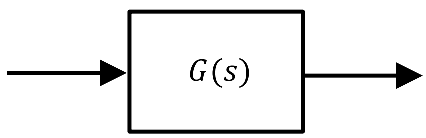
LECTURE 9: SYSTEM IDENTIFICATION FROM THE STEP RESPONSE

System Identification

The basic idea of system identification is to derive a mathematical model from the response of a system to a known input.

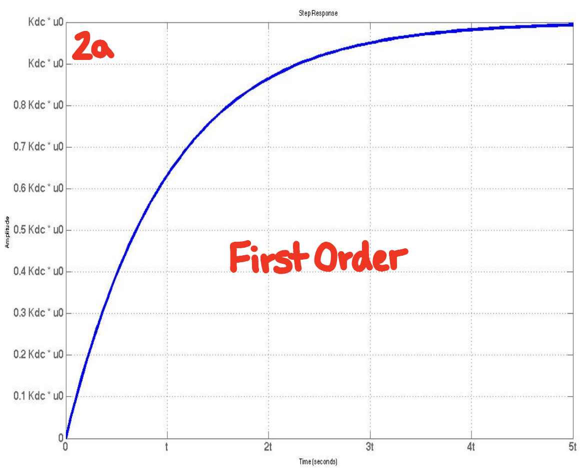
There are two parts to system identification from the step response. The first task is to classify the system as first order, second order, or higher order. If the system is first order or second order, then one proceeds to the second task, which is matching features of the step response to parameters from the canonical forms. We will assume that the systems are all stable, that is, that all poles are negative or have negative real part.

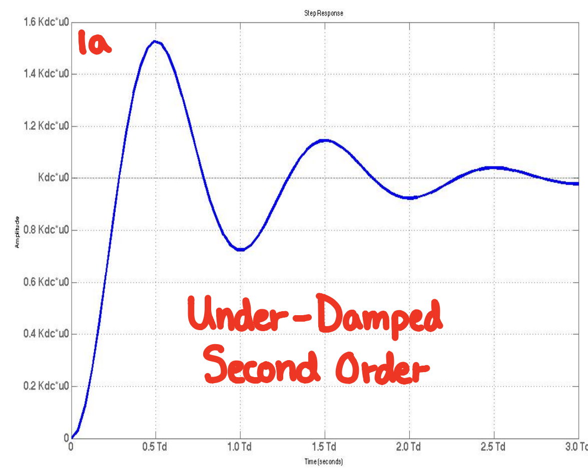


Classification

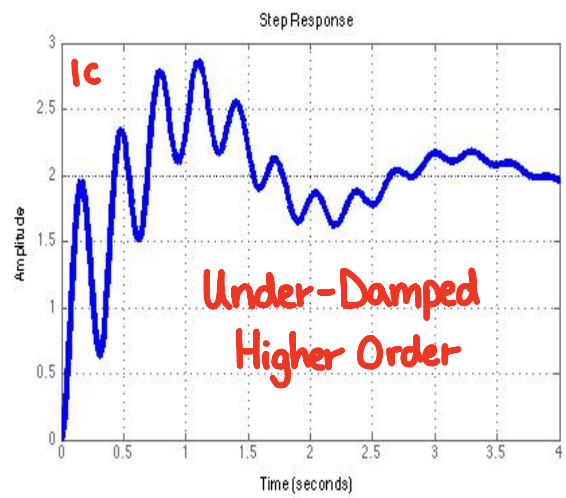
Classification proceeds of a step response as follows:

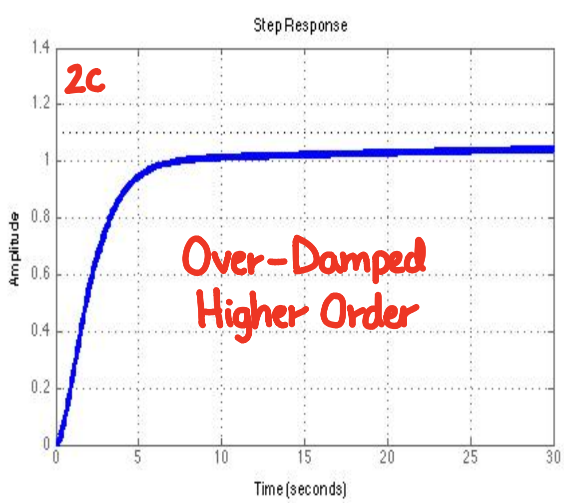
1. *Look for oscillations*
    a. *If there are oscillations present with a single frequency and settles to a steady-state value at a single exponential rate, then the system is an* ***UNDER-DAMPED SECOND ORDER SYSTEM****.*
    b. *If there are oscillations present with a single frequency and settles to a steady-state value at a multiple exponential rates, then the system is an* ***UNDER-DAMPED HIGHER ORDER SYSTEM****.*
    c. *If there are oscillations present with a multiple frequencies, then the system is an* ***UNDER-DAMPED HIGHER ORDER SYSTEM****.*
    d. *If there are no oscillations present, then the system is either first order or an over damped second order system or an over damped higher order system. Go to step 2.*
2. *Look for an inflection point.*
    a. *If there is no inflection point then the system is* ***FIRST ORDER****.*
    b. *If there is a single inflection point then the system is an* ***OVER-DAMPED SECOND ORDER SYSTEM*** *or an* ***OVER-DAMPED HIGHER ORDER SYSTEM****. Determining which of these is correct requires the system parameter identification procedure for over-damped second order systems.*
    c. *If there are multiple inflection points then the system is an* ***OVER DAMPED HIGHER ORDER SYSTEM****.*

