1 Fisher Information

The Fisher information (for one parameter) is defined as

$$\mathcal{I}(\theta) = E\left[\left(\frac{d}{d\theta}\log(f(X|\theta))\right)^2\right]$$

where the expectation is taken with respect to X which has PDF $f(x|\theta)$. This quantity is useful in obtaining estimators for θ with good properties, such as low variance. It is also the basis for the Jeffreys prior.

Example: Let $X \mid \theta \sim \mathrm{N}(\theta, 1)$. Then we have

$$\begin{aligned}
f(x|\theta) &= \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}(x-\theta)^2\right] \\
\log(f(x|\theta)) &= -\frac{1}{2}\log(2\pi) - \frac{1}{2}(x-\theta)^2 \\
\frac{d}{d\theta}\log(f(x|\theta)) &= -\frac{2}{2}(x-\theta)(-1) = x - \theta \\
\left(\frac{d}{d\theta}\log(f(x|\theta))\right)^2 &= (x-\theta)^2
\end{aligned}$$

and so $\mathcal{I}(\theta) = E[(X-\theta)^2] = Var(X) = 1$.