# Homework 2

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## Part 1: Cavendish's Experiment

Cavendish's experiment consists of a torsion balance (originally created by John Michell, who died without completing his experiments) that tests the balance between gravitational and torsional forces. Cavendish did not initially use the machine to measure the gravitational constant, but his first measurements were aimed at determining the mass of the Earth.

The fundamental equilibrium equation present in the experiment is Newton's law of universal gravitation. The equilibrium condition is reached when the torque due to the gravitational force between the large and small masses balances the torque due to the torsion wire. In this case, the equation that describes the equilibrium is:

$$\kappa\theta = LF$$

Where $L$ is the length of the torsion balance beam, $F$ is the gravitational force between the large and small masses, $\theta$ is the angular deflection of the torsion balance, and $\kappa$ is the torsion coefficient of the wire. If we measure the oscillation period $T$ of the Cavendish torsion balance, we can find the following relationship between the moment of inertia $I$ and the torsion coefficient. We write the next equation:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

For the two small masses the moment of inertia at a distance $L/2$ from the axis of rotation is:

$$I = 2m\left(\frac{L}{2}\right)^2 = \frac{mL^2}{2}$$

Substituting $I$

$$\kappa = \frac{2\pi^2 mL^2}{T^2}$$

Using the fundamental force balance equation, and Newton's law of gravitation:

$$F = \frac{GMm}{r^2}$$

Then

$$\kappa\theta = L\frac{GMm}{r^2}$$

$$\frac{2\pi^2 mL^2}{T^2}\theta = L\frac{GMm}{r^2}$$

Finally,

$$G = \frac{2\pi^2 Lr^2}{MT^2}\theta$$

## Part 2: Obtain the free-air and Bouguer gravity anomalies

The surface gravity at a measuring site is $9.803243\ m/s^2$. The site has a latitude $43º32'16''N$ and an elevation of $542.3\ m$. Obtain the free air and Bouguer gravity anomalies. Ignore the tidal and terrain corrections.

Resolution:

In both anomalies we need the value of normal gravity:

$$g_n = g_e(1 + \beta_1 \sin^2 \lambda + \beta_2 \sin^2 \lambda)$$

Where $\beta_1$ is equal to $5.30244 \times 10^{-3}$, $\beta_2$ is equal to $5.8 \times 10^{-6}$, $g_e$ correspond to the equator gravity that is $9.780327 m/s^2$ and $\lambda$ is the latitud in radians. For this specific case the value of $\lambda$ is given by:

$$43°32'16''N = 43 + \frac{32}{60} + \frac{16}{3600} = 43.53777778° \times \frac{\pi}{180°} = 0.7598 rad$$

Then

$$g_n = \frac{9.780327m}{s^2}(1 + 5.30244 \times 10^{-3} \sin^2 0.7598 rad + 5.80 \times 10^{-6} \sin^2 0.7598 rad)$$

$$g_n = 9.80493 m/s^2$$

Another value that we need in both anomalies is the free air correction which is given by:

$$\Delta g_{FA} = \frac{2h}{R} g_0$$

Where $g_0$ is the mean gravity, $R$ is the Earth's radius, and $h$ is altitude at a given point. The factor $\frac{2}{R} g_0$ has the value of $3.086 \times 10^{-6} m/s^2$ per meter of elevation.

$$\Delta g_{FA} = 3.086 \times 10^{-6} m/s^2 (542.3\ m)$$

$$\Delta g_{FA} = 1.6735 \times 10^{-3} m/s^2$$

**Free-air anomaly**

The equation for this anomaly is:

$$\Delta g_F = g_m + (\Delta g_{FA} + \Delta g_T + \Delta g_{tide}) - g_n$$

Following the directions we will ignore the tidal and terrain corrections.

