

7. maths

HCF and LCM

What is factors?

HCF \rightarrow Highest common factor ...

LCM \rightarrow Lowest common multiple ...

① $a \times b = p$

② a, b are factor of p

factor
 $12 = \{ 1, 2, 3, 4, 6, 12 \}$
 $n \leq 12$
Inke kahi 12
ata hai
Inke kahi be 10 ata hai
 $n \leq 10$

factor 10 $= \{ 1, 2, 5, 10 \}$
 $\begin{array}{r} 10 \\ 1 \overline{) 10} \\ \underline{10} \\ 0 \end{array} \quad \begin{array}{r} 10 \\ 2 \overline{) 10} \\ \underline{10} \\ 0 \end{array} \quad \begin{array}{r} 10 \\ 5 \overline{) 10} \\ \underline{10} \\ 0 \end{array}$

30 $= \{ 1, 2, 3, 5, 6, 10, 15, 30 \}$
 $\begin{array}{l} 1 \times 30 \\ 2 \times 15 \\ 3 \times 10 \\ 5 \times 6 \end{array}$
Trick

$a \times b = p \rightarrow$ means a and b are factor of p

★ $1 \rightarrow$ Smallest factor \rightarrow Sabka factor

★ Every num have minimum two factor
one and the number itself...

$$15 = \{1, 5, 3, 15\}$$

TCS Question

Every number will have minimum two factors 1 and number pt self

0 TRUE



0 FALSE



~~100~~ $15 \rightarrow 15$

don't have two factors

HCF / GCD

Divisor
Common

$$HCF = \{60, 75\}$$

$$60 = \{1, 2, 3, 5, 6, 10, 12, 20, 30, 60\}$$

$$\begin{array}{l} 1 \times 60 \\ 2 \times 30 \\ 3 \times 20 \\ 5 \times 12 \\ 6 \times 10 \end{array}$$

$$75 = \{1, 3, 5, 15, 25, 75\}$$

$$\begin{array}{l} 1 \times 75 \\ 3 \times 25 \\ 5 \times 15 \end{array}$$

Common factors = $\{1, 3, 5, 15\}$

Highest Common factor = $\{15\}$

Lowest common factor = $\{1\}$
LCF

Multiples of 5 = $\{5, 10, 15, 20, 25, 30, \dots\}$

multiples of 8 = $\{8, 16, 24, 32, \dots\}$

multiples of 9 = $\{9, 18, 27, 36, \dots\}$

$$\frac{12}{4} = 3 \text{ times}$$

multiples divided by num

Example \rightarrow multiples = 24

24

8 (Divided by number)

LCM (5, 8, 9)

Least
Common
multiple

5 = $\{5, 10, 15, 20, 25, 30, 40, 45, 50, 55, \dots\}$

8 = $\{8, 16, 24, 32, 40, 48, 56, 64, 72, 80, \dots\}$

10 = $\{10, 20, 30, 40, 50, 60, 70, 80, 90, 100, \dots\}$

LCM = $\{40\}$

HCM (Highest common multiple) — ~~not possible~~
To find...

$$\boxed{HCF \leq (m, n) \leq LCM}$$

~~(HCF)~~ $(5, 8, 10)$

$$\boxed{HCF \leq 5}$$

$$\boxed{LCM \geq 10}$$

Q HCF (12, 18)

$$12 = 2 \times 1, 2, 3, 6, 12$$

$$18 = 2 \times 1, 2, 3, 6, 9, 18$$

$$CF = \{1, 2, 3, 6\}$$

$$\boxed{HCF = 6}$$

Q HCF (1822, 1526) ?

Q LCM (5, 15, 20, 30)

$$5 = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60\}$$

$$15 = \{15, 30, 45, 60\}$$

$$20 = \{20, 40, 60, 80, 100, 120, 140, 160, 180\}$$

$$30 = \{30, 60, 90\}$$

$$\boxed{LCM = 60}$$

LCM = {5, 15, 20, 30} ^{largest number}
 LCM C {5, 15, 20, 30} ^{for kalle malya hai 30}
 → Check 30 next multiple
 30 = 30, 60, 90
 → next multiple
 60 w kalle malya 20 and 30 dono
 atee hai
 LCM
 Ans

LCM {8, 8, 10}
 10, 20, 30, 40
 comes in
 Band to
 kalle
 → 40 → ans

HCF = {12, 18} ^{Bada num jisme 12 or 18 dono ke honge}
 No → yes → jisme 18 ataa hai
 12 = {1, 2, 3, 4, 6, 12}
 → check comes in 18 → yes
 → check if 6 not equal
 HCF = {12, 20} ^{Ans = 4}
 12 = {1, 2, 3, 4, 6, 12} ^{HCF ≤ 12}

$$HCF = \{60, 75\}$$

$$\frac{60}{2} = 30 \text{ --- doesn't come in 75}$$

$$\frac{30}{2} = 15 \text{ --- comes in 75}$$

Ans

2 divide 30 bcz left ka next num dik gya mtlb us number k ek phle cell hcf me wo chote hote hai but 15 me toh 2 hote hai that's why usko 2x krte hai...

