B4. Create a program that fits a mixture of Gaussians to a dataset of handwritten digit features and clusters them into distinct groups. Use the Expectation Maximization method to estimate the parameters of the Gaussian mixture model.

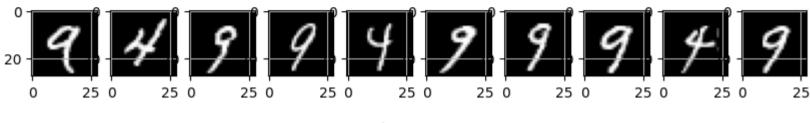
```
In [1]: import numpy as np
         import matplotlib.pyplot as plt
         from sklearn.mixture import GaussianMixture
         from sklearn.datasets import fetch_openml
         from sklearn.decomposition import PCA
         from sklearn.metrics import accuracy_score, confusion_matrix
         from scipy.optimize import linear_sum_assignment
In [2]: mnist = fetch_openml('mnist_784', version=1, as_frame=False)
         X = mnist.data.astype(np.float32) / 255.0
         y = mnist.target.astype(int)
In [3]: pca = PCA(n_components=50, random_state=42)
         X_reduced = pca.fit_transform(X)
In [4]: num_clusters = 10
         gmm = GaussianMixture(n_components=num_clusters, covariance_type='full', random_state=42)
         gmm.fit(X_reduced)
Out[4]:
                          GaussianMixture
         GaussianMixture(n_components=10, random_state=42)
         clusters = gmm.predict(X_reduced)
In [10]: plt.figure(figsize=(8, 6))
         plt.scatter(X_reduced[:, 0], X_reduced[:, 1], c=clusters, cmap='tab10', s=15)
         plt.title("Digit Clustering using GMM (EM Algorithm)")
         plt.xlabel("PCA Component 1")
         plt.ylabel("PCA Component 2")
         plt.colorbar(label="Cluster")
         plt.show()
                         Digit Clustering using GMM (EM Algorithm)
             6
             4
             2
                                                                                              6
        PCA Component 2
             0
           -2
                                                                                              3
                                                                                              2
           -4
                                                                                              . 1
                                         PCA Component 1
In [11]: def cluster_accuracy(y_true, y_pred):
             cm = confusion_matrix(y_true, y_pred)
             row_ind, col_ind = linear_sum_assignment(-cm)
             accuracy = cm[row_ind, col_ind].sum() / y_true.size
             return accuracy
         acc = cluster_accuracy(y, clusters)
         print(f"GMM Clustering Accuracy: {acc:.4f}")
        GMM Clustering Accuracy: 0.6080
In [ ]: #OPTIONAL
         def plot_cluster_images(cluster_number, num_samples=10):
             indices = np.where(clusters == cluster_number)[0][:num_samples]
             plt.figure(figsize=(10, 2))
             for i, idx in enumerate(indices):
```

plt.subplot(1, num_samples, i+1)

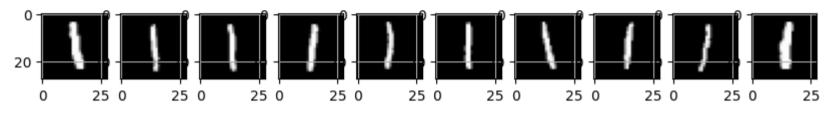
```
plt.imshow(X[idx].reshape(28, 28), cmap='gray')
    plt.grid("off")
    plt.suptitle(f"Cluster {cluster_number}")
    plt.show()

for i in range(5):
    plot_cluster_images(i)
```

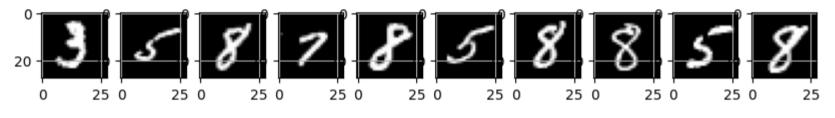
Cluster 0



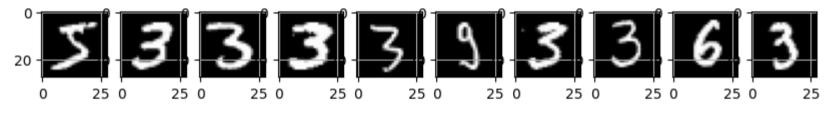
Cluster 1



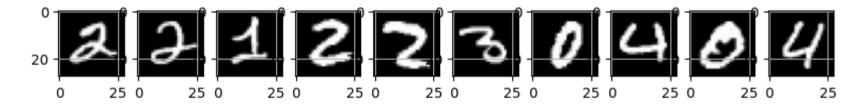
Cluster 2



Cluster 3



Cluster 4



In []