CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Retake Token Quiz- Standard 3

Due Date ..... April 28
Name ..... **Your Name**
Student ID ..... **Your Student ID**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 3- Exchange Arguments

2.1 Problem 1

Problem 1. Let A be a finite set, and let w be the weight function assigning each element in A a positive integer weight. Let $M > 0$. Our goal is to find the minimum number of boxes, with each box having capacity M, to place the elements of A. Here, the weight of a box is the sum of the weights of the elements it stores; that is, if B is a box, then:

$$w(B) = \sum_{a \in B} w(a).$$

As each box has capacity M, we note that each box B must satisfy $w(B) \leq M$. Suppose that we have a solution where at least two boxes B_i, B_j such that $w(B_i) \leq M/2$ and $w(B_j) \leq M/2$. Using an exchange argument, prove that such a solution is not optimal.

Proof. Let 2M be the total amount of weight we want to fit in the two boxes B_i and B_j. Using the formula $w(B_i) \leq M/2$ and $w(B_j) \leq M/2$ we get that $w(B_i) \leq M$ and $w(B_j) \leq M$.
When either $w(B_i)$ or $w(B_j)$ is less than M you cannot fit the total amount of weight in both of the boxes. Therefore this is not the optimal solution. To get an optimal solution the formula should be $w(B_i) = M/2$ and $w(B_j) = M/2$ that way each box carries the same amount of weight and all the weight will be able to fit in the boxes. □