

## BENCHMARK PROBLEM

Identification	16-A1	Source Situation: ID.16
Date Submitted:	July 1980	By: H. L. Dodds, Jr. (Univ. of Tennessee)
Date Accepted:	October 1985	By: R. M. Westfall (ORNL) C. E. Lee (Consult.)

Descriptive Title: *Delayed Supercritical Transient; One-Dimensional, Two-Group Neutron Transport Problem in a Fast Reactor*

## Reduction of Source Situation:

1. One-dimensional (slab with azimuthal symmetry), two-group neutron transport theory
2. Isotropic scattering
3. Zero return current boundary conditions on external surfaces
4. Steady-state critical initial conditions
5. Six delayed neutron precursor groups

The equations for this mathematical model are<sup>1</sup>

$$\begin{aligned}
 (1) \quad \frac{1}{v_g} \frac{\partial}{\partial t} \phi_g(x, \mu, t) + \mu \frac{\partial}{\partial x} \phi_g(x, \mu, t) + \Sigma_t^g(x, t) \phi_g(x, \mu, t) = \\
 \frac{1}{2} \sum_{g'=1}^2 \left[ \chi_g^p (1-\beta) v \Sigma_f^{g'}(x, t) + \Sigma_s^{g' \rightarrow g}(x, t) \right] \phi_{g'}(x, t) \\
 + \frac{1}{2} \sum_{i=1}^6 \lambda_i C_i(x, t) \chi_g^i + Q_g(x, \mu, t); \quad g = 1, 2
 \end{aligned}$$

$$(2) \quad \frac{\partial}{\partial t} C_i(x, t) + \lambda_i C_i(x, t) = \beta_i \sum_{g=1}^2 v \Sigma_f^g \phi_g(x, t); \quad i = 1, \dots, 6$$

where  $\phi_g(x, t)$  = scalar group flux (neutrons/cm<sup>2</sup>-sec)

$$\phi_g(x, t) = \int_{-1}^{+1} \phi_g(x, \mu, t) d\mu$$

$\phi_g(x, \mu, t)$  = angular group flux (neutrons/cm<sup>2</sup>-sec-unit cosine)

$$\phi_g(x, \mu, t) = \int_0^{2\pi} \phi_g(x, \mu, \varphi, t) d\varphi, \quad \varphi = \text{azimuthal angle}$$

$C_i(x, t)$  = concentration of type  $i$  precursor (nuclei/cm<sup>3</sup>)

and the remaining terms are defined in detail in References 1, 2, or 3.

For the problem,  $x_g^p = x_g^i = x_g$  and  $Q_1 = Q_2 = 0$ .

Date: Initial two-group constants shown in Table I. Delayed neutron parameters, fission spectra, and  $1/v$  values shown in Table II.

The initial configuration is made critical by dividing the production cross sections by  $k$ -eff, and the initial precursor concentrations are in equilibrium with the initial critical flux distribution.

Initial Perturbation:

At time  $t = 0.0$  sec, the density of the material in Zone 2 is increased by 5% and the density of the material in Zone 6 is decreased by 5% (resulting in a step insertion of reactivity at time  $t = 0.0$  sec).

Expected Primary Results:

1. Initial  $k$ -eff and initial scalar flux distribution for each group
2. Convergence requirements on flux (and eigenvalue) if an iterative solution is used
3. Total reactor power versus time (normalized such that the total neutron production rate at  $t = 0.0$  is 1.0 neutron/sec)
4. Time-dependent group flux (i.e., scalar flux) distributions
5. Sensitivity of results to time step size
6. CPU time and core storage requirements

Table I. Initial Two-Group Constants<sup>a</sup> (cm<sup>-1</sup>)

Zone	Group g	$v\Sigma_f^g$	$\Sigma_t^g$	$\Sigma_s^{g\rightarrow g}$	$\Sigma_s^{g\rightarrow g'}$
1,7 <sup>b</sup>	1	8.3441-4	2.411-1	2.33644-1	3.598-3
	2	3.2776-4	4.172-1	4.07004-1	0.0
2,4,6 <sup>c</sup>	1	7.4518-3	1.849-1	1.77711-1	2.085-3
	2	1.10612-2	3.668-1	3.53721-1	0.0
3,5 <sup>d</sup>	1	0.0	9.432-2	8.571-2	1.7168-3
	2	0.0	1.8762-1	1.7131-1	0.0

<sup>a</sup> $\Sigma_a^g = \Sigma_t^g - \Sigma_s^g$  and  $\Sigma_s^g = \Sigma_s^g + \Sigma_s^{g\rightarrow g'}$ .

<sup>b</sup>Blanket material in Zones 1 and 7.

<sup>c</sup>Core material in Zones 2, 4, and 6.

<sup>d</sup>Mixture of sodium and control rod material in Zones 3 and 5.

Table II. Delayed Neutron Parameters<sup>a</sup>

Type	$\beta_i$	$\lambda$ (sec <sup>-1</sup> )
1	0.81-4	0.0129
2	6.87-4	0.0311
3	6.12-4	0.134
4	11.38-4	0.331
5	5.12-4	1.26
6	1.70-4	3.21

<sup>a</sup>Prompt and delayed neutron spectra are identical with  $x_1 = 1.0$  and  $x_2 = 0.0$ , Also,  $1/v_1 = 1.851-9 \frac{\text{sec}}{\text{cm}}$  and  $1/v_2 = 1.088-8 \frac{\text{sec}}{\text{cm}}$ .

Possible Additional Results:

7. Zone-averaged power fractions versus time
8. Sensitivity of results to spatial mesh size
9. Sensitivity of results to angular quadrature (if solution is obtained by the discrete ordinates method)

Based on preliminary static k-eff calculations of the initial critical configuration using ANISN,<sup>4</sup> an  $S_4$  quadrature with 114 spatial intervals (as defined in Tables III and IV) is a sufficiently accurate representation for the angular and spatial discretization. Specifically, by using a finer spatial mesh (i.e., 228 intervals), k-eff changed by 0.0001 and using an  $S_8$  and  $S_{16}$  quadrature, k-eff changed by 0.0003 and 0.0003 respectively. Therefore, it is suggested that the "Expected Primary Results" requested above be obtained initially using the discretization indicated in Tables III and IV if the conventional discrete ordinates method is used to obtain solutions. Also, if a discrete ordinates method is used, it may be worthwhile to note that the units for angular flux in most discrete ordinates codes are per "unit weight" rather than per "unit cosine," as indicated earlier in the definitions following Eq. (2).

Table III.  $S_4$  Angular Quadrature

Cosine ( $\mu$ )	Weight
-1.0	0.0
-0.788675	0.25
-0.211325	0.25
+0.211325	0.25
+0.788675	0.25

Table IV. Spatial Mesh

Zone	Number of Equal Intervals
1	20
2	24
3	5
4	16
5	5
6	24
7	20

## REFERENCES

1. G. I. Bell and S. Glasstone, Nuclear Reactor Theory, Van Nostrand Reinhold Co., p. 466 (1970).
2. H. L. Dodds, Jr. et al., Nucl. Sci. Eng., 47, p. 264 (1972).
3. T. R. Hill and W. H. Reed, "TIMEX: A Time-Dependent Explicit Discrete Ordinates Program for the Solution of Multigroup Transport Equations with Delayed Neutrons," LA-6201-MS, Los Alamos Scientific Laboratory (1976).
4. W. W. Engle, Jr., "A User's Manual for ANISN, A One-Dimensional Discrete Ordinates Transport Code with Anisotropic Scattering," ORGDP-K-1693, Oak Ridge Gaseous Diffusion Plant (1967).

## BENCHMARK PROBLEM

Identification:	16-A1	Source Situation:	ID.16
Date Submitted:	July 1980	By:	H. L. Dodds, Jr. (Univ. of Tennessee)
Date Accepted:	October 1985	By:	R. M. Westfall Oak Ridge National Laboratory C. E. Lee (Consult.)

Acceptance is based upon agreement between independent solutions obtained with the TIMEX and TDA computer programs (solutions 16-A1.1 and 16-A1.2). In comparing the solutions it should be borne in mind that the power is normalized to one neutron per second at time equals zero. Also it should be noted that the tabulated values of the scalar flux are mesh-interval-average values for TDA and mesh-interval-boundary values for TIMEX. With regard to the power, solutions by TIMEX and TDA disagree by 2% or less for  $0 \leq t \leq 10^{-1}$  seconds and for  $t > 10^{-1}$  seconds, the disagreement varies from 2 to 4%. Therefore, the reviewers find the two solutions acceptable as computational benchmark solutions for the time interval  $0 \leq t \leq 10^{-1}$  seconds.

BENCHMARK PROBLEM SOLUTION

Identification: 16-A1-1

Benchmark Problem ID.16-A1

Date Submitted: November 1981

By: T. R. Hill (Los Alamos National  
Laboratory)

Date Accepted: October 1985

By: R. M. Westfall (ORNL)  
C. E. Lee (Consult.)Descriptive Title: Multigroup Discrete Ordinates Solution with TIMEX<sup>1</sup>

Mathematical Model: Multigroup energy approximation, discrete ordinates angular approximation, linear discontinuous spatial approximation,<sup>2</sup> first-order semi-implicit time difference scheme<sup>1</sup> with fine-mesh rebalance and exponential extrapolation.

