Implicit coupling over exchange grid

Fei Liu

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1 Introduction

Implicit coupling improves model stability, allows flexible time step, conserves flux exchange when implemented correctly.

1. Explicit time stepping algorithm for state variable tends to be unstable and oscillatory.

2. Explicit time stepping algorithm is constrained by CFL condition while implicit time stepping is unconditionally stable.

3. flux exchange is conserved by updating components with identical flux calculated on exchange grid

2 Description of the algorithm

At the surface boundary layer, on the i-th exchange grid cell, using Atm and Lnd flux to update temperature tendency:

$$\rho_A h_{bot} c_{p,A} \frac{T_{bot}^{n+1} - T_{bot}^{n}}{\Delta t_A} = F(T_A^{n+1}, T_L^{n+1}) \tag{1}$$

$$\rho_L h_{top} c_{p,L} \frac{T_{top}^{n+1} - T_{top}^{n}}{\Delta t_L} = -F(T_A^{n+1}, T_L^{n+1}) \tag{2}$$

where the variables are summarized in table

ρ_A	bottom level (boundary layer if present) air density	kg/m^3
ρ_L	top level (boundary layer if present) soil density	kg/m^3
h_{bot}	bottom level height	m
h_{top}	top level height	m
$c_{p,A}$	air specific heat	$J/(kgK)$
$c_{p,L}$	soil specific heat	$J/(kgK)$
$\Delta T_A = T_{bot}^{n+1} - T_{bot}^{n}$	Air temperature change	K
$\Delta T_L = T_{top}^{n+1} - T_{top}^{n}$	Soil temperature change	K
$F(T_A^{n+1}, T_L^{n+1})$	Total heat flux (sensible, latent, radiative, advective) using updated air/soil temperature	W/m^2

Among other things the flux term depends on temperature of the bottom level of the air ($T_A = T_{bot}$) and top level of the soil ($T_L = T_{top}$). Without losing generality, the flux term can be linearized at the boundary layer

$$F = F_0 + \frac{\partial F}{\partial T_A}\Delta T_A + \frac{\partial F}{\partial T_L}\Delta T_L \tag{3}$$

where ΔT_A and ΔT_L are the same temperature change listed in the table. Note that this linearization does not yield an semi-implicit approach where one of the equations is chosen to be explicit and conservation is not exact, the linearized equations remain implicit and flux conserving. The disadvantage of such an linearization is that it's no longer completely prognostic.

$$\rho_A h_{bot} c_{p,A} \frac{\Delta T_A}{\Delta t_A} = F_0 + \frac{\partial F}{\partial T_A}\Delta T_A + \frac{\partial F}{\partial T_L}\Delta T_L \tag{4}$$

$$\rho_L h_{top} c_{p,L} \frac{\Delta T_L}{\Delta t_L} = -(F_0 + \frac{\partial F}{\partial T_A}\Delta T_A + \frac{\partial F}{\partial T_L}\Delta T_L) \tag{5}$$

Let $\mu = \rho h c_p$, in the limit of hydrostatic approximation, $\rho h = \frac{\Delta p}{g}$,

$$\mu_A \frac{\Delta T_A}{\Delta t_A} = F_0 + \frac{\partial F}{\partial T_A}\Delta T_A + \frac{\partial F}{\partial T_L}\Delta T_L \tag{6}$$

$$\mu_L \frac{\Delta T_L}{\Delta t_L} = -(F_0 + \frac{\partial F}{\partial T_A}\Delta T_A + \frac{\partial F}{\partial T_L}\Delta T_L) \tag{7}$$

Equ. 6 can be rearrange into

$$(\frac{\mu_A}{\Delta t_A} - \frac{\partial F}{\partial T_A})\Delta T_A = \frac{\partial F}{\partial T_L}\Delta T_L + F_0$$

let $\nu = \frac{\mu_A}{\Delta t_A} - \frac{\partial F}{\partial T_A}$, then

$$\Delta T_A = \frac{\frac{\partial F}{\partial T_L}}{\nu}\Delta T_L + \frac{1}{\nu}F_0$$

let $e = \frac{\frac{\partial F}{\partial T_L}}{\nu}$ and $f = \frac{1}{\nu}$, then

$$\Delta T_A = e\Delta T_L + f \tag{8}$$

Substitute Equ. 8 into Equ. 7 leads to

$$\frac{\mu_L}{\Delta t_L}\Delta T_L = [-F_0 - \frac{\partial F}{\partial T_A}f] + [-\frac{\partial F}{\partial T_L} - \frac{\partial F}{\partial T_A}e]\Delta T_L$$

let $\alpha = F_0 + \frac{\partial F}{\partial T_A}f$ and $\beta = \frac{\partial F}{\partial T_L} + \frac{\partial F}{\partial T_A}e$,

$$\frac{\mu_L}{\Delta t_L}\Delta T_L = -\alpha - \beta\Delta T_L \tag{9}$$

3 Exchange grid consideration

The previous discussion assumes Atm and Lnd grids align perfectly which in general is not true. Thus the flux equations 6 and 7 can only be performed exactly on an exchange grid cell. Temperature tendency update on either the Atm or Lnd grid requires coupled flux calculation involves horizontal mixing that can lead to artificial propogation of local disturbance. In fully implicit scheme, local disturbance can affect model state across the entire model domain which is not reasonable. To constrain local disturbance in flux coupling, GFDL has chosen to calculate temperature change based on Lnd temperature change ΔT_L uniformly over Lnd grid cell and Atm temperature change ΔT_A^* varying over exchange grid cell.

$$\mu_A\frac{\Delta T_A^*}{\Delta t_A} = F_0 + \frac{\partial F}{\partial T_A}\Delta T_A^* + \frac{\partial F}{\partial T_L}\Delta T_L \tag{10}$$

$$\mu_L\frac{\Delta T_L}{\Delta t_L} = -(F_0 + \frac{\partial F}{\partial T_A}\Delta T_A^* + \frac{\partial F}{\partial T_L}\Delta T_L) \tag{11}$$

The strategy of time integration (specific to GFDL implementation) is thus

1. Starting from initial and boundary condition, calculate flux and their derivatives on the exchange grid cell (flux_calculation)

2. Perform a downward sweep and calculate e, f, α, β in the vertical atmospheric layers on Atm grid cell and boundary layer by using flux and their derivatives on the exchange grid cell. (flux_down_from_atmos)

3. Instruct land to update temperature change ΔT_L on Lnd grid cell based on Equ. 9 using averaged e, f, α, β

4. Using updated ΔT_L, calculate atmospheric temperature change ΔT_A^* by Equ. 8, average the change onto the Atm grid and update atmospheric temperature upwards successively (flux_up_to_atmos)