BENCHMARK PROBLEM

Identification: 11-A2 Source Situation ID.11

Date Submitted: June 1976 By: R. R. Lee (CE)
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Date Accepted: June 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
M. V. Gregory (SRL)

Descriptive Title: Two-dimensional LWR Problem,
also 2D IAEA Benchmark Problem

Reduction of Source Situation

1. Two-group diffusion theory
2. Two-dimensional (x,y)-geometry

Two-Group Diffusion Equations:

$$- \nabla D_1 \nabla \Phi_1 + (\Sigma_{a1} + \Sigma_{1\to 2} + D_1 B_{z1}^2)\Phi_1 = \frac{1}{\lambda}\nu\Sigma_{f2}\Phi_2$$

$$- \nabla D_2 \nabla \Phi_2 + (\Sigma_{a2} + D_2 B_{z2}^2)\Phi_2 = \Sigma_{1\to 2}\Phi_1$$

Data

Two-group Constants

Region	D_1	D_2	$\Sigma_{1\to 2}$	Σ_{a1}	Σ_{a2}	$\nu\Sigma_{f2}$	Material
1	1.5	0.4	0.02	0.01	0.08	0.135	Fuel 1
2	1.5	0.4	0.02	0.01	0.085	0.135	Fuel 2
3	1.5	0.4	0.02	0.01	0.13	0.135	Fuel 2 + Rod
4	2.0	0.3	0.04	0	0.01	0	Reflector

Axial buckling $B^2_{z,g} = 0.8 \cdot 10^{-4}$ for all regions and energy groups.

Note: This 2D IAEA Benchmark Problem represents the midplane z = 190 cm of the 3D IAEA Benchmark Problem

Boundary Conditions:

$J_g^{in} = 0$ No incoming current at external boundaries.

For finite difference diffusion theory codes the following form is considered equivalent

$$\frac{\partial \Phi_g}{\partial n} = -\frac{0.4692}{D_g} \Phi_g ,$$

where n the outward directed normal to the surface.
At symmetry boundaries:

$$\frac{\partial \Phi_g}{\partial n} = 0$$

Expected Primary Results:

1. Maximum eigenvalue

2. Fundamental flux distributions

   2.1 Radial flux traverses $\phi_g(x,o)$ and $\phi_g(x,x)$

Note: The fluxes shall be normalized such that

$$\frac{1}{V_{Core}} \int_{V_{Core}} \sum_g \nu \Sigma_{fg} \Phi_g \, dV = 1$$

   2.2 Value and location of maximum power density. This corresponds to maximum of ϕ_2 in the core. It is recommended that the maximum values of ϕ_2 both in the inner core and at the core/reflector interface be given.

3. Average subassembly powers P_k

$$P_k = \frac{1}{V_k} \int_{V_k} \sum_g \nu\Sigma_{fg} \Phi_g \, dV$$

where V_k volume of the k-th subassembly and k designates the fuel subassemblies as shown in lower octant of Fig. 1

4. Number of unknowns in the problem, number of iterations, total and outer

5. Total computing time, iteration time, IO-time, computer used

6. Type and numerical values of convergence criteria

7. Table of average group fluxes for a square mesh grid of 20 x 20 cm

8. Dependence of results on mesh spacing

Best Solution Available: Extrapolated finite difference solution described in 11-A2-1

Solutions

1. Finite Difference Method: 11-A2-1
2. Finite Element Method: 11-A2-2
3. Nodal Expansion Method: 11-A2-3
4. Finite Difference Method: 11-A2-4

BENCHMARK PROBLEM SOLUTION

Identification: 11-A2-1 Benchmark Problem 11.A2

Date Submitted: June 1, 1976 By: D. R. Vondy, T. B. Fowler (ORNL)

Date Accepted: June 1, 1977 By: H. L. Dodds, Jr. (U. of Tenn.)
M. V. Gregory (SRL)

Descriptive Title: Two-dimensional PWR Problem (IAEA)

Mathematical Model: Diffusion theory, various difference formulations

Computer: IBM-369/91, 1973-76, ORNL
IBM-360/195, 1976 UC-CTC

Program:
(1) VENTURE, ORNL-5062 Report
(2) EXTERMINATOR-2, ORNL-4078
(3) VANCER, to be documented (ORNL)

Note: To produce acceptable solutions for benchmarking, tighter convergence of the iterative process was required than is common practice in application, maximum relative flux change on outer iterations = 10^{-5}.