Pertinent Features of Techniques Used:  $S_4$  angular quadrature, suggested 114 spatial intervals, 6 delayed neutron groups, exponential extrapolation of the time variable, with the following time steps used in each time zone:

Time Zone (sec)	$\Delta t$ (sec)
0 to $10^{-3}$	$2 \times 10^{-8}$
$10^{-3}$ to $10^{-1}$	$2 \times 10^{-6}$
$10^{-1}$ to 1.0	$2 \times 10^{-5}$
1.0 to 10.0	$2 \times 10^{-4}$

Computer: CDC 7600

Codes: ONETRAN, TIMEX (single precision, 14 decimal digits accuracy)

Date Solved: June 1981 at Los Alamos National Laboratory

## References

1. T. R. Hill and Wm. H. Reed, "TIMEX: A Time-Dependent Explicit Discrete Ordinates Program for the Solution of the Multigroup Transport Equations with Delayed Neutrons," Los Alamos Scientific Laboratory report LA-6201-MS (February 1976)
2. T. R. Hill, "ONETRAN: A Discrete Ordinates Finite Element Code for the Solution of the One-Dimensional Multigroup Transport Equation," Los Alamos Scientific Laboratory report LA-5990-MS (June 1975)



**Primary Results:**

Calculated  $k_{\text{eff}}$  by ONETRAN for initial two group constants of  
Table I : 1.000198

Table I  $v\Sigma_f^8$  were divided by this  $k_{\text{eff}}$  for all TIMEX calculations.

TIMEX uses a fixed time step in each time zone and requires no inner or outer iterations. CPU time is approximately 0.040 seconds per time step.

Exhibit A : Total power vs. time (relative to an initial total neutron production rate of 1.0) for the reference time step (dt) and for two and ten times the reference time step. Estimated accuracy of the time discretization : less than 0.01% at  $t = 10.0$  seconds.

Exhibit B : Plot of total power vs. time for the reference time step size.

Exhibit C : Scalar flux vs. spatial position at  $t = 0.0$ ,  $0.01$ , and  $1.0$  seconds. TIMEX uses a linear discontinuous representation for the spatial distribution of the flux in each mesh interval. For each of the 114 spatial intervals, the scalar flux on the left and right boundaries, respectfully, of each mesh cell is given.

Exhibit D : Plot of data in Exhibit C.

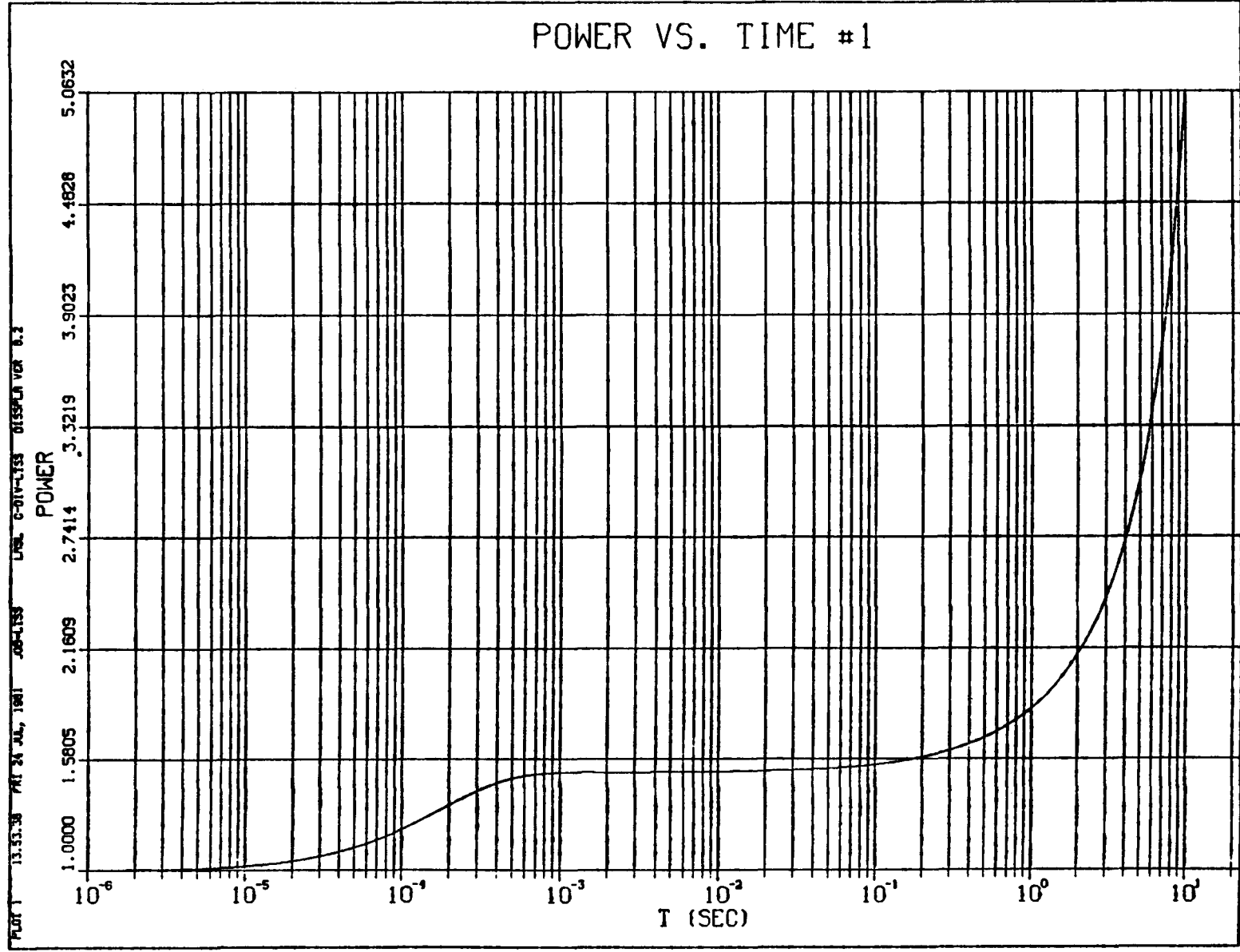
## EXHIBIT A

POWER VS. TIME

PROBLEM 16-A1

T(SEC)	TIME STEP DT(SEC)	POWER(DT)	POWER(2*DT)	POWER(10*DT)
.0		1.000000E+00		
.00001		1.020201E+00	1.020081E+00	1.013946E+00
.00002		1.046314E+00		
.00003		1.071326E+00		
.00004		1.095005E+00		
.00005		1.117408E+00		
.00006		1.138603E+00		
.00007		1.158656E+00		
.00008		1.177628E+00		
.00009		1.195578E+00		
.0001	2.E-8	1.212560E+00	1.212584E+00	1.21351E+00
.0002		1.339370E+00		
.0003		1.412250E+00		
.0004		1.454142E+00		
.0005		1.478231E+00		
.0006		1.492090E+00		
.0007		1.500071E+00		
.0008		1.504674E+00		
.0009		1.507337E+00		
.001		1.508886E+00	1.508890E+00	1.504053E+00
.002		1.511313E+00		
.003		1.511832E+00		
.004		1.512171E+00		
.005		1.512608E+00		
.006		1.513028E+00		
.007		1.513448E+00		
.008		1.513869E+00		
.009		1.514289E+00		
.01	2.E-6	1.514708E+00	1.514715E+00	1.501655E+00
.02		1.518887E+00		
.03		1.523033E+00		
.04		1.527147E+00		
.05		1.531229E+00		
.06		1.535282E+00		
.07		1.539305E+00		
.08		1.543299E+00		
.09		1.547265E+00		
.1		1.551203E+00	1.551204E+00	1.544958E+00
.2		1.589212E+00		
.3		1.625115E+00		
.4		1.659353E+00		
.5		1.692272E+00		
.6	2.E-5	1.724138E+00		
.7		1.755160E+00		
.8		1.785505E+00		
.9		1.815304E+00		
1.0		1.844662E+00	1.844656E+00	1.861842E+00
2.0		2.127482E+00		
3.0		2.411400E+00		
4.0		2.700651E+00		
5.0		3.028485E+00		
6.0	2.E-4	3.372046E+00		
7.0		3.743840E+00		
8.0		4.147277E+00		
9.0		4.585890E+00		
10.0		5.063440E+00	5.063146E+00	5.063405E+00
CPU TIME (MIN)		126.4	63.1	12.7
CDC 7600				
CORE STORAGE :		4,900 WORDS SMALL CORE	5,600 WORDS LARGE CORE	

