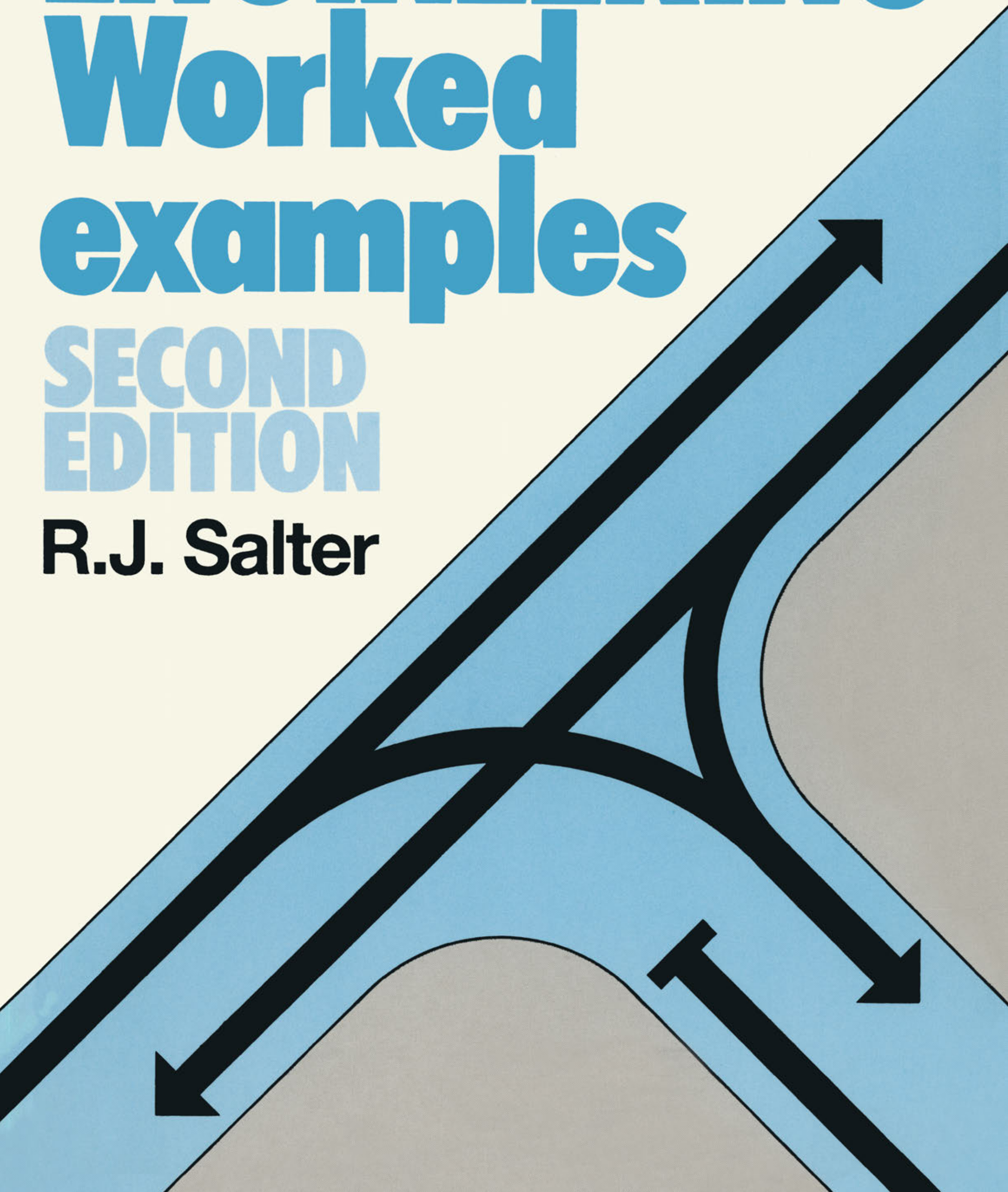
TRAFFIC ENGINEERING Worked examples

SECOND EDITION

R.J. Salter



TRAFFIC ENGINEERING

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TRAFFIC ENGINEERING

Worked examples

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PREFACE

Current and projected increases in the number of vehicles on the highway systems of developed and developing countries, together with a realisation of the limited resources available for construction, have made the efficient use of road space by traffic engineering techniques of increasing importance. In response to this demand educational institutions throughout the world now include the study of highway traffic engineering in their curriculum.

This book is intended to be of use to students on these courses by presenting a set of worked examples in a wide range of highway traffic engineering problems designed to illustrate the principles of highway traffic flow and the practical design of highway elements.

The second edition of Traffic Engineering has been considerably revised to include examples of current United Kingdom design methods and will be of considerable use to engineers who are seeking an introduction to current highway traffic engineering or who wish to update their knowledge on this important area of highway design.

R.J. Salter

EXAMPLE 1 Measurement of Highway Traffic Stream Speed, Time and Space Mean Speeds

(a) Define and discuss the ways by which the speed of a highway traffic stream may be described.

(b) The following observations of the speed of a highway traffic stream were obtained using a radar speed meter. Determine the distributions of speeds in time and in space, the time and space mean speeds and illustrate the relationship between time and space mean speed.

Solution

It has been shown by Wardrop (1) that when the traffic stream is divided into subsidiary streams with flows q_1, $q_2 \ldots q_c$ and speeds v_1, v_2 ..., v_c so that the total flow is given by

$$Q = q_1 + q_2 + \ldots + q_c$$

$$= \sum_{i=1}^{c} q_i$$

and if $f_1 = q_1/Q$, $f_2 = q_2/Q$, ..., and $f_c = q_c/Q$, then f_1, $f_2 \ldots f_c$ are the frequencies in time of vehicles whose speeds are v_1, $v_2 \ldots v_c$ and

$$\sum_{i=1}^{c} f_i = 1$$

Considering the subsidiary stream q_1 with speed v_1, the average time interval between its vehicles is $1/q_1$ and the distance travelled in this time is v_1/q_1. The density or concentration of this stream in space, i.e. the number of vehicles per unit length of road at any instant is given by

$$d_i = q_i/v_i,\ i = 1, 2, \ldots, c \tag{1.1}$$

The quantities d_1, d_2, ..., d_c represent the densities of vehicles in each individual stream and the total density is given by

$$D = \sum_{i=1}^{c} d_i$$

$$\text{Also } f_i^1 = d_j/D \tag{1.2}$$

the frequencies f_1^1, f_2^1, ... f_c^1, of v_1, v_2, ..., v_c in space.

Each of these two speed distributions has a mean value. The time mean speed may be obtained from

$$\bar{v}_t = \sum_{i=1}^{c} q_i v_i / Q = \sum_{i=1}^{c} f_i v_i \qquad (1.3)$$

The space mean speed may be obtained from

$$\bar{v}_s = \sum_{i=1}^{c} d_i v_i / D = \sum_{i=1}^{c} f_i^I v_i \qquad (1.4)$$

giving

$$\bar{v}_s = \sum_{i=1}^{c} d_i v_i / D$$

and from equation (1.1)

$$d_i = q_i / v_i$$

so that

$$\bar{v}_s = \sum_{i=1}^{c} q_i / D = Q/D$$

This is a fundamental relationship for traffic flow, that flow equals space mean speed times density.

The usual method of obtaining time mean speed is by the use of radar speed meter or by timing vehicles over a short length of road and calculating the speed from the observed time of travel and known base length. If the individual observed speeds are $v_1, \ldots, v_n$

$$\bar{v}_t = \sum^{n} v_1 / n$$

If the space mean speed is required then if the individual travel times are $t_1, \ldots, t_n$

$$\bar{v}_s = \sum^{n} n / \bar{t}$$

where $\bar{t}$ is the mean travel time.

Alternatively the space mean speed may be calculated from two aerial photographs taken at a short interval t apart and measuring the distance ℓ covered by each vehicle and computing gives

$$v = \ell / t, \text{ then } \bar{v}_s = \sum^{n} v / n$$

It has also been shown (1) that the general relationship between $\bar{v}_t$ and $\bar{v}_s$ is given by

$$\bar{v}_t = \bar{v}_s + \sigma_s^2/\bar{v}_s \tag{1.5}$$

where σ_s is the standard deviation of the space distribution given by

$$\sigma_s^2 = \sum_{i=1}^{c} d_i (v_i - \bar{v}_s)^2/D \tag{1.6}$$

Breiman (2) has also shown that the relationship between the space and time distributions of speed is given by

$$\frac{f(v)\ \text{space}}{f(v)\ \text{time}} = \frac{\bar{v}_s}{v} \tag{1.7}$$

where $\bar{v}_s$ is the space mean speed and v is the mid-point of a class for which f(v) is the relative frequency.