Primary Results:*

a. Primary results obtained in 1973-74 are shown in Table 1. The larger problems were initialized with the flux solution from the smaller problems, and an early version of the VENTURE code used obsolete procedures, so compute times are not representative. Apparent finite-difference error in the multiplication factor is displayed in Fig. 1. Tables 2 and 3 present zone average flux values. Normalization is to one neutron produced for the problem.+

b. Recent results with the VANCER code using the mesh-edge formulation parameterized to admit different approximations are shown in Table 4 from calculations on the IBM-360/91 done in 1976. These calculations were done with rather obsolete procedures oriented to use of an extended, slow memory, so representative computation times are not available.

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* Extrapolation of results is done on the basis of error dependence on the square of the mesh spacing.

\+ To obtain the proper normalization, results in Tables 2 and 3 should be multiplied by 1.78.

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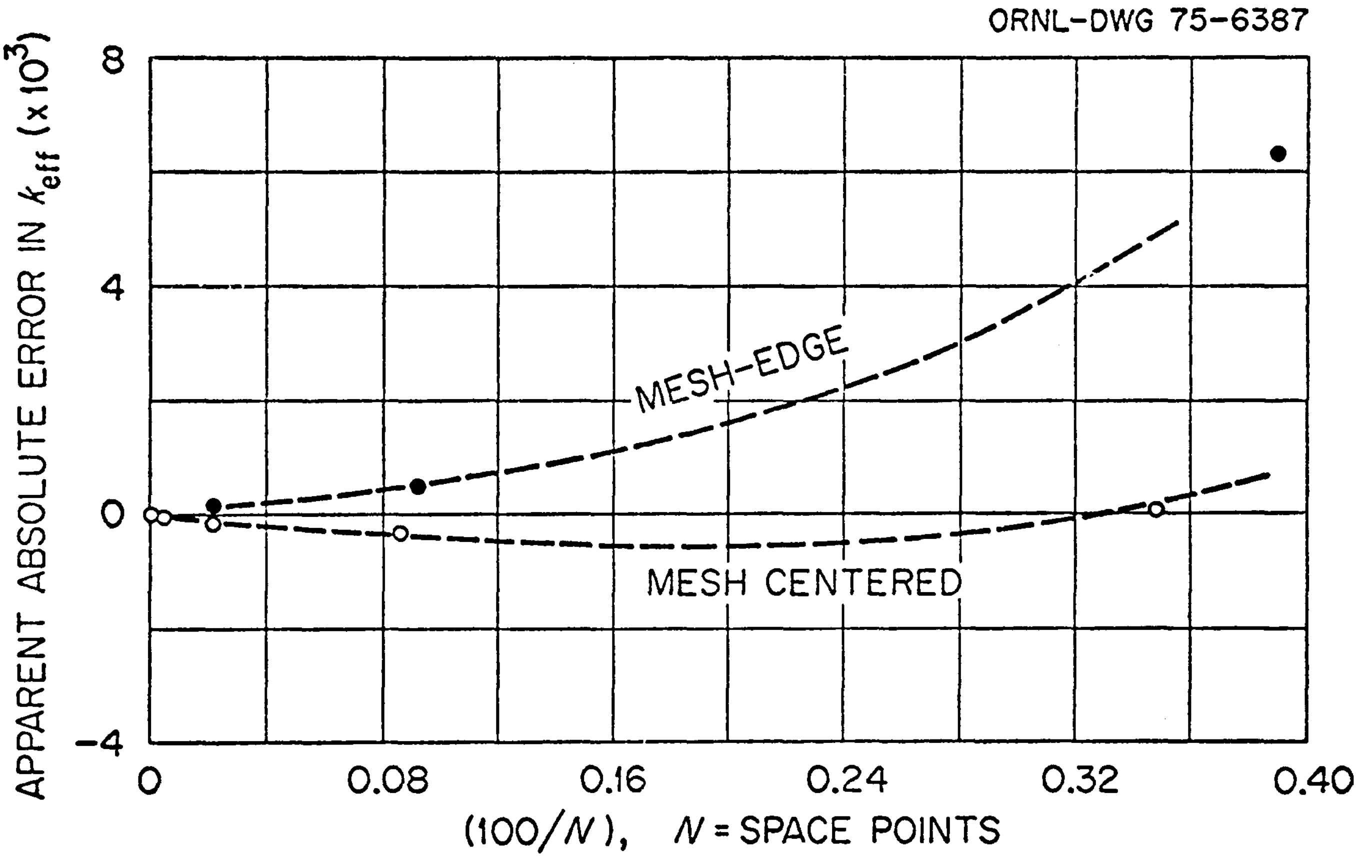
Table 1