# EXHIBIT B



## EXHIBIT C

## PROBLEM 16 SCALAR FLUX (GROUP 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29)

1	9.3314E-03	1.8360E-02	30	8.6223E-01	8.6408E-01	59	8.2279E-01	9.1910E-01	87	8.3675E-01	8.0767E-01
2	1.8367E-02	2.6515E-02	31	8.8408E-01	9.0221E-01	60	8.1911E-01	9.0804E-01	88	8.0769E-01	7.7513E-01
3	2.6531E-02	3.4962E-02	32	9.0220E-01	9.1653E-01	61	8.0805E-01	8.9260E-01	89	7.7515E-01	7.3922E-01
4	3.4981E-02	4.4020E-02	33	9.1654E-01	9.2705E-01	62	8.9261E-01	8.7269E-01	90	7.3923E-01	7.0001E-01
5	4.4040E-02	5.3682E-02	34	9.2704E-01	9.3368E-01	63	8.7271E-01	8.4781E-01	91	7.0003E-01	6.9757E-01
6	5.3904E-02	6.4734E-02	35	9.3368E-01	9.3643E-01	64	8.4781E-01	8.1553E-01	92	6.9759E-01	6.1181E-01
7	6.4758E-02	7.6777E-02	36	9.3643E-01	9.3829E-01	65	8.1549E-01	7.6577E-01	93	6.1182E-01	5.6213E-01
8	7.6804E-02	9.0232E-02	37	9.3829E-01	9.3026E-01	66	7.6126E-01	7.3924E-01	94	5.6210E-01	5.0602E-01
9	9.0263E-02	1.0535E-01	38	9.3027E-01	9.2138E-01	67	7.3893E-01	7.2898E-01	95	5.0325E-01	4.3395E-01
10	1.0538E-01	1.2240E-01	39	9.2139E-01	9.0867E-01	68	7.2889E-01	7.2910E-01	96	4.3386E-01	3.7755E-01
11	1.2244E-01	1.4171E-01	40	9.0868E-01	8.9212E-01	69	7.2919E-01	7.3955E-01	97	3.7744E-01	3.2894E-01
12	1.4176E-01	1.6364E-01	41	8.9214E-01	8.7166E-01	70	7.3986E-01	7.6230E-01	98	3.2884E-01	2.8656E-01
13	1.6369E-01	1.8858E-01	42	8.7166E-01	8.4664E-01	71	7.6653E-01	8.1454E-01	99	2.8647E-01	2.4951E-01
14	1.8864E-01	2.1700E-01	43	8.4662E-01	8.1464E-01	72	8.1464E-01	8.4662E-01	100	2.4943E-01	2.1707E-01
15	2.1707E-01	2.4943E-01	44	8.1454E-01	7.6653E-01	73	8.4664E-01	8.7166E-01	101	2.1700E-01	1.8864E-01
16	2.4951E-01	2.8647E-01	45	7.6230E-01	7.3986E-01	74	8.7166E-01	8.9214E-01	102	1.8858E-01	1.6369E-01
17	2.8656E-01	3.2884E-01	46	7.3955E-01	7.2919E-01	75	8.9213E-01	9.0868E-01	103	1.6364E-01	1.4176E-01
18	3.2894E-01	3.7744E-01	47	7.2910E-01	7.2889E-01	76	9.0867E-01	9.2139E-01	104	1.4171E-01	1.2244E-01
19	3.7755E-01	4.3386E-01	48	7.2898E-01	7.3893E-01	77	9.2138E-01	9.3027E-01	105	1.2240E-01	1.0538E-01
20	4.3395E-01	5.0325E-01	49	7.3924E-01	7.6126E-01	78	9.3026E-01	9.3829E-01	106	1.0535E-01	9.0263E-02
21	5.0602E-01	5.6210E-01	50	7.6577E-01	8.1549E-01	79	9.3829E-01	9.3643E-01	107	9.0232E-02	7.6804E-02
22	5.6213E-01	6.1182E-01	51	8.1553E-01	8.4781E-01	80	9.3643E-01	9.3368E-01	108	7.6777E-02	6.4758E-02
23	6.1181E-01	6.9759E-01	52	8.4781E-01	8.7271E-01	81	9.3168E-01	9.2704E-01	109	6.4734E-02	5.3904E-02
24	6.9757E-01	7.0003E-01	53	8.7269E-01	8.9261E-01	82	9.2703E-01	9.1654E-01	110	5.3882E-02	4.4040E-02
25	7.0001E-01	7.3923E-01	54	8.9260E-01	9.0804E-01	83	9.1653E-01	9.0220E-01	111	4.4020E-02	3.4981E-02
26	7.3922E-01	7.7515E-01	55	9.0804E-01	9.1911E-01	84	9.0221E-01	8.9260E-01	112	3.4962E-02	2.6531E-02
27	7.7513E-01	8.0769E-01	56	9.1910E-01	9.2705E-01	85	8.9261E-01	8.7269E-01	113	2.6531E-02	1.8367E-02
28	8.0767E-01	8.3675E-01	57	9.2705E-01	9.3368E-01	86	8.7271E-01	8.4781E-01	114	1.8360E-02	1.3314E-02
29	8.3674E-01	8.6223E-01	58	9.2707E-01	9.3643E-01						

## PROBLEM 16 SCALAR FLUX (GROUP 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29)

1	2.8774E-03	7.2825E-03	30	1.3154E-01	1.3308E-01	59	1.3673E-01	1.3567E-01	87	1.2876E-01	1.2570E-01
2	7.2577E-03	1.1202E-02	31	1.3400E-01	1.3605E-01	60	1.3567E-01	1.3391E-01	88	1.2571E-01	1.2247E-01
3	1.1204E-02	1.5240E-02	32	1.3605E-01	1.3767E-01	61	1.3391E-01	1.3144E-01	89	1.2247E-01	1.1916E-01
4	1.5243E-02	1.9428E-02	33	1.3767E-01	1.3881E-01	62	1.3144E-01	1.2827E-01	90	1.1917E-01	1.1591E-01
5	1.9430E-02	2.3815E-02	34	1.3881E-01	1.3943E-01	63	1.2827E-01	1.2438E-01	91	1.1592E-01	1.1288E-01
6	2.3817E-02	2.8448E-02	35	1.3944E-01	1.3955E-01	64	1.2439E-01	1.1977E-01	92	1.1290E-01	1.1029E-01
7	2.8450E-02	3.3369E-02	36	1.3955E-01	1.3911E-01	65	1.1982E-01	1.1361E-01	93	1.1031E-01	1.0841E-01
8	3.3370E-02	3.8612E-02	37	1.3911E-01	1.3811E-01	66	1.1314E-01	1.1049E-01	94	1.0840E-01	1.0825E-01
9	3.8613E-02	4.4204E-02	38	1.3811E-01	1.3659E-01	67	1.1049E-01	1.0935E-01	95	1.0906E-01	1.0785E-01
10	4.4204E-02	5.0159E-02	39	1.3659E-01	1.3438E-01	68	1.0935E-01	1.0939E-01	96	1.0785E-01	1.0660E-01
11	5.0158E-02	5.6474E-02	40	1.3438E-01	1.3164E-01	69	1.0939E-01	1.1061E-01	97	1.0663E-01	9.7932E-02
12	5.6472E-02	6.3128E-02	41	1.3164E-01	1.2831E-01	70	1.1062E-01	1.1335E-01	98	9.7956E-02	9.1382E-02
13	6.3124E-02	7.0066E-02	42	1.2831E-01	1.2437E-01	71	1.1338E-01	1.1984E-01	99	9.1399E-02	8.4367E-02
14	7.0061E-02	7.7198E-02	43	1.2438E-01	1.1980E-01	72	1.1980E-01	1.2439E-01	100	8.4380E-02	7.7189E-02
15	7.7189E-02	8.4380E-02	44	1.1984E-01	1.1380E-01	73	1.2437E-01	1.2831E-01	101	7.7198E-02	7.0061E-02
16	8.4367E-02	9.1399E-02	45	1.1335E-01	1.1062E-01	74	1.2831E-01	1.3164E-01	102	7.0066E-02	6.3124E-02
17	9.1382E-02	9.7956E-02	46	1.1061E-01	1.0939E-01	75	1.3164E-01	1.3438E-01	103	6.3128E-02	5.6472E-02
18	9.7932E-02	1.0363E-01	47	1.0939E-01	1.0935E-01	76	1.3438E-01	1.3659E-01	104	5.6474E-02	5.0158E-02
19	1.0360E-01	1.0785E-01	48	1.0935E-01	1.1048E-01	77	1.3659E-01	1.3911E-01	105	5.0159E-02	4.4204E-02
20	1.0785E-01	1.0906E-01	49	1.1049E-01	1.1314E-01	78	1.3911E-01	1.3911E-01	106	4.4204E-02	3.8613E-02
21	1.0825E-01	1.0840E-01	50	1.1361E-01	1.1982E-01	79	1.3911E-01	1.3955E-01	107	3.8612E-02	3.3370E-02
22	1.0841E-01	1.1031E-01	51	1.1977E-01	1.2439E-01	80	1.3955E-01	1.3944E-01	108	3.3369E-02	2.8450E-02
23	1.1029E-01	1.1290E-01	52	1.2438E-01	1.2827E-01	81	1.3945E-01	1.3881E-01	109	2.8448E-02	2.3817E-02
24	1.1288E-01	1.1592E-01	53	1.2827E-01	1.3144E-01	82	1.3881E-01	1.3767E-01	110	2.3815E-02	1.9430E-02
25	1.1591E-01	1.1917E-01	54	1.3144E-01	1.3391E-01	83	1.3767E-01	1.3605E-01	111	1.9428E-02	1.5243E-02
26	1.1916E-01	1.2247E-01	55	1.3391E-01	1.3567E-01	84	1.3605E-01	1.3400E-01	112	1.5240E-02	1.1202E-02
27	1.2247E-01	1.2571E-01	56	1.3567E-01	1.3673E-01	85	1.3400E-01	1.3154E-01	113	1.1202E-02	7.2577E-03
28	1.2570E-01	1.2876E-01	57	1.3673E-01	1.3708E-01	86	1.3155E-01	1.2876E-01	114	7.2825E-03	2.8774E-03
29	1.2876E-01	1.3155E-01	58	1.3708E-01	1.3673E-01						