1	*2*	*3*	*4*	*5*	*6*
speed class km/h	*flow veh/h*	*percentage in time*	*density veh/km*	*percentage in space*	$\bar{V}_s$
4-11	2	0.2208	0.2667	1.7875	60.72
12-19	8	0.8830	0.5161	3.4590	60.72
20-27	2	0.2208	0.0851	0.5704	60.71
28-35	14	1.5453	0.4444	2.9785	60.71
36-43	40	4.4150	1.0127	6.7874	60.73
44-51	89	9.8234	1.8737	12.5581	60.72
52-59	160	17.6600	2.8829	19.3220	60.72
60-67	164	18.1015	2.5827	17.3100	60.72
68-75	159	17.5497	2.2238	14.9045	60.72
76-83	100	11.0375	1.2579	8.4308	60.72
84-91	72	7.9470	0.8229	5.5153	60.73
92-99	53	5.8499	0.5550	3.7198	60.73
100-107	20	2.2075	0.1932	1.2949	60.71
108-115	18	1.9868	0.1614	1.0817	60.71
116-123	5	0.5519	0.0418	0.2804	60.72
	Σ906		Σ14.9203		60.73

Table 1.1

(b) The problem is solved in a tabular manner in Table 1.1. Column 1 gives the speed classes, all speeds being rounded to the nearest km/h. Column 2 gives the frequency with which these speeds were observed during a period of observation of one hour and hence the frequency represents the flow of vehicles in each speed class in terms of vehicles per hour.

Column 3 is the calculated percentage frequency distribution obtained by dividing individual values in column 2 by the sum of column 2.

Column 4 is the calculated density of vehicles in terms of vehicles per km for vehicles in each speed class. It is obtained from equation 1.1 using values in column 2 divided by the mid-point of the classes in column 1.

Column 5 is the calculated space distribution of speeds obtained from equation 1.2 using individual values in column 4 divided by the sum of column 4.

Column 6 is the calculated value of the space mean speed $\bar{v}_s$ for each speed class obtained from equation 1.3 using values in column 5 divided by column 3 and multiplied by the mid-point of the classes in column 1. It can be seen that apart from rounding errors the relationship given by Breiman is shown to be correct and can be compared with the $\bar{v}_s$ calculated from the speed/flow/density relationship using the sum of column 2 divided by the sum of column 4.

The time and space distributions are plotted in Figure 1.1 and, as is expected, the higher frequencies of observed space speeds in the lower class intervals can be clearly seen.

Table 1.2

7	8	9	10	
mid-point speed class km/h	$\frac{\text{col 3 x col 7}}{100}$	$\frac{\text{col 5 x col 7}}{100}$	col 7 − $\bar{v}_s$	(col 5 x col 10)^2
7.5	0.01656	0.13406	-53.225	50.638
15.5	0.13687	0.53615	-45.225	70.747
23.5	0.05189	0.13404	-37.225	7.904
31.5	0.48677	0.93823	-29.225	25.439
39.5	1.74393	2.68102	-21.225	30.577
47.5	4.66612	5.96510	-13.225	21.964
55.5	9.90130	10.72371	-5.225	5.275
63.5	11.49445	10.99185	2.775	1.333
71.5	12.54804	10.65672	10.775	17.304
79.5	8.77481	6.70249	18.775	29.719
87.5	6.95363	4.82589	26.775	39.539
95.5	5.58665	3.55241	34.775	44.984
103.5	2.28476	1.34022	42.775	23.693
111.5	2.21528	1.20610	50.775	27.887
119.5	0.65952	0.33508	58.775	9.686
	Σ 67.42058	Σ 60.72507		Σ 406.689

The time mean speed $\bar{v}_t$ may be calculated from equation 1.3 where f_i is the proportion of vehicles in each class (column 3 divided by 100) and v_i is the mid-point of the speed class (column 7). The calculation is tabulated in Table 1.2 and gives $\bar{v}_t = \Sigma\ q_i v_i / D$ = 67.4 km/h.

The space mean speed $\bar{v}_s$ may be calculated from equation 1.4 where f_i^1 is the proportion of vehicles in each class (column 5 divided by 100) and v_i is the mid-point of the speed class (column 7). The calculation is tabulated in Table 1.2 and gives $\bar{v}_s = \Sigma d_i v_i / D$ = 60.7 km/h.

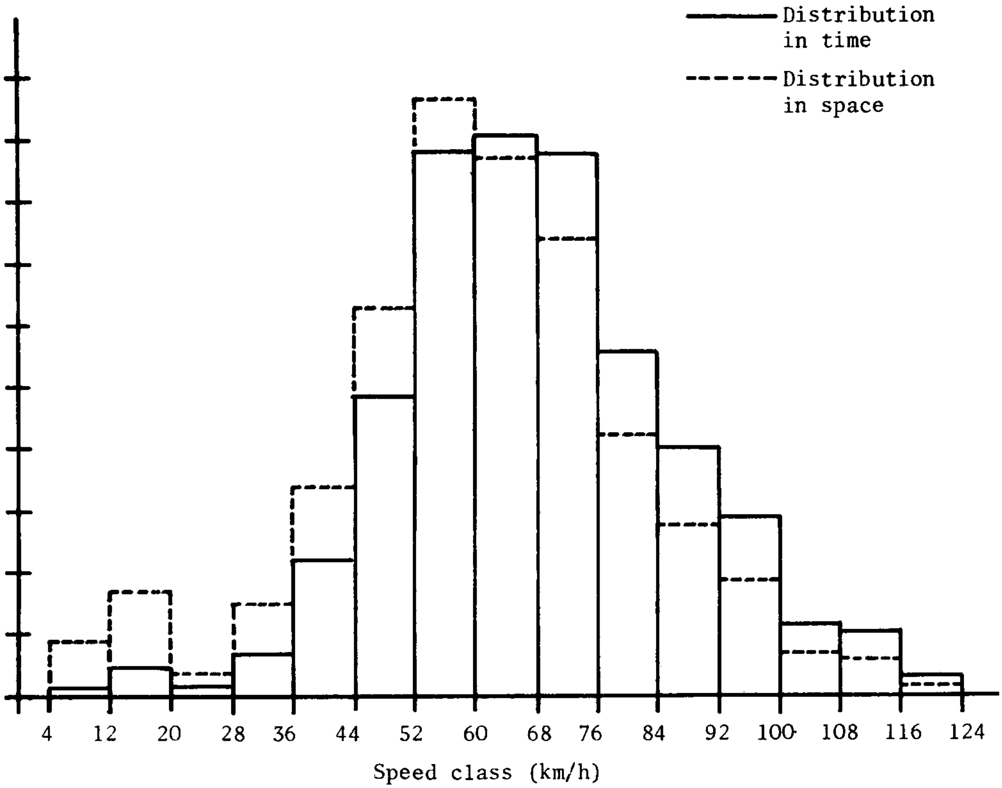


Figure 1.1

speed class km/h	4-11	12-19	20-27	28-35	36-43	44-51	52-59
frequency/h	2	8	2	14	40	89	160

60-67	68-75	76-83	84-91	92-99	100-107	108-115	116-123
164	159	100	72	53	20	18	5

The space mean speed may also be calculated using equation 1.7 and this is given in column 6 where the mean values is also noted to be 60.7 km/h.

The standard deviation of the distribution of speeds in space may be calculated from equation 1.6. The value of d_i/D is given by column 5 divided by 100 and $v_i - \bar{v}_s$ is given by column 7 minus $\bar{v}_s$, this value is tabulated in column 10. The sum of column 11 is σ^2 and equals 406.689 $(km/h)^2$.

Then from equation 1.5

$$\bar{v}_t = 60.725 + 406.689/60.725$$

$$= 67.4 \text{ km/h}$$

This can be compared with the previously calculated value.

References

1. J.G. Wardrop. Some theoretical aspects of road traffic research. Proc. Institution of Civil Engineers 1. Part II 325-362. 1952.

2. L. Breiman. Space time relationships in one-way traffic flow. Transportation Research, 3(3), 365-376, 1969.

EXAMPLE 2 Distribution of Highway Traffic Speeds, Fitting to a Normal Distribution

Observations were made of the speeds of vehicles passing a point on the highway using a radar speedmeter. Calculate the mean speed, the standard deviation of the observed speeds, and the 85 percentile speed. Show whether the distribution of speeds can be represented by a normal distribution. The observations are given in Table 2.1 in the form of a list of speed classes (column 1) and the observed number of speeds in each class (column 2).

Solution

The cumulative frequency distribution is inserted in column 3. This is the cumulative arithmetic sum of column 2 and represents the number of vehicles travelling at a speed equal to or lower than the upper class limit. For example there are 79 vehicles travelling at a speed of 54.9 km/h or less. In column 4 this cumulative frequency is shown as a percentage cumulative frequency. For example a cumulative frequency of 269 represents a cumulative frequency of 269 divided by 394 expressed as a percentage.

