

GRE Quantitative Reasoning

Bo Wang

University of Virginia

Aug 2015

WEICHEN EDUCATION

GRE数学基本假设

- ① All numbers used are real numbers. 所有的数都是实数
- ② All figures lie on a plane unless otherwise indicated. 除非题目中有专门指出，假设所有图形都在同一平面内
- ③ All angle measures are positive 所有的测量角度都是正数
- ④ All line shown as straight are straight. On the computer-based test, lines that appear “jagged” can also be assumed to be straight. 所有显示为直线的线都可以当作直线来处理
- ⑤ Figures are intended to provide useful information for answering the questions. However, except where a figure is accomplished by a “Note” stating that the figure is drawn to scale, solve the problem using your knowledge of mathematics, not by visual measurement or estimation. 伴随问题的图形将为解题提供有用的信息。但是，只有在问题中指出相应图形是按比例画出(**drawn to scale**)时，才可以用目测或者估计而得到的信息去解题。否则只能运用你的数学知识去回答问题。

Example

n is an integer.

Quantity A

$$(-1)^n(-1)^{n+2}$$

Quantity B

$$1$$

Solution: 指数

$(-1)^n(-1)^{n+2} = (-1)^{2n+2}$, 因为 $2n+2$ 一定是个偶数, 所以 $(-1)^{2n+2} = 1$, 选C

WEICHEN EDUCATION

Practice

Of the following values of n , the value of $(\frac{1}{3})^n$ will be greatest for $n =$

- A. -3
- B. -2
- C. 0
- D. 2
- E. 3

Solution: $\frac{1}{3} < 1$, 所以指数越大数越小, 反之越大, 选A

WEICHEN EDUCATION

Practice

If n is an integer and $5^n > 4,000$, what is the least possible value of n ?

- A. 3
- B. 4
- C. 5
- D. 6
- E. 7

Solution: $5^5 = 3125$, $5^6 = 15625$ 所以选D

WEICHEN EDUCATION

Practice

$$3^x + 3^x + 3^x =$$

- A. 9^x
- B. 3^{x+1}
- C. 9^{3x}
- D. 3^{2x}
- E. 3^{3x}

Solution: $3^x + 3^x + 3^x = 3 * 3^x = 3^{x+1}$, 选B

WEICHEN EDUCATION

Exponents指数式

形如 x^a 的代数式称之为指数式，其中 $x \neq 0$ 称为base底， a 称为exponent指数

当 a, b 为整数的时候，以下成立。否则请谨慎

运算法则

① $x^{-a} = \frac{1}{x^a}$

② $(x^a)(x^b) = x^{a+b}$

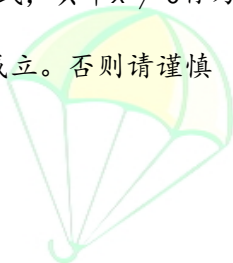
③ $\frac{x^a}{x^b} = x^{a-b}$

④ $x^0 = 1$

⑤ $(xy)^a = (x^a)(y^a)$

⑥ $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$

⑦ $(x^a)^b = x^{ab}$



WEICHEN EDUCATION

Example

What is the sum of the integers between -90 and 95 , inclusive?

- A. 5
- B. 185
- C. 465
- D. 4275
- E. 4560

Solution: 整数性质

等差数列求和公式: $(\text{首项} + \text{末项}) * \text{项数} / 2$

其中项数 = $(\text{末项} - \text{首项}) / \text{公差} + 1$

在这里首项为 -90 , 末项为 95 , 项数为 196 (不是 $195!!!$ 不是 $195!!!$ 不是 $195!!!$)

于是和为 $(-90 + 95) * 196 / 2 = 465$, 选C

Example

A certain sequence has 25 terms, all of which are positive. The first term in the sequence is an even integer, and the sum of any two consecutive terms is an odd integer.

Quantity A
number of even integers
in the sequence

Quantity B
number of odd integers
in the sequence

Solution: 奇偶性

第一项是偶数，之后任意连续两项都是奇数，于是任意连续两项都是一奇一偶，于是数列：偶奇偶奇偶...奇偶。总13偶数，12奇数，A大

WEICHEN EDUCATION

Practice

Which of the following could be the sum of three consecutive integers?

- A. 29
- B. 46
- C. 57
- D. 92
- E. 100

Solution: (首项+末项) * 项数 / 2, 其中项数为3, 于是一定是3的倍数, 只有C满足

WEICHEN EDUCATION

Practice

If x is an integer and $k = (x - 1)(x + 2) - (x - 1)(x - 2)$, then which of the following must be true?

- A. k is odd only when x is odd.
- B. k is odd only when x is even.
- C. k is even only when x is odd.
- D. k is even only when x is even.
- E. None of the above

Solution: $k = 4(x - 1)$ 所以 k 永远是偶数, 跟 x 无关, 选E

WEICHEN EDUCATION

Example

$$|2y - 5| < 1$$

Quantity A

y

Quantity B

1

Solution: 绝对值

$$|2y - 5| < 1$$

$$-1 < 2y - 5 < 1$$

$$4 < 2y < 6$$

$$2 < y < 3$$

于是肯定 y 大, 选A

WEICHEN EDUCATION

Example

Which of the following is equivalent to $0 < x < 2$?

