**Homework 4**

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| Part 1: Electric field flux |

**Part a)**

We know:

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First, we need to find :

For the flux, we use gauss’s law:

Where is the permittivity in the vacuum.

With the gauss’s law the flux only depends on the enclosed charge, so it does not matter if the cube is not at the center of the sphere.

**Part b)**

The flux only depends on the enclosed charge. However, we have the same charge as in part a), for that reason:

**Part c)**

The vector potential of a dipole is:

Equation 6.10 from Griffiths, (2017). Where is the permeability of the free space, is the magnetic dipole moment, is the radial distance and is the unit vector in the radial direction.

In cartesian coordinates:

Then, we calculate ,

Now, we have:

The magnetic field is obtained by:

In spherical coordinates, curl operator is:

For the magnetic potential vector, first we need to transform into spherical coordinates

We have:

To convert, we need to use:

Now

Expanding each term for r component:

Expanding each term for

Expanding each term for

We know that

Now

Calculating

Then

Calculating

Now

Now, we need to calculate the magnetic field flux through a sphere using:

In spheric coordinates:

Now, we use , because only the radial component contributes to the flux.

For this case

For

And

We obtained

The magnetic field flux through the surface of a sphere is zero if the magnetic field is generated by a dipole.

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| Part 2 |

**Part a)**

A single charge ion, accelerated from rest by an electric potential difference, kinetic energy is given by:

We know that

For this case

Then

We have , , correspond to the electron charge, , mass of 87 ion.

**Part b)**

Once the ion enters a vacuum chamber with a uniform magnetic field , it follows a circular path due to the Lorentz force, which equals the centripetal force.

Solving for the radius

We have , , correspond to the electron charge, , mass of 87 ion, and .

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| Part 3: inclination and latitude |

**Part a)**

The equation for the inclination of the magnetic field in an axial geocentric dipole is given by:

where is the latitude

For this case

The latitude where the field inclination is is

**Part b)**

Using the same equation, but for the geocentric axial dipole field at latitude .

The inclination where the latitude is is

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| Part 4 |

**Part a)**

We have measurements of the three cartesian components of the Erath’s magnetic field at a geomagnetic observatory.

Because is negative, the field in pointing upward rather than downward. In the northern earth’s hemisphere, the vertical component is downward . In the southern hemisphere is typically negative. For that reason, the observatory is in the southern hemisphere.

**Part b)**

The total field intensity is given by:

**Part c)**

For this case, the inclination is given by:

Substituting

The negative indicates the field points upward from the horizontal plane, typically of the southern hemisphere.

For the declination, we must use:

Substituting

The declination means the horizontal field vector is about west of true north

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| Part 5 |

**Part a)** In Jupiter Notebooks

**Part b)** In Jupiter Notebooks

**Part c)** In Jupiter Notebooks

**Part d)** In Jupiter Notebooks

**Part e)** Calculation in In Jupiter Notebooks

We have the scalar potential in spheric coordinates:

Where is the Earth’s radius, is the radial distance, is the colatitude, is the Legendre Polynomials, and are the gauss’s coefficients. The magnetic field is given by:

Equation 5.67 from Griffiths, (2017).

In spherical coordinates

The derivate of respect to

Since

Then

For

Now, we have

Also, we have

Calculating

We have

Substituting and remember, and

For

Since

Now, we have

Substituting

**Jupiter Notebooks:** [**https://github.com/ESDelgado/ceri8211\_S2025/tree/main/Hw4**](https://github.com/ESDelgado/ceri8211_S2025/tree/main/Hw4)

**References**

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