$$\Delta g_F = g_m + (\Delta g_{FA}) - g_n$$

$$\Delta g_F = 9.803243\, m/s^2 + 1.6735 \times 10^{-3} m/s^2 - 9.80493 m/s^2$$

$$\Delta g_F = -1.35 \times 10^{-5}$$

$$\Delta g_F = -1.35 mGal$$

**Bouguer anomaly**

In addition to the values, we already have, we must calculate the Bouguer plate correction:

$$\Delta g_{BP} = 2\pi G\rho h$$

The average density of the crust will be used, $\rho = 2800\ kg/m3$

$$\Delta g_{BP} = 2\pi(6.67 \times 10^{-11} Nm^2/kg^2)(2800\ kg/m3)542.3\ m$$

$$\Delta g_{BP} = 6.3636 \times 10^{-4}$$

Now, the calculation of the Bouguer anomaly is as follows:

$$\Delta g_B = g_m + (\Delta g_{FA} - \Delta g_{BP} + \Delta g_T + \Delta g_{tide}) - g_n$$

Ignoring tidal and terrain corrections.

$$\Delta g_B = g_m + (\Delta g_{FA} - \Delta g_{BP}) - g_n$$

$$\Delta g_B = 9.803243\, m/s^2 + 1.6735 \times 10^{-3} m/s^2 - 6.3636 \times 10^{-4} - 9.80493 m/s^2$$

$$\Delta g_B = -6.4986 \times 10^{-4} m7s^2$$

$$\Delta g_B = -64.986 mGal$$

## Part 3: Qualitatively description of the profiles of geoid

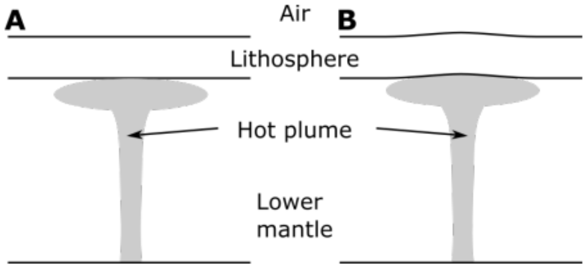



Figure 1: **A**. A mantle plume impinging on the rigid lithosphere from below. **B**. Same with A but the lithosphere is deformable.

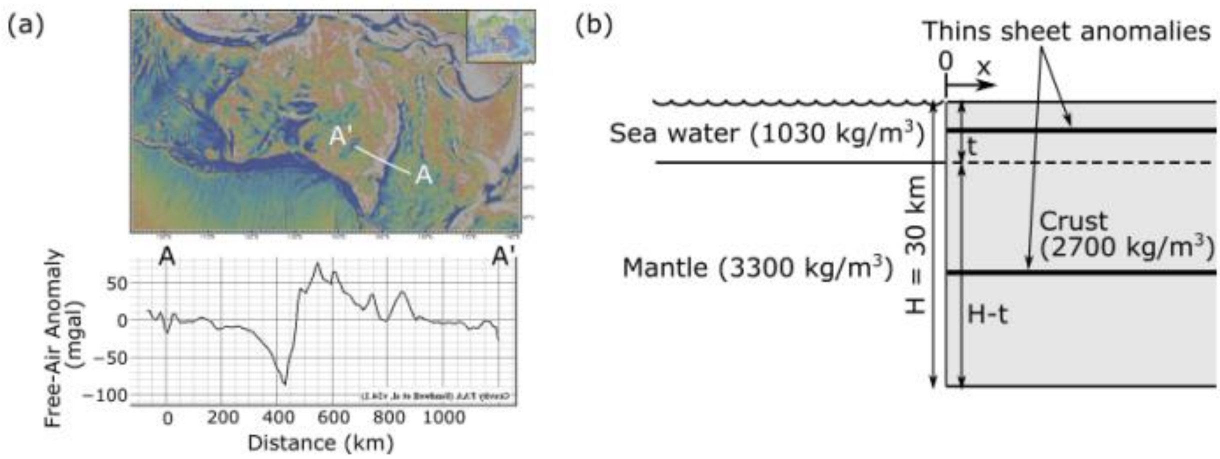
If there is a plume, which we know is hot material that rises due to the difference in density compared to the colder material in the mantle, gravity anomaly may occur in that area. This is broadly one of the ideas proposed by Fowler (2012) in section 5.6.2.

