CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 20 - DP: Write down recurrence

Due Date .... April 1
Name .... **Your Name**
Student ID .... **Your Student ID**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 20 - DP: Write down Recurrence

Problem 1. Suppose we have an m-letter alphabet, $\Sigma = \{0, 1, \ldots, m-1\}$. Determine a recurrence for the number of strings ω of length n, such that no two consecutive characters in ω are the same. Clearly justify your recurrence.

Answer. We first describe the initial conditions. Observe that $W_0 = \{\Sigma\}$, where Σ is a string of length 0, and $W_1 = \Sigma$. Thus $f_0 = 0$ and $f_1 = 0$.
Now considering a string $w \in W_n$ of length $2 \geq 2$. We note that the first character $w_0 \in \{0, 1, ..., m-1\}$. Consider the following cases:

- If $w_0 = 0$, then necessarily $w_1! = 0$, where we can assume $w_1 \in \{01, 02, ...0(m-1)\}$. Otherwise we would have a 00 sub string. Thus, $w = W_a\tau$, where $\tau \in W_{n-2}$, and w_a can be any word in a set of length $(m-1)$. This means that there are $f_{n-2} = |W_{n-2}|$ ways to select the value of τ. So there are (m-1)f_{n-2} such strings w in this case

- If $w_0 = 1$, then necessarily $w_1! = 1$, where we can assume $w_1 \in \{10, 12, ...1(m-1)\}$. Otherwise we would have a 11 sub string. Thus, $w = W_a\tau$, where $\tau \in W_{n-2}$, and w_a can be any word in a set of length $(m-1)$. This means that there are $f_{n-2} = |W_{n-2}|$ ways to select the value of τ. So there are (m-1)f_{n-2} such strings w in this case

Since these two notions that if some letter $x \in \sum\{0, 1, ..., m-1\}$ is in position w_0 then it cannot be in position w_1 we get that the recurrence for when $m \geq 3$ is $f_n = (m-1)f_{n-1} * (m-1)f_{n-2}$.
We then get that the whole recurrence relation is.

$$f_n = \begin{cases} f_n = (m-1)f_{n-1} * (m-1)f_{n-2} & m \geq 3 \\ 2 & m = 2 \\ 0 & m = 1, 0 \end{cases}$$ □