satisfy (3a) and (3b), and the primal objective will equal the dual objective

$$rhs^{T}y + lb^{T}zl - ub^{T}zu. (5)$$

(Entries for which -1b or ub is infinity are understood to be dropped from the sum.)

1.3 Crossover

The crossover method recovers an optimal basis from the interior solution. A basis is defined by variable and constraint statuses

$$\verb|vbasis||j| \in \{\verb|IPX_basic|, \verb|IPX_nonbasic_ub|, \verb|IPX_nonbasic_ub|, \verb|IPX_superbasic|\}, \qquad (6)$$

$$\mathtt{cbasis}[i] \in \{\mathtt{IPX_basic}, \mathtt{IPX_nonbasic}\}. \tag{7}$$

The columns of A for which $vbasis[j] = IPX_basic$ and the columns of the identity matrix for which $cbasis[i] = IPX_basic$ form a square, nonsingular matrix of dimension num_constr . The corresponding basic solution (x, slack, y, z) is obtained by setting

$$\mathbf{z}[j] = 0$$
 if $vbasis[j] = IPX_basic$, (8a)

$$x[j] = lb[j]$$
 if $vbasis[j] = IPX_nonbasic_lb$, (8b)

$$x[j] = ub[j]$$
 if $vbasis[j] = IPX_nonbasic_ub$, (8c)

$$x[j] = 0$$
 if $vbasis[j] = IPX_superbasic,$ (8d)

$$y[i] = 0$$
 if cbasis[i] = IPX_basic, (8e)

$$slack[i] = 0$$
 if $cbasis[i] = IPX_nonbasic$ (8f)

and computing the remaining components such that Ax + slack = rhs and $A^Ty + z = obj$. The basis is primal feasible if $lb \le x \le ub$ and (4) hold; the basis is dual feasible if (2) holds and

$$z[j] \ge 0$$
 if $vbasis[j] = IPX_nonbasic_lb,$ (9a)

$$z[j] \le 0$$
 if $vbasis[j] = IPX_nonbasic_ub$, (9b)

$$z[j] = 0$$
 if $vbasis[j] = IPX_superbasic. (9c)$

The IPX crossover consists of a primal and dual push phase. Depending on the accuracy of the interior solution and the numerical stability of the LP problem, the obtained basis may not be primal and/or dual feasible. In this case reoptimization with an external simplex code is required.

2 Usage

2.1 C++ Interface

User code written in C++ must include the header file src/lp_solver.h. Both the src/and include/ directories must be in the compiler's search path for header files.

The following code snippet illustrates the use of the C++ interface. The complete example program can be found in example/afiro.cc and can be compiled by calling make in the example/ directory.

```
#include <cmath>
#include <iostream>
#include "lp_solver.h"
using Int = ipxint;
constexpr Int num_var = 12;
constexpr Int num_constr = 9;
const double obj[] = { -0.2194, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0
                                         -0.5564, 0.6, -0.48 };
const double lb[num_var] = { 0.0 };
const double ub[] = { 80.0, 283.303, 283.303, 312.813, 349.187, INFINITY,
                                       INFINITY, INFINITY, 57.201, 500.0, 500.501, 357.501};
// Constraint matrix in CSC format with 0-based indexing.
const Int Ap[] = { 0, 2, 6, 10, 14, 18, 20, 22, 24, 26, 28, 30, 32 };
const Int Ai[] = { 0, 5, ... };
const double Ax[] = \{ -1.0, 0.301, ... \};
const double rhs[] = { 0.0, 80.0, 0.0, 0.0, 0.0, 0.0, 0.0, 44.0, 300.0 };
int main() {
       ipx::LpSolver lps;
       // Solve the LP.
       Int status = lps.Solve(num_var, obj, lb, ub, num_constr, Ap, Ai, Ax, rhs,
                                                constr_type);
       if (status != IPX_STATUS_solved) {
              // no solution
              // (invalid input, time/iter limit, numerical failure, out of memory)
              std::cout << " status: " << status << ','
                                << " errflag: " << lps.GetInfo().errflag << '\n';</pre>
              return 1;
       // Get solver and solution information.
       ipx::Info info = lps.GetInfo();
       // Get the interior solution (available if IPM was started).
       double x[num_var], xl[num_var], xu[num_var], slack[num_constr];
       double y[num_constr], zl[num_var], zu[num_var];
       lps.GetInteriorSolution(x, x1, xu, slack, y, z1, zu);
       // Get the basic solution (available if crossover terminated without error).
       double xbasic[num_var], sbasic[num_constr];
       double ybasic[num_constr], zbasic[num_var];
       Int cbasis[num_constr], vbasis[num_var];
       lps.GetBasicSolution(xbasic, sbasic, ybasic, zbasic, cbasis, vbasis);
       return 0;
}
```