## EXHIBIT C (Continued)

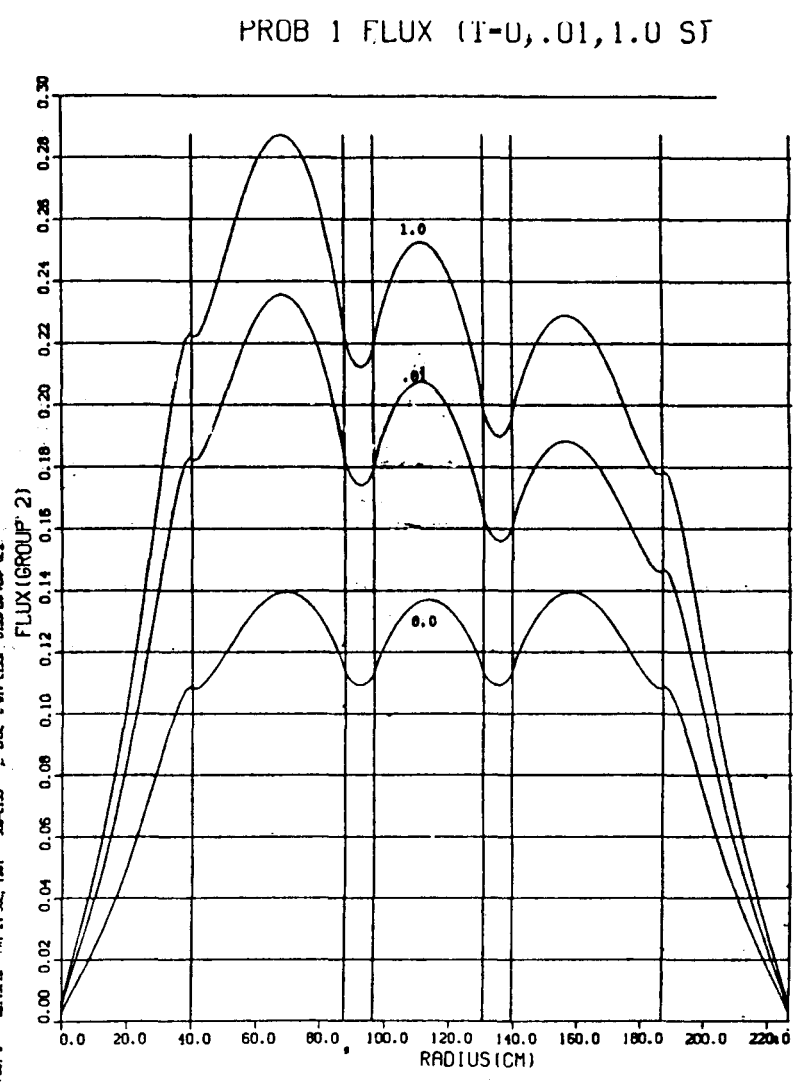
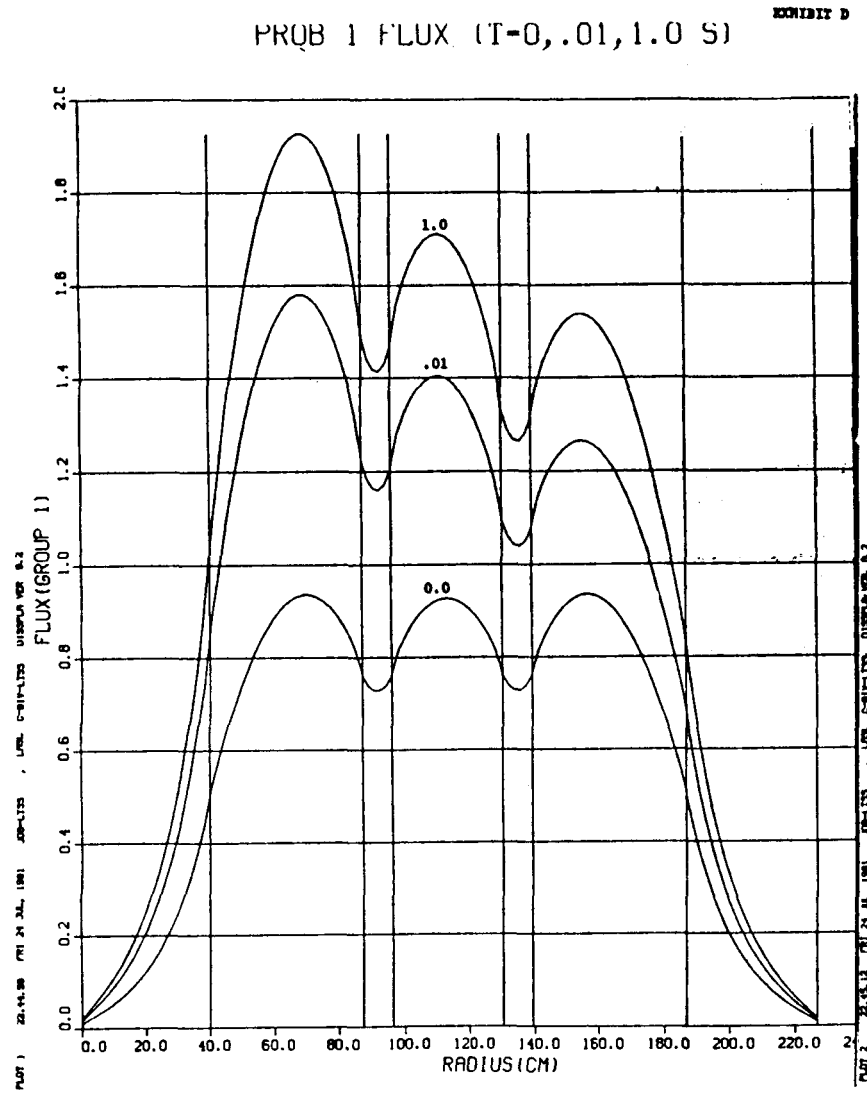
PROBLEM 16-A1 SCALAR FLUX (GROUP 1, 1.01 SEC)									
1	1.5699E-02	3.0889E-02	30	1.4698E+00	1.8097E+00	87	1.1141E+00	1.0751E+00	
2	3.0901E-02	4.4610E-02	31	1.5057E+00	1.8348E+00	88	1.0751E+00	1.0319E+00	
3	4.4636E-02	5.8822E-02	32	1.5346E+00	1.8583E+00	89	1.0319E+00	9.8458E-01	
4	5.8853E-02	7.4062E-02	33	1.5565E+00	1.8713E+00	90	9.8460E-01	9.8333E-01	
5	7.4096E-02	9.0655E-02	34	1.5713E+00	1.8789E+00	91	9.3335E-01	8.7816E-01	
6	9.0692E-02	1.0891E-01	35	1.5789E+00	1.8793E+00	92	8.7819E-01	8.1892E-01	
7	1.0996E-01	1.2918E-01	36	1.5793E+00	1.8729E+00	93	8.1891E-01	7.5476E-01	
8	1.2922E-01	1.5182E-01	37	1.5723E+00	1.8585E+00	94	7.5470E-01	6.8255E-01	
9	1.5187E-01	1.7725E-01	38	1.5585E+00	1.8375E+00	95	6.7891E-01	5.8542E-01	
10	1.7731E-01	2.0595E-01	39	1.5375E+00	1.8095E+00	96	5.8530E-01	5.0933E-01	
11	2.0602E-01	2.3845E-01	40	1.5095E+00	1.4748E+00	97	5.0919E-01	4.4376E-01	
12	2.3852E-01	2.7334E-01	41	1.4746E+00	1.4826E+00	98	4.4362E-01	3.8658E-01	
13	2.7343E-01	3.1731E-01	42	1.4326E+00	1.3828E+00	99	3.8646E-01	3.3659E-01	
14	3.1741E-01	3.6514E-01	43	1.3828E+00	1.3213E+00	100	3.3649E-01	2.9282E-01	
15	3.6525E-01	4.1971E-01	44	1.3212E+00	1.2225E+00	101	2.9273E-01	2.5447E-01	
16	4.1984E-01	4.8206E-01	45	1.2254E+00	1.1857E+00	102	2.5439E-01	2.2081E-01	
17	4.8220E-01	5.5336E-01	46	1.1851E+00	1.1648E+00	103	2.2074E-01	1.9123E-01	
18	5.5352E-01	6.3314E-01	47	1.1646E+00	1.1604E+00	104	1.9117E-01	1.6517E-01	
19	6.3332E-01	7.3008E-01	48	1.1606E+00	1.1726E+00	105	1.6512E-01	1.4215E-01	
20	7.3024E-01	8.4685E-01	49	1.1730E+00	1.2041E+00	106	1.4211E-01	1.2176E-01	
21	8.4616E-01	9.5035E-01	50	1.2113E+00	1.2811E+00	107	1.2172E-01	1.0360E-01	
22	9.5037E-01	1.0376E+00	51	1.2812E+00	1.3234E+00	108	1.0356E-01	8.7353E-02	
23	1.0376E+00	1.1178E+00	52	1.3234E+00	1.3540E+00	109	8.7320E-02	7.2711E-02	
24	1.1178E+00	1.1919E+00	53	1.3540E+00	1.3769E+00	110	7.2681E-02	5.9405E-02	
25	1.1919E+00	1.2601E+00	54	1.3769E+00	1.3929E+00	111	5.9378E-02	4.7184E-02	
26	1.2600E+00	1.3221E+00	55	1.3929E+00	1.4022E+00	112	4.7160E-02	3.5787E-02	
27	1.3221E+00	1.3779E+00	56	1.4022E+00	1.4049E+00	113	3.5769E-02	2.4775E-02	
28	1.3779E+00	1.4272E+00	57	1.4049E+00	1.4008E+00	114	2.4765E-02	1.2587E-02	
29	1.4272E+00	1.4698E+00	58	1.4008E+00	1.3801E+00				