Calculation of the mean speed and the standard deviation of the speeds may be carried out using a calculator with these facilities. If such a calculator is not available then some of the arithmetic tedium may be eliminated by the use of coding. A class is selected which appears to contain the mean value and this is assigned a zero value, lower and higher value classes are assigned progressively increasing negative and positive values respectively as shown in column 5.

The code number is then multiplied by the respective frequency as shown in column 6 and summed. The mean value is then given by

$$\begin{array}{l}\text{mid-class mark}\\ \text{of selected class}\end{array} + \frac{\text{class interval} + \Sigma\ \text{column 6}}{\text{column 2}}$$

$$\text{mean speed} = 67.5 + \frac{5 \times 33}{394}\ \text{km/h}$$

$$= 67.9\ \text{km/h}$$

The standard deviation is given by

$$\text{class interval}\left[\frac{\Sigma\ (\text{frequency x deviation}^2)}{\Sigma\ \text{column 2}} - \left(\frac{\text{frequency x deviation}}{\Sigma\ \text{column 2}}\right)^2\right]^{1/2}$$

Before this can be evaluated it is necessary to calculate the frequency x deviation^2 and this is tabulated in column 7.

Table 2.1

1	2	3	4	5	6	7	8	9	10	11	12	13
speed class km/h	*observed frequency*	*cumulative frequency*	*percentage cumulative frequency*	*code*	*col 2 x col 5*	*col 2 x col 5^2*		$\frac{\text{column 8}}{\text{st. dev.}}$	*normal area*	*probability*	*theoretical frequency*	$\frac{(\text{col 12} - \text{col 2})^2}{\text{col 12}}$
30-34.9	3	3	0.76	-7	-21	147	-33	-2.21	.0136			
35-39.9	9	12	3.05	-6	-54	324	-28	-1.88	.0301	.0165	6.5)	
40-44.9	14	26	6.60	-5	-70	350	-23	-1.54	.0618	.0317	12.5)	0.84
45-49.9	21	47	11.93	-4	-84	336	-18	-1.21	.1131	.0513	20.2	0.03
50-54.9	32	79	20.05	-3	-96	288	-13	-0.87	.1922	.0791	31.2	0.02
55-59.9	40	119	30.20	-2	-80	160	-8	-0.54	.2946	.1024	40.3	0.02
60-64.9	48	167	42.39	-1	-48	48	-3	-0.20	.4207	.1261	49.7	0.06
65-69.9	55	222	56.35	0	0	0	2	0.13	.5517	.1310	51.6	0.22
70-74.9	47	269	68.27	1	47	47	7	0.47	.6808	.1291	50.9	0.30
75-79.9	39	308	78.17	2	78	156	12	0.81	.7910	.1102	43.4	0.45
80-84.9	33	341	86.55	3	99	297	17	1.14	.8729	.0819	32.3	0.02
85-89.9	23	364	92.39	4	92	368	22	1.48	.9306	.0577	22.7	0
90-94.9	15	379	96.19	5	75	375	27	1.81	.9649	.0343	13.5	0.17
95-99.9	10	389	98.73	6	60	360	32	2.15	.9842	.0193	7.6	1.29
100-104.9	5	394	100	7	35	245	37	2.48	.9934	.0092	3.6	
	Σ 394				Σ 33	Σ 3501						Σ 3.42

$$\text{Standard deviation} = 5\left[\frac{3501}{394} - \frac{33^2}{394}\right]^{\frac{1}{2}}$$

$$= 14.90 \text{ km/h}$$

The theoretical frequency, assuming the speeds are normally distributed, may be calculated using tables of normal areas which may be found in most statistical textbooks. First the mean speed is deducted from the upper class limit and entered in column 8. Secondly the deviation from the mean in column 8 is divided by the standard deviation and entered in column 9.

The corresponding area between the class limit and the mean can then be obtained from statistical tables and is entered in column 10. The differences between successive values in this column is the probability of a speed being observed between the corresponding class limits, these values are placed in column 11. When this value is multiplied by the theoretical frequency then the theoretical number of observed speeds in each class is obtained (column 12). A chi-squared test is then applied in column 13 and indicates that with 8 degrees of freedom, 12 classes minus the constraints plus one (total number of observations, the mean and the standard deviation) a value of χ^2 of 3.42 indicates that no significant difference exists between the observed sample and a population with a normal distribution at the 10 per cent level of significance, but that there is a significant difference at the 5 per cent level of significance.

EXAMPLE 3 Highway Journey Speeds, Moving Car Observer Method

(a) Describe how the journey speed of a traffic stream may be measured.

(b) Six runs were made in each direction along a two way highway between Smith Avenue and Ginn Square. Flows were measured both with and against the moving car and the following notes obtained.

Car travelling from Smith Avenue to Ginn Square

Trip		*Number of Vehicles*		
Commences	*Ends*	*Overtaking*	*Overtaken*	*Met*
16.05	16.16	2	1	401
16.34	16.44	3	2	360
17.05	17.17	4	1	419
17.35	17.44	5	3	397
18.05	18.18	2	1	406
18.35	18.45	2	3	412

Car travelling from Ginn Square to Smith Avenue

16.19	16.31	3	2	320
16.50	17.03	7	3	319
17.20	17.32	4	2	307
17.50	17.59	4	3	331
18.20	18.33	5	2	317
18.50	19.01	7	1	305

Distance from Smith Avenue to Ginn Square 6.4 km.
Calculate the flow and stream speed in each direction.

Solution

(a) In practical traffic engineering it is frequently the traffic stream journey speed or average speed for a traffic stream over a section of highway which is required. This cannot be obtained by averaging the spot speeds taken at one point on a highway but is usually obtained by the travelling car or moving observer method. The method has been described by Wardrop and Charlesworth (1) and gives both the flow and the speed of the stream.

An observer travels over a known length of highway and the travel times both with (t_w) and against (t_a) the stream are noted. Whilst travelling along the highway with the traffic stream under study the observer notes the number of vehicles which overtake the observer minus the number of vehicles overtaken by the observer (y). The observer then travels against the traffic stream under study and notes the number of vehicles met (x).

The flow is then given by

$$q = \frac{x + y}{t_a + t_w} \tag{3.1}$$

and the mean stream journey time is given by

$$\bar{t} = t_w - y/q \tag{3.2}$$

Frequently on two-way highways when the speed and flow in both directions is required then the number of journeys may be halved by the use of two observers one of whom notes the flow in the stream in which the observing vehicle is travelling whilst the other observer notes the flow in the opposing flow.

Several runs are normally made over each route to increase the accuracy of the observations.

(b) Stream flow and journey time for the traffic stream travelling from Smith Avenue to Ginn Square is calculated in Table 3.1. The flow q and average journey time $\bar{t}$ are calculated from equations 3.1 and 3.2. The average stream speed is then calculated using the given length of the test run. A similar tabulation is carried out in Table 3.2 for the traffic stream travelling from Ginn Square to Smith Avenue.

The calculated values are, 859.9 vehicles/h and 35.7 km/h; 1085.9 vehicles/h and 33.4 km/h for the traffic streams travelling from Smith Avenue to Ginn Square and Ginn Square to Smith Avenue respectively.

Reference

(1) J.G. Wardrop and G. Charlesworth. A method of estimating speed and flow of traffic from a moving vehicle. J. Inst. Civ. Engs 3 (Pt 2) 1954, 158-71.

Stream from Smith Avenue to Ginn Square

journey time start	x (vehicles)	y (vehicles)	t_a (min)	t_w (min)	q (veh)	$\bar{t}$ (min)
16.05 16.19	320	1	12	11	837.39	10.93
16.34 16.50	319	1	13	10	834.78	9.93
17.05 17.20	307	3	12	12	775.00	11.77
17.35 17.50	331	2	9	9	1110.00	8.89
18.05 18.20	317	1	13	13	733.85	12.92
18.35 18.50	305	-1	11	10	868.57	10.07
			mean values		859.93	10.75

Table 3.1

Stream from Ginn Square to Smith Avenue

journey time start	x (vehicles)	y (vehicles)	t_a (min)	t_w (min)	q (veh)	$\bar{t}$ (min)
16.05 16.19	401	1	11	12	1048.70	11.94
16.34 16.50	360	4	10	13	949.57	12.75
17.05 17.20	419	2	12	12	1052.50	11.89
17.35 17.50	397	1	9	9	1326.67	8.95
18.05 18.20	406	3	13	13	948.35	12.81
18.35 18.50	412	6	10	11	1194.29	10.70
			mean values		1085.93	11.51

Table 3.2

EXAMPLE 4 Theoretical Basis of the Moving Car Observer Method

Show how the space mean speed of a traffic stream and the flow may be estimated by an observer travelling with, and against the traffic stream. Prove any formulae which are used. The following observations were taken by an observer travelling with and against the traffic stream; calculate the mean traffic stream speed and also the flow.