All classes and functions belonging to IPX are declared in namespace ipx. There are three classes that are intended for direct use from external code: ipx::LpSolver, ipx::Parameters and ipx::Info. The Parameters and Info classes have no methods (except for a constructor that initializes them to default values). They are used to pass control parameters to IPX and to return information about the execution of the solver. They are documented in Sections 3 and 4.

The LpSolver class provides the user interface to the LP solver. Its methods are documented through source code comments in src/lp_solver.h. The method

requires the LP problem in the form (1) with the constraint matrix A given in compressed sparse column (CSC) format. That means, Ap is an array of size $num_var + 1$ and Ai and Ax are arrays of size equal to the number of entries in A. They must be set such that Ai[Ap[j]..Ap[j+1]-1] and Ax[Ap[j]..Ap[j+1]-1] hold the row indices and nonzero values of the entries in column j of A. The entries within each column can be in any order, but there must be no duplicates. The input is checked prior to running the solver and an error code is returned if it is invalid (see Section 4).

2.2 C Interface

C wrapper functions for the methods of class LpSolver are provided to allow using the full functionality of IPX from C code. User code written in C must include the header file include/ipx_c.h.

The following code snippet illustrates the use of the C interface. The complete example program can be found in example/afiro_c.c and can be compiled by calling make in the example/ directory.

```
#include <math.h>
#include <stdio.h>
#include "ipx_c.h"
typedef ipxint Int;
#define NUM_VAR 12
#define NUM_CONSTR 9
-0.5564, 0.6, -0.48 };
const double lb[NUM_VAR] = { 0.0 };
const double ub[] = { 80.0, 283.303, 283.303, 312.813, 349.187, INFINITY,
                  INFINITY, INFINITY, 57.201, 500.0, 500.501, 357.501};
// Constraint matrix in CSC format with 0-based indexing.
const Int Ap[] = { 0, 2, 6, 10, 14, 18, 20, 22, 24, 26, 28, 30, 32 };
const Int Ai[] = { 0, 5, ... };
const double Ax[] = \{ -1.0, 0.301, ... \};
const double rhs[] = { 0.0, 80.0, 0.0, 0.0, 0.0, 0.0, 0.0, 44.0, 300.0 };
int main() {
   void *lps = NULL;
```

```
// Create new solver instance. This allocates a tiny amount of memory.
   ipx_new(&lps);
   if (!lps) return 1;
   // Solve the LP.
   Int status = ipx_solve(lps, NUM_VAR, obj, lb, ub, NUM_CONSTR, Ap, Ai, Ax,
                           rhs, constr_type);
   if (status != IPX_STATUS_solved) {
        // no solution
        // (invalid input, time/iter limit, numerical failure, out of memory)
        struct ipx_info info = ipx_get_info(lps);
        printf(" status: %ld, errflag: %ld\n", (long) status,
               (long) info.errflag);
       return 2;
   }
   // Get solver and solution information.
   struct ipx_info info = ipx_get_info(lps);
   // Get the interior solution (available if IPM was started).
   double x[NUM_VAR], x1[NUM_VAR], xu[NUM_VAR], slack[NUM_CONSTR];
   double y[NUM_CONSTR], z1[NUM_VAR], zu[NUM_VAR];
   ipx_get_interior_solution(lps, x, xl, xu, slack, y, zl, zu);
   // Get the basic solution (available if crossover terminated without error).
   double xbasic[NUM_VAR], sbasic[NUM_CONSTR];
   double ybasic[NUM_CONSTR], zbasic[NUM_VAR];
   Int cbasis[NUM_CONSTR], vbasis[NUM_VAR];
   ipx_get_basic_solution(lps, xbasic, sbasic, ybasic, zbasic, cbasis, vbasis);
   // Must call ipx_free() to deallocate memory in solver instance.
   ipx_free(&lps);
   return 0;
}
```