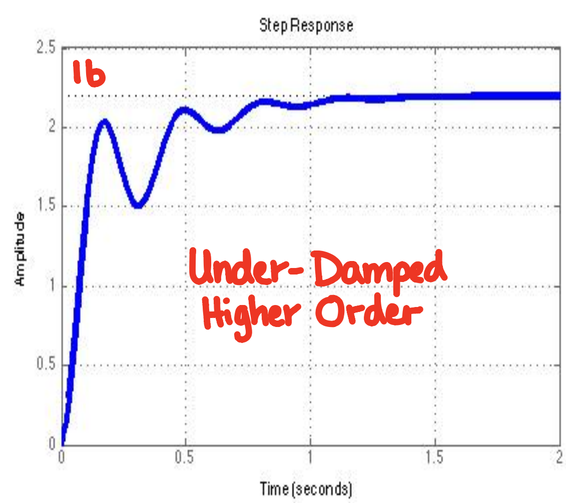

2a
First Order
Kdc * u0
0.8 Kdc * u0
0.7 Kdc * u0
0.6 Kdc * u0
0.5 Kdc * u0
0.4 Kdc * u0
0.3 Kdc * u0
0.2 Kdc * u0
0.1 Kdc * u0
0
t
2t
3t
4t
5t

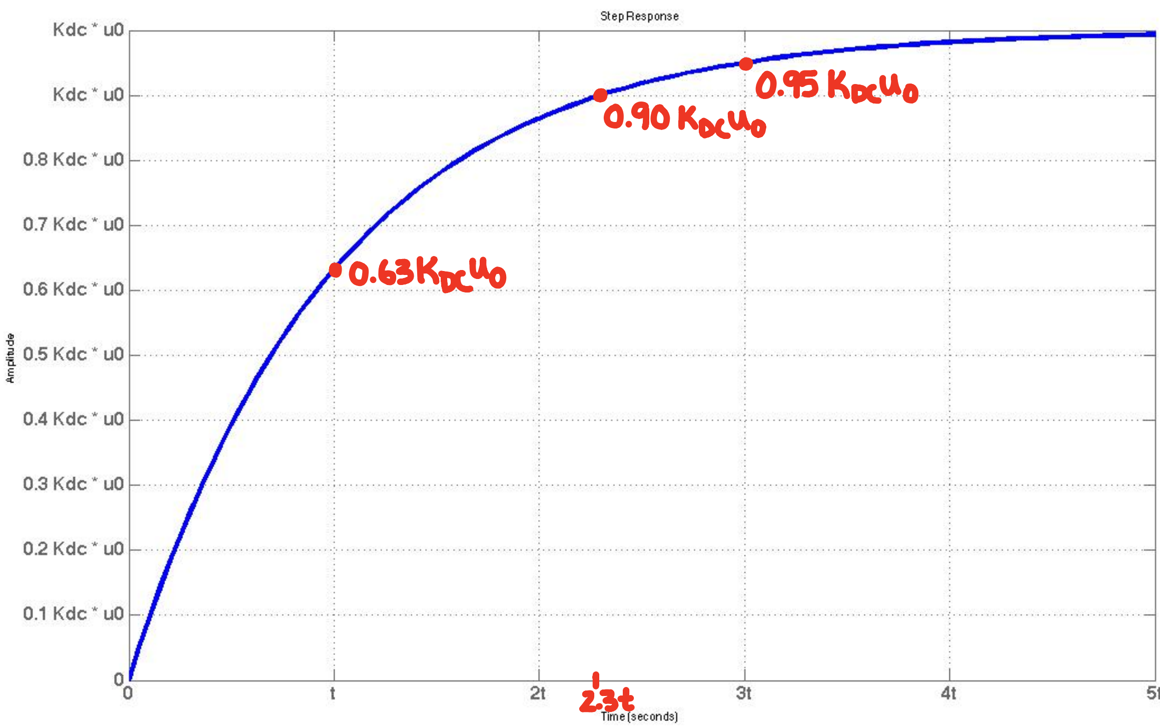

1a
Under-Damped
Second Order
1.6 Kdc*u0
1.4 Kdc*u0
1.2 Kdc*u0
Kdc*u0
0.8 Kdc*u0
0.6 Kdc*u0
0.4 Kdc*u0
0.2 Kdc*u0
0
0.5 Td
1.0 Td
1.5 Td
2.0 Td
2.5 Td
3.0 Td


Step Response
2b
Over-Damped
Second or Higher Order
Amplitude
Time (seconds)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9


Step Response
1c
Under-Damped
Higher Order
Amplitude
Time (seconds)
3
2.5
2
1.5
1
0.5
0
0.5
1
1.5
2
2.5
3
3.5
4


Step Response
2c
Over-Damped
Higher Order
Amplitude
Time (seconds)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
5
10
15
20
25
30


Step Response
1b
Under-Damped
Higher Order
Amplitude
Time (seconds)
2.5
2
1.5
1
0.5
0
0.5
1
1.5
2



System Identification for First Order Systems

Canonical Form

$$G(s)=\frac{K_{DC}}{\tau s+1}=\frac{b}{s+a} \Longrightarrow a=\frac{1}{\tau}\ ;\ b=\frac{K_{DC}}{\tau}$$