Observer travelling to north (average of 10 runs)		
observer's travel time (mins)	no. of vehicles which pass observer	no. of vehicles passed by observer
2.6	5.4	3.5

Observer travelling to south (average of 10 runs)		
	observer's travel time (mins)	no. of vehicles met whilst travelling against the stream
	2.9	76.9

length of travel path 1.1 km.

Solution

From the fundamental equation of traffic flow

flow = speed x density

When a vehicle travels with or against a traffic stream then an observer in the vehicle can note the traffic flow relative to himself.

The speed of the traffic stream relative to the observer is then the stream speed minus the observer's speed.

Considering relative flows and speeds, then relative stream flow

= relative stream speed x stream density
= (stream speed - observer's speed) stream density
= stream speed x stream density - observer's speed x stream density
= stream flow - observer's speed x stream density (4.1)

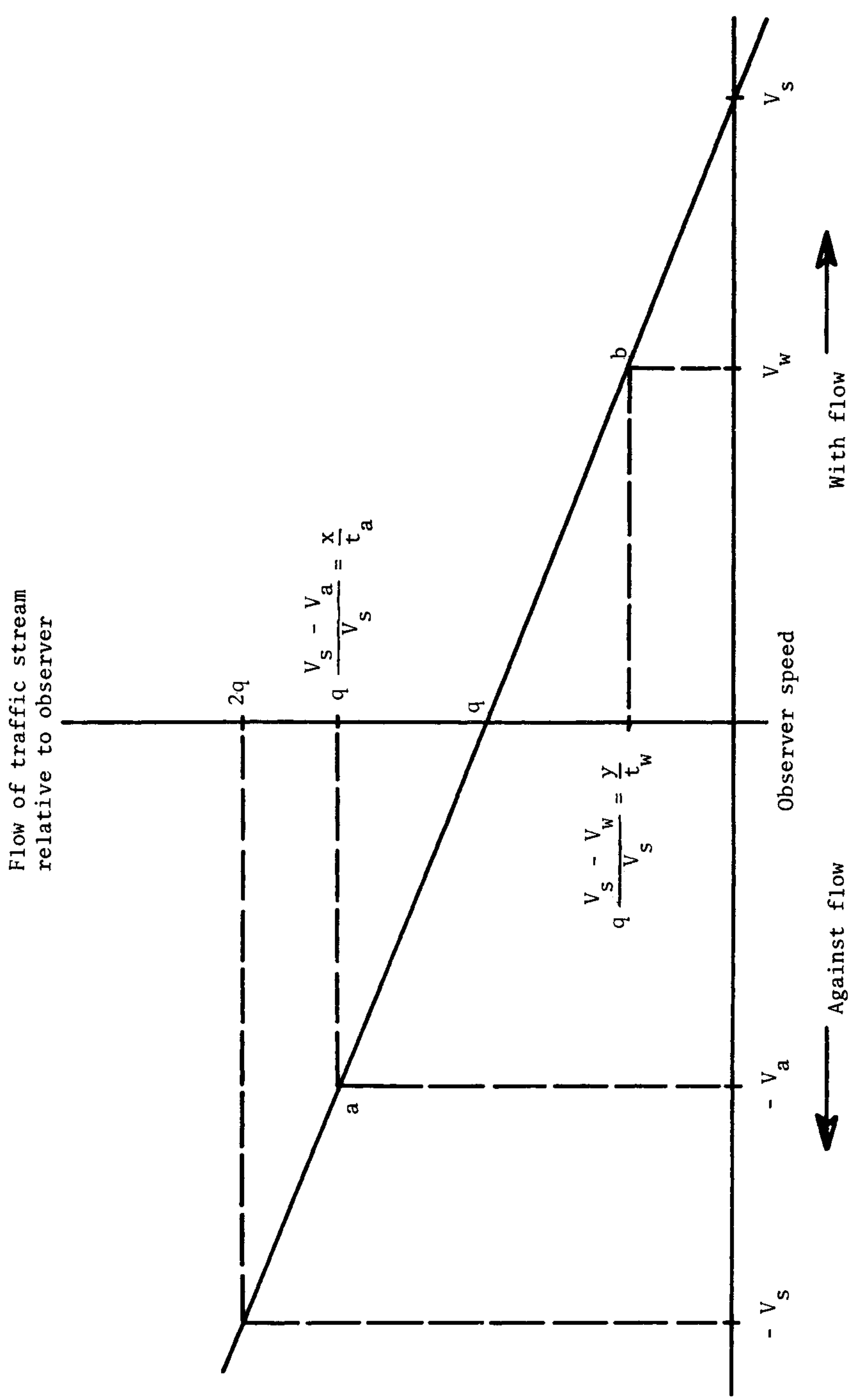

Flow of traffic stream
relative to observer
2q
$q \frac{V_s - V_a}{V_s} = \frac{x}{t_a}$
q
$q \frac{V_s - V_w}{V_s} = \frac{y}{t_w}$
a
b
$- V_s$
$- V_a$
V_w
V_s
Observer speed
Against flow
With flow

Figure 4.1

This relationship may be shown diagrammatically as in Figure 4.1. When the relative stream flow is zero then the observer is travelling at the stream speed and when the observer's speed is zero, then the relative stream flow is equal to the stream flow.

When the observer is travelling against the traffic stream at a speed equal to that of the traffic stream then relative flow is equal to twice the stream flow.

If the average values of observer's speeds and relative flows V_a, V_w, x and y represented by points a and b in Figure 4.1 for travel against and with the traffic stream are known, then the relative flow when the observer was travelling against the stream would be

$$\frac{q\ (V_s - V_a)}{V_s}$$

and when the observer was travelling with the stream would be

$$\frac{q\ (V_s - V_w)}{V_s}$$

where, q is the stream flow
V_s is the stream speed
V_a is the speed of the observer against the flow (negative)
and V_w is the speed of the observer with the flow.

Also the observer relative flow is xV_a/ℓ and yV_w/ℓ where x is the number of vehicles met and the net number passing the observer respectively and ℓ is the travel path length.

Then $$\frac{q\ (V_s - V_w)}{V_s} = \frac{y\ V_w}{\ell}$$

giving $$V_s = \frac{V_w}{1 - \frac{y}{q}\frac{V_w}{\ell}}$$

This is equivalent to the value of

$$t_s = t_w - \frac{y}{q}$$

derived by Wardrop and Charlesworth (1).

Also from similar triangles $$\frac{q - \frac{yV_w}{\ell}}{V_w} = \frac{\frac{xV_a}{\ell} - q}{V_a}$$

$$\text{giving } q = \frac{V_w V_a (x + y)}{\ell (V_a + V_w)}$$

which is equivalent to the value of

$$q = \frac{x + y}{t_a + t_w}$$

derived by Wardrop and Charlesworth (1).

From the observations given in the example,
vehicles met when travelling against the stream (x) = 76.9
vehicles met when travelling with the stream (y) = 1.9
length of travel path = 1.1 km
time of travel against the stream = 2.9 min
time of travel with the stream = 2.6 min
speed of travel against the stream = 22.759 km/h
speed of travel with the stream = 25.385 km/h

$$\text{Giving } q = \frac{V_w V_a (x + y)}{1 (V_a + V_w)} = 859.65 \text{ veh/h}$$

$$\text{and} \quad V_s = \frac{V_w}{1 - \frac{y}{q} \frac{V_w}{1}} = 26.75 \text{ km/h}$$

Reference

(1) J.G. Wardrop and G. Charlesworth. A method of estimating speed and flow of traffic from a moving vehicle. J. Inst. Civ. Engs 3 (Part 2) 1954. 158-171.

EXAMPLE 5 Car Following Theory Illustrated by an Example

(a) Show how car following theory may be used to estimate the maximum flow of vehicles in a single stream of vehicles.

(b) Vehicles in a traffic stream may be assumed to have an average spacing at rest of 5 m, a maximum difference in deceleration ability of 0.5 m/s^2 when the preceding vehicle decelerates at 2.5 m/s^2, and vehicle drivers a maximum reaction time of 1.0 s. Assuming that drivers space themselves so as to be able to stop safely should the preceding vehicle execute an emergency stop determine the maximum single lane flow and the speed at which it occurs.

Solution

(a) The flow of vehicles along a highway can be considered in a microscopic manner if the behaviour of a following vehicle is related to the actions of the vehicle in front. This is in contrast to macroscopic approaches when the whole traffic stream is considered using characteristics which are averages for all vehicles.