- A. $x = 1$
- B. $|x| < 1$
- C. $|x| < 2$
- D. $|x - 1| < 1$
- E. $|x + 1| > 1$

Solution:

$$\begin{aligned} |x - 1| &< 1 \\ -1 &< x - 1 < 1 \\ 0 &< x < 2 \\ \text{于是选D} \end{aligned}$$

WEICHEN EDUCATION

Example

$$s = |t - 2|$$

Quantity A

$$s + 2$$

Quantity B

$$t$$

Solution:

$$s = |t - 2|$$

$$s = t - 2 \text{ or } s = -t + 2$$

所以选D, 可大可小

WEICHEN EDUCATION

Practice

x is some negative number

Quantity A

$$|x|$$

Quantity B

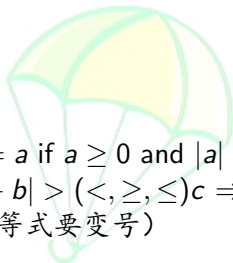
$$\sqrt{x^2}$$

Solution:

$$\sqrt{x^2} = -x = |x|, \text{ 选C}$$

WEICHEN EDUCATION

Absolute Value 绝对值



absolute value 绝对值: $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$

解带绝对值的不等式: $|ax + b| > (<, \geq, \leq)c \Rightarrow -c < ax + b < c$ 然后
解不等式 (记得乘除负数不等式要变号)

WEICHEN EDUCATION

Example

n is an odd integer between 2 and 10, and n is not a prime number.

Quantity A

n

Quantity B

9

Solution: 质数

n 是奇数而且在2到10之间，那么只有3,5,7,9，其中只有9不是质数。于是 $n = 9$ ，选C

WEICHEN EDUCATION

Example

If p and n are prime numbers, $p - n = 4$, and $\frac{3}{2} < \frac{p}{n} < 2$, what is the value of p ?

Solution:

一般而言做质数的题最简单有效的方法就是把前几项质数列举出来，然后按照条件甄选。2, 3, 5, 7, 11, 13, 17足矣。此题带入发现唯一可能是 $p = 11$, $n = 7$ 。

另，带入 $p - n = 4$, $\frac{3}{2} < \frac{n+4}{n} < 2$ 解得 $4 < n < 8$, 只有5, 7满足，其中只有 $n = 7$ 才有 p 也是质数。

WEICHEN EDUCATION

Example

Quantity A

the number of different
prime factors of 500

Quantity B

the number of different
prime factors of 360

Solution: 因数分解

$500 = 2^2 * 5^3$, 质因数只有2,5. $360 = 2^3 * 3^2 * 5$, 质因数有2,3,5. 选B

WEICHEN EDUCATION

Practice

If p, q are prime numbers, how many divisors does the product p^3q^6 have?

- A. 9
- B. 12
- C. 18
- D. 28
- E. 36

Solution: 因数分解, 排列组合

p^3q^6 的因子一定也具有 p^aq^b 的形式, 其中 a 可以从0, 1, 2, 3里选择, b 可以从0, 1, 2, 3, 4, 5, 6里选择, 而且互相不影响, 共 $4 * 7 = 28$ 种。

推广: 如果一个数的质因数分解为 $p_1^{k_1} \cdots p_n^{k_n}$, 那么它有 $(k_1 + 1) \cdots (k_n + 1)$ 个不同的因数, n 和不同的质因数

Example

If x , y , and z are integers greater than 1 and $xyz = 483$, which of the following could be the value of xy ?

- A. 15
- B. 22
- C. 69
- D. 91
- E. 144

Solution: 因数分解

$483 = 3 * 7 * 23$ 所以 x, y, z 里面某一个数是3, 一个是7, 一个是23, 于是 xy 可能为 21, 69, 161, 选C

WEICHEN EDUCATION

Example

k is a positive integer and 225 and 216 are both divisors of k . If $k = 2^a 3^b 5^c$, where a , b and c are positive integers, what is the least possible value of $a + b + c$?

- A. 4
- B. 5
- C. 6
- D. 7
- E. 8

Solution: 因数分解

$225 = 3^2 5^2$, $216 = 2^3 3^3$, 所以 a 至少是3, b 至少是3, c 至少是2, 选E

Practice

If k is an integer, and $\frac{35^2-1}{k}$ is an integer, then k could be each of the following, EXCEPT?

- A. 8
- B. 9
- C. 12
- D. 16
- E. 17

Solution: 因数分解

$35^2 - 1 = 1224$, 将五个选项带入尝试, 选D

另: $1224 = 2^3 * 3^2 * 17$, 只有16不行

Example

x is an integer greater than 3. Quantity A

Quantity B

the number of even
factors of $2x$

the number of odd
factors of $3x$

Solution:

帶入 $x = 4, 7$, 即可判斷為 D

WEICHEN EDUCATION

Example

Quantity A

$$x^2$$

Quantity B

$$x(x + 5)$$

Solution: 代数式比较

$B = x(x + 5) = x^2 + 5x$, $A - B = -5x$ 无法判断正负性, 于是选D

推广: 一般比较两个代数式的大小, 最好的办法是代数字进去算, 如 $x = 0, 1, -1$ 即可判断为D

如果不放心或者代数依然无法判断, 先将两个代数式化简, 然后做减法 $A - B$, 如结果 > 0 , 那么选A, 如结果 < 0 , 那么选B, $= 0$, 那么选C, 其余选D