For the two cases presented here, we can make the following analysis:

First case: In this case, the ascending plume does not push the lithosphere. As mentioned by Fowler (2012), in some instances, topography helps counteract the density difference caused by the lower density material. However, in this case, there is no compensation, so a negative gravity anomaly would be observed.

Second case: in this case, a bulge is observed due to the upward push of the mantle column. We can see that the density deficit due to the hot mantle material is maintained, there is also a contribution from topography, which can lead to higher gravity values near the topographic bulge. However, the Bouguer anomaly could still be negative, depending on whether the topographic effect is able to completely counteract the low-density effect in the area.

## Part 4:




**Solving a)**

According to the Airy isostasy model, the pressure at the base of the continental crust is equal to the pressure at the base of the oceanic crust, so the equilibrium equation for pressures could be written as:

$$P_{continental\ crust} = P_{oceanic\ crust}$$

$$\rho_1 g h_1 = \rho_2 g h_2$$

$$\rho_c g H = \rho_w g t + \rho_m g(H - t)$$

Where $\rho_c = 2700 kg/m^3$ is the crust density, $\rho_w = 1030 kg/m^3$ correspond to the density sea water, $\rho_m = 3300 kg/m^3$ is the density of the mantle, $H$ is the total crustal thickness of 30 $km$, and $t$ is the thickness of crust above seafloor. Solving the equation for $t$.

$$\rho_c H = \rho_w t + \rho_m (H - t)$$

$$\rho_c H = \rho_w t + H\rho_m - t\rho_m$$

$$\rho_c H = (\rho_w - \rho_m)t + H\rho_m$$

$$\rho_c H - H\rho_m = (\rho_w - \rho_m)t$$

$$(\rho_c - \rho_m)H = (\rho_w - \rho_m)t$$

$$t = \frac{(\rho_c - \rho_m)H}{(\rho_w - \rho_m)}$$

Replacing the values

$$t = \frac{(2700kg/m^3 - 3300kg/m^3)30\ km}{(1030kg/m^3 - 3300kg/m^3)}$$

$$t = 7.9295km$$

**Solving b)**

Two expressions will be written, one for the “above" thin sheet and another for "below" thin sheet. To do this the formula that will be used is:

$$\Delta g_z = 2G\Delta\rho h\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{z_0}\right)\right]$$

Where $\Delta g_z$ is the gravity anomaly, $\Delta\rho$ is the density contrast, $G = 6.67 \times 10^{-11} Nm^2/kg^2$, $h$ is the thickness of the sheet, $x$ is horizontal distance from the observation point, and $z_0$ correspond to the depth of the sheet. Above seafloor level $z_0 = t$, and below seafloor level $z_0 = H - t$

**For the “above" thin sheet**

$$\Delta g_{above} = 2G(\rho_c - \rho_w)h\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{t/2}\right)\right]$$