Usually all vehicles in a traffic stream cannot be considered to be following each other; it has been shown that in many situations when vehicles are separated by a time headway greater than approximately 6 s then the following vehicle is not influenced by the preceding vehicle.

The separation between vehicles may be measured in two ways, either as the time interval between successive vehicles passing a point on the highway or as the instantaneous distances between successive vehicles as they travel along the highway. The former is the time headway and is measured at one point in space over a period of time, whilst the latter is measured over space at one point in time. The inverse of the time headway is the flow and the inverse of the distance headway is the concentration or density of vehicles.

It is frequently assumed that when the flow is a maximum then vehicles space themselves at a minimum headway consistent with safety. This headway is usually considered to be related to the behaviour of the preceding vehicle. A simple theory is that the following driver responds to the speed of the preceding vehicle, and adjusts his speed and following distance so as to be able to stop safely should the preceding vehicle execute an emergency stop. This is shown in Figure 5.1 where H is the instantaneous distance headway (all measurements are bumper to bumper). X_1 is the distance needed by vehicle 1 to execute an emergency stop from speed V_1, X_2 is the distance needed by vehicle 2 to stop from speed V_2 and A is

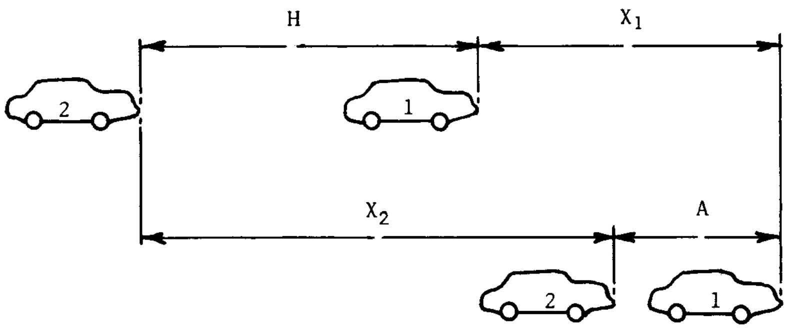


Figure 5.1

the zero speed headway between vehicles 1 and 2 at rest, which is the length of vehicle 1 plus some small separation distance.

From Figure 5.1 it is obvious that

$$H + X_1 = X_2 + A \tag{5.1}$$

From the laws of motion (assuming constant deceleration)

$$X_1 = V_1{}^2/(2\ F_1) \tag{5.2}$$

where F_1 is the maximum deceleration of vehicle 1.

The distance X_2 can be assumed to consist of two parts, the distance travelled at speed V_2 during the reaction time of the driver and the distance needed to bring the vehicle to test hence

$$X_2 = B\ V_2 + V_2{}^2/(2\ F) \tag{5.3}$$

where B is reaction time (assumed constant)

F_2 is the maximum deceleration of vehicle 2.

Hence from equation 5.3 we get

$$H = A + B\ V_2 + V_2{}^2/(2\ F_2) - V_1{}^2/(2\ F_1) \tag{5.4}$$

For the general case we can write

$$1/(2\ F_2) = C \text{ and } 1/(2\ F_1) = D$$

where C and D are constants from which equation 5.4 becomes

$$H = A + B\ V_2 + CV_2{}^2 - DV_1{}^2 \tag{5.5}$$

Equation 5.5 represents the basic car following equation based on the concept that the following driver responds to the speed of the preceding vehicle. This equation covers the case of simple car following and the case in which the following vehicle is 'catching up' on the preceding vehicle. It should be noted that the two constants C and D in equation 5.5 do not normally have the same value due to the fact that the driver of the following vehicle does not wish to employ the extreme deceleration of the preceding vehicle. For conditions of high concentration the difference between V_1 and V_2 is usually small and it is reasonable to let

$$V = (V_1 + V_2)/2 \tag{5.6}$$

From which equation 5.5 becomes

$$H = A + B\,V + V^2\,(C - D) \tag{5.7}$$

Many observations of the speeds and headways of following vehicles have been made of traffic flow to obtain the best fit values of the constants. An interesting review of safe following distances and the resulting maximum values of flow and the speed at which it occurs are given in ref. (1).

(b) From equation 5.7

$$H = A + B\,V + V^2\,(C - D)$$

Assuming a maximum value of 1.0 s for the reaction time B

$$C = 1/2\,F_2 = 1/(2 \times 2.0)$$

$$D = 1/2\,F_1 = 1/(2 \times 2.5)$$

$$\text{Minimum distance headway } H = 5 + V + V^2\,(0.25 - 0.20)$$

$$= 5 + V + 0.05V^2$$

$$\text{Minimum time headway } = \frac{5 + V + 0.05V^2}{V}$$

$$\text{Flow} = \frac{V}{5 + V + 0.05V^2}\ \text{veh/s}$$

$$\frac{d(\text{flow})}{dV} = 0 \text{ for a maximum value}$$

$$\frac{(5 + V + 0.5V^2) - V(1 + 0.1V)}{(5 + V + 0.05V^2)^2} = 0$$

$$V^2 = 100$$

$$V = 10\ \text{m/s}$$

$$= 36\ \text{km/h}$$

References

1. Normann, O.K. and W.P. Walker. Highway Capacity. Public Roads, Vol. 25, No. 10. October 1949.

EXAMPLE 6 The Negative Exponential Distribution Applied to Headways on Highways

(a) Describe the use of the negative exponential distribution to describe highway traffic flow.

(b) The time headways between vehicles in a single traffic stream were measured and classified into one second intervals and are given below. Demonstrate the goodness of fit of the negative exponential distribution to the observed headways.

Observed traffic flow 500 vehicles/hour

Time headway class(s)	0-2.9	3-5.9	6-8.9	9-11.9	12-14.9	15-17.9	18-20.9	21-23.9	24-26.9	27-29.9	30-32.9	> 33
Observed number of headways in class	37	36	26	11	9	5	5	1	1	1	2	0

(c) The traffic flow along a one way street system is free flowing with ample opportunities for overtaking. If pedestrians require at least 8 s to cross the carriageway what is the maximum possible traffic flow if there are to be 60 opportunities per hour to cross the highway between successive vehicles.

Solution

(a) Uniform time headways between vehicles do not occur in actual traffic flow except in undesirably congested traffic conditions. If headways at less congested levels of flow are measured then very wide variations in their sizes will be found. Because it is important to be able to mathematically describe the nature of this variation in headways when producing mathematical models of traffic lane it is usual to attempt to fit statistical distributions to the observed headways.

One of the simplest distributions to describe headways which is applicable only when traffic is non-congested and free-flowing is the negative exponential distribution. This states that the percentage of headways greater or equal to t seconds is given by

$$100 \exp(-qt) \tag{6.1}$$

where q is the rate of vehicle arrivals or the reciprocal of the mean headway.

(b) The example can be tabulated as shown in Table 6.1. Column 2 gives the observed frequency in the classes given in column 1, column 3 is the cumulative total of column 2, column 4 is column 3 expressed as a percentage. Column 5 is calculated from equation 6.1 where q is 500/3600 vehicles per second. Column 6 is the theoretical cumulative headway distribution obtained from column 5.

The similarity between observed and theoretical frequencies (columns 2 and 6) is clearly evident. A graphical plot is shown in Figure 6.1 where the observed values are compared with the theoretical distribution.

