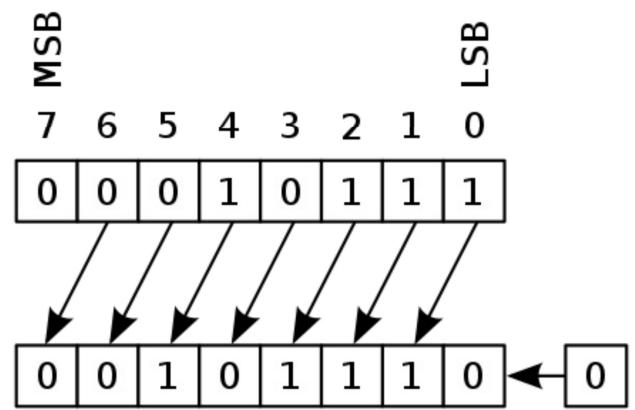


Multiplication

- ▶ Each shift to the left doubles the number, therefore each left shift multiplies the original number by 2.
- ▶ Example multiply 011 by 010 and 100

$$\begin{array}{r} 011 \\ \times \\ 010 \\ \hline 000 \\ 011 \\ 000 \\ \hline 110 \end{array}$$

$$\begin{array}{r} 011 \\ \times \\ 100 \\ \hline 000 \\ 000 \\ 011 \\ \hline 1100 \end{array}$$



Signed Multiplication

- Requires special consideration for negative (2's complement) numbers

signed

3 bit multiplication

101	-3
$\times \quad 011$	$\times \quad 3$
$\underline{101}$	$\underline{-\ 9}$
101	↑ ↓ ERROR
000	
$\underline{001111}$	15



Signed Multiplication

- In 2's complement you must **sign extend** to the product bit width

4b x 4b

$\begin{array}{r} 0 & 1 & 1 & 1 \\ \times 0 & 1 & 1 & 0 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 6 \\ \hline \end{array}$
$\begin{array}{r} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \times 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ + 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline x & x & x & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{array}$	
$\begin{array}{r} 7 \\ \times 6 \\ \hline 42 \end{array}$	

Must carry out
all the bits

Stop after 8 bits



Signed Multiplication

- In 2's complement you must **sign extend** to the product bit width

4b x 4b

$$\begin{array}{r} \begin{array}{r} 1 & 0 & 0 & 1 \\ \times & 1 & 0 & 1 & 0 \\ \hline \end{array} & \begin{array}{r} -7 \\ \times & -6 \\ \hline \end{array} \\ \begin{array}{r} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ \times & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ + & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 \\ \hline \end{array} & \begin{array}{r} -7 \\ \times & -6 \\ \hline \end{array} \\ \begin{array}{r} x | 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{array} & 42 \end{array}$$

Must carry out all the bits

Stop after 8 bits



Signed Multiplication

- In 2's complement you must **sign extend** to the product bit width
- When doing it by hand - where possible multiply by the positive value

4b x 4b

$ \begin{array}{r} 0 & 1 & 1 & 1 \\ \times & 1 & 0 & 1 & 0 \\ \hline \end{array} $	$ \begin{array}{r} 7 \\ \times -6 \\ \hline \end{array} $
$ \begin{array}{r} 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \times & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} $	
$ \begin{array}{r} 7 \\ \times -6 \\ \hline \end{array} $	
$ \begin{array}{r} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ \times & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{array} $	
$ \begin{array}{r} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ \times & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{array} $	
$ \begin{array}{r} 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ + & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \end{array} $	

Must carry out all the bits

Common

Stop after 8 bits

-42

Others are 0

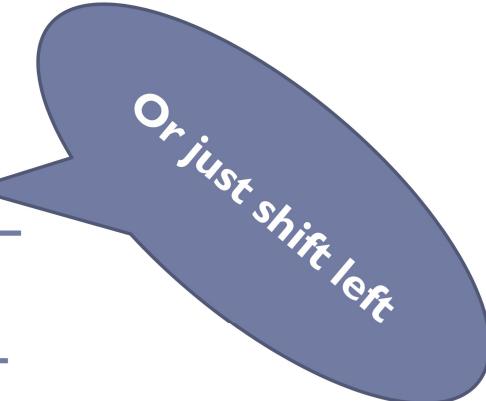
Stop after 8 bits

-42

© tj

Example

- ▶ The 6-bit integer L in the U2 code has the following form: 010011 , Calculate the number $-L*2$, add 1 to it, and present the result in **eight-bit** format.
- ▶ Solution : $-L = 101101$
- ▶ $-L * 2$


$$\begin{array}{r} \text{111111101101} \\ \text{000000000010} \\ \hline \text{000000000000} \\ \text{111111101101} \\ \hline \text{111111011010} \\ + \\ \text{000000000001} \\ \hline \text{111111011011} \end{array}$$

