Evaluation of Effective Diameter for Radiation in the NCAR Climate Models

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The concept of effective diameter D_e (or effective radius r_e) is described in Section 2 of Mitchell (2002, JAS). Although D_e was originally defined by Hansen and Travis (1974, Space Sci. Rev.) in the context of liquid water clouds as the ratio of the 3rd to 2nd moment of the droplet size distribution, a general definition was not given for all particle shapes. The general definition given in Mitchell (2002) first describes the effective radiative path for a single particle as d_e = V/A where V is the particle volume and A is the projected area of the particle size distribution (PSD). The question then comes "what density should be used to define V?" The radiative properties for liquid and solid water depend ultimately on wavelength λ and complex refractive index *M*, and *M* measurements are made at bulk density (1.0 g cm^{-3} for liquid and 0.917 g cm^{-3} for solid water). V can thus be defined for ice particles as $V = m/\rho_i$ where $\rho_i = 0.917$ g cm^{-3} and m = ice particle mass. This gives $d_e = m/(\rho_i A)$, which is one of the foundation concepts for the Modified Anomalous Diffraction Approximation (MADA) used in NCAR climate models. In the case of spherical geometry (where $m = \rho_i(\pi D^3/6)$ and $A = \pi D^2/4$), the diameter D of a sphere is related to d_e as

$$D = (3/2)\, d_e = (3/2)\, m/(\rho_i\, A). \qquad (1)$$

It is argued in Mitchell (2002) that the conceptual relationship describing the effective photon path for a single particle also applies to a PSD containing ice particles of arbitrary shape:

$$D_e = (3/2)\, IWC/(\rho_i\, A_{PSD}), \qquad (2)$$

where IWC = ice water content (e.g., kg m^{-3}) and A_{PSD} = concentration of PSD projected area (e.g., m^2 m^{-3}). This is demonstrated by showing the equivalence between (2) and the original definition of r_e given in Hansen and Travis (1974) and Slingo (1989, QJRMS):

$$r_e = \int r^3\, N(r)\, dr \,/ \int r^2\, N(r)\, dr \qquad (3)$$

where N(r) is the PSD and r is radius for spherical liquid water droplets. For convenience, this simple derivation is repeated here. Defining r_e as ½ D_e, r_e is given from (2) for ice clouds as

$$r_e = (3/4)\, IWC/(\rho_i\, A_{PSD}). \qquad (4)$$

Defining IWC and A_{PSD} for spherical ice particles as

$$IWC = \int \rho_i\, (4/3)\, \pi r^3\, N(r)\, dr, \qquad (5)$$

$$A_{PSD} = \int \pi r^2 N(r)\, dr, \quad (6)$$

then substituting (5) and (6) into (4) yields Eqn. (3), which is the original definition of r_e. Note that spherical particles of density ρ_i must be assumed in (5) so that cancellation occurs with ρ_i in (4). For this reason, it is evident that r_e defined by (3) implicitly assumes ρ_i = 0.917 g cm^{-3}, since the fundamental concept for r_e or D_e is expressed in (1). While (3) is only valid for spherical particles, (2) is valid for any particle shape and can be viewed as a universal definition for D_e.

Treatment of r_e and D_e in the NCAR Climate Models

The effective radius is calculated in micron units in version 2 of the Morrison-Gettelman cloud physics module (MG2), used in the NCAR climate models, as:

effi(i,k) = 1.5_r8/lami(i,k)*1.e6_r8 (7)

where effi = r_e and lami is the PSD slope parameter for cloud ice (1.e6_r8 converts meters to microns). This equation is derived from (3) based on an exponential gamma distribution. Some lines below the calculation for effi, one finds "! ice effective diameter for david mitchell's optics" and D_e is given as:

deffi(i,k)=effi(i,k)*rhoi/rhows*2._r8 (8)

where rhows = density of solid water (i.e., bulk ice) = 0.917 g cm^{-3} and rhoi = density of cloud ice = 0.500 g cm^{-3}. This appears to be correcting effi for density differences between cloud ice and bulk ice, where bulk ice density is implicit for D_e used in the ice optics scheme. As mentioned, the PSD used to calculate effi from (3) implicity assume spheres having density rhows. In MG2, the PSD used to calculate effi from (3) assume spheres having density rhoi where lami is calculated via the MG2 utility subroutine as

$$\text{lami} = (\pi * \text{rhoi} * N/Q)^{1/3} \quad (9)$$

where N = cloud ice number concentration and Q = mass mixing ratio, and lami affects effi via (3) or (7). To correct for this mismatch in density, a correction factor of the form $1/(\text{rhows}/\text{rhoi})^{1/3}$ could be used in (8) (instead of rhoi/rhows that is currently used). The former has a value of 0.817 while rhoi/rhows = 0.545.

To summarize, what is passed to the radiation module as an effective diameter must have a density of 917 kg/m^3 or the radiation calculation is corrupted.