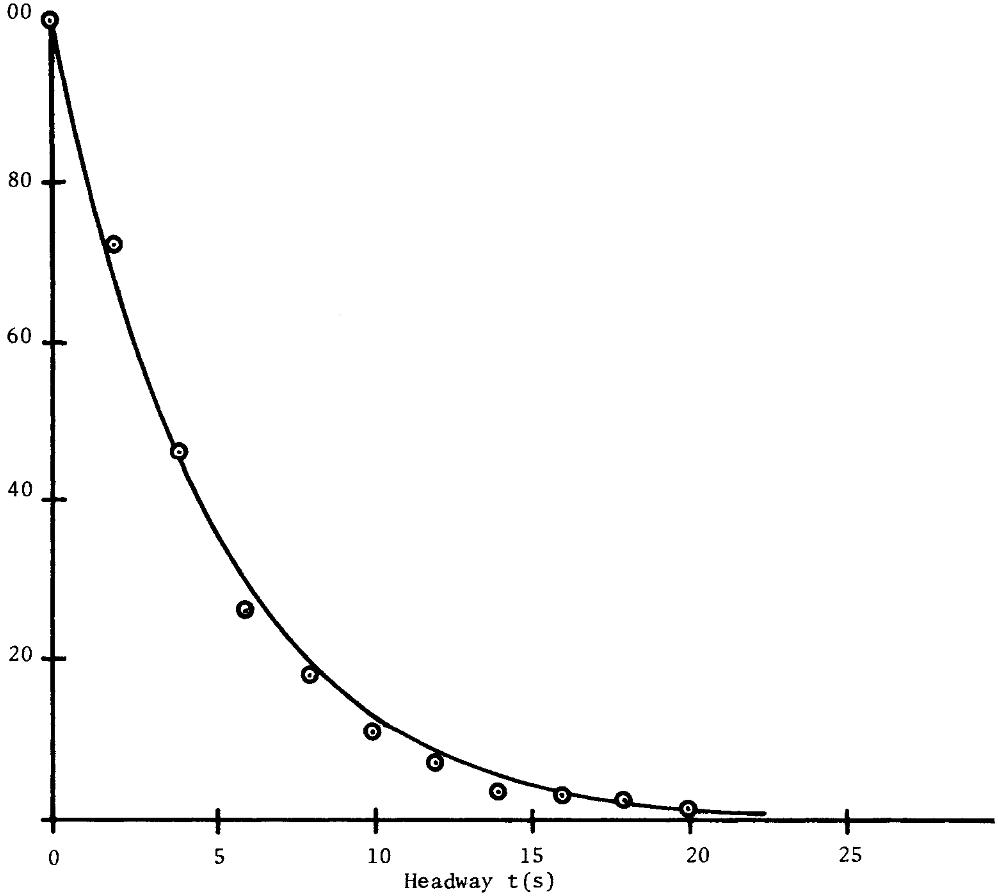


Figure 6.1

Table 6.1

1	2	3	4	5	6
Time headway class (s)	*Observed frequency*	*Observed frequency of headways > lower class limit*	*Column 4 as percentage*	*Theoretical percentage of headways lower class limit*	*Theoretical frequency*
0-2.9	37	134	100	100	134
3-5.9	35	97	72.4	65.6	87.9
6-8.9	26	61	45.5	42.8	57.4
9-11.9	11	35	26.1	28.7	38.5
12-14.9	9	24	17.9	18.8	25.2
15-17.9	5	15	11.2	12.5	16.8
18-20.9	5	10	7.5	8.2	11.0
21-23.9	1	5	3.7	5.4	7.2
24-26.9	1	4	3.0	3.6	4.8
27-29.9	1	3	2.2	2.4	3.2
30-32.9	2	2	1.5	1.6	2.1
>33	0	0	0	0.8	1.1

(c) It will be assumed that one crossing opportunity exists when the headway between successive vehicles is equal to or greater than 8 s.

From equation 6.1 the percentage headways greater than or equal to t seconds is given by 100 exp(-qt). This is specified to be (60/hourly volume) x 100 per cent since the number of headways per hour is equal to the traffic flow, and the hourly volume is 3600q where q is the vehicle arrival rate/second.

$$\text{Giving } \frac{60}{3600q} = \exp(-8q)$$

This expression can be rewritten

$$f(q) = \exp(8q) - 60q$$

and this has to be solved for the values of q which make f(q) zero.

By consideration of the problem it can be seen that there are likely to be two solutions. When traffic flow is low then there is a high probability of headways greater than 8s and the product of a high probability and a low volume will give the required number of headways per hour. Conversely when the flow is high then there will be a low probability of headways greater than 8s and the product of a low probability and a high volume will again give the required number of headways per hour.

Newton's Method of approximation to the solutions will be used and this requires an initial approximation to the values of q which make f(q) zero. The two values of flow which are assumed are 60 veh/h (q = 0.0167 veh/s) and 1400 veh/h (q = 0.3889).

Newton's Method states that a second approximation to the solution can be found from

$$q = Z - \frac{f(z)}{f^1(z)}$$

where Z is the first approximation.

$$f(z) = \exp(8z) - 60z$$

$$f^1(z) = 8\exp(8z) - 60$$

Solution 1: let z = 0.0167, f(z) = 0.1409, $f^1(z)$ = -50.8565

$$q = 0.0167 + \frac{0.1409}{50.8565}$$

$$= 0.0195$$

let z = 0.0195, f(z) = 0.012, $f^1(z)$ = -50.6494

$$q = 0.0195 - \frac{0.0012}{50.6494}$$

$$q = 0.0193$$

Solution 2: let z = 0.3889, f(z) = -0.8860, $f^1(z)$ = 119.5837

$$q = 0.3889 + \frac{0.8860}{119.5837}$$

$$= 0.3963$$

let z = 0.3963, f(z) = 0.0390, $f^1(z)$ = 130.5361

$$q = 0.3963 - \frac{0.0390}{130.5361}$$

$$q = 0.3960$$

Check:

where q = 0.0193

probability of headways > 8 = exp (-0.0193 x 8)

= 0.8569

No. of headways > 8 = 0.8569 x 0.0193 x 3600

= 60.15

when q = 0.3960

no. of headways > 8 = 60.00

EXAMPLE 7 The Double Exponential Distribution Applied to Headways on Congested Highways

(a) Describe a headway distribution suitable for use in congested traffic flow conditions.

(b) Show that the observed distribution of headways of a single stream of urban traffic given below can be represented by the double exponential distribution and determine the parameters of the distribution.

Observed traffic flow 987 veh/h

time headway class (s)	0-0.9	1-1.9	2-2.9	3-3.9	4-4.9	5-5.9	6-6.9
observed number of headways in class	70	150	91	62	40	29	22

7-7.9	8-8.9	9-9.9	10-10.9	11-11.9	12-12.9	13-13.9	14-14.9
12	8	8	7	8	3	7	5

15-15.9	16-16.9	17-17.9	18-18.9	19-19.9	20-20.9	21-21.9
2	0	2	0	1	0	0

22-22.9	23-23.9	24-24.9	25-25.9	>26
0	1	0	1	0

Solution

(a) Whilst it is convenient to use such a simple form of distribution as the negative exponential distribution it is not usual to find free flowing traffic conditions in urban areas where many traffic problems are to be found.

Many differing headway distributions have been proposed for a variety of traffic conditions. One of these which is able to describe traffic flows which vary from congested to free flowing conditions is the double exponential distribution. The form of this distribution is

$$\text{percentage of headways} > t = 100\ (r \exp -(t - c)/(t_1 - c)$$
$$+ (1 - r) \exp -(t/t_2)) \qquad (7.1)$$
$$t > c$$

where r is the proportion of vehicles in the traffic stream which are prevented from overtaking the preceding vehicle i.e. restrained vehicles,
c is the minimum time headway between following or restrained vehicles,
t_1 is the mean headway between restrained vehicles,
t_2 is the mean headway between free flowing or unrestrained vehicles.

By the use of the two terms, one of which represents restrained vehicles and the other free flowing or unrestrained vehicles it is possible for this distribution to represent a wide variety of traffic flow conditions.

To fit a double exponential distribution to the observations the observed percentage of headways greater than or equal to the lower class limit is calculated and placed in column 3 of Table 7.1. A graphical method of fitting is used and the observed cumulative headway values are plotted as shown in Figure 7.1 using semi-log paper. That portion of the negative exponential distribution which represents the free flowing or unrestrained vehicles is first fitted to the observed values by assessing which observed points lie on a straight line. In Figure 7.1 it can be seen that a straight line can be fitted from A to B as shown and this is represented by the following portion of equation 7.1.

$$\text{percentage of headways} > t = 100\ (1 - r) \exp -(t/t_2)$$

The value of r is given by the intersection of the line A-B with the vertical axis, the point of intersection gives (100 - r) per cent or r = 0.46. The value of t_2 can be calculated by selecting a point on the line noting the value of time and percentage of headways and substituting as below

$$10 = 100\ (1 - 0.46) \exp -(8.1/t_2)$$

giving $t_2 = 4.8$ s

The arithmetic difference between the observed percentage headways and the straight line AB is also plotted on Figure 7.1 and this line CD represents the restrained vehicles and the following portion of equation 7.1,

$$\text{percentage of headways} \geqslant t = 100\ r \exp -(t - c)/(t_1 - c)$$

Table 7.1

1 *time headway class (s)*	2 *observed frequency*	3 *observed percentage headway ≥ lower class limit*	4 *theoretical percentage headway ≥ lower class limit*	5 *theoretical frequency*
0-0.9	70	100	100	54.3
1-1.9	150	86.77	89.74	168.6
2-2.9	91	58.41	57.87	96.0
3-3.9	62	41.21	39.73	58.1
4-4.9	40	29.49	28.73	37.6
5-5.9	29	21.93	21.62	25.9
6-6.9	22	16.45	16.72	18.8
7-7.9	12	12.29	13.17	14.2
8-8.9	8	10.02	10.50	11.0
9-9.9	8	8.51	8.42	8.6
10-10.9	7	6.99	6.79	6.9
11-11.9	8	5.67	5.49	5.6
12-12.9	3	4.16	4.44	4.4
13-13.9	7	3.59	3.60	3.6
14-14.9	5	2.27	2.92	2.9
15-15.9	2	1.32	2.37	2.4
16-16.9	0	0.95	1.92	1.9
17-17.9	2	0.95	1.56	1.5
18-18.9	0	0.57	1.26	1.3
19-19.9	1	0.57	1.02	1.0
20-20.9	0	0.38	0.83	0.8
21-21.9	0	0.38	0.67	0.7
22-22.9	0	0.38	0.55	0.5
23-23.9	1	0.38	0.44	0.4
24-24.9	0	0.19	0.36	0.4
25-25.9	1	0.19	0.29	0.3
>26	0	0	0.10	0.3
	Σ 529			

The value of c is the minimum time headway between following vehicles, it is frequently assumed to be 1 s and unless headways are grouped into classes of less than one second there is no point in assuming a smaller value. Some indication of its value can be obtained by the headway at which the curve changes shape, i.e. point E.