The solver is accessed through a void pointer, which must be allocated by <code>ipx_new</code> and finally deallocated by <code>ipx_free</code> (see <code>include/ipx_c.h</code> for documentation of these functions). The remaining functions of the C interface are simply wrappers around the methods of class <code>LpSolver</code>, which require the pointer to the solver as their first argument.

Parameters and solver information are passed in and out through data types struct ipx_parameters and struct ipx_info, which are the same as ipx::Parameters and ipx::Info in the C++ interface and are documented in Sections 3 and 4. In contrast to the C++ interface, the struct members are uninitialized when a data type is defined. A struct ipx_parameters object with default parameter values is returned by ipx_default_parameters().

3 Parameters

display

Type: ipxint Default: 1

If nonzero, then solver log is printed to standard output.

logfile

Type: const char*

Default: NULL

Name of file to which solver log is appended. The file is created if it does not exist. If logfile is NULL or the empty string (""), file logging is turned off.

print_interval

Type: double Default: 5.0

Frequency (in seconds) for printing progress reports during construction of the starting basis and crossover. If negative, progress reports are turned off. If zero, a progress line is printed after each basis update.

time_limit

Type: double Default: -1.0

Time limit (in seconds) for the solver. If negative, no time limit is imposed.

dualize

Type: ipxint Default: -1

Controls dualization of the LP model by the preprocessor.

 $\begin{array}{ll} 0 & \text{model is not dualized} \\ \geq 1 & \text{model is dualized} \end{array}$

< 0 an automatic choice is made

scale

Type: ipxint Default: 1

Controls the automatic scaling of the LP model by the preprocessor.

 ≤ 0 no scaling is applied

1 recursive equilibration of rows and columns of the constraint matrix

> 1 currently the same as 1, but reserved for future options

ipm_maxiter

Type: ipxint Default: 300

Maximum number of interior point iterations.

$ipm_feasibility_tol$

Type: double Default: 1e-6

The interior point solver terminates when a feasibility and an optimality criterion are satisfied. The feasibility criterion requires that the relative primal and dual residuals are not larger than ipm_feasibility_tol.

ipm_optimality_tol

Type: double Default: 1e-8

The interior point solver terminates when a feasibility and an optimality criterion are satisfied. The optimality criterion requires that the relative gap between the primal and dual objective values is not larger than ipm_optimality_tol.

ipm_drop_primal

Type: double Default: 1e-9

Controls handling of primal degeneracies in the interior point solve. If a degenerate variable $\mathtt{xl}[j]$ or $\mathtt{xu}[j]$ is not larger than $\mathtt{ipm_drop_primal}$, then its dual variable is fixed at its current value and eliminated from the optimization; $\mathtt{x}[j]$ will then be at its bound in the solution. If $\mathtt{ipm_drop_primal}$ is zero or negative, no primal variables are dropped. If the model was dualized by the preprocessor, then this option affects dual degeneracies in the input model.

ipm_drop_dual

Type: double Default: 1e-9

Controls handling of dual degeneracies in the interior point solve. If the dual variables $\mathtt{zl}[j]$ and $\mathtt{zu}[j]$ are degenerate and not larger than $\mathtt{ipm_drop_dual}$, then $\mathtt{x}[j]$ is fixed at its current value and eliminated from the optimization. The dual variables will then be zero in the solution. If $\mathtt{ipm_drop_dual}$ is zero or negative, no dual variables are dropped. If the model was dualized by the preprocessor, then this option affects primal degeneracies in the input model.