Stop after 8 bits



Division

- ▶ Each shift to the right reduces the number by half

▶ 0 → 1000 → $\xrightarrow{\text{SR}}$ 0100 → $\xrightarrow{\text{SR}}$ 0010

- Let's say we want to do $1001 \div 11$
- Step 1: Set up the problem $11\overline{)1001}$
- Step 2: Can we get a 11 out of 1? No, put a 0.

$$\begin{array}{r} 0??? \\ 11\overline{)1001} \end{array}$$

- Step 3: Can we get a 11 out of 10? No, put a 0.

$$\begin{array}{r} 00?? \\ 11\overline{)1001} \end{array}$$



Division

- Step 4: Can we get 11 out of 100?
Yes, so it is a 1. So then we multiply
 $11 \times 1 = 11$ and we subtract that
from 100 ($100 - 11$) and we get 1.

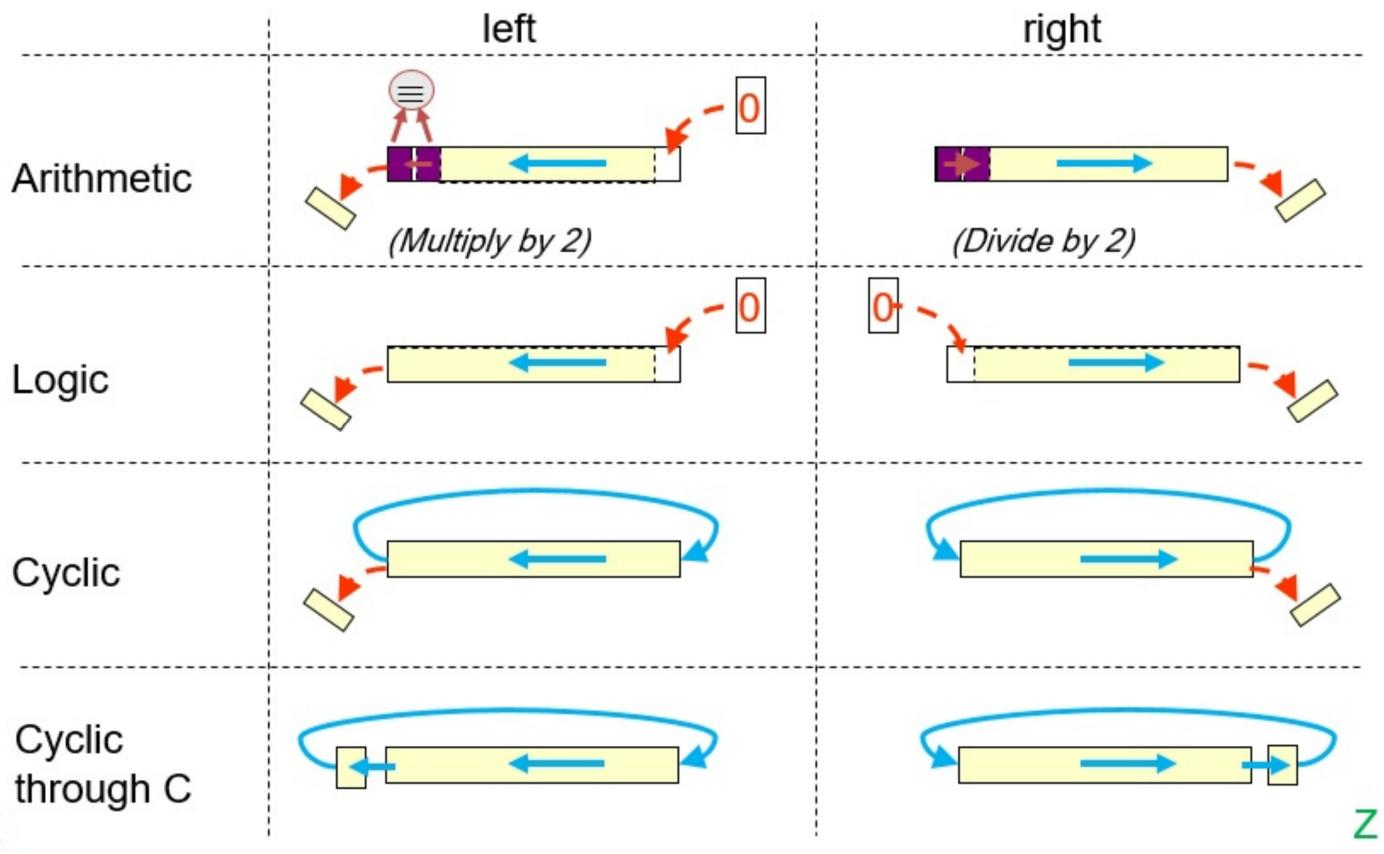
$$\begin{array}{r} 001? \\ 11 \overline{)1001} \\ -0011 \\ \hline 00001 \end{array}$$