WEICHEN EDUCATION

Example

$$0 < x < 2$$

Quantity A

$$2x + 3$$

Quantity B

$$3x + 2$$

Solution:

帶入0,1即可判断为D

另, $A - B = -x + 1$ 无法判断正负, 选D

WEICHEN EDUCATION

Example

$$a > 0$$

Quantity A

$$(a + a^{-1})^2$$

Quantity B

$$a^2 + a^{-2}$$

Solution: $A = a^2 + 2 * a * a^{-1} + a^{-2} = a^2 + 2 + a^{-2}$, 于是 $A - B = 2 > 0$, 选A

WEICHEN EDUCATION

Example

$$x > 1, y > 1$$

Quantity A

$$x^3y^2$$

Quantity B

$$(xy)^5$$

Solution: $\frac{A}{B} = \frac{1}{x^2y^3} < 1$ 所以B大

推广：将两个代数式化简，然后做除法法 $A \div B$, 如结果 > 1 , 那么选A, 如结果 < 1 , 那么选B, $= 1$, 那么选C, 其余选D

WEICHEN EDUCATION

Example

$$(x - y)^3 + (x - y) = 0$$

Quantity A Quantity B

x y

Solution: 解方程: $(x - y)^3 + (x - y) = (x - y)[(x - y)^2 + 1] = 0$ 因为后一项 $(x - y)^2 + 1 > 0$ 所以一定有 $x - y = 0$ 于是选C

WEICHEN EDUCATION

Example

$$x^2 + 6x = 7$$

Quantity A

$$(x + 3)^2$$

Quantity B

$$16$$

Solution: 解方程: $x^2 + 6x = 7$ 那么 $x = 1, -7$, 于是带入 $(x + 3)^2$, 选C
另: $(x + 3)^2 = x^2 + 6x + 9 = 7 + 9 = 16$

WEICHEN EDUCATION

Solving quadratic equations解一元二次方程

形如 $ax^2 + bx + c = 0$ 其中 $a \neq 0$ 的方程我们成为一元二次方程

方程的解我们可以用quadratic formula一元二次方程求根公式求

得: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta = b^2 - 4ac > 0$ 则方程有俩不同的解

$\Delta = b^2 - 4ac = 0$ 则方程有一个解

$\Delta = b^2 - 4ac < 0$ 则方程无解。

WEICHEN EDUCATION

常用公式

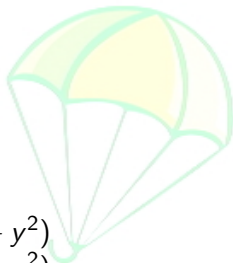
$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$



WEICHEN EDUCATION

Example

Sam and Tara together spent a total of \$1,400. Tara spent \$400 less than twice the amount Sam spent.

Quantity A

Quantity B

The amount that Sam spent

The amount that Tara spent

Solution: 列方程, 解方程: $s + t = 1400$, $t = 2s - 400$

解得 $s = 600$, $t = 800$, 选B

WEICHEN EDUCATION

Example

If x is 4 more than half of y and if y is 10 more than half of x , what is the value of x ?

Solution: 列方程, 解方程: $x = \frac{1}{2}y + 4$, $y = 10 + \frac{1}{2}x$
解得 $x = 12$, $y = 16$, 答案是12

WEICHEN EDUCATION

Example

A pianist agreed to perform one concert at a fee $12\frac{1}{2}$ percent less than her usual fee and a second concert at a fee 20 percent greater than the first fee. The fee for the second concert was what percent greater than her usual fee?

- A. 5%
- B. 7.5%
- C. 15%
- D. 16.25%
- E. 32.5%

Solution: 百分数: $(1 - 12.5\%) * (1 + 20\%) = 105\%$ 答案选A

Example

140.6 is approximately what percent less than 181.8?

- A. 22.7%
- B. 24.1%
- C. 26.5%
- D. 27.2%
- E. 29.3%

Solution: 百分数: $(181.8 - 140.6)/181.8 = 22.7\%$ 答案选A

WEICHEN EDUCATION

Example

For each of the last 5 years, the population of a colony of beetles increased by 8 percent of the preceding year's population. If P represents the current population of the colony, which of the following best represents the population 5 years ago, in terms of P ?

- A. $(5)(1.08P^{-1})$
- B. $(1.08)^{-5}P^{-1}$
- C. $(1.08P)^{-5}$
- D. $(1.08)^{-5}P$
- E. $(1.08)^{-5}P^5$

Solution:

每年上涨8%，涨5年就是 $(1.08)^5$ ，现在是 P ，那么五年前就是 $\frac{P}{(1.08)^5} = (1.08)^{-5}P$ ，选D

Example

The population of Country X for 1980 was P . The population of Country X increased by 3.8 percent in each of the next two years.

Quantity A

Quantity B

The population of Country X for 1982

$1.076P$

Solution:

每年上涨8%，涨5年就是 $(1.08)^5$ ，现在是 P ，那么五年前就是 $\frac{P}{(1.08)^5} = (1.08)^{-5}P$ ，选D

WEICHEN EDUCATION

Example

If 32 is 40 percent of x , then x is what percent of 320?