PROBLEM 16-A1 SCALAR FLUX (GROUP 2, 1.01 SEC)									
1	4.8411E-03	1.2253E-02	30	2.2365E-01	2.2774E-01	87	1.7216E-01	1.6812E-01	
2	1.2211E-02	1.8847E-02	31	2.2773E-01	2.3105E-01	88	1.6813E-01	1.6392E-01	
3	1.8851E-02	2.5642E-02	32	2.3105E-01	2.3352E-01	89	1.6393E-01	1.5967E-01	
4	2.5646E-02	3.2687E-02	33	2.3352E-01	2.3508E-01	90	1.5968E-01	1.5594E-01	
5	3.2691E-02	4.0069E-02	34	2.3508E-01	2.3569E-01	91	1.5596E-01	1.5173E-01	
6	4.0073E-02	4.7866E-02	35	2.3568E-01	2.3530E-01	92	1.5175E-01	1.4850E-01	
7	4.7869E-02	5.6146E-02	36	2.3530E-01	2.3389E-01	93	1.4852E-01	1.4461E-01	
8	5.6149E-02	6.4970E-02	37	2.3389E-01	2.3144E-01	94	1.4461E-01	1.4007E-01	
9	6.4972E-02	7.4380E-02	38	2.3144E-01	2.2793E-01	95	1.4017E-01	1.3550E-01	
10	7.4381E-02	8.4402E-02	39	2.2793E-01	2.2337E-01	96	1.3550E-01	1.3076E-01	
11	8.4401E-02	9.5031E-02	40	2.2337E-01	2.1774E-01	97	1.3081E-01	1.2322E-01	
12	9.5028E-02	1.0623E-01	41	2.1774E-01	2.1103E-01	98	1.2321E-01	1.1328E-01	
13	1.0622E-01	1.1791E-01	42	2.1103E-01	2.0324E-01	99	1.1323E-01	1.1382E-01	
14	1.1790E-01	1.2991E-01	43	2.0325E-01	1.9433E-01	100	1.1383E-01	1.0413E-01	
15	1.2990E-01	1.4200E-01	44	1.9441E-01	1.8301E-01	101	1.0414E-01	9.4514E-02	
16	1.4198E-01	1.5382E-01	45	1.8226E-01	1.7731E-01	102	9.4521E-02	8.5156E-02	
17	1.5379E-01	1.6485E-01	46	1.7729E-01	1.7477E-01	103	8.5160E-02	7.6181E-02	
18	1.6481E-01	1.7441E-01	47	1.7477E-01	1.7412E-01	104	7.6184E-02	6.7662E-02	
19	1.7435E-01	1.8152E-01	48	1.7413E-01	1.7535E-01	105	6.7663E-02	5.9630E-02	
20	1.8152E-01	1.8356E-01	49	1.7537E-01	1.7899E-01	106	5.9629E-02	5.2087E-02	
21	1.8216E-01	1.8255E-01	50	1.7974E-01	1.8825E-01	107	5.2085E-02	4.5014E-02	
22	1.8256E-01	1.8606E-01	51	1.8917E-01	1.9417E-01	108	4.5012E-02	3.8276E-02	
23	1.8603E-01	1.9078E-01	52	1.9417E-01	1.9902E-01	109	3.8274E-02	3.2126E-02	
24	1.9076E-01	1.9625E-01	53	1.9902E-01	2.0278E-01	110	3.2123E-02	2.6209E-02	
25	1.9623E-01	2.0206E-01	54	2.0278E-01	2.0544E-01	111	2.6205E-02	2.0560E-02	
26	2.0205E-01	2.0794E-01	55	2.0544E-01	2.0703E-01	112	2.0557E-02	1.5113E-02	
27	2.0792E-01	2.1362E-01	56	2.0703E-01	2.0755E-01	113	1.5110E-02	9.7895E-03	
28	2.1360E-01	2.1891E-01	57	2.0755E-01	2.0700E-01	114	9.8229E-03	3.8811E-03	
29	2.1890E-01	2.2366E-01	58	2.0700E-01	2.0539E-01				

## EXHIBIT C (Continued)

PROBLEM 16-A1 SCALAR FLUX (GROUP 1) T=10 SECT											
1	1.9142E-02	3.7663E-02	30	1.7920E+00	1.8357E+00	59	1.8928E+00	1.8671E+00	87	1.8950E+00	1.3074E+00
2	3.7678E-02	5.4392E-02	31	1.8357E+00	1.8709E+00	60	1.8715E+00	1.8423E+00	88	1.3075E+00	1.2548E+00
3	5.4425E-02	7.1721E-02	32	1.8709E+00	1.8976E+00	61	1.8424E+00	1.8053E+00	89	1.2549E+00	1.1974E+00
4	7.1759E-02	9.0303E-02	33	1.8976E+00	1.9155E+00	62	1.8053E+00	1.7602E+00	90	1.1974E+00	1.1350E+00
5	9.0345E-02	1.1054E-01	34	1.9155E+00	1.9247E+00	63	1.7602E+00	1.7063E+00	91	1.1350E+00	1.0679E+00
6	1.1058E-01	1.3280E-01	35	1.9247E+00	1.9252E+00	64	1.7063E+00	1.6394E+00	92	1.0679E+00	9.9986E-01
7	1.3289E-01	1.5751E-01	36	1.9252E+00	1.9168E+00	65	1.6393E+00	1.5420E+00	93	9.9986E-01	9.1783E-01
8	1.5756E-01	1.8511E-01	37	1.9169E+00	1.8998E+00	66	1.5441E+00	1.4291E+00	94	9.1777E-01	8.3003E-01
9	1.8517E-01	2.1612E-01	38	1.8998E+00	1.8741E+00	67	1.4290E+00	1.2689E+00	95	8.2560E-01	7.1192E-01
10	2.1620E-01	2.5112E-01	39	1.8741E+00	1.8399E+00	68	1.2688E+00	1.2647E+00	96	7.1177E-01	6.1938E-01
11	2.5120E-01	2.9074E-01	40	1.8399E+00	1.7972E+00	69	1.2648E+00	1.2784E+00	97	6.1921E-01	5.3964E-01
12	2.9083E-01	3.3572E-01	41	1.7973E+00	1.7460E+00	70	1.2789E+00	1.3132E+00	98	5.3948E-01	4.7011E-01
13	3.3582E-01	3.8689E-01	42	1.7460E+00	1.6852E+00	71	1.3202E+00	1.3907E+00	99	4.6996E-01	4.0931E-01
14	3.8701E-01	4.4521E-01	43	1.6852E+00	1.6102E+00	72	1.3910E+00	1.4355E+00	100	4.0919E-01	3.5609E-01
15	4.4535E-01	5.1175E-01	44	1.6101E+00	1.5019E+00	73	1.4356E+00	1.4686E+00	101	3.5598E-01	3.0945E-01
16	5.1191E-01	5.8776E-01	45	1.4933E+00	1.4447E+00	74	1.4687E+00	1.4945E+00	102	3.0935E-01	2.6852E-01
17	5.8794E-01	6.7470E-01	46	1.4441E+00	1.4192E+00	75	1.4945E+00	1.5143E+00	103	2.6844E-01	2.3254E-01
18	6.7490E-01	7.7442E-01	47	1.4190E+00	1.4139E+00	76	1.5143E+00	1.5282E+00	104	2.3247E-01	2.0086E-01
19	7.7464E-01	8.9018E-01	48	1.4141E+00	1.4287E+00	77	1.5282E+00	1.5365E+00	105	2.0079E-01	1.7287E-01
20	8.9037E-01	1.0326E+00	49	1.4292E+00	1.4671E+00	78	1.5365E+00	1.5390E+00	106	1.7281E-01	1.4806E-01
21	1.0384E+00	1.1587E+00	50	1.4758E+00	1.5607E+00	79	1.5390E+00	1.5538E+00	107	1.4801E-01	1.2599E-01
22	1.1588E+00	1.2652E+00	51	1.5608E+00	1.6121E+00	80	1.5538E+00	1.5268E+00	108	1.2594E-01	1.0623E-01
23	1.2651E+00	1.3629E+00	52	1.6121E+00	1.6493E+00	81	1.5268E+00	1.5121E+00	109	1.0619E-01	8.8420E-02
24	1.3629E+00	1.4533E+00	53	1.6493E+00	1.6771E+00	82	1.5121E+00	1.4917E+00	110	8.8384E-02	7.2240E-02
25	1.4532E+00	1.5364E+00	54	1.6771E+00	1.6965E+00	83	1.4917E+00	1.4657E+00	111	7.2206E-02	5.7379E-02
26	1.5363E+00	1.6120E+00	55	1.6965E+00	1.7078E+00	84	1.4657E+00	1.4342E+00	112	5.7348E-02	4.3518E-02
27	1.6120E+00	1.6800E+00	56	1.7078E+00	1.7109E+00	85	1.4342E+00	1.3972E+00	113	4.3492E-02	3.0127E-02
28	1.6799E+00	1.7401E+00	57	1.7109E+00	1.7059E+00	86	1.3972E+00	1.3549E+00	114	3.0116E-02	1.9386E-02
29	1.7400E+00	1.7928E+00	58	1.7059E+00	1.6927E+00	87	1.3549E+00				