IAEA BENCHMARK PROBLEM

(Two-Group, Two-Dimensional, $B_1^2=8\times10^{-5}$)

Meshpoints (Total Unknowns)	k_{eff}	Peak-to-Average Power Density	IBM-360/91 Processor Time (min)
Old Code Exterminator-2 (mesh-edge points, zero flux external boundary)			
16x16(512)	1.03651	2.2404	0.20
33x33(2,178)	1.03065	1.6538	1.0
67x67(8,978)	1.03033	1.5314	(10.)
Extrapolated	1.03022	1.491	
VENTURE (mesh-centered points, non-return external boundary)			
9x9(162)	1.03208	1.5493	0.05
17x17(578)	1.02965	1.6486	0.06
34x34(2,312)	1.02924	1.5985	0.32
68x68(9,248)	1.02944	1.5442	3.4
136x136(36,992)	1.02954	1.5217	13.5
272x272(147,968)	1.02958	1.5149	80
Extrapolated	1.02959	1.5126	

Figure 1. Two-Dimensional Finite-Difference Results



Two-Dimensional Finite-Difference Results.

Table 2. Zone Average Fast Group Flux - Mesh Centered VENTURE

(Results below should be multiplied by 1.78 for proper normalization)

Location/ Intervals	9x9	17x17	34x34	68x68	272x272	Extrapolated
1	17.35	19.28	19.37	18.77	18.37	18.34
2	25.20	26.42	25.30	24.27	23.71	23.67
3	27.12	28.43	27.46	26.49	25.96	25.92
4	23.48	23.87	23.06	22.30	21.89	21.86
5	13.14	14.92	15.32	15.16	15.02	15.01
6	17.53	17.09	17.01	16.95	16.92	16.92
7	16.19	15.80	16.15	16.45	16.61	16.62
8	11.55	10.37	10.76	11.21	11.47	11.49
9	0.954	1.202	1.531	1.772	1.898	1.906
10	27.14	28.46	27.28	26.23	25.65	25.61
11	27.61	28.87	27.90	26.94	26.42	26.39
12	24.73	25.45	24.63	23.90	23.49	23.46
13	19.56	20.39	19.96	19.56	19.35	19.34
14	18.77	18.62	18.54	18.51	18.49	18.49
15	16.45	15.98	16.34	16.67	16.85	16.86
16	11.23	10.06	10.46	10.93	11.20	11.22
17	0.921	1.144	1.458	1.690	1.813	1.821
18	27.28	28.38	27.53	26.99	26.23	26.18
19	24.82	25.59	24.96	24.35	24.01	23.99
20	21.14	21.77	21.50	21.20	21.04	21.03
21	19.02	18.85	18.93	19.00	19.03	19.03
22	16.96	15.53	15.80	16.19	16.42	16.43
23	9.222	8.176	8.616	9.114	9.394	9.413
24	0.733	0.856	1.088	1.272	1.373	1.380
25	22.21	22.50	22.00	21.55	21.30	21.28
26	17.86	17.97	17.75	17.57	17.46	17.45
27	16.14	15.70	15.83	16.00	16.09	16.10
28	12.77	11.62	12.01	12.48	12.74	12.76
29	1.682	2.104	2.686	3.115	3.346	3.361
30	0.188	0.239	0.3136	0.3704	0.4012	0.4033
31	9.374	10.79	11.32	11.48	11.52	11.52
32	12.26	11.18	11.31	11.57	11.72	11.73
33	8.075	7.136	7.482	7.903	8.141	8.157
34	0.712	0.849	1.080	1.263	1.362	1.369
35	8.206	7.071	7.320	7.690	7.904	7.918
36	1.217	1.417	1.789	2.083	2.244	2.255
37	0.150	0.190	0.2511	0.2975	0.3226	0.3243
38	0.190	0.221	0.2844	0.3343	0.3616	0.3634