time constant

Step Response

$$y(t)=K_{DC}u_0(1-e^{-t/\tau})$$

Determine the DC Gain

$$K_{DC}=\frac{\text{Steady-State Value}}{\text{Magnitude of Step}}$$

Determine the Time Constant

$$\tau=T_{63\%}$$

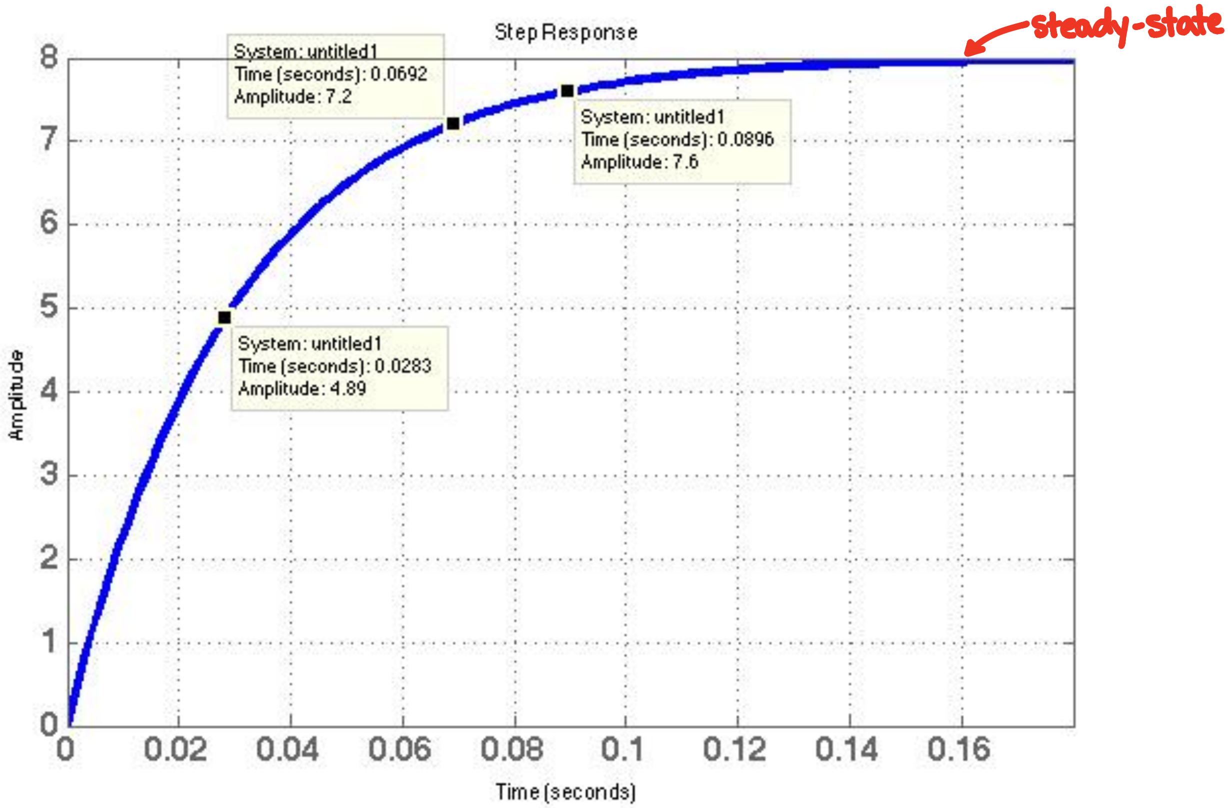
or

$$\tau=T_{90\%}/2.3$$

or

$$\tau=T_{95\%}/3$$

Example of a First Order System



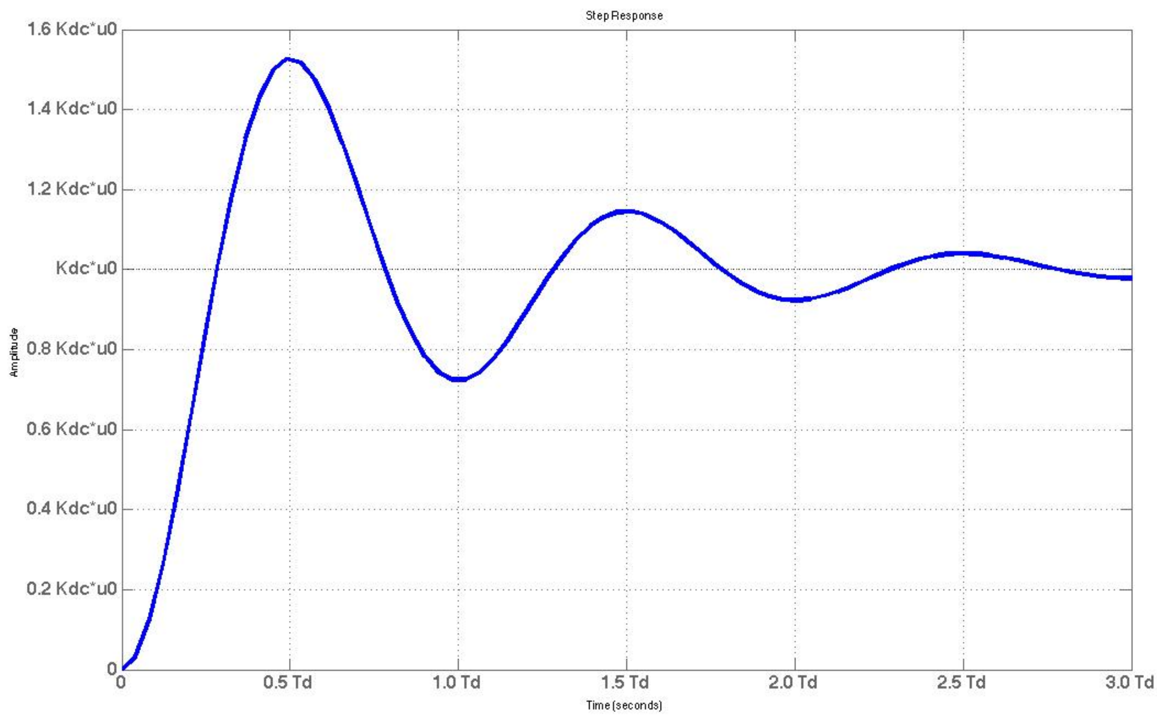
DC Gain
$u_0 = 0.4$

$$K_{DC} = \frac{8}{0.4} = 20$$

Time Constant

$$\tau = T_{63\%} \approx 0.03$$
$$\tau = T_{90\%}/2.3 = \frac{0.07}{2.3} \approx 0.03$$
$$\tau = T_{95\%}/3 = \frac{0.09}{3} \approx 0.03$$

System Identification for Under-Damped Second Order Systems



Canonical Form

$$G(s) = K_{DC}\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Step Response

imaginary part of pole

$$y(t) = K_{DC}u_0\left(1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_d t}cos(\omega_d t - \phi)\right)$$

where

period of damped oscillation

$$\omega_d = \omega_n\sqrt{1-\zeta^2} \implies T_d = 2\pi/\omega_d$$

damped natural frequency

$$\phi = \arccos\left(\sqrt{1-\zeta^2}\right)$$

Determine the DC Gain

$$K_{DC} = \frac{\text{steady-state}}{\text{magnitude of step}}$$

Determine the Damping Ratio

Use the percent overshoot formula

$$OSR = overshoot\ ratio = \frac{\text{Peak Value} - \text{Steady State Value}}{\text{Steady State Value}}$$

$$\zeta = \frac{-\ln(OSR)}{\sqrt{\pi^2 + \ln(OSR)^2}}$$

Determine the Natural Frequency

First determine the period of the damped oscillation.