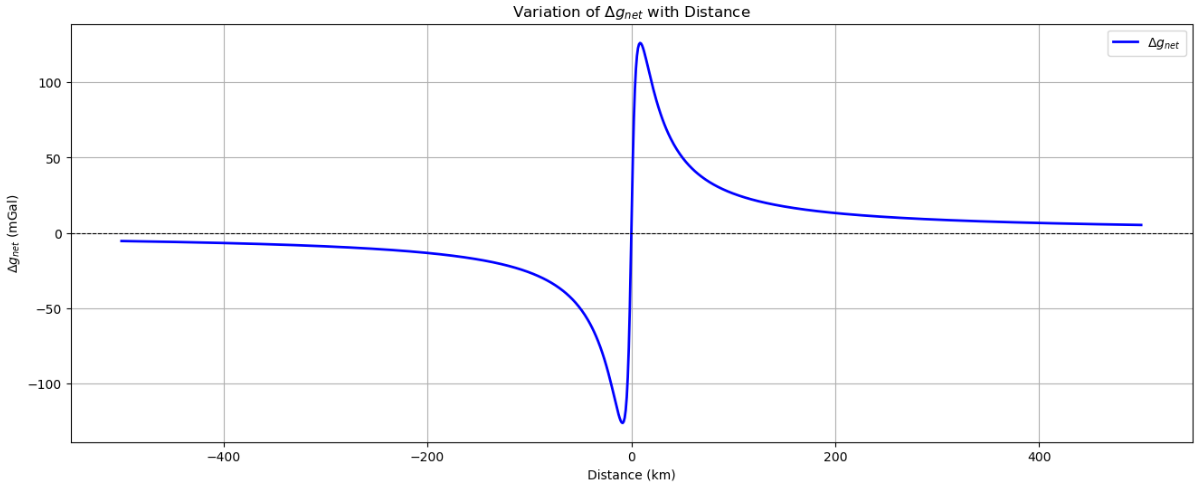
We use $t/2$ because the sheet is located at the middle depth, and $t$ corresponds to the thickness.

**For the “below" thin sheet**

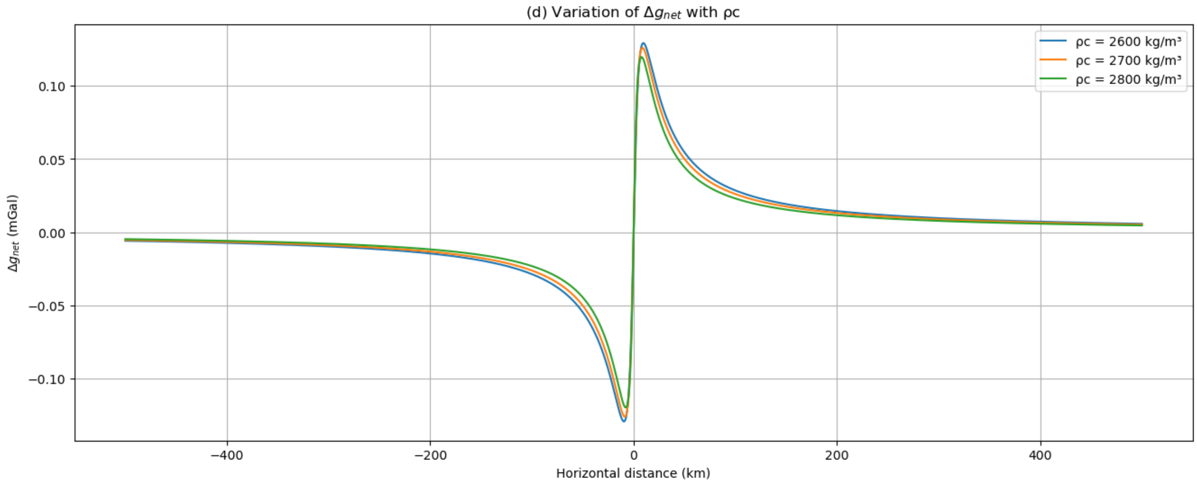
$$\Delta g_{below} = 2G(\rho_c - \rho_m)h\left[\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{t + \left(\frac{H-t}{2}\right)}\right)\right]$$

In this case we use $t + \left(\frac{H-t}{2}\right)$ because we need the portion in the middle depth of the portion below seafloor that is $\frac{H-t}{2}$, but it is necessary add $t$ to have the distance from the seafloor to sea level.