PROBLEM 16-A1 SCALAR FLUX (GROUP 2) T=10 SECT											
1	5.9027E-03	1.4939E-02	30	2.7266E-01	2.7788E-01	59	2.5010E-01	2.4684E-01	87	2.0938E-01	2.0447E-01
2	1.4889E-02	2.2980E-02	31	2.7764E-01	2.8188E-01	60	2.4684E-01	2.4230E-01	88	2.0448E-01	1.9935E-01
3	2.2985E-02	3.1265E-02	32	2.8167E-01	2.8468E-01	61	2.4230E-01	2.3650E-01	89	1.9936E-01	1.9418E-01
4	3.1270E-02	3.9856E-02	33	2.8468E-01	2.8698E-01	62	2.3650E-01	2.2944E-01	90	1.9420E-01	1.8916E-01
5	3.9860E-02	4.8856E-02	34	2.8698E-01	2.8731E-01	63	2.2944E-01	2.2111E-01	91	1.8918E-01	1.8433E-01
6	4.8860E-02	5.8362E-02	35	2.8731E-01	2.8683E-01	64	2.2112E-01	2.1150E-01	92	1.8435E-01	1.8039E-01
7	5.8366E-02	6.8459E-02	36	2.8683E-01	2.8510E-01	65	2.1150E-01	1.9917E-01	93	1.8036E-01	1.7777E-01
8	6.8462E-02	7.9217E-02	37	2.8510E-01	2.8211E-01	66	1.9935E-01	1.9306E-01	94	1.7776E-01	1.7784E-01
9	7.9219E-02	9.0691E-02	38	2.8211E-01	2.7783E-01	67	1.9304E-01	1.8904E-01	95	1.7783E-01	1.7695E-01
10	9.0692E-02	1.0291E-01	39	2.7783E-01	2.7223E-01	68	1.8904E-01	1.8981E-01	96	1.7695E-01	1.6996E-01
11	1.0291E-01	1.1587E-01	40	2.7225E-01	2.6538E-01	69	1.8981E-01	1.8912E-01	97	1.6992E-01	1.6087E-01
12	1.1587E-01	1.2952E-01	41	2.6538E-01	2.5719E-01	70	1.8912E-01	1.8953E-01	98	1.6071E-01	1.4992E-01
13	1.2952E-01	1.4376E-01	42	2.5719E-01	2.4769E-01	71	1.8909E-01	2.0475E-01	99	1.4995E-01	1.3841E-01
14	1.4375E-01	1.5840E-01	43	2.4769E-01	2.3682E-01	72	2.0469E-01	2.1096E-01	100	1.3843E-01	1.2663E-01
15	1.5838E-01	1.7314E-01	44	2.3691E-01	2.2301E-01	73	2.1095E-01	2.1621E-01	101	1.2665E-01	1.1494E-01
16	1.7311E-01	1.8759E-01	45	2.2210E-01	2.1609E-01	74	2.1621E-01	2.2055E-01	102	1.1495E-01	1.0356E-01
17	1.8751E-01	2.0100E-01	46	2.1603E-01	2.1295E-01	75	2.2055E-01	2.2399E-01	103	1.0356E-01	9.2642E-02
18	2.0095E-01	2.1265E-01	47	2.1295E-01	2.1216E-01	76	2.2399E-01	2.2655E-01	104	9.2645E-02	8.2282E-02
19	2.1259E-01	2.2132E-01	48	2.1216E-01	2.1363E-01	77	2.2655E-01	2.2824E-01	105	8.2283E-02	7.2514E-02
20	2.2132E-01	2.2381E-01	49	2.1367E-01	2.1807E-01	78	2.2825E-01	2.2909E-01	106	7.2513E-02	6.3341E-02
21	2.2211E-01	2.2257E-01	50	2.1808E-01	2.2934E-01	79	2.2909E-01	2.2911E-01	107	6.3339E-02	5.4740E-02
22	2.2259E-01	2.2666E-01	51	2.2925E-01	2.3654E-01	80	2.2912E-01	2.2834E-01	108	5.4738E-02	4.6668E-02
23	2.2682E-01	2.3261E-01	52	2.3653E-01	2.4243E-01	81	2.2834E-01	2.2680E-01	109	4.6665E-02	3.9068E-02
24	2.3258E-01	2.3927E-01	53	2.4243E-01	2.4699E-01	82	2.2680E-01	2.2453E-01	110	3.9064E-02	3.1871E-02
25	2.3924E-01	2.4636E-01	54	2.4699E-01	2.5023E-01	83	2.2454E-01	2.2159E-01	111	3.1868E-02	2.5003E-02
26	2.4634E-01	2.5352E-01	55	2.5023E-01	2.5215E-01	84	2.2160E-01	2.1803E-01	112	2.4999E-02	1.8378E-02
27	2.5350E-01	2.6044E-01	56	2.5215E-01	2.5276E-01	85	2.1804E-01	2.1393E-01	113	1.8374E-02	1.1905E-02
28	2.6043E-01	2.6689E-01	57	2.5276E-01	2.5208E-01	86	2.1394E-01	2.0937E-01	114	1.1945E-02	4.7197E-03
29	2.6688E-01	2.7267E-01	58	2.5208E-01	2.5010E-01						



Identification: 16-A1-2                      Benchmark Problem ID.16-A1

Date Submitted: November 1981                By: W. W. Engle, Jr. (ORNL)

    By: R. M. Westfall (ORNL)

Date Accepted: October 1985                   C. E. Lee (Consult.)

Descriptive Title: Multigroup Discrete Ordinates Solution with TCA

Mathematical Model: Multigroup energy approximation, discrete ordinates  
angular approximation, weighted difference spatial and  
time approximation with automatic coarse mesh rebalance.

Pertinent Features of Technique Used: S4 angular quadrature, suggested  
114 interval spatial mesh, 6 delayed neutron groups, and  
the following time mesh:

Computer: IBM 370/3033  
Codes: ANISN , TDA (single precision, 6 decimal digits accuracy)  
Date solved: August 1981 at Oak Ridge National Laboratory

1. W. W. Engle, Jr. et.al., "One Dimensional Time Dependent Discrete Ordinates," Trans. Am. Nucl. Soc., 12, p 400 (1969)
2. E. T. Tomlinson, W. A. Rhoades and W. W. Engle, Jr., "Flux Extrapolation Models Used in the DOT IV Discrete Ordinates Neutron Transport Code," ORNL/TM-7033 (May 1980). (see p.15, The Theta-Weighted Model)
3. W. W. Engle, Jr., "A Users Manual for ANISN," K-1693 (March 1967).



## Primary Results:

k-eff calculated by ANISN for the initial condition  
constants of Table I: 1.00000

Exhibit A: Total power vs. time (relative to an initial total  
neutron production rate of 1.0) for the reference  
time step ( $\Delta t$ ) and for two and ten times the  
reference time step.

Exhibit B: Plot of total power vs. time for the reference time  
step size.

Exhibit C: Average interval scalar flux vs. position at  $t=0$ , 0.01,  
and 1.0 sec.

Exhibit D: Plot of data in Exhibit C.

## Exhibit A

## Power vs. Time

<u>t (sec)</u>	<u>power(delta-t)</u>	<u>power(2*delta-t)</u>	<u>power(10*delta-t)</u>
0	1.0		
1 x 10 <sup>-5</sup>	1.0221	1.0209	
2	1.0476		
4	1.0938		
8	1.1737		
1 x 10 <sup>-4</sup>	1.2130	1.2055	1.2023
2	1.3282		
4	1.4362		
8	1.4929		
1 x 10 <sup>-3</sup>	1.5003	1.5022	1.4948
2	1.5041		
4	1.5055		
8	1.5087		
1 x 10 <sup>-2</sup>	1.5089	1.5080	1.5089
2	1.5120		
4	1.5149		
8	1.5192		
1 x 10 <sup>-1</sup>	1.5215	1.5320	1.5432
2	1.5535		
4	1.6129		
8	1.7195		
1.0	1.7710	1.9170	1.8300
CPU time (min)	117.2	58.6	12.0