Pick a point where the plot crosses the steady state value. Move to the next crossing of the steady state value. The difference between these two times, ΔT, is one-half the period of the damped oscillation, i.e. $T_d/2$.

$$\Delta T = \frac{T_d}{2} \Rightarrow 2\Delta T = T_d$$

$$\omega_d = \frac{2\pi}{T_d} = \frac{2\pi}{2\Delta T} = \frac{\pi}{\Delta T}$$

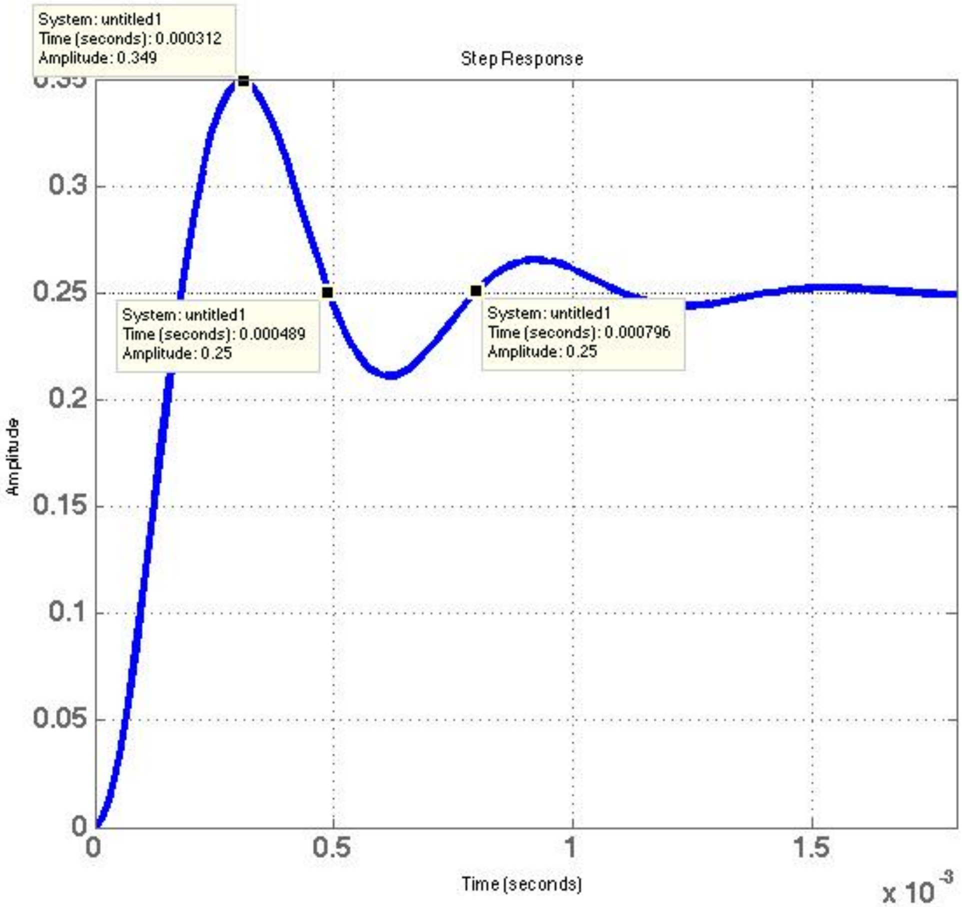
Solve for ω_n using the relation for T_d and the value of the damping ratio already determined.

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}} = \frac{\pi/\Delta T}{\sqrt{1-\zeta^2}} = \frac{\pi}{\Delta T\sqrt{1-\zeta^2}}$$

Or all at once

$$\omega_n = \frac{\pi}{\Delta T\sqrt{1-\zeta^2}}$$

Example of An Under-Damped Second Order System



DC Gain

$u_0 = 2.5$

$$K_{DC} = \frac{0.25}{2.5} = 0.1$$

Damping Ratio

$$OSR = \frac{0.1}{0.25} = 0.4$$

$$\zeta = \frac{-\ln(0.4)}{\sqrt{\pi^2 + \ln(0.4)^2}} = 0.3$$

Natural Frequency

$$\Delta T = 0.00032 = 3.2 \times 10^{-4}$$

$$\omega_n = \frac{\pi}{3.2 \times 10^{-4} \sqrt{1 - 0.3^2}} = 10700$$

