ECE 351 - Lab 8 - Fourier Series Approximation of a Square Wave

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Contents

1 Purpose

The purpose of this lab was to gain familiarity using Fourier series to approximate periodic time-domain signals.

2 Procedure

This lab began by examining the square wave function shown in Figure 1 shown below and the general equation for it shown in Equation 1 in the Equations section. In the preliminary work, the equations for a_k and b_k were calculated, and they are also shown in the Equations section.

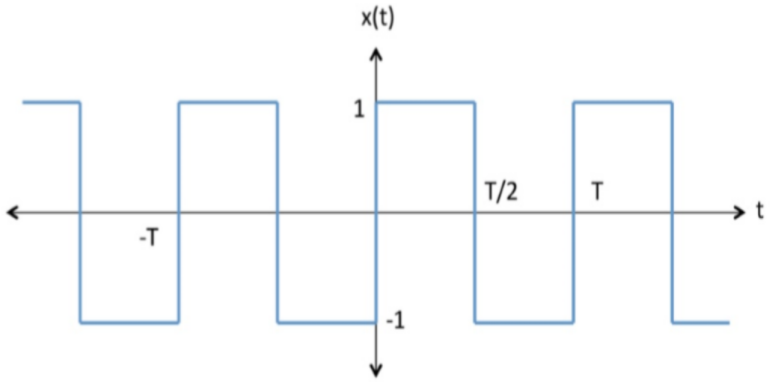

Figure 1: Given Square Wave Plot

With the a_k and b_k functions entered into Python, the values of a_0, a_1, b_1, b_2, and b_3 were calculated, as shown in the Appendix at the end of this report.

Using the functions for a_k and b_k, the Fourier series approximation was plotted with a period of T = 8s for a range of t = [0, 20]s. These plots are shown for values of N = 1, 3, 15, 50, 150, 1500 in the Results section.

3 Equations

$$x(t) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty} a_k cos(k\omega_0 t) + b_k sin(k\omega_0 t) \tag{1}$$

$$a_k = 0 \tag{2}$$

$$b_k = \frac{2}{k\pi}(1 - cos(k\pi)) \tag{3}$$

4 Results

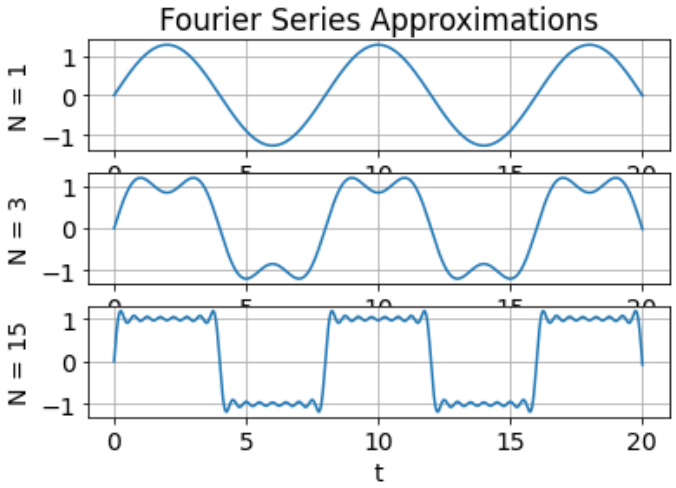

Figure 2: Fourier Series Approximations for N = 1, 3, 15

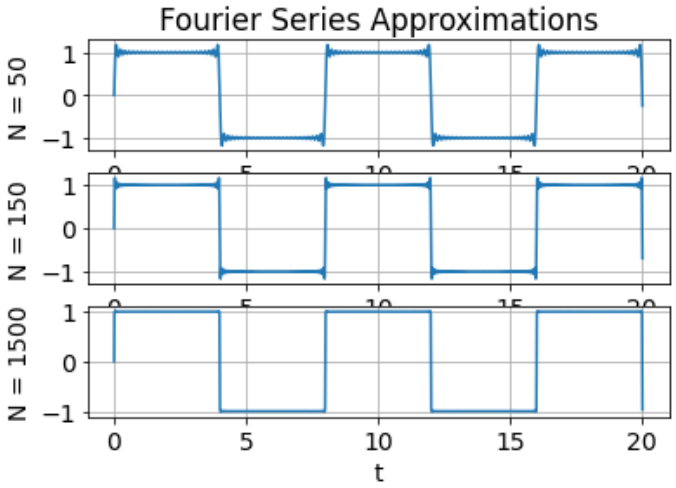

Figure 3: Fourier Series Approximations for N = 50, 150, 1500

5 Questions

Is x(t) an even or an odd function? Explain why.

x(t) is an odd function. This is because f(-t) = -f(t), which is the definition of an odd function.

Based on your results from Task 1, what do you expect the values of a_2, a_3, ... , a_n to be? Why?

All values of a_k will be 0 due to the fact that the function is odd, and the a_k coefficient inherently models an even function.

How does the approximation of the square wave change as the value of N increases? In what way does the Fourier series struggle to approximate the square wave?

The approximation gets closer to the desired square wave as N increases, as can be seen as N changes from 1 to 1500. Even when N is 1500, the Fourier series can never stay a constant value. This is best illustrated in Figure 2 when N=15. The value still oscillates up and down, but the oscillation magnitude decreases.

What is occurring mathematically in the Fourier series summation as the value of N increases?

As N increases, the series is having more and more functions to sum together to get a closer approximation. This is why when N=1, the approximation is just a cosine function, but as N increases, the approximation gets closer and closer to the square wave given in the lab handout.

Leave any feedback on the clarity of lab tasks, expectations, and deliverables.

The lab was clear and the expectations were understood.

6 GitHub Link

https://github.com/EReeder35

7 Conclusion

In conclusion, this lab was instrumental in developing my understanding of how Fourier series work. It was helpful to get more practice coding functions in Python, as well as to get some visual explanations of how Fourier series calculate things, as well as how they change as the N value increases.

8 Appendix

```
a_0:  [0.]
a_1:  [0.]
b_1:  [1.27323954]
b_2:  [0.]
b_3:  [0.42441318]
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