- A. 10%
- B. 25%
- C. 40%
- D. 65%
- E. 80%

Solution:

$40\%x = 32$ 所以 $x = 80$, $80/320 = 25\%$, 选A

WEICHEN EDUCATION

Practice

What percent of 15 is 15 percent of 1?

- A. 0.001
- B. 0.01
- C. 0.1
- D. 1
- E. 10

Solution:

15 percent of 1就是0.15, 也就是15的1%, 选D

WEICHEN EDUCATION

Example

According to a tax rate formula for a certain year, the amount of tax owed by an individual whose annual income was between \$31,850 and \$77,100 was equal to a base of \$4,386 plus 24 percent of the annual income that exceeded \$31,850. According to this formula, what was the amount of tax owed by an individual whose annual income that year was \$42,000?

Solution:

又臭又长的阅读，其实内容很简单，答案
 $4386 + 0.24 * (42000 - 31850) = 6822$

WEICHEN EDUCATION

percent 百分数

Example 1

What percent of 150 is 12.9?

Solution: $\frac{12.9}{150} = 0.086 = 8.6\%$

Example 2

Find 30% of 350

Solution: $30\% * 350 = 105$

Example 3

15 is 60% of what number?

Solution: $15 \div 60\% = 25$

Amount A increased by p percent is $A(1 + p\%)$

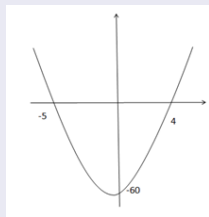
Amount A decreased by p percent is $A(1 - p\%)$

A is what percent larger (smaller) than B:

$|A - B| / B * 100\%$, 两个数的差(大的减小的)除以than后面的数

Example

Which of the following could be the equation of the graph in the xy -plane as shown?



- A. $y = x^2 + x - 60$
- B. $y = x^2 + x - 20$
- C. $y = x^2 + 3x - 60$
- D. $y = 3x^2 + x - 60$
- E. $y = 3x^2 + 3x - 60$

Solution: 函数图像：图中可以看出x截距为-5和4, y截距为-60, 带入方程只有D满足

另：分别对5个选项求根，BD满足根为-5和4，但是D满足过(0, -60)，B不满足

Example

In the xy -plane, the point $(t, t - 1)$ lies on the line with equation $y = -\frac{1}{2}x + \frac{1}{3}$. What is the value of t ?
Give your answer as a fraction.

Solution: 函数图像: 将 $(t, t - 1)$ 带入方程 $y = -\frac{1}{2}x + \frac{1}{3}$,
得 $t - 1 = -\frac{1}{2}t + \frac{1}{3}$, 解得 $t = 8/9$

WEICHEN EDUCATION

Example

In the xy -plane, the point (c, c) lies on the graph of the equation $0.3x + 0.3y = 12$

Quantity A

Quantity B

The value of c

20

Solution: 函数图像: 将 (c, c) 带入方程得 $0.3c + 0.3c = 12$, 解得 $c = 20$, 选C

WEICHEN EDUCATION

Example

In the rectangular coordinate system, (x, y) is a point on a circle that has center $(3, 2)$ and is tangent to x -axis at $(3, 0)$

Quantity A

Quantity B

The least possible value of x

0

Solution: 函数图像：这个圆的圆心在 $(3, 2)$ ，半径是2，于是 x 最小应该是1，选A

WEICHEN EDUCATION

Example

In the rectangular coordinate system, the point $(3, 1)$ is on a circle with center $(0, -3)$. What is the area of the circle?

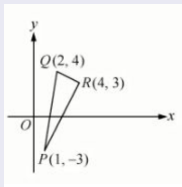
- A. 5π
- B. 7π
- C. 10π
- D. 25π
- E. $y\sqrt{7}\pi$

Solution: 函数图像: 半径就是两点的距离 $r = \sqrt{3^2 + 4^2} = 5$, 面积就是D

WEICHEN EDUCATION

Example

Which of the following statements about triangle PQR shown in the xy-plane are true?



- A. PQR is a right triangle.
- B. The area of PQR is $15/2$.
- C. PQR is an isosceles triangle.

Solution: 垂直与平行：两直线斜率乘积为-1时两直线垂直。
QR斜率为 $-\frac{1}{2}$, PR斜率为2, 于是R为直角, A正确。QR长度为 $\sqrt{5}$, PR长度为 $\sqrt{45} = 3\sqrt{5}$, 于是面积为 $15/2$, B正确, C不对。

graphs of functions 函数图像

$y = ax + b$ 的图像是一条直线，斜率的 a , y 截距为 b 。

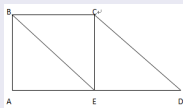
$y = ax^2 + bx + c$ 其中 $a \neq 0$ 的图像是一个抛物线 parabola
如 $y = x^2 - 2x - 3$ 的顶点 vertex 是 $(1, -4)$, $x = 1$ 是对称轴 line of symmetry, 开口向上 opens upward

$(x - a)^2 + (y - b)^2 = r^2$ 的图像是一个圆 circle, 圆心 center 为 (a, b) , radius 半径为 r

WEICHEN EDUCATION

Example

ABCE is a square, and BCDE is a parallelogram.



