Homework 3

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Part 1: Population equations

Part a) In Jupiter Notebooks

Part b)

We know that, for a sphere we have:

$$V = \frac{4}{3}\pi r^3$$

And

$$A = 4\pi r^2$$

The variation in volume with time is proportional to the surface area

$$\frac{dV}{dt} = -kA \ \ or \ \ \frac{dV}{dt} = -k4\pi r^2$$

Where k is a positive proportionality constant. We know that the volume also depends on the radius, so we can use:

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$$

We know that

$$\frac{dV}{dr} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right) = 4\pi r^2$$

We can rewrite the equation as

$$4\pi r^2 \frac{dr}{dt} = -k4\pi r^2$$

Simplifying

$$\frac{dr}{dt} = -k$$

Solving the differential equation

$$dr = -kdt$$
$$\int dr = -\int kdt$$

We have

$$r = -kt + C$$

Considering the initial condition as

$$r(0) = r_0$$

Finally

$$r(t) = -kt + r_0$$

The units of k are

$$\frac{dr}{dt} = k$$

$$\frac{dr}{dt} = \frac{m}{s}$$

This means that k represents an evaporation rate in terms of change in radius per unit time.

Part c)

We have a lake with $V = 10^6 m^3$ and water flowing into and out of the lake. A pollutant is introduced into the lake, the variation of its mass with time is

$$M_p(t)$$

We can describe the concentration of the pollutant in the lake as

$$C(t) = \frac{M_p(t)}{M_w}$$

Where M_w is the mass of water in tons, $M_w = 10^6$ tons.

Water input rate:

$$Q_{in} = 157680\ tons/year$$

Evaporation rate

$$Q_{evap} = 60000\ tons/year$$

Total water output rate (outflow + evaporation)

$$Q_{out} = Q_{in} - Q_{evap} = 97680\ tons/year$$

Pollutant input rate

$$F_{in} = 40\ tons/year$$

Pollutant output rate (the amount of pollutant that leaves is proportional to the concentration in the water)

$$F_{out} = Q_{out}C(t) = \frac{97680M_p}{10^6} = 0.09768M_p$$

The change in mass of the pollutants in the lake is modelled with

$$\frac{dM_p}{dt} = entry\ rate - exit\ rate$$

$$\frac{dM_p}{dt} = 40 - 0.09768M_p$$

This equation is of the form

$$\frac{dX}{dt} = a + bx$$

We rewrite the equation

$$\frac{dM_p}{dt} + 0.09768M_p = 40$$

$$\frac{dM_p}{40 - 0.09768M_p} = dt$$

Integrating

$$u = 40 - 0.09768M_p\ \ and\ \ du = -0.09768M_p$$

$$\int \frac{dM_p}{40 - 0.09768M_p} = \int -\frac{du}{0.09768u}$$

$$\int \frac{dM_p}{u} = \int -\frac{du}{0.09768u}$$

$$-\frac{1}{0.09768}\int \frac{du}{u} = -\frac{1}{0.09768}\int \frac{du}{u}$$

We know

$$\int \frac{du}{u} = \ln{(u)}$$

Then

$$-\frac{1}{0.09768}\ln{(40 - 0.09768M_p)}$$

The integral of the right side

$$\int dt = t + C$$

We have

$$-\frac{1}{0.09768}\ln\left(40 - 0.09768M_p\right) = t + C$$

$$\frac{1}{0.09768}\ln\left(40 - 0.09768M_p\right) = -t - C$$
$$40 - 0.09768M_p = e^{\frac{-t+C}{0.09768}}$$

Now $e^{-\frac{C}{10}} = C'$

$$40 - 0.09768M_p = C'e^{-0.09768t}$$

$$M_p = \frac{40 - C'e^{-0.09768t}}{0.09768}$$

Now $\frac{C'}{0.1} = C_1$. The initial condition where $t = 0$, and $M_p(0) = 0$

$$0 = 400C_1e^0$$
$$400 = C_1$$

Finally

$$M_p = 400 - 400e^{-0.09768t}$$
$$M_p = 400(1 - e^{-0.09768t})$$

Plot in Jupiter notebooks

Part 2: U-Pb

Part a) In Jupiter Notebooks

Part b) In Jupiter Notebooks

Part 3:

Part a) In Jupiter Notebooks

Part b) In Jupiter Notebooks

Part c) In Jupiter Notebooks

Part 4: Rb - Sr

Part a) In Jupiter Notebooks

Part b) In Jupiter Notebooks

Part c)

We assume an undifferentiated Earth with an initial ratio $^{87}Sr/^{86}Sr=0.699$ and a ratio of $^{87}Rr/^{86}Sr=0.1$, 4550 million years ago. The batholith proposed in the exercise has an initial ratio of $^{87}Sr/^{86}Sr=0.7074$ (82.79 Ma) which corresponds to the moment when the rock's isotopic system closed (crystallization or metamorphism).

Considering Figure 6.2 (Fowler, 2005), mantle rocks typically exhibit initial ratios between 0.7000 and 0.7040. This means that the initial ratio of the batholith is slightly higher, which could indicate that its formation (or isotopic reset) was more influenced by crustal material rather than being entirely mantle-derived.

It could have formed through a magma derived from the mantle that interacted with crustal rocks due to its initial ratio proximity to those values, or it may have formed through the partial melting of the crust, generating a magma with no direct mantle contribution.

References

Lowrie, W. (2007). Fundamentals of geophysics (2nd ed.). Cambridge University Press.

Fowler, C. M. R. (2005). The solid Earth: An introduction to global geophysics (2nd ed.). Cambridge University Press.