Table 3. Zone Average Thermal Group Flux - Mesh Centered VENTURE

(Results below should be multiplied by 1.78 for proper normalization)

Location/ Intervals	9x9	17x17	34x34	68x68	272x272	Extrapolated
1	2.796	3.131	3.202	3.155	3.122	3.120
2	5.898	6.173	5.891	5.633	5.491	5.482
3	6.367	6.673	6.446	6.218	6.093	6.085
4	5.501	5.581	5.372	5.181	5.075	5.068
5	2.113	2.422	2.531	2.548	2.552	2.552
6	4.105	3.990	3.957	3.921	3.915	3.914
7	3.803	3.712	3.797	3.869	3.908	3.911
8	2.900	2.670	2.863	3.050	3.153	3.160
9	3.286	3.706	4.105	4.368	4.505	4.514
10	6.372	6.678	6.401	6.151	6.015	6.006
11	6.483	6.777	6.548	6.342	6.202	6.193
12	5.806	5.972	5.779	5.604	5.510	5.503
13	4.580	4.766	4.647	4.541	4.481	4.477
14	4.407	4.371	4.351	4.342	4.336	4.336
15	3.865	3.759	3.848	3.930	3.974	3.977
16	2.818	2.587	2.781	2.969	3.072	3.079
17	3.163	3.510	3.891	4.149	4.284	4.293
18	6.406	6.662	6.462	6.265	6.157	6.149
19	5.826	6.005	5.860	5.714	5.635	5.630
20	4.964	5.111	5.047	4.977	4.938	4.935
21	4.468	4.430	4.450	4.469	4.479	4.480
22	4.201	3.847	3.918	4.019	4.076	4.080
23	2.369	2.220	2.494	2.746	2.887	2.896
24	2.498	2.637	2.944	3.176	3.299	3.307
25	5.214	5.280	5.162	5.055	4.996	4.992
26	4.185	4.201	4.138	4.082	4.049	4.047
27	3.794	3.690	3.726	3.768	3.792	3.794
28	3.223	3.016	3.224	3.423	3.533	3.540
29	5.641	6.040	6.518	6.844	7.013	7.024
30	0.9503	1.122	1.336	1.497	1.584	1.590
31	1.506	1.755	1.878	1.937	1.967	1.969
32	3.031	2.756	2.780	2.836	2.867	2.869
33	2.071	1.930	2.156	2.371	2.490	2.498
34	2.543	2.724	3.041	3.280	3.406	3.414
35	2.115	1.929	2.129	2.327	2.440	2.448
36	4.045	4.038	4.330	4.570	4.698	4.707
37	0.7636	0.889	1.059	1.190	1.261	1.266
38	0.9518	1.045	1.222	1.364	1.443	1.448