Quantity A

The area of ABCE

Quantity B

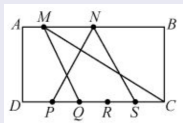
The area of BCDE

Solution: 几何图形的面积：正方形和平行四边形的底是一样的(BC), 高也是一样的(AB), 于是面积也一样, 选C

WEICHEN EDUCATION

Example

In rectangle ABCD, side DC is divided into five equal segments by points P, Q, R and S.



Quantity A

The area of MCQ

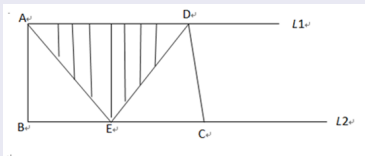
Quantity B

The area of NSP

Solution: 几何图形的面积: 两个三角形的底 $PS = CQ$, 高都是 AD , 于是面积一样, 选C

WEICHEN EDUCATION

Example



Quantity A

The area of shaded region

Quantity B

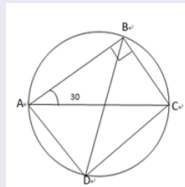
The sum of the two
unshaded triangular regions

Solution: 几何图形的面积: 没说平行没说长度相同, 不能确定, 选D

Example

In the figure, quadrilateral ABCD is inscribed in a circle with radius 10. The lengths of which of the following line segments can be determined from the information given? Indicates all such line segments.

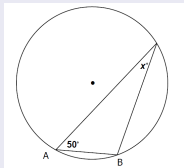
- | | |
|-------|-------|
| A. AB | B. AC |
| C. AD | D. BC |
| E. BD | F. CD |



Solution: 圖：ABC为直角，于是AC为直径。BAC为30度，所以AB,BC可求得。其他都不能确定。

Example

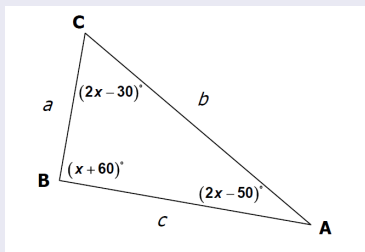
If the radius of the circle above is equal to the chord AB , then what is the value of x ?



- A. 25
- B. 30
- C. 40
- D. 45
- E. 50

Solution: 圆: 圆心称为 O , 那么 OAB 就是等边三角形, 于是圆心角是 60 度, 那么对应的圆周角就是一半, 30 度。所以 $x = 30$, 选B
注: 50 度的条件没用

Example



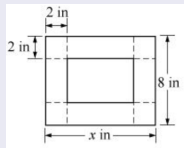
Quantity A
 $a^2 + c^2$

Quantity B
 b^2

Solution: 勾股定理：根据给定的三个角度求得 $x = 40$ ，于是角B为100度，此时两边平方和小于斜边。

Example

The thin rectangular sheet of metal shown in the figure is 8 inches wide and x inches long. An open box is to be made by cutting a 2-inch square from each corner of the sheet of metal and then folding up the sides. If the volume of the box is to be 48 cubic inches, what is the value of

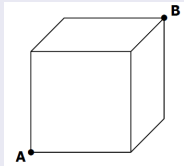


- x ?
- A. 6 B. 8
C. 10 D. 12
E. 14

Solution: **立体图形**: 这是一个长方体, 长为 $x - 4$, 宽4, 高2, 体积为48. 于是长为6, $x = 10$, 选C

Example

If the volume of the cube above is 64 cubic centimeters, what is the shortest distance, in centimeters, from point A to point B?



- A. $4\sqrt{2}$
- B. $4\sqrt{3}$
- C. $4\sqrt{6}$
- D. $8\sqrt{2}$
- E. $8\sqrt{3}$

Solution: 立体图形：长方体的对角线长度为 $\sqrt{a^2 + b^2 + c^2}$ ，此立方体体积为64，于是长宽高都是4，对角线长 $4\sqrt{3}$ ，选B

Example

The average of three different positive integers is 6.

Quantity A

Quantity B

The product of the three integers

25

Solution: **平均数**: 三个不同的正整数的平均是6, 那么他们三个乘积最小就是 $1 * 2 * 15 = 30 > 25$, 选A

注: 如果没有说positive那么选D

WEICHEN EDUCATION

Example

From a set of 100 numbers, half were selected to form group I, and 60 percent of the remaining numbers were selected to form group II. The average (arithmetic mean) of the numbers in group I is 24.4, and the average of the numbers in group II is 31.5. Which of the following is closest to the average of the numbers in groups I and II combined?

- A. 27.1
- B. 27.6
- C. 27.8
- D. 28.0
- E. 28.3

Solution: 平均数: group1 有50个数, 平均是24.4. group2 有30个数, 平均是31.5. 那么合在一起总共有80个数, 平均是 $(24.4 * 50 + 31.5 * 30) / 80 = 27.06$, 选A

Example

List L consists of 7 numbers u , $-2u$, $3u$, $-4u$, $5u$, $-6u$, and $7u$, where $u \neq 0$

Quantity A

Quantity B

The median of L

u

Solution: 中位数: 如果 $u > 0$, 那么从小到大排列是 $-6u$, $-4u$, $-2u$, u , $3u$, $5u$, $7u$

如果 $u < 0$, 那么从小到大排列是 $7u$, $5u$, $3u$, u , $-2u$, $-4u$, $-6u$

总之无论那种可能, 中位数都是 u , 选C

WEICHEN EDUCATION

Example

List L consists of 7 numbers. The range of the numbers in List L is 0.