HCF by Prime factorization

$$HCF (60, 75)$$

$$60 = 2 \times 30$$

$$2 \times 3 \times 10$$

$$2 \times 3 \times 2 \times 5$$

$$2 \times 2 \times 3 \times 5$$

$$60 = 2^2 \times 3 \times 5$$

$$70 = 3 \times 5 \times 5$$

$$70 = 3 \times 5^2$$

$$\text{Intersection} = \{3, 5\}$$

$$\text{min power of } 3 = 1 \quad = 3 \times 5$$

$$\text{min power of } 5 = 1 \quad = 15$$

Ans

$$3 \quad \text{HCF}(36, 24, 12)$$

$$\begin{aligned} 36 &= 2 \times 18 \\ &= 2 \times 3 \times 6 \\ &= 2 \times 3 \times 2 \times 3 \\ &= 2^2 \times 3^2 \end{aligned}$$

$$\begin{aligned} 24 &= 2 \times 12 \\ &= 2 \times 6 \times 2 \\ &= 2 \times 2 \times 2 \times 3 \\ &= 2^3 \times 3 \end{aligned}$$

$$\begin{aligned} 12 &= 2 \times 6 \\ &= 2 \times 2 \times 3 \\ &= 2^2 \times 3 \end{aligned}$$

$$\begin{aligned} \text{Common} &= 2 \times 3 \\ &= 2^2 \times 3 \\ &= 4 \times 3 = 12 \end{aligned}$$

$$\text{My trick } (36, 24, 12)$$

$$12 = \{1, 2, 3, 4, 6, \underline{12}\} \quad \text{Common in } 24, 12$$

$$\Rightarrow 12 \text{ Ans}$$

$$\text{Example HCF } 36, 27, 80$$

$$\begin{aligned} 36 &\rightarrow 6 \times 6 \\ &= 2 \times 3 \times 2 \times 3 \\ &= 2^2 \times 3^2 \end{aligned}$$

$$\begin{aligned} 27 &\rightarrow 3 \times 9 \\ &= 3 \times 3 \times 3 \\ &= 3^3 \end{aligned}$$

$$80 \Rightarrow 2^4 \times 5$$

$\boxed{HCF = 1}$ — If no common
then HCF = 1

$$\begin{array}{l} 27, 36, 80 \\ 27 - \\ \underline{27} \quad 27 = 9 \times 3 \\ \underline{27} \quad 3 = 3 \times 1 \end{array}$$

Method can only be used for smaller numbers
and numbers whose prime factors can be found
easily

Q \rightarrow find the HCF of

$$\begin{array}{l} 2^3 \times 3^2 \times 5 \times 7^4 \times 13^4 \\ 2^2 \times 3^5 \times 5^2 \times 7^8 \\ 2^3 \times 5^3 \times 7^2 \times 13^3 \end{array}$$

$$\begin{aligned} &= 2^2 \times 5 \times 7^2 \\ &= 4 \times 5 \times 49 \end{aligned}$$

$$\begin{array}{r} 49 \\ \times 20 \\ \hline 980 \end{array}$$

HCF = 980

LCM = 100800

Q LCM(12, 18)

$$\begin{aligned} 12 &= 2^2 \times 3 &= 2 \times 6 \\ 18 &= 2 \times 3^2 &= 2 \times 9 \end{aligned}$$

$$\begin{aligned} &= 2 \times 3 \rightarrow \text{Take big power} \\ &= 2^2 \times 3^2 = \underline{\underline{36}} \end{aligned}$$

LCM using prime factorization method

The LCM of $2^6 \times 3^2 \times 5 \times 7$, $2^3 \times 3^5 \times 7$
and $2 \times 3^4 \times 5 \times 7$

~~(a) $2^6 \times 3^5$~~ ~~(b) $2^6 \times 3^5 \times 5$~~ \times

(c) $2^6 \times 3^5 \times 7$ \times (d) None of these \checkmark

$$\text{Ans} = 2 \times 3 \times 5 \times 7$$

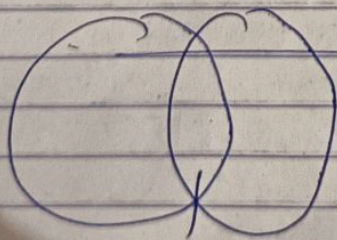
$$= 2^6 \times 3^5 \times 5^1 \times 7^1$$