Table 4. TWO-DIMENSIONAL, TWO-GROUP IAEA BENCHMARK PROBLEM RESULTS

Formulation (Near Neighbors)	Mesh Intervals	k_{eff}	Peak Relative Power Density: Internal	Peak Relative Power Density: Near Reflector
Meshpoint Centered, VENTURE (4)	9^2	1.03208	1.549	
	17^2	1.02965	1.649	
	34^2	1.02924	1.599	
	68^2	1.02944	1.544	
	136^2	1.02954	1.522	
	272^2	1.02958	1.515	
Extrapolated	(∞)	1.02959	1.513	
Mesh Edge, VANCER				
Usual Finite-Difference (4)	9^2	1.07647	none	4.28
	17^2	1.03733	0.962	2.231
	34^2	1.03077	1.364	1.660
	68^2	1.02983	1.475	1.546
	(∞)	1.02952	1.512	1.508
Taylor Series (8)	34^2	1.03080	1.364	1.652
Higher Order Taylor Series (8)	17^2	1.03442	1.095	2.043
	34^2	1.03036	1.405	1.629
	68^2	1.02975	1.485	1.544
	(∞)	1.02955	1.512	1.516
Linear Finite-Element (8)*	17^2	1.03109	1.309	1.779
	34^2	1.02985	1.462	1.605
	68^2	1.02965	1.499	1.545
	(∞)	1.02958	1.511	1.525
Linear Finite-Difference (8)*	17^2	1.03236	1.214	1.887
	34^2	1.03006	1.437	1.614
	68^2	1.02969	1.493	1.544
	(∞)	1.02957	1.512	1.521
Compromise (8)	17^2	1.03390	1.123	2.009
	34^2	1.03028	1.412	1.625
	68^2	1.02973	1.487	1.544
	(∞)	1.02955	1.512	1.517
Simple Compromise (4)	34^2	1.03051	1.389	1.645
	68^2	1.02978	1.481	1.544
	(∞)	1.02954	1.512	1.510
Compensated Difference (4)*	17^2	1.03206	1.228	1.900
	34^2	1.03002	1.438	1.628
	68^2	1.02968	1.493	1.547
	(∞)	1.02957	1.511	1.520
Local Source				
H-O Taylor Series (8)	34^2	1.03162	1.393	1.724
Linear Finite-Element (8)	34^2	1.03229	1.402	1.792
Linear Finite-Difference (8)	34^2	1.03280	1.422	1.860
Compromise (8)	34^2	1.03178	1.387	1.737
Simple Compromise (4)	34^2	1.03126	1.375	1.700
ompensated Difference (4)	34^2	1.03224	1.403	1.799
pparent Solution		1.02958	1.51	1.52

*Results for 9^2 mesh inadequate, resulting flux skewed; the only clue of inadequate solution is a neutron balance k.

BENCHMARK PROBLEM SOLUTION

Identification: 11-A2-2 — Benchmark Problem ID.11-A2

Date Submitted: June 1976 — By: Ib Misfeldt (Risø-Denmark)

Date Accepted: June 1977 — By: H. L. Dodds, Jr. (U. of Tenn.), M. V. Gregory (SRL)

Descriptive Title: Two-dimensional PWR Problem

Mathematical Model: FEM (2nd order Lagrange interpolation, rectangular elements)

Pertinent Features of Solution Method: The grid had 36 x 36 meshes and 73 x 73 flux points.

Computer: B 6700 — Date Solved: August 25, 1975 at: Risø, Denmark

Program: FEMB

References

1. Ib Misfeldt, "Solution of the multigroup neutron diffusion equations by the finite element method," Risø-M-1809 (1975).

2. G. K. Kristiansen, "Investigation of the accuracy of centerpoint-, cornerpoint-, and finite-element-methods for solution of the neutron diffusion equation," NEACRP-L-149 (1976).

Results

1. Maximum eigenvalue: $k_{eff} = 1.0296$

2. Fundamental flux distributions

   2.1. See Tables 2A and 2B of flux traverses.

   2.2. Maximum power density

   Uninterpolated values are given

   $(\phi_2)_{max,1} = 11.31$ at $(x,y) = (130,55)$

   $(\phi_2)_{max,2} = 11.18$ at $(x,y) = (30,30)$

3. Average subassembly powers

   See Table 2C.

4. Number of unknowns and iteration number

   73 x 73 x 2 unknowns; 120 iterations.

5. Computing times

   2½ hours cp=time; ½ hour io-time, on B 6700.

6. Convergence criteria

   Maximal flux-error-estimate less than 0.01% of ϕ_{max} in each group.

7. Average group-fluxes for 20 x 20 cm grid

   See Table 2D.

8. Dependence of results on mesh spacing

   See Refs. 1 and 2.