Quantity A

Quantity B

The mean of L

0

Solution: **极差range**: 极差就是数据集里最大的减去最小的。这里极差是0, 于是L里面的7个数都是一样的。但是依然没办法确定mean和0哪个大。选D

WEICHEN EDUCATION

Example

List K consists of 16 positive numbers. List M is obtained from list K by multiplying each number in list K by -1

Quantity A

The standard deviation of K
deviation of M

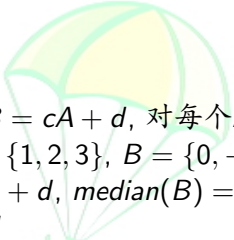
Quantity B

The standard

Solution: 标准差: $M = -K$, $d(M) = |-1|d(K) = d(K)$, 选C

WEICHEN EDUCATION

数据集变化，统计量的随之变化



如果A,B两个数据集，其中 $B = cA + d$ ，对每个A的元素x进行运算 $cx + d$ 就组成了B。如 $A = \{1, 2, 3\}$, $B = \{0, -2, -4\}$ ，那么 $B = 2 - 2A$ 。
那么 $mean(B) = c * mean(A) + d$, $median(B) = c * median(A) + d$,
 $mode(B) = c * mode(A) + d$
standard deviation $d(B) = |c| * d(A)$, range $range(B) = |c| * range(A)$

WEICHEN EDUCATION

Example

A jar contains exactly 10 dimes and x quarters and no other coins. If a coin is randomly selected from the jar, the probability that a quarter is selected is 0.6. What is the value of x ?

- A. 5
- B. 6
- C. 8
- D. 12
- E. 15

Solution: 概率: 总共有 $10 + x$ 硬币, 其中 x 是 quarter, 那么选出 quarter 的概率为 $\frac{x}{10+x} = 0.6$, 解得 $x = 15$, 选 E

Example

There are 30 balls in an urn, each of which is either red or yellow. If one ball is to be selected at random from the urn, the probability that the ball will be red is greater than $\frac{1}{3}$. Which of the following could be the number of yellow balls in the urn? Indicates all such number.

- A. 10
- B. 14
- C. 19
- D. 22
- E. 26

Solution: **概率**: 总共有30个球, 选出红球的概率大于 $\frac{1}{3}$, 说明红球多于10, 那么黄球就少于20.选ABC

Example

In how many ways can Ann, Bob, Chuck, Don and Ed be seated in a row such that Ann and Bob are not seated next to each other?

- A. 24
- B. 48
- C. 56
- D. 72
- E. 96

Solution: **排列组合**: 方法一: 随便排列的总共有 $A_5^5 = 120$ 种, 当 Ann 和 Bob 坐在一起总共有 $2 * A_4^4 = 48$ 种, 那么不坐在一起就有 72 种, 选 D

方法二: 这俩人太麻烦, 先把别人排好, 有 $A_3^3 = 6$ 种, 这三个排好之后有 4 个空位可以坐。选出其中两个, 其中一个给 Ann, 另一个给 Bob 就好, 总共有 $A_4^2 = 12$ 种, 乘起来就有 72 种。

Example

In how many ways can 3 boys and 3 girls be seated in a row of 6 chairs such that the girls are not separated and the boys are not separated?

- A. 24
- B. 36
- C. 72
- D. 144
- E. 288

Solution: **排列组合**: 既然男生要在一起, 那就把他们捆在一起, 捆的方式有 $A_3^3 = 6$ 种, 同理把女生全部捆在一起也有6种。当把男生女生分别捆住之后, 我们只需要考虑到底男生放在前面还是女生放在前面, 共2种, 全部乘起来就是72种。选C

Example

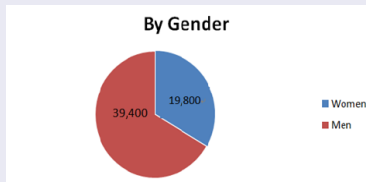
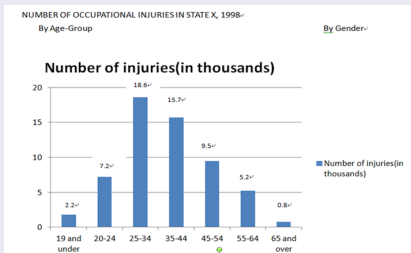
If there are 16 people to choose from, what is the ratio of the number of possible 7-person committees to the number of possible 8-person committees?

- A. 7:8
- B. 8:7
- C. 7:9
- D. 8:9
- E. 9:8

Solution: 排列组合: 7人的committees有 C_{16}^7 种, 8人的committees有 C_{16}^8 种, 比例为D

Example

How many of the age-groups each accounted for more than 15 percent of the total number of occupational injuries in State X in 1998?