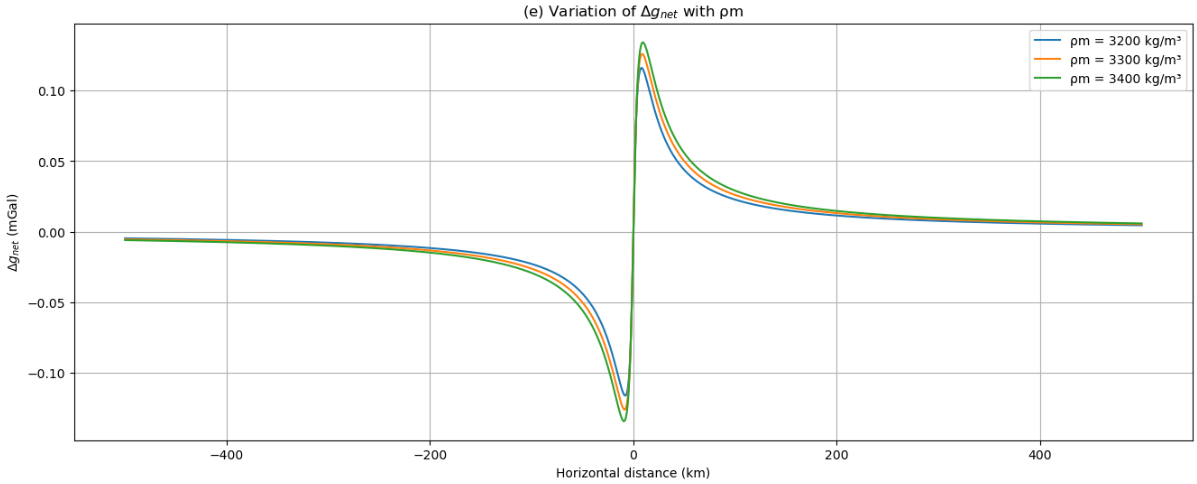
**Solving c) (code in Jupiter Notebook)**



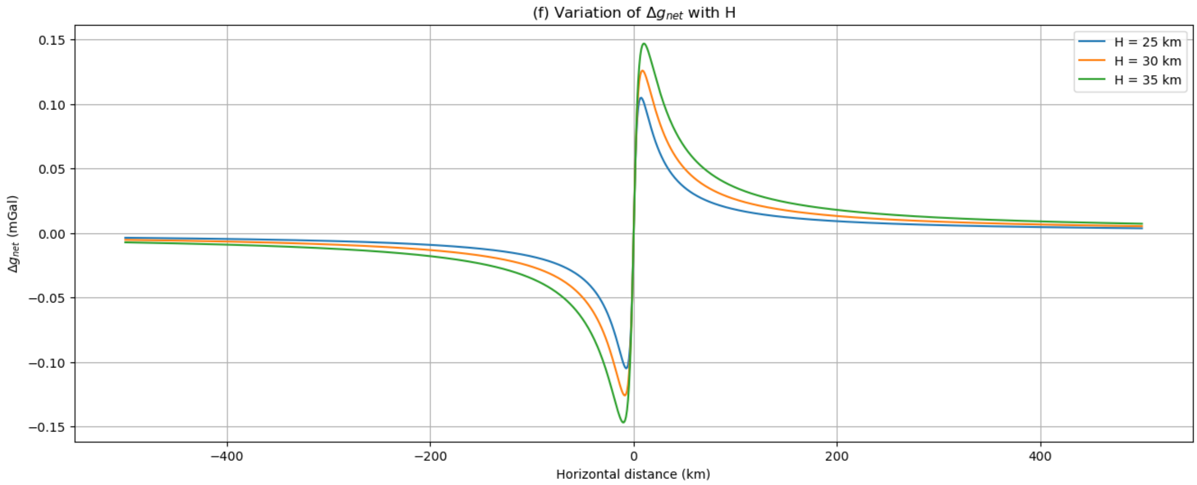

Solving d) **(code in Jupiter Notebook)**



## Solving e) (code in Jupiter Notebook)




## Solving f) (code in Jupiter Notebook)




## References


Lowrie, W. (2007). Fundamentals of geophysics (2nd ed.). Cambridge University Press.

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