- Step 5: bring down the next 1, to
make 11 and we can get one 11 out
of 11.

$$\begin{array}{r} 0011 \\ 11 \overline{)1001} \\ -0011 \\ \hline 000011 \\ -00011 \\ \hline 000000 \end{array}$$



binary shift operations by 1 bit



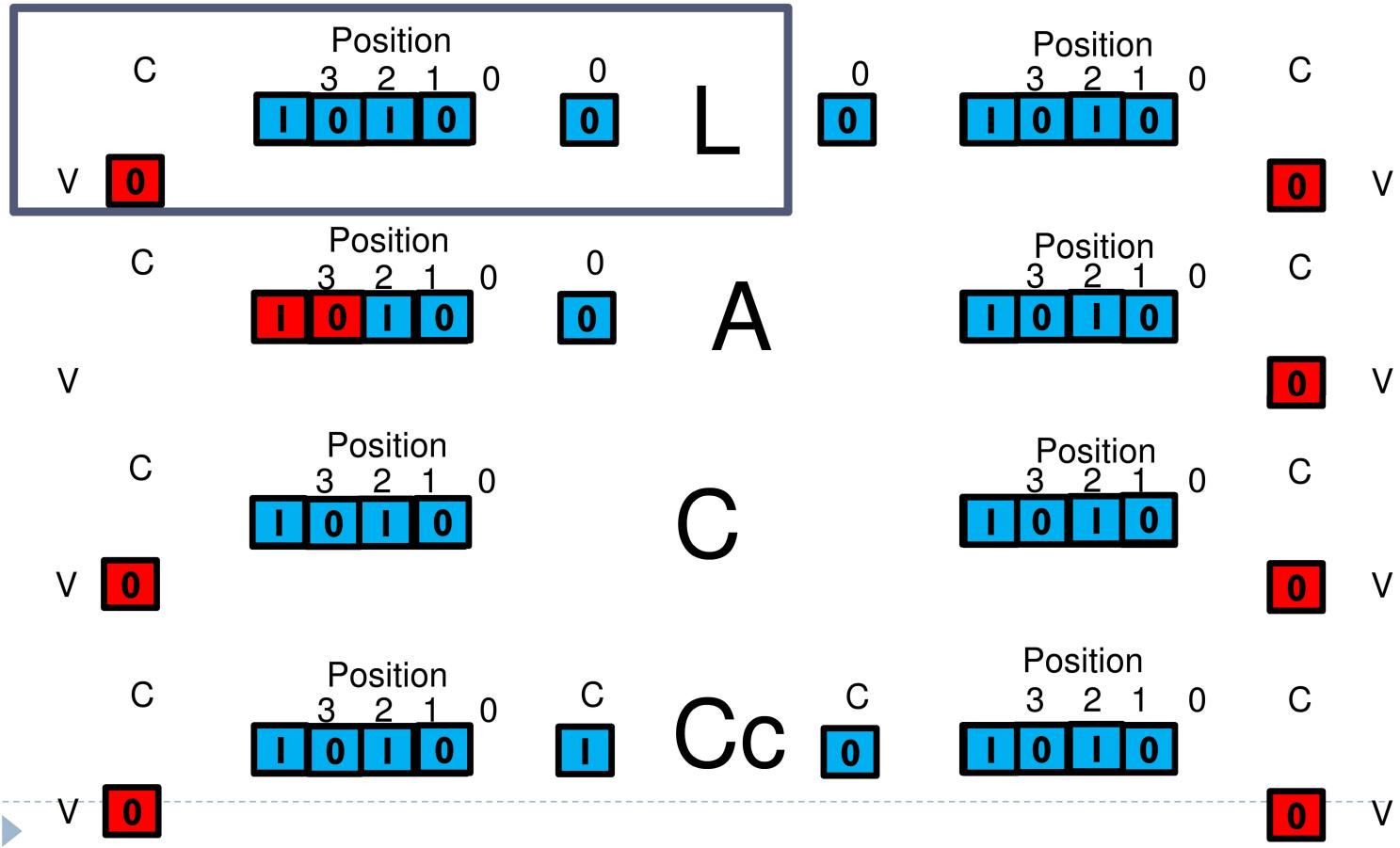
Z

Z

SHIFT

RIGHT

LEFT



Example

- ▶ Enter the result of the LSL (logical shift left) shift of the 4-bit register: **0111**
- ▶ Before shifting conditions C=1, V=1.
- ▶ Where C and V are the condition bits after shift (carry and overflow)

- ▶ **Solution :**
- ▶ **0111**  **1110** C=0 V=0