Table 2A

Flux along x-axis

X	PHI1	PHI2
0.000	29.350	4.620
1.250	29.422	4.633
2.500	29.641	4.675
3.750	30.009	4.750
5.000	30.531	4.868
6.250	31.211	5.049
7.500	32.052	5.336
8.750	33.046	5.809
10.000	34.166	6.646
12.500	36.512	8.164
15.000	38.666	8.945
17.500	40.501	9.468
20.000	42.030	9.853
22.500	43.280	10.155
25.000	44.282	10.393
27.500	45.061	10.577
30.000	45.635	10.712
32.500	46.019	10.802
35.000	46.223	10.850
37.500	46.255	10.857
40.000	46.120	10.826
42.500	45.819	10.755
45.000	45.355	10.646
47.500	44.725	10.498
50.000	43.926	10.310
52.500	42.950	10.081
55.000	41.788	9.808
57.500	40.426	9.486
60.000	38.845	9.107
62.500	37.021	8.656
65.000	34.934	8.085
67.500	32.576	7.289
70.000	30.063	5.846
72.500	27.763	4.618
75.000	25.984	4.157
77.500	24.760	3.909
80.000	24.042	3.787
82.500	23.801	3.756
85.000	24.035	3.840
87.500	24.765	4.107
90.000	25.939	5.020
92.500	27.277	6.087
95.000	28.463	6.579
97.500	29.398	6.871
100.000	30.097	7.055
102.500	30.582	7.176
105.000	30.879	7.247
107.500	31.010	7.279
110.000	30.991	7.274
112.500	30.836	7.238
115.000	30.557	7.173

Table 2A (cont'd)

117.500	30.164	7.081
120.000	29.664	6.964
122.500	29.064	6.827
125.000	28.371	6.676
127.500	27.585	6.525
130.000	26.689	6.434
132.500	25.629	6.300
135.000	24.355	6.025
137.500	22.854	5.677
140.000	21.128	5.292
142.500	19.178	4.920
145.000	16.945	4.746
147.500	14.273	5.182
150.000	10.430	7.976
152.500	7.301	12.063
155.000	5.107	12.443
157.500	3.565	11.132
160.000	2.480	9.136
162.500	1.711	6.976
165.000	1.162	4.838
167.500	0.762	2.743
170.000	0.459	0.580

Table 2B

Flux along the diagonal x = y

x	y	PHI1	PHI2
0.000	0.000	29.350	4.620
1.250	1.250	29.494	4.647
2.500	2.500	29.927	4.730
3.750	3.750	30.642	4.876
5.000	5.000	31.628	5.101
6.250	6.250	32.865	5.440
7.500	7.500	34.317	5.961
8.750	8.750	35.921	6.778
10.000	10.000	37.562	8.030
12.500	12.500	40.453	9.391
15.000	15.000	42.671	10.000
17.500	17.500	44.343	10.406
20.000	20.000	45.590	10.701
22.500	22.500	46.493	10.913
25.000	25.000	47.108	11.058
27.500	27.500	47.480	11.145
30.000	30.000	47.642	11.183
32.500	32.500	47.621	11.178
35.000	35.000	47.436	11.135
37.500	37.500	47.106	11.057
40.000	40.000	46.640	10.948
42.500	42.500	46.046	10.808
45.000	45.000	45.325	10.639
47.500	47.500	44.476	10.440
50.000	50.000	43.493	10.209
52.500	52.500	42.363	9.944
55.000	55.000	41.073	9.641
57.500	57.500	39.601	9.295
60.000	60.000	37.922	8.901
62.500	62.500	35.998	8.448
65.000	65.000	33.776	7.916
67.500	67.500	31.177	7.241
70.000	70.000	28.120	6.013
72.500	72.500	24.843	4.306
75.000	75.000	21.997	3.572
77.500	77.500	19.869	3.146
80.000	80.000	18.479	2.914
82.500	82.500	17.783	2.815
85.000	85.000	17.681	2.873
87.500	87.500	18.019	3.145
90.000	90.000	18.447	4.081
92.500	92.500	18.398	4.495
95.000	95.000	17.696	4.385
97.500	97.500	16.444	4.103
100.000	100.000	14.747	3.746
102.500	102.500	12.688	3.400
105.000	105.000	10.293	3.301
107.500	107.500	7.555	3.832
110.000	110.000	4.283	6.272
112.500	112.500	2.399	6.920