The value of t_1 can be obtained by selecting a point Y on CD, noting the value of the percentage of headways and the value of the headway and substituting as below

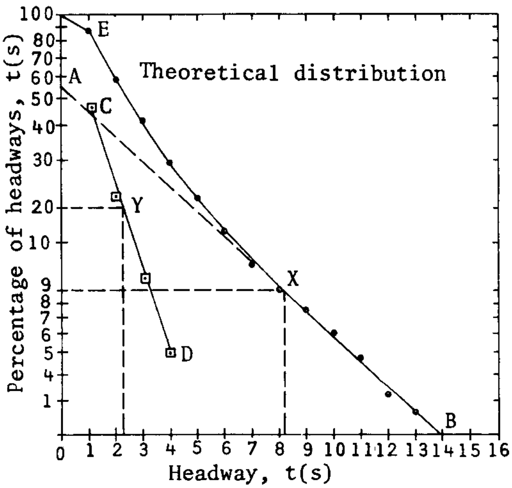
$$20 = 100 \times 0.46 \exp - (2.15 - 1)/(t_1 - 1)$$

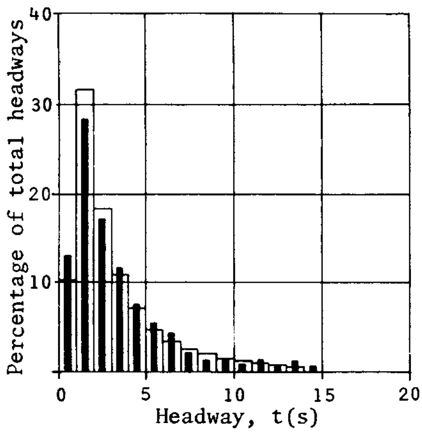
giving $t_1 = 2.4$ s

The values of all the parameters in equation 7.1 have now been determined and the theoretical percentage of headways can now be calculated and entered in column 4 of Table 7.1.

The theoretical frequency is then obtained by subtraction of successive theoretical percentages in column 4 and the multiplication of these differences by the total number of observed headways. For example the theoretical percentage of headways in the class 3-3.9 s is 57.87 - 39.73 or 18.14 per cent. The theoretical frequency is then 18.14 x 529 = 96.0 as given in column 5.

A comparison of columns 4 and 5 indicates a reasonable fit between observed and theoretical values. The theoretical distribution calculated in column 4 is plotted in Figure 7.2 and its fit to the observed values can be clearly seen.





EXAMPLE 8 Flow, Speed and Density Relationships for Highway Flow

(a) For a highway traffic stream describe the relationship between flow, speed and density.

(b) On a two lane carriageway roadworks restrict the width of both traffic lanes forming a bottleneck to traffic flow. The maximum flow per lane on the unobstructed carriageway is 2500 vehicles per hour whilst on the section under repair the maximum flow per lane is 2000 vehicles per hour. When stationary, vehicles are spaced at average distance headways of 8m. It may be assumed that there is a linear relationship between speed and density.

When the traffic flow approaching the roadworks is 4500 veh/h, calculate

(i) the speed of the traffic stream a considerable distance in advance of the bottleneck

(ii) the speed of the traffic stream immediately before the commencement of the bottleneck

(iii) the speed of the shockwave formed by the bottleneck.

Solution

(a) The speed, flow and density of a traffic stream are connected by the fundamental equation

$$\text{flow} = \text{space mean speed x density} \qquad (8.1)$$

If the relationship between any two of these characteristics are known then the remaining variable can be obtained. For simplicity it is often assumed that speed and density are connected in a linear manner with the maximum speed v_f at zero density and zero speed at maximum density d_j. These two constants are referred to as the free speed and the jam density. The three relationships are shown in Figure 8.1 where it can be seen that the maximum flow occurs when the density is half the jam density and when the speed is half the free speed.

The speed density relationship is of the form

$$v_s = v_f - \frac{v_f}{d_j} d \qquad (8.2)$$

where v_s is the space mean speed,
v_f is the free speed,
d_j is the jam density,
d is the density.

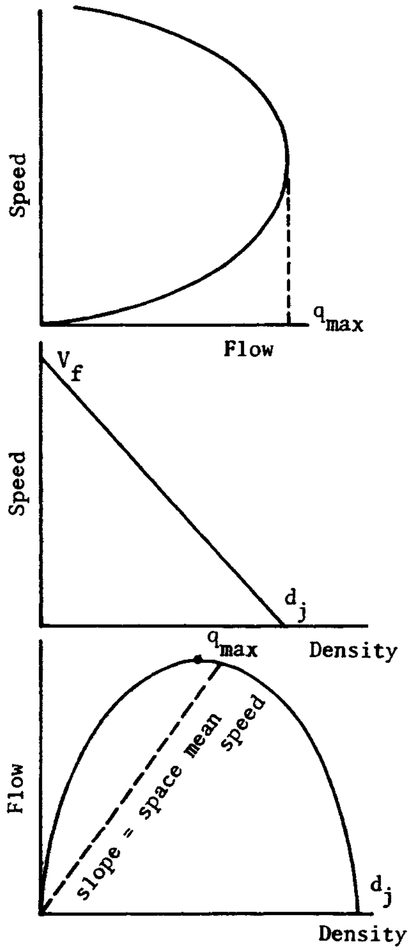


Figure 8.1

The relationships between speed, flow and density

Rearranging equation 8.2

$$d = (v_f - v_s)\,\frac{d_j}{v_f}$$

From equation 8.1

$$q = v_s\, d$$

so that $q = d_j\, v_s - \dfrac{d_j}{v_s}\, v_s^{\,2}$ (8.3)

For a maximum value

$$\frac{dq}{dv_s} = 0$$

giving $d_j - 2v_s \frac{d_j}{v_f} = 0$

or $v_s = \frac{v_f}{2}$

The maximum flow can be found by substituting in equation 8.3

$$q \max = \frac{d_j v_f}{2} - \frac{d_j}{v_f}\left(\frac{v_f}{2}\right)^2$$

$$= \frac{d_j v_f}{4} \tag{8.4}$$

The relationship between speed and flow obtained from equation 8.3 gives two values of speed for every value of flow, except at the maximum flow. These two values of speed represent differing flow conditions, on that portion of the curve where the speed varies from v_f to the speed for maximum flow $v_f/2$ the flow is increasing until it reaches its maximum value. On the remaining portion of the curve speed is dropping and also the flow is falling, indicating unstable traffic flow conditions.

The diagram relating flow and density of a traffic stream is frequently referred to as the fundamental diagram of traffic flow. Flow increases as density increased until the point of maximum flow is reached after which flow decreases with increasing density until the jam density is reached. If a line is drawn from the origin to any point on the curve then the slope of the line is the space mean speed corresponding to the traffic flow represented by the point on the curve.

(b) When there is a linear relationship between speed and density then from equation 8.4

$$Q \max = \frac{d_j v_f}{4}$$

where d_j and v_f are the jam density and the free speed respectively.

For the unobstructed section the jam density d_j is 2 x 1000/8 = 250 veh/km and from equation 8.4

$$v_f = \frac{4 \times 5000}{250} = 80 \text{ km/h}$$

Also when there is a linear relationship between speed and density then,

$$Q = v_f d - \frac{v_f}{d_j} d^2$$

$$\text{or } Q = 80d - \frac{80}{250} d^2 \qquad (8.5)$$

When the flow is 4500 vehicles per hour then

$$4500 = 80d - \frac{80}{250} d^2$$

and d = 85.5 or 164.5 veh/km.

The former value is appropriate for the traffic conditions given (point A Figure 8.2).