kkt_tol

Type: double Default: 0.3

Controls the accuracy to which the KKT linear equation systems are solved by an iterative method. A smaller value reduces the number of interior point iterations but increases the computational cost per iteration. Typical values are within the interval [0.05, 0.5].

precond_dense_cols

Type: ipxint Default: 1

In combination with diagonal preconditioning, controls handling of "dense" columns (i. e. columns with a relatively large number of entries). If nonzero, dense columns are treated separately by a low rank update.

crash_basis

Type: ipxint
Default: 1

Controls the construction of the starting basis for the preconditioner.

- ≤ 0 slack basis
 - 1 crash method that prefers variables with a larger interior point scaling factor
- > 1 currently the same 1, but reserved for future options

The chosen procedure (slack basis, crash) is followed by a sequence of basis updates that makes free variables basic and fixed (slack) variables nonbasic.

dependency_tol

Type: double Default: 1e-6

Controls the detection of linearly dependent rows and columns while constructing the starting basis. If possible, columns corresponding to free variables are pivoted into the basis, and slack columns corresponding to equality constraints are pivoted out of the basis. Hereby a nonbasic variable cannot replace a basic variable if the pivot element is less than or equal to dependency_tol. A negative value is treated as 0.0.

$volume_tol$

Type: double Default: 2.0

Controls the update of the basis matrix from one interior point iteration to the next. An entry of the scaled tableau matrix is used as pivot element if it is larger than volume_tol in absolute value. Increasing the parameter usually leads to fewer basis updates but more iterations of the linear solver. Typical values are in the interval [1.1, 10.0]. A value smaller than 1.0 is treated as 1.0.

rows_per_slice

Type: ipxint Default: 10000

Controls the update of the basis matrix from one interior point iteration to the next. The search for pivot elements partitions the tableau matrix into slices, each slice containing approximately rows_per_slice rows. A smaller value leads a finer pivot search and possibly a better preconditioner, but makes the update procedure more expensive.

maxskip_updates

Type: ipxint Default: 10

Controls the update of the basis matrix from one interior point iteration to the next. For each slice of the tableau matrix, the update search is terminated after computing maxskip_updates columns of the tableau matrix that do not contain eligible pivot elements. Decreasing the parameter makes the update procedure faster, but may affect the quality of the basis.

lu_kernel

Type: ipxint Default: 0

Chooses the method/package for computing and updating the LU factorization of basis matrices.

- ≤ 0 BASICLU for factorization and update
 - 1 BASICLU for factorization, Forrest-Tomlin update without hypersparsity
- > 1 currently the same as 1, but reserved for future options

$lu_pivottol$

Type: double Default: 0.0625

Partial pivoting tolerance for the LU factorization. The tolerance is tightened automatically if a factorization is detected unstable.

crossover

Type: ipxint Default: 1

If nonzero, crossover is used for recovering an optimal basis.

$crossover_start$

Type: double Default: 1e-8

Tightens the IPM termination criterion for crossover. At the beginning of crossover, the final IPM iterate is dropped to complementarity (i.e. for each pair of variables either the primal is set to its bound or the dual is set to zero). In addition to the standard IPM termination criterion, it is required that the relative primal or dual residual caused by dropping any variable is not larger than <code>crossover_start</code>. A nonpositive value means that the standard criterion is used. This parameter has no effect if crossover is turned off.

$pfeasibility_tol$

Type: double Default: 1e-7

A basic solution is considered primal feasible if the primal variables do not violate their bounds by more than pfeasibility_tol.

dfeasibility_tol

Type: double Default: 1e-7

A basic solution is considered dual feasible if the dual variables do not violate their sign condition by more than dfeasibility_tol.

4 Info

Class ipx::Info (for C++ users) and struct ipx_info (for C users) provide information about the input problem, the solver operations and the computed solution. Most of their members are almost self-explanatory and can be found in include/ipx_info.h.