Table 2B (cont'd)

115.000	115.000	1.382	5.429
117.500	117.500	0.803	3.767
120.000	120.000	0.463	2.387
122.500	122.500	0.260	1.360
125.000	125.000	0.138	0.652
127.500	127.500	0.066	0.211
130.000	130.000	0.025	0.010
132.500	132.500	0.000	0.000

Table 2C

Average subassembly powers

170.00									
	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
150.00									
	0.7571	0.7379	0.6952	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
130.00									
	0.9350	0.9512	0.9761	0.8485	0.5997	0.0000	0.0000	0.0000	0.0000
110.00									
	0.9345	1.0359	1.0706	0.9068	0.6859	0.5874	0.0000	0.0000	0.0000
90.00									
	0.6099	1.0684	1.1783	0.9661	0.4712	0.6860	0.5998	0.0000	0.0000
70.00									
	1.2083	1.3128	1.3433	1.1916	0.9662	0.9070	0.8487	0.0000	0.0000
50.00									
	1.4509	1.4772	1.4671	1.3434	1.1785	1.0708	0.9764	0.6954	0.0000
30.00									
	1.3068	1.4323	1.4773	1.3130	1.0686	1.0362	0.9516	0.7382	0.0000
10.00									
	0.7443	1.3068	1.4510	1.2084	0.6100	0.9348	0.9354	0.7574	0.0000
0.00	10.00	30.00	50.00	70.00	90.00	110.00	130.00	150.00	170.00

Table 2D, group 1

Flux averages

Group 1

170.00	3.3944	3.2431	2.4578	0.7188	0.0215	0.0040	0.0000	0.0000	0.0000
150.00	20.3675	19.8959	16.7075	5.9875	2.4387	0.5778	0.0100	0.0000	0.0000
130.00	29.4320	29.8661	29.1220	22.6278	14.4753	4.0186	0.6479	0.0100	0.0000
110.00	29.9296	32.7203	33.6976	28.5075	20.7859	14.0517	4.0194	0.5781	0.0040
90.00	26.5281	34.1781	37.1833	30.8752	20.3850	20.7890	14.4796	2.4397	0.0215
70.00	38.6266	41.4685	42.3909	37.6359	30.8782	28.5131	22.6343	5.9895	0.7190
50.00	45.7876	46.6162	46.2972	42.3941	37.1893	33.7055	29.1304	16.7128	2.4586
30.00	41.8016	45.2435	46.6185	41.4731	34.1849	32.7299	29.8766	19.9035	3.2444
10.00	32.3916	41.8032	45.7911	38.6309	26.5336	29.9397	29.4437	20.3760	3.3959

0.00 10.00 30.00 50.00 70.00 90.00 110.00 130.00 150.00 170.00

Table 2D, group 2

Group 2

170.00	8.0138	7.6236	5.8770	2.8301	0.0293	0.0053	0.0000	0.0000	0.0000
150.00	5.6078	5.4661	5.1497	12.4623	6.0648	2.2525	0.0143	0.0000	0.0000
130.00	6.9263	7.0461	7.2307	6.2849	4.4419	8.3528	2.5767	0.0143	0.0000
110.00	6.9224	7.6736	7.9302	6.7171	5.0810	4.3515	8.3545	2.2534	0.0054
90.00	4.5176	7.9143	8.7280	7.1565	3.4904	5.0817	4.4432	6.0674	0.0293
70.00	8.9502	9.7246	9.9503	8.8264	7.1572	6.7184	6.2867	12.4665	2.8309
50.00	10.7476	10.9421	10.8673	9.9511	8.7293	7.9321	7.2328	5.1513	5.8789
30.00	9.6798	10.6093	10.9427	9.7256	7.9158	7.6758	7.0485	5.4682	7.6265
10.00	5.5137	9.6801	10.7484	8.9512	4.5185	6.9247	6.9290	5.6102	8.0174
0.00	10.00	30.00	50.00	70.00	90.00	110.00	130.00	150.00	170.00