CSCI 3104 Spring 2022
Instructor: Profs. Chen and Layer

Problem Set 3

Due Date ..... **February 8, 2022**
Name ..... **Your Name**
Student ID ..... **Your Student ID**
Collaborators ..... **List Your Collaborators Here**

Contents

1 Instructions

- The solutions **must be typed**, using proper mathematical notation. We cannot accept hand-written solutions. Here's a short intro to LaTeX.
- You should submit your work through the **class Canvas page** only. Please submit one PDF file, compiled using this LaTeX template.
- You may not need a full page for your solutions; pagebreaks are there to help Gradescope automatically find where each problem is. Even if you do not attempt every problem, please submit this document with no fewer pages than the blank template (or Gradescope has issues with it).
- You are welcome and encouraged to collaborate with your classmates, as well as consult outside resources. You must **cite your sources in this document. Copying from any source is an Honor Code violation. Furthermore, all submissions must be in your own words and reflect your understanding of the material.** If there is any confusion about this policy, it is your responsibility to clarify before the due date.
- Posting to **any** service including, but not limited to Chegg, Reddit, StackExchange, etc., for help on an assignment is a violation of the Honor Code.
- You **must** virtually sign the Honor Code (see Section 2). Failure to do so will result in your assignment not being graded.

2 Honor Code (Make Sure to Virtually Sign)

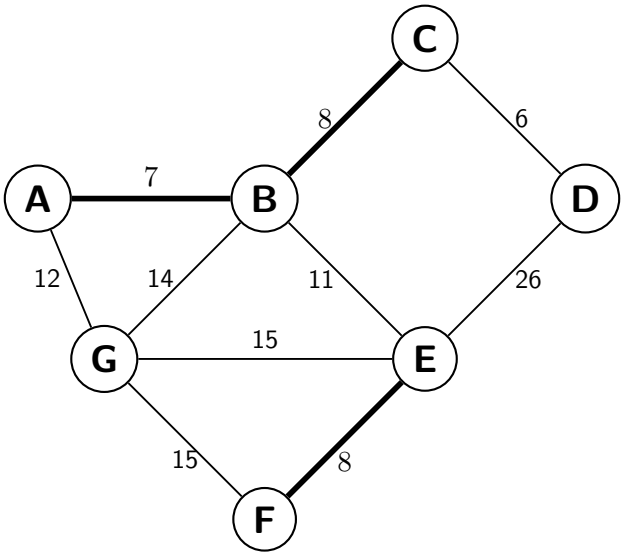
Problem 1.

- My submission is in my own words and reflects my understanding of the material.
- Any collaborations and external sources have been clearly cited in this document.
- I have not posted to external services including, but not limited to Chegg, Reddit, StackExchange, etc.
- I have neither copied nor provided others solutions they can copy.

Agreed (signature here). I agree to the above. Ethan Richman □

3 Standard 7 - MST: safe and useless edges

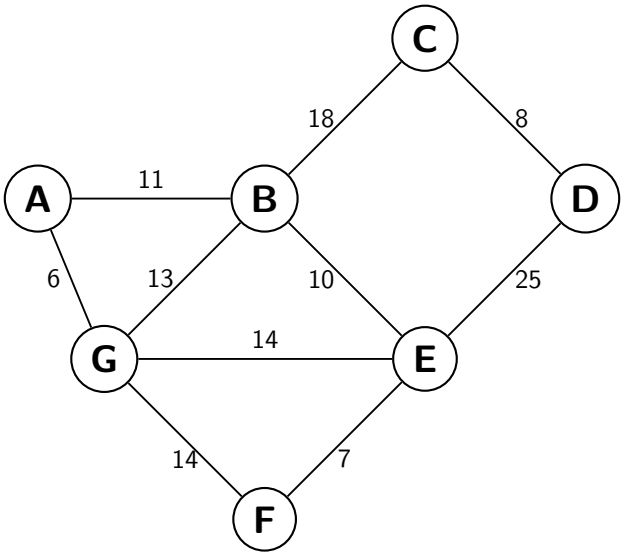
Problem 2. Consider the weighted graph $G(V, E, w)$ below. Let $\mathcal{F} = \{\{A, B\}, \{B, C\}, \{E, F\}\}$ be an intermediate spanning forest (indicated by the thick edges below). Label each edge that is **not** in $\mathcal{F}$ as safe, useless, or undecided. Provide a 1-2 sentence explanation for each such edge.