- A. One B. Two C. Three D. Four E. Five

Solution: 总共有 $0.2 + 7.2 + 18.6 + 15.7 + 9.5 + 5.2 + 0.8 = 59.2$ 受伤, 15%就是8.88, 多于它的有3个, 选C

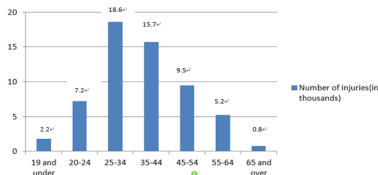
Example

In 1998, if one-half of the occupational injuries in the combined 34-and-under age-groups were incurred by men, what was the number of occupational injuries incurred by men in the combined 35-and-over age-groups?

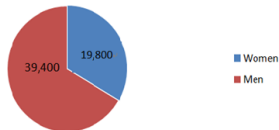
NUMBER OF OCCUPATIONAL INJURIES IN STATE X, 1998
By Age-Group

By Gender

Number of injuries(in thousands)



By Gender



- A. 33, 500 B. 31900 C. 30500 D. 25400 E. 21700

Solution: 34和以下的总共有 $2.2 + 7.2 + 18.6 = 28$, 一半就是14,
 $39400 - 14000 = 25400$ 选D

Example

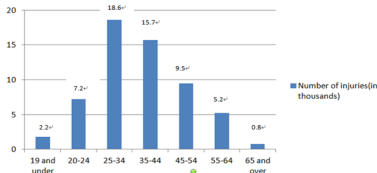
For the 55-64 age-group in 1998, the average (arithmetic mean) number of work-hours lost per occupational injury was 48.5. If a workweek is 40 work-hours, which of the following is closest to the total number of workweeks lost due to occupational injuries in the 55-64 age-group in 1998?

NUMBER OF OCCUPATIONAL INJURIES IN STATE X, 1998

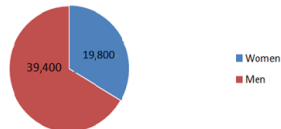
By Age-Group

By Gender

Number of injuries(in thousands)



By Gender

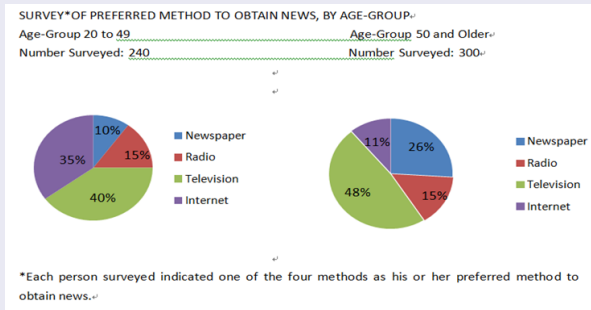


- A. 4500 B. 5200 C. 5500 D. 5900 E. 6300

Solution: 总共有5.2，每个损失时间48.5小时，那么总共损失星期数为 $5.2 * 48.5 / 40 = 6.305$ ，选E

Example

What fraction of the people in the age-group 20 to 49 indicated newspaper or the Internet as their preferred method to obtain news?

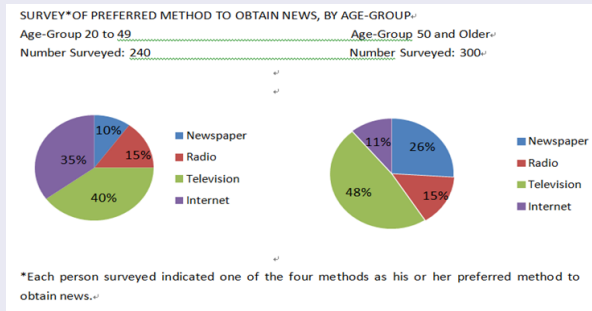


Write the answer as a quotient.

Solution: 一个10%，一个35%，总共45%，答案写成45/100，化简乘9/20也对。

Example

Which of the following is close to the percent of all the people surveyed who indicated the Internet as their preferred method to obtain news?

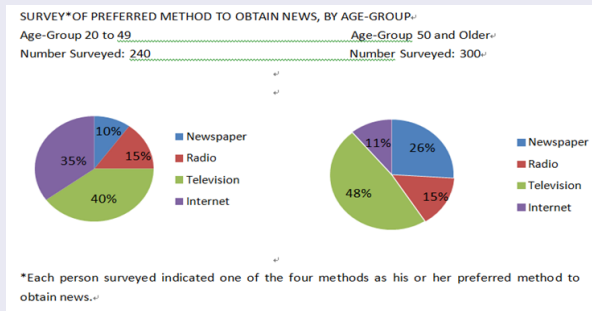


- A. 18.8% B. 21.7% C. 23.0% D. 33.3% E. 46.0%

Solution: 年轻人总240, 35%就是84, 老头子们总300, 11%就是33, 总共117人。总共540个人参与问卷, 比例为21.7%, 选B

Example

For the age-group 50 and older, the number of people who indicated the Internet as their preferred method to obtain news is approximately what percent less than the number of people who indicated radio?



- A. 12% B. 27% C. 36% D. 45% E. 50%

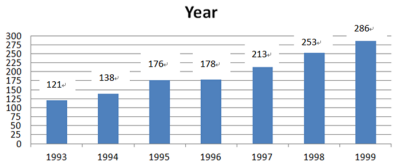
Solution: internet是11%, radio是15%, internet比radio少多少个百分点? $(15 - 11)/15 = 26.7\%$, 选B

Example

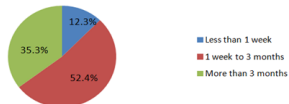
Which of the following is closest to the percent increase in the number of temporary employees from 1993 to 1999?

