CSCI 3104 Spring 2022
Instructors: Profs. Chen and Layer

Quiz 10 - Asymptotics I

Due Date .... March 4
Name .... **Your Name**
Student ID .... **Your Student ID**

Contents

1 Instructions

- The solutions **should be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You **may not collaborate with other students**. **Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Discord, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.

2 Standard 10 - Asymptotics I

Problem 1. Let $f(n) = n^3$ and $g(n) = 2n^3 + 5n^2 + 4n\log(n)$. Determine the relationship that **best** applies: $f(n) \in O(g(n)), f(n) \in \Theta(g(n))$, or $f(n) \in \Omega(g(n))$. Prove your answer. Note that you are expected to spell out **all Calculus details** at the level of Calculus I-II.

Answer. Using the limit comparison test we test.//

$$\begin{aligned}
\lim_{n\to\infty} \frac{n^3}{2n^3 + 5n^2 + 4n\log(n)} &= \lim_{n\to\infty} \frac{n^3}{2n^3 + 20n^3\log(n)} \\
&= \lim_{n\to\infty} \frac{n^3}{2n^3(1 + 10\log(n))} \\
&= \lim_{n\to\infty} \frac{1}{2(1 + 10\log(n))} \\
&= \frac{1}{2} \frac{\lim_{n\to\infty} 1}{\lim_{n\to\infty}(10\log(n) + 1)} \\
&= \frac{1}{2} \frac{1}{\infty} \\
&= 0
\end{aligned}$$

By the limit comparison test we can say that $f(n) \in O(g(n))$ $\square$