## Exhibit B

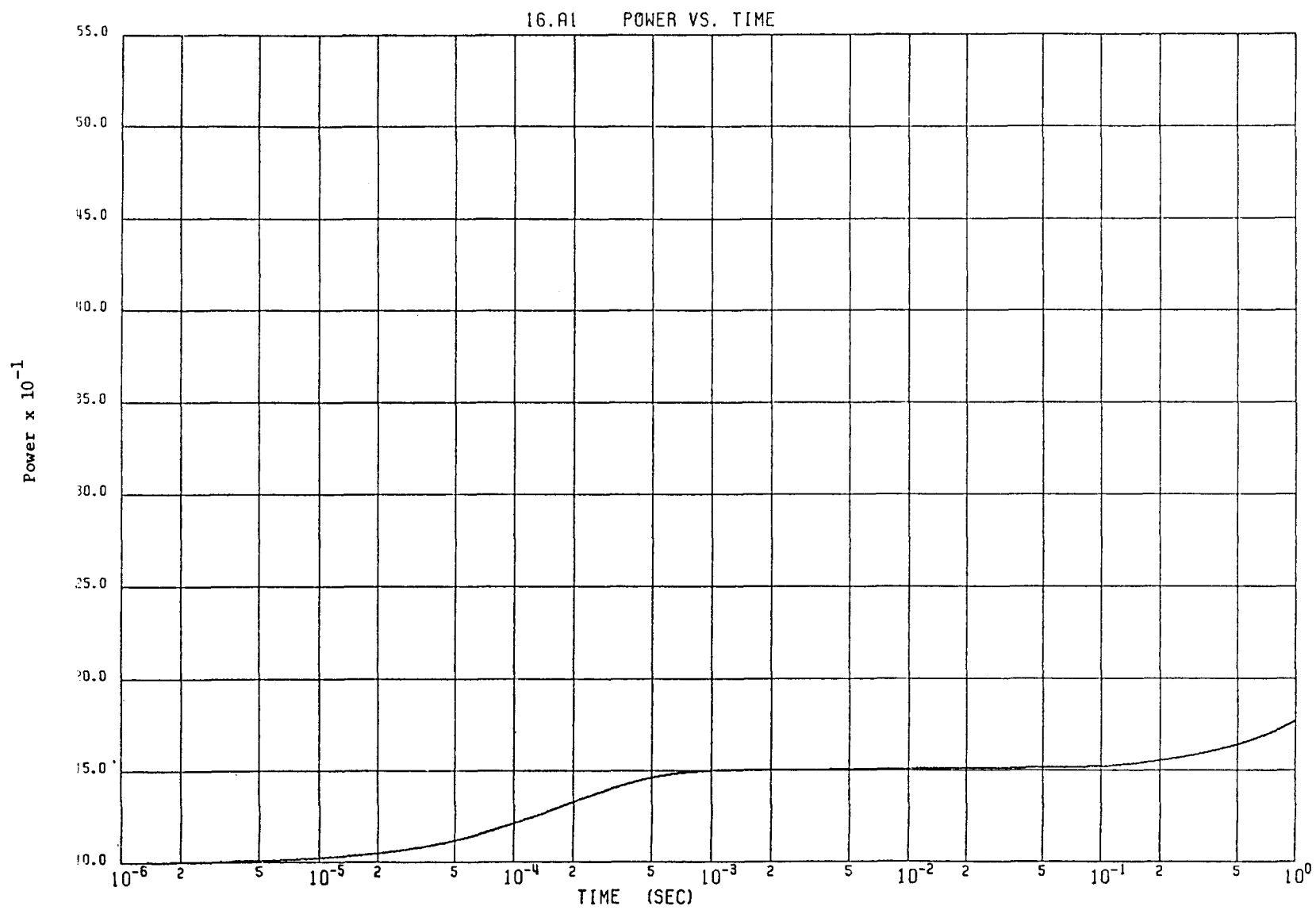


Exhibit C  
Problem 16-A1 Scalar Flux

T=0 SEC.

INT.	GRP. 1	GRP. 2	INT.	GRP. 1	GRP. 2
1	1.37294E-02	5.04370E-03	58	9.26557E-01	1.36850E-01
2	2.24807E-02	9.27842E-03	59	9.22127E-01	1.36145E-01
3	3.07236E-02	1.32175E-02	60	9.13237E-01	1.34735E-01
4	3.94552E-02	1.73451E-02	61	9.00066E-01	1.32620E-01
5	4.89082E-02	2.16354E-02	62	8.82482E-01	1.29802E-01
6	5.92630E-02	2.61520E-02	63	8.60312E-01	1.26261E-01
7	7.07110E-02	3.09363E-02	64	8.32039E-01	1.22106E-01
8	8.34630E-02	3.60272E-02	65	7.99313E-01	1.16604E-01
9	9.77575E-02	4.14549E-02	66	7.51030E-01	1.11915E-01
10	1.13856E-01	4.72395E-02	67	7.34302E-01	1.09959E-01
11	1.32059E-01	5.33868E-02	68	7.29226E-01	1.09412E-01
12	1.52702E-01	5.98834E-02	69	7.34727E-01	1.10046E-01
13	1.76169E-01	6.66903E-02	70	7.51700E-01	1.12090E-01
14	2.02896E-01	7.37136E-02	71	7.89374E-01	1.16728E-01
15	2.33361E-01	8.08932E-02	72	8.31019E-01	1.22133E-01
16	2.68192E-01	8.79868E-02	73	8.59347E-01	1.26308E-01
17	3.07980E-01	9.47570E-02	74	8.81912E-01	1.29951E-01
18	3.53514E-01	1.00811E-01	75	9.00340E-01	1.32987E-01
19	4.05937E-01	1.05781E-01	76	9.14931E-01	1.35433E-01
20	4.69663E-01	1.09265E-01	77	9.25720E-01	1.37294E-01
21	5.33752E-01	1.09441E-01	78	9.32671E-01	1.38579E-01
22	5.87045E-01	1.09316E-01	79	9.35745E-01	1.39297E-01
23	6.34699E-01	1.11592E-01	80	9.34940E-01	1.39461E-01
24	6.78734E-01	1.14371E-01	81	9.30244E-01	1.39087E-01
25	7.19525E-01	1.17494E-01	82	9.21667E-01	1.38195E-01
26	7.57669E-01	1.20763E-01	83	9.09246E-01	1.36813E-01
27	7.91286E-01	1.24030E-01	84	8.93017E-01	1.34972E-01
28	8.22063E-01	1.27173E-01	85	8.73030E-01	1.32716E-01
29	8.49356E-01	1.30095E-01	86	8.49362E-01	1.30095E-01
30	8.73024E-01	1.32716E-01	87	8.22086E-01	1.27174E-01
31	8.93011E-01	1.34972E-01	88	7.91291E-01	1.24030E-01
32	9.09246E-01	1.36812E-01	89	7.57074E-01	1.20763E-01
33	9.21667E-01	1.38195E-01	90	7.19530E-01	1.17494E-01
34	9.30244E-01	1.39087E-01	91	6.78730E-01	1.14372E-01
35	9.34940E-01	1.39460E-01	92	6.34703E-01	1.11593E-01
36	9.35737E-01	1.39296E-01	93	5.87049E-01	1.09317E-01
37	9.32663E-01	1.38578E-01	94	5.33755E-01	1.08442E-01
38	9.25713E-01	1.37293E-01	95	4.69666E-01	1.08266E-01
39	9.14924E-01	1.35432E-01	96	4.05940E-01	1.05782E-01
40	9.00334E-01	1.32985E-01	97	3.53516E-01	1.00812E-01
41	8.81906E-01	1.29950E-01	98	3.07982E-01	9.47580E-02
42	8.59342E-01	1.26307E-01	99	2.68194E-01	8.79878E-02
43	8.31015E-01	1.22132E-01	100	2.33383E-01	8.08942E-02
44	7.89371E-01	1.16727E-01	101	2.02898E-01	7.37348E-02
45	7.51898E-01	1.12090E-01	102	1.76170E-01	6.66913E-02
46	7.34727E-01	1.10045E-01	103	1.52703E-01	5.98845E-02
47	7.29234E-01	1.09411E-01	104	1.32060E-01	5.33877E-02
48	7.34303E-01	1.09759E-01	105	1.13357E-01	4.72404E-02
49	7.51030E-01	1.11914E-01	106	9.77535E-02	4.14557E-02
50	7.89319E-01	1.16603E-01	107	8.34649E-02	3.60280E-02
51	8.32039E-01	1.22105E-01	108	7.07119E-02	3.09371E-02
52	8.60310E-01	1.26260E-01	109	5.92637E-02	2.61526E-02
53	8.82481E-01	1.29801E-01	110	4.89099E-02	2.16360E-02
54	9.00066E-01	1.32619E-01	111	3.94553E-02	1.73456E-02
55	9.13286E-01	1.34735E-01	112	3.07240E-02	1.32179E-02
56	9.22127E-01	1.36144E-01	113	2.24811E-02	9.27871E-03
57	9.26557E-01	1.36850E-01	114	1.37296E-02	5.04387E-03

Exhibit C (Con't)  
PROBLEM 16-A1 SCALAR FLUX

T=0.01 SEC.