There are four members holding status and error codes that are documented below. Possible values are macro-defined constants that have the prefix IPX_STATUS_ and IPX_ERROR_. They are defined in include/ipx_status.h.

status

Termination status of the solver. Possible values are:

IPX_STATUS_not_run Solver has not been called.

IPX_STATUS_solved Solver terminated successfully. The solution sta-

tus (optimal, imprecise, infeasible) is given by sta-

tus_ipm and status_crossover.

IPX_STATUS_stopped Solver stopped without having reached its termina-

tion criterion (time/iteration limit, numerical failure). status_ipm or status_crossover hold further

information.

IPX_STATUS_invalid_input Solver was not started because the input to Solve()

was invalid. errflag holds further information.

IPX_STATUS_out_of_memory Memory allocation failed.

IPX_STATUS_internal_error An internal error occured. This is a bug in IPX.

$status_ipm$

Status of the interior solution. Possible values are:

IPX_STATUS_not_run IPM has not been started.

IPX_STATUS_optimal IPM terminated successfully and the computed solu-

tion satisfies the feasibility and optimality tolerances.

IPX_STATUS_imprecise IPM reached its termination criterion, but the solu-

tion after postprocessing does not satisfy the feasi-

bility or optimality tolerance.

IPX_STATUS_primal_infeas The LP problem is primal infeasible because equality

constraints are inconsistent.

IPX_STATUS_dual_infeas The LP problem is dual infeasible because columns of

the constraint matrix corresponding to free variables are linearly dependent and their objective coefficients

cause an unbounded ray in primal space.

IPX_STATUS_time_limit IPM stopped because of time limit.
IPX_STATUS_iter_limit IPM stopped because of iteration limit.

IPX_STATUS_no_progress IPM stopped because no progress was achieved over

a number of iterations.

IPX_STATUS_failed Linear system solve or building the preconditioner

failed. errflag holds further information.

status_crossover

Status of the basic solution obtained by crossover. Possible values are:

IPX_STATUS_not_run Crossover has not been started.

IPX_STATUS_optimal Crossover terminated successfully and the computed

basic solution satisfies the feasibility and optimality

tolerances.

lution does not satisfy the feasibility or optimality tolerance. Reoptimization with a simplex solver is

needed.

IPX_STATUS_time_limit Crossover stopped because of time limit.

IPX_STATUS_failed Crossover failed due to linear algebra issues. errflag

holds further information.

errflag

Possible values when status = IPX_STATUS_invalid_input:

 ${\tt IPX_ERROR_invalid_matrix} \qquad \qquad {\tt Constraint\ matrix\ was\ invalid\ } ({\tt Ap}[0] \, \neq \, 0, \ {\tt Ap}[j] \, > \,$

Ap[j+1] for some j, non-finite entry in Ax, out-of-

range or duplicate index in Ai).

IPX_ERROR_invalid_vector One of the vectors obj, lb, ub, rhs and constr_type

contained an invalid entry, or $\mathtt{lb}[j] > \mathtt{ub}[j]$ for some

j.

Possible values when status_ipm or status_crossover = IPX_STATUS_failed:

IPX_ERROR_cr_iter_limit

IPX_ERROR_cr_matrix_not_posdef
IPX_ERROR_cr_precond_not_posdef
The iterative method for solving the linear systems

IPX_ERROR_cr_no_progress in the IPM did not converge.

IPX_ERROR_cr_inf_or_nan

IPX_ERROR_basis_singular

IPX_ERROR_basis_almost_singular
IPX_ERROR_basis_update_singular
IPX_ERROR_basis_repair_overflow
Factorizing or updating the basis matrix failed.

IPX_ERROR_basis_repair_search

IPX_ERROR_basis_too_ill_conditioned

IPX_ERROR_lapack_chol Cholesky factorization of dense Schur complement

matrix broke down.

In all cases the failure is caused by numerical difficulties. The specific error codes are meant for developers.