Answer. {C,D}: Safe, next minimum weight edge with exactly one end point in F.
{A,G}: Safe, next minimum weight edge with exactly one end point in F.
{B,E}: Safe, connects two components of F and does not form a cycle.
{D,E}: Undecided, connects the two components so is not useless but forms a cycle so is not safe
{E,G}: Undecided, connects two components of F so isn't useless but not safe because it forms a cycle.
{B,G}: Undecided connects two components so is not useless but creates a cycle so is not safe.
{G,F}: Undecided isn't useless because it connects two components but creates a cycle. □

4 Standard 8- Kruskal's Algorithm

Problem 3. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Kruskal's algorithm adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Kruskal's algorithm takes as it selects the first **three** edges.

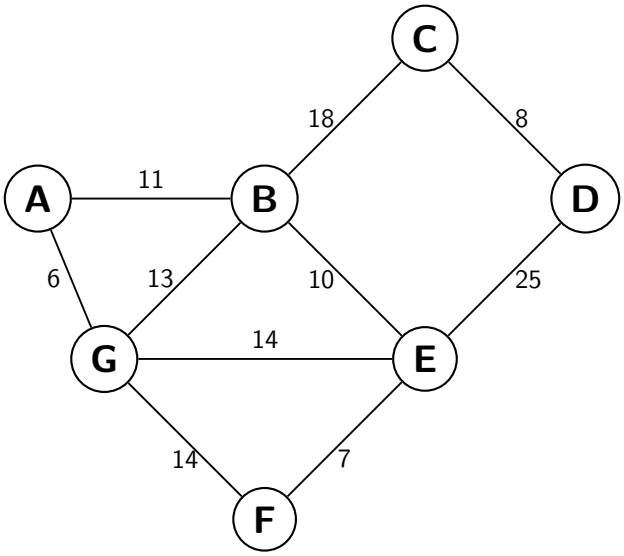


Answer. Order of Edges Added: [{A,G},{E,F},{C,D},{B,E},{A,B},{C,B}]

To select the first three edges of the graph Kruskals first puts the edges into a priority queue based on the weights of the edges. It then pulls the first edge out which in this case is {A,G} it then determines if the edge should be added or not in this case it is so now the graph consists of the edge {A,G}. After that process is done the edge just analyzed is removed from the priority queue. The next edge that is pulled is {E,F}. The algorithm goes through the same process and since {E,F} connects two disjoint components it is added to the tree. {E,F} is then removed from the queue. The next edge pulled from the queue is {C,D} that is then added to the tree because it connects two disjoint components. □

5 Standard 9- Prim's Algorithm

Problem 4. Consider the weighted graph $G(V, E, w)$ below. Clearly list the order in which Prim's algorithm, **using the source vertex** A, adds edges to a minimum-weight spanning tree for G. Additionally, clearly articulate the steps that Prim's algorithm takes as it selects the first **three** edges.



Answer. Order Edges are added: {A,G},{A,B},{B,E},{E,F},{B,C},{C,D}
To start the algorithm a priority queue is created with the edges coming from the source vertex A. So the priority queue looks like Q=[({A,G},6),({A,B},11)]. It then selects the first edge in the queue which is {A,G} and checks if it meets the requirements to be added and then adds it to the tree. Then it polls all the edges of A and G and creates a priority queue that looks like Q=[({A,B},11),({G,B},13),({G,E},14),({G,F},14)]. It then pulls {A,B} and assess if it can be added and since it has only one endpoint in the existing tree it is added. Then a new priority queue is created with the edges from G and B. The next thing that is assessed is the edge {B,E} and since it has one endpoint in the existing tree it is added. □