INT.	GRP. 1	GRP. 2	INT.	GRP. 1	GRP. 2
1	2.29390E-02	8.42700E-03	58	1.38374E-00	2.04445E-01
2	3.75620E-02	1.55042E-02	59	1.36978E-00	2.02316E-01
3	5.13347E-02	2.20945E-02	60	1.34930E-00	1.99145E-01
4	6.59242E-02	2.89938E-02	61	1.32238E-00	1.94938E-01
5	8.17195E-02	3.61530E-02	62	1.29997E-00	1.89703E-01
6	9.90216E-02	4.37005E-02	63	1.24911E-00	1.83420E-01
7	1.18151E-01	5.16955E-02	64	1.20030E-00	1.76248E-01
8	1.39461E-01	6.02028E-02	65	1.11106E-00	1.67151E-01
9	1.63346E-01	6.92731E-02	66	1.07050E-00	1.59570E-01
10	1.90248E-01	7.99402E-02	67	1.04313E-00	1.56246E-01
11	2.20665E-01	8.92135E-02	68	1.03229E-00	1.54928E-01
12	2.55162E-01	1.00072E-01	69	1.03645E-00	1.55283E-01
13	2.94379E-01	1.11450E-01	70	1.05792E-00	1.57626E-01
14	3.39044E-01	1.23223E-01	71	1.10273E-00	1.63146E-01
15	3.89980E-01	1.35191E-01	72	1.15185E-00	1.69363E-01
16	4.48165E-01	1.47050E-01	73	1.18336E-00	1.73958E-01
17	5.14659E-01	1.59360E-01	74	1.22716E-00	1.77879E-01
18	5.90757E-01	1.68492E-01	75	1.22565E-00	1.81054E-01
19	6.78368E-01	1.76905E-01	76	1.23339E-00	1.83503E-01
20	7.84863E-01	1.80966E-01	77	1.24846E-00	1.85240E-01
21	8.94181E-01	1.91308E-01	78	1.25288E-00	1.86279E-01
22	9.67227E-01	1.82976E-01	79	1.25262E-00	1.86637E-01
23	1.07019E-00	1.97111E-01	80	1.24760E-00	1.86333E-01
24	1.14666E-00	1.92110E-01	81	1.23807E-00	1.85393E-01
25	1.21722E-00	1.27696E-01	82	1.22381E-00	1.83845E-01
26	1.28191E-00	2.03472E-01	83	1.20495E-00	1.81726E-01
27	1.34025E-00	2.09194E-01	84	1.18155E-00	1.79080E-01
28	1.39237E-00	2.14633E-01	85	1.15367E-00	1.75962E-01
29	1.43797E-00	2.19612E-01	86	1.12140E-00	1.72438E-01
30	1.47690E-00	2.23992E-01	87	1.08484E-00	1.68588E-01
31	1.50902E-00	2.27660E-01	88	1.04409E-00	1.64513E-01
32	1.53419E-00	2.30527E-01	89	9.99258E-01	1.60332E-01
33	1.55234E-00	2.32524E-01	90	9.56455E-01	1.56195E-01
34	1.56334E-00	2.33595E-01	91	8.97760E-01	1.52282E-01
35	1.56731E-00	2.33690E-01	92	8.41133E-01	1.48024E-01
36	1.56409E-00	2.32801E-01	93	7.89940E-01	1.44023E-01
37	1.55375E-00	2.30882E-01	94	7.11967E-01	1.44999E-01
38	1.53635E-00	2.27922E-01	95	6.28052E-01	1.44738E-01
39	1.51197E-00	2.23911E-01	96	5.42835E-01	1.41462E-01
40	1.48072E-00	2.18423E-01	97	4.72720E-01	1.34814E-01
41	1.44253E-00	2.12711E-01	98	4.11836E-01	1.26716E-01
42	1.39735E-00	2.05491E-01	99	3.58628E-01	1.17661E-01
43	1.34247E-00	1.97338E-01	100	3.12077E-01	1.08173E-01
44	1.26519E-00	1.87104E-01	101	2.71311E-01	9.85981E-02
45	1.19763E-00	1.78569E-01	102	2.35569E-01	8.91786E-02
46	1.16650E-00	1.74743E-01	103	2.04183E-01	8.00756E-02
47	1.15394E-00	1.73162E-01	104	1.76584E-01	7.13879E-02
48	1.15810E-00	1.73451E-01	105	1.52243E-01	6.31673E-02
49	1.18061E-00	1.75957E-01	106	1.30716E-01	5.54323E-02
50	1.23444E-00	1.82380E-01	107	1.11603E-01	4.81744E-02
51	1.29263E-00	1.89720E-01	108	9.45497E-02	4.13670E-02
52	1.32826E-00	1.94949E-01	109	7.92418E-02	3.49694E-02
53	1.35440E-00	1.99233E-01	110	6.53960E-02	2.89299E-02
54	1.37352E-00	2.02409E-01	111	5.27562E-02	2.31931E-02
55	1.38600E-00	2.04513E-01	112	4.10809E-02	1.76739E-02
56	1.39187E-00	2.05549E-01	113	3.00593E-02	1.24067E-02
57	1.39111E-00	2.05524E-01	114	1.83577E-02	6.74419E-03

T=1.0 SEC.

INT.	GRP. 1	GRP. 2	INT.	GRP. 1	GRP. 2
1	2.69223E-02	9.89110E-03	58	1.62237E-00	2.39705E-01
2	4.40832E-02	1.91957E-02	59	1.60593E-00	2.37200E-01
3	6.02469E-02	2.59207E-02	60	1.53185E-00	2.33472E-01
4	7.73693E-02	3.40154E-02	61	1.45021E-00	2.28529E-01
5	9.59067E-02	4.24240E-02	62	1.51109E-00	2.22342E-01
6	1.16213E-01	5.12866E-02	63	1.46417E-00	2.15036E-01
7	1.30663E-01	6.06694E-02	64	1.40699E-00	2.06548E-01
8	1.63673E-01	7.06531E-02	65	1.32566E-00	1.95915E-01
9	1.91725E-01	8.12978E-02	66	1.25463E-00	1.97721E-01
10	2.23277E-01	9.26431E-02	67	1.22248E-00	1.83120E-01
11	2.58974E-01	1.04701E-01	68	1.20978E-00	1.81570E-01
12	2.99459E-01	1.17444E-01	69	1.21461E-00	1.81992E-01
13	3.45484E-01	1.30797E-01	70	1.23869E-00	1.84722E-01
14	3.97925E-01	1.44613E-01	71	1.29220E-00	1.91194E-01
15	4.57691E-01	1.58659E-01	72	1.39769E-00	1.98460E-01
16	5.25966E-01	1.72576E-01	73	1.38655E-00	2.03936E-01
17	6.04002E-01	1.85861E-01	74	1.41438E-00	2.08422E-01
18	6.93305E-01	1.97741E-01	75	1.43599E-00	2.12134E-01
19	7.96125E-01	2.07496E-01	76	1.45234E-00	2.14996E-01
20	9.21110E-01	2.12380E-01	77	1.46262E-00	2.17024E-01
21	1.04941E-00	2.12782E-01	78	1.46775E-00	2.18235E-01
22	1.15860E-00	2.14739E-01	79	1.46740E-00	2.18648E-01
23	1.25596E-00	2.19591E-01	80	1.46157E-00	2.18287E-01
24	1.34571E-00	2.25456E-01	81	1.45028E-00	2.17181E-01
25	1.42851E-00	2.31992E-01	82	1.43354E-00	2.15363E-01
26	1.50430E-00	2.38787E-01	83	1.41142E-00	2.12876E-01
27	1.57288E-00	2.45500E-01	84	1.38388E-00	2.09773E-01
28	1.63403E-00	2.51980E-01	85	1.35130E-00	2.06116E-01
29	1.68753E-00	2.57721E-01	86	1.31349E-00	2.01985E-01
30	1.73320E-00	2.62857E-01	87	1.27064E-00	1.97473E-01
31	1.77086E-00	2.67158E-01	88	1.22244E-00	1.92696E-01
32	1.80036E-00	2.70519E-01	89	1.17037E-00	1.87797E-01
33	1.82164E-00	2.72858E-01	90	1.11319E-00	1.82949E-01
34	1.83458E-00	2.74110E-01	91	1.05147E-00	1.78364E-01
35	1.83914E-00	2.74225E-01	92	9.85141E-01	1.74312E-01
36	1.83532E-00	2.73167E-01	93	9.13594E-01	1.71030E-01
37	1.82314E-00	2.70908E-01	94	8.33853E-01	1.69830E-01
38	1.80268E-00	2.67428E-01	95	7.35574E-01	1.69582E-01
39	1.77403E-00	2.62715E-01	96	6.35768E-01	1.65586E-01
40	1.73730E-00	2.56760E-01	97	5.53660E-01	1.57899E-01
41	1.69255E-00	2.49559E-01	98	4.82342E-01	1.48414E-01
42	1.63937E-00	2.41280E-01	99	4.20024E-01	1.37907E-01
43	1.57491E-00	2.31505E-01	100	3.65503E-01	1.26695E-01
44	1.48419E-00	2.19491E-01	101	3.17759E-01	1.15490E-01
45	1.40408E-00	2.09471E-01	102	2.75898E-01	1.04448E-01
46	1.36834E-00	2.04978E-01	103	2.39144E-01	9.37856E-02
47	1.35357E-00	2.03118E-01	104	2.06814E-01	8.36101E-02
48	1.35842E-00	2.03452E-01	105	1.78306E-01	7.39822E-02
49	1.38478E-00	2.06397E-01	106	1.53094E-01	6.49226E-02
50	1.44786E-00	2.13919E-01	107	1.30708E-01	5.64222E-02
51	1.51605E-00	2.22510E-01	108	1.10736E-01	4.84493E-02
52	1.55775E-00	2.28633E-01	109	9.28074E-02	4.09565E-02
53	1.58833E-00	2.33646E-01	110	7.65913E-02	3.38930E-02
54	1.61067E-00	2.37359E-01	111	6.17876E-02	2.71640E-02
55	1.62524E-00	2.39817E-01	112	4.81136E-02	2.06998E-02
56	1.63205E-00	2.41021E-01	113	3.52052E-02	1.45308E-02
57	1.63109E-00	2.40982E-01	114	2.15094E-02	7.89887E-03

## Exhibit D