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$$60 = 2 \times 3 \times 5 = 2^2 \times 3 \times 5$$

$$\text{HCF} = 2^2 \times 3^1 = 12$$

$$\text{LCM} = 2^3 \times 3 \times 5 = 120$$



$\rightarrow \text{LCM} = \text{union of maximum}$

HCF \rightarrow Region of intersection

Let $a = 24$
 $b = 60$

- Test time:-
- Shaam:-
- Example + input + output
- LCM VS Code:-

Product of number = HCF \times LCM
 $= 12 \times 120$
 $= 1440$

$24 \times 60 = 1440$

① TCS 2020

[If HCF of 189 and 297 is 27, find their LCM]

$189 \times 297 = \text{HCF} \times \text{LCM}$

$189 \times 297 = 27 \times \text{LCM}$

$$\begin{array}{r} 63 \\ 33 \\ \hline 189 \\ 189 \times \\ \hline 2079 \end{array}$$

$$\begin{array}{r} 189 \\ 7 \\ \hline 1323 \\ 6 \\ \hline 2079 \end{array}$$

$\text{LCM} = \frac{189 \times 297}{27}$

$\text{LCM} = 2079$

$= 7 \times 3 \times 3 \times 11$

$= 7 \times 3^3 \times 11$

$= 3^3 \times 11 \times 7$

$\text{LCM} = 3^3 \times 7 \times 11$

Infosys 2019, 2021

Q Find the smallest number that leaves
a remainder of 4 on division by 5, 5
on division by 6, 6 on division by 7,
7 on division by 8 and 8 on division by 9
(A) 2519 (B) 5039 (C) 1079 (D) 979

Remainder 4 — on division by 5

$$R=5 \text{ — } \div 6$$

$$R=6 \text{ — } \div 7$$

$$R=7 \text{ — } \div 8$$

$$R=8 \text{ — } \div 9$$

Smallest num \log HCF m

AP^T

$$4 \times 6 \times 8$$

$$a = 48$$

$$a \div 5 = 4$$

$$a \div 6 = 5$$

$$a \div 7 = 6$$

$$a \div 8 = 7$$

$$a \div 9 = 8$$

$$A = \frac{2519}{5}$$

Understanding

Let n be number

$$n \% 5 = 4$$

$$n \% 6 = 5$$

$$n \% 7 = 6$$

$$n \% 8 = 7$$

$$n \% 9 = 8$$

$$\boxed{n \% n-1 = n}$$

find the smallest num which divided by
5, 6, 7, 8, 9

option > $\frac{\text{sum}}{\text{num}} = \text{LCM}$

$$\text{LCM}(5, 6, 7, 8, 9)$$

$$\text{HCF}(5, 6, 7, 8, 9)$$

$$\boxed{\text{HCF} \leq 5}$$

$$\boxed{\text{LCM} \geq 9}$$

$$\rightarrow \underline{\underline{2520}}$$

$$9 = 9, 18, 27, 36, 45, 54, 63, 72, 81, 90$$

$$2520 \dots$$

$$\boxed{5, 2 \times 3, 7, 2^3, 3}$$

$$\rightarrow 2519 \checkmark \underline{\underline{\text{Ans}}}$$

Q How many pair of positive integer x, y exist such that HCF of $x, y = 35$

Sum of x and $y = 1085$

(A) 12

(B) 18

(C) 15

(D) Cannot be determined

$$\text{HCF}(x, y) = 35$$

$$x + y = 1085$$

Abhi kitni pair banayin jiska HCF 35 aur sum 1085 hota hai

$$x \times y = \text{HCF} \times \text{LCM}$$

$$x + y = 1085$$

$$x = 1085 - y$$

$$1085 - y^2 = 35 \times \text{LCM}$$

$$12 = 1, 2, 3, 4, 6, 12$$

4 ki taluk me 12 badi hai

$$4 \times h = 12$$

$$4h = 12$$

35 \rightarrow ~~factor~~ factor h to usme x and y alu ho ke hog

Similarly $x = 35a$
 $y = 35b$

35 is a factor of x and y

$$\text{HCF}(a, b) = 1$$

highest common factor is 35 so a and b have nothing common in between

let $a = b = 2$ $x = 70$ $y = 70$ \rightarrow not

$$x = 35a$$

$$y = 35b$$

$$x + y = 1085$$

$$35(a + b) = 1085$$

$$a + b = 31$$