TEMPORARY EMPLOYMENT GERMANY, 1993-1999

Number of Temporary Employees



Percent of Temporary Employees
by Length of Employment Contract,
1999



- A. 36% B. 58% C. 136% D. 158% E. 236%

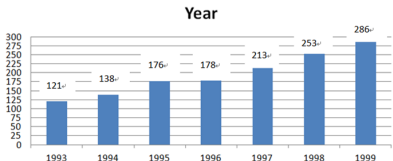
Solution: 从121到286, 增长了 $(286 - 121)/121 = 136\%$, 选C

Example

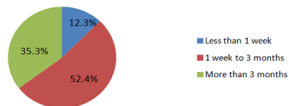
In 1999 approximately how many of the temporary employees had an employment contract with a length of at most 3 months?

TEMPORARY EMPLOYMENT GERMANY, 1993-1999

Number of Temporary Employees



Percent of Temporary Employees
by Length of Employment Contract,
1999



- A. 185,000 B. 150,000 C. 101,000 D. 35,000 E. 19,000

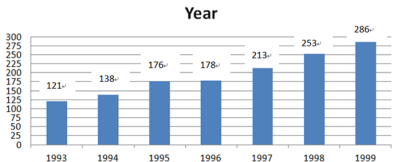
Solution: 总286，其中占12.3 + 52.4%，所以有185042，选A

Example

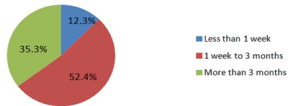
In 1998 the ratio of the number of female temporary employees to the number of male temporary employees was 1 to x , where $x > 0$. In terms of x , what was the number, in thousands, of female temporary employees in 1998?

TEMPORARY EMPLOYMENT GERMANY, 1993-1999

Number of Temporary Employees



Percent of Temporary Employees
by Length of Employment Contract,
1999



A. $253(x - 1)$

B. $253(x + 1)$

C. $253/x$

D. $253/(x - 1)$

E. $253/(x + 1)$

Solution: $F : m = 1 : x$ 所以 $F : (M + F) = 1 : (1 + x)$, 选E

Example

For how many of the four incentives listed does each of the companies surveyed that offers this incentive also offer a health-insurance benefit?

BENEFITS AND INCENTIVES SURVEY RESULTS FOR 600 COMPANIES

						Incentive	Number of Companies Offering Incentive
						Bonus Plan	482
						Profit Sharing	246
						Stock Ownership	78
						Stock Options	60
Benefit	Health Insurance	Flex-time	Disability Insurance	Tuition	Tele-commuting		
Number of Companies Offering Benefit	588	426	387	303	207		

Note: Each shaded cell is the intersection of a benefit column and an incentive row and contains the number of companies (out of the 600 surveyed) that offer both that benefit and that incentive. For example, 195 of the companies surveyed offer both a disability-insurance benefit and a profit-sharing incentive.

A. None B. One C. Two D. Three E. Four

Solution: 对于Bonus plan, 有482公司提供, 其中有481提供health insurance。其他的三种incentives都满足。选D

Example

A certain benefits and incentives package consists of 2 benefits to be chosen from the benefits offered by more than $1/2$ of all the companies surveyed and 1 incentive to be chosen from the incentives offered by more than $1/3$ of all the companies surveyed. How many such packages are possible?

BENEFITS AND INCENTIVES SURVEY RESULTS FOR 600 COMPANIES

						Incentive	Number of Companies Offering Incentive
	481	327	274	225	198	Bonus Plan	482
	246	103	195	186	112	Profit Sharing	246
	78	60	59	55	40	Stock Ownership	78
	60	41	44	41	24	Stock Options	60
Benefit	Health Insurance	Flex-time	Disability Insurance	Tuition	Tele-commuting		
Number of Companies Offering Benefit	588	426	387	303	207		

- A. 4 B. 6 C. 8 D. 10 E. 12

Solution: 总共600个公司，一半以上就是300个，health insurance, flex-time, disability insurance 和tuition都满足，需要从四个里面选两个，有6种选择。1/3以上的公司提供的incentive有Bonus plan 和profit sharing, 选一个有2种。乘起来有12种，选E。

Example

The ratio of the number of companies offering both a stock-options incentive and one of the benefits listed to the number of companies offering that benefit is greatest for which of the five benefits?

BENEFITS AND INCENTIVES SURVEY RESULTS FOR 600 COMPANIES

						Incentive	Number of Companies Offering Incentive
	481	327	274	225	198	Bonus Plan	482
	246	103	195	186	112	Profit Sharing	246
	78	60	59	55	40	Stock Ownership	78
	60	41	44	41	24	Stock Options	60
Benefit	Health Insurance	Flex-time	Disability Insurance	Tuition	Telecommuting		
Number of Companies Offering Benefit	588	426	387	303	207		

- A. Health insurance B. Flex-time C. Disability insurance D. Tuition E. Telecommuting

Solution: 五个选项比例分别为 $60 : 588$, $41 : 426$, $44 : 387$, $41 : 303$ 和 $24 : 207$, 最大的是 $41 : 303$, 选D



WEICHEN EDUCATION