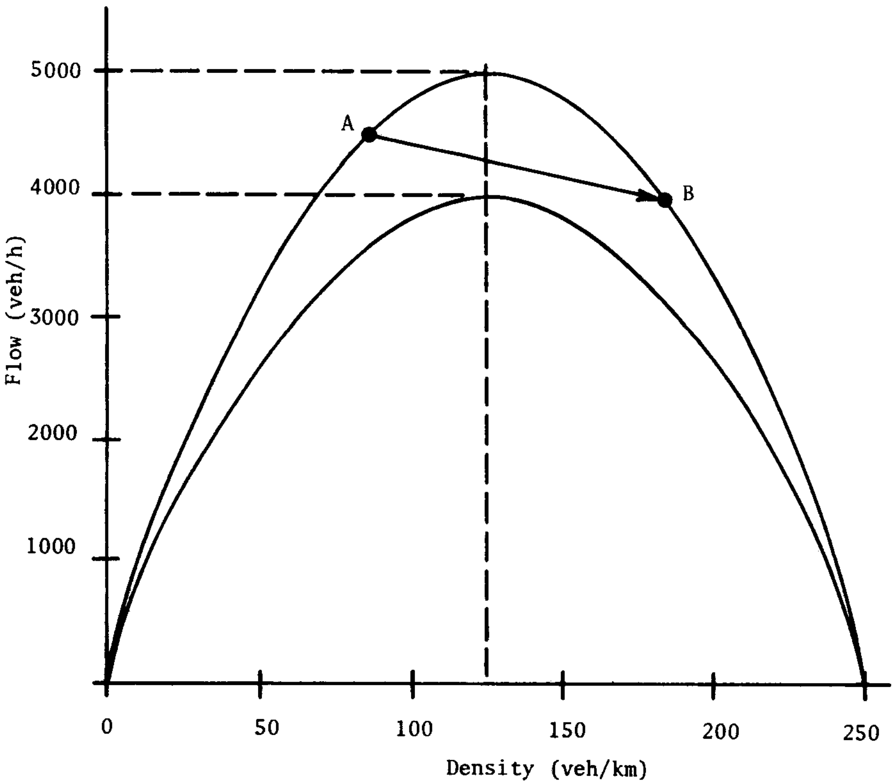


Figure 8.2

When the flow is 4000 veh/h then

$$4000 = 80d - \frac{80}{250} d^2$$

and d = 69.1 or 180.9 veh/km.

The latter value is appropriate for the traffic conditions given (point B Figure 8.2).

Also when there is a linear relationship between speed and density then from equation 8.2

$$v = v_f - \frac{v_f}{d_j} d$$

(i) When the flow is 4500 veh/h then the speed of the traffic stream a considerable distance in advance of the bottleneck is given by

$$v = 80 - \frac{80}{250} 85.5 \text{ km/h}$$

$$= 52.7 \text{ km/h}$$

(ii) When the flow is 4000 veh/h then the speed of the traffic stream immediately before the commencement of the bottleneck is given by

$$v = 80 - \frac{80}{250} 180.9 \text{ km/h}$$

$$= 22.1 \text{ km/h}$$

(iii) The speed of the shockwave formed by the bottleneck is given by the slope of the line AB

$$v = \frac{4500 - 4000}{85.5 - 180.9} \text{ km/h}$$

$$= -5.2 \text{ km/h}$$

EXAMPLE 9 Flow, Speed and Density Relationships Applied to a Highway Bottleneck

The capacity of a highway link is suddenly reduced by a width restriction at roadworks to a maximum flow of 1000 veh/h and the speed of all vehicles to 5 km/h.

During off-peak periods the flow is less than 1000 veh/h and the peak hour flow may be represented by a block of demand which increases instantaneously to a flow of 1500 veh/h and which before it reaches the width restriction has an average speed of 80 km/h. The flow continues for a period of 20 minutes and then falls instantaneously to the off-peak level of flow.

Calculate the maximum length of queue which occurs at the restriction during peak periods. Derive any formulae used from first principles.

Solution

Let the space mean speed and flow of vehicles before and after they join the queue be v_1, q_1 and v_2, q_2 respectively.

During the off-peak periods the flow is less than the capacity of the restricted width section and at the commencement of the peak period a queue will not exist at the bottleneck. During the peak period the queue length at the bottleneck will increase and let the queue length be ℓ at time t.

Consider this length of road ℓ before the commencement of the bottleneck. At zero time the queue has not yet commenced to form and the number of vehicles in road length ℓ, is $k_1\ell$ where k_1 is the density q_1/v_1.

At time t the length of road contains $k_2\ell$ where k_2 is q_2/v_2.

And $(k_2\ell - k_1\ell)$ is the difference between the number of vehicles joining and leaving the queue, or $(q_1 - q_2)t$.

The rate at which the queue length is changing is given by

$$\frac{d\ell}{dt} = \frac{q_1 - q_2}{k_2 - k_1} = (q_1 - q_2)/(\frac{q_2}{v_2} - \frac{q_1}{v_1})$$

The maximum queue length occurs at the end of the peak period and as its initial length is zero, the maximum length is given by

$$(q_1 - q_2)/ (\frac{q_2}{v_2} - \frac{q_1}{v_1}) \text{ x duration of peak period}$$

For the given values

$$(1500 - 1000) \left(\frac{1000}{50} - \frac{1500}{80}\right) \frac{20}{60} \text{ km}$$

$$= 0.92 \text{ km}$$

EXAMPLE 10 Queueing Theory Applied to Highways

A toll collection system has four toll booths and it is observed that at each toll booth the time required to pay the toll had an exponential distribution with a mean time of 5 s. Vehicles arrive at the toll facility at random and at a rate of 2400 veh/h.

Initially the flow was divided into four equal streams and vehicles in each stream could not move to another stream, passing through the toll booth on a first come first served basis. The toll system was then revised so that vehicles could move to the first vacant toll booth.

Calculate the percentage improvement in the following system characteristics,

(a) the average time spent in the system,
(b) the average waiting time spent in the queue,
(c) the average queue length.

which result from the change in the method of operation.

Solution

Vehicles approach the toll bridge in a free flowing manner and the queueing system can be represented by the standard system of Poisson arrivals and exponential service.

The parameters are,

the average number of vehicle arrivals at each toll booth per unit time

$$(\lambda) = \frac{600}{3600} \text{ veh/s}$$

and the average number of vehicles serviced at each toll booth per unit time

$$(\mu) = \frac{1}{5} \text{ veh/s.}$$

(i) From queueing theory the following relationships apply for the first come first served system.

(a) The average time spent in the system $= \dfrac{1}{\mu - \lambda} = 30.0$ s

(b) The average waiting time in the queue $= \dfrac{\lambda}{\mu(\mu - \lambda)} = 25.0$ s

(c) The average queue length = $\frac{\lambda^2}{\mu(\mu - \lambda)}$ = 4.2 vehicles.

(ii) From queueing theory the following relationships apply when vehicles move to the first vacant toll booth.

(a) The average time spent in the system = $\frac{\mu(\lambda/\mu)^k}{(k - 1)!(k - \lambda)^2}\, p(0) + \frac{1}{\mu}$

(b) The average waiting time spent in the queue = $\frac{\mu(\lambda/\mu)^k}{(k - 1)!(k\mu - \lambda)^2}\, p(0)$

(c) The average queue length = $\frac{\lambda\mu(\lambda/\mu)^k}{(k - 1)!(k\mu - \lambda)^2}\, p(0)$

Where the probability of having zero vehicles in the system p(0) is given by

$$p(0) = \frac{1}{\sum_{n=0}^{k-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu - \lambda}}$$

and k is the number of toll booths
and λ is the total arrival rate for all the toll booths.

The parameters are

average number of vehicle arrivals at all toll booths per unit time

$$(\lambda) = \frac{2400}{3600} \text{ veh/s}$$

average number of vehicles serviced at each toll booth per unit time

$$(\mu) = \frac{1}{5} \text{ veh/s}$$

number of toll booths (k) = 4.

The probability p(0) of having zero vehicles in the system will first be calculated.

(a) Let $y = \frac{1}{n!} \times \left(\frac{\lambda}{\mu}\right)^n$

when n = 0, $y = 1$

when n = 1, $y = (\frac{2}{3} \times \frac{5}{1}) = \frac{10}{3}$

when n = 2, $$y = \frac{1}{2 \times 1} \left(\frac{2}{3} \times \frac{5}{1}\right)^2 = \frac{100}{18}$$

when n = 3, $$y = \frac{1}{3 \times 2 \times 1} \left(\frac{2}{3} \times \frac{5}{1}\right)^3 = \frac{1000}{162}$$

therefore $$\sum_{n=0}^{k-1} y = 16.0617.$$

(b) Let $$z = \frac{1}{k!} \left(\frac{\lambda}{\mu}\right)^k \frac{k\mu}{k\mu - \lambda}$$

$$= \frac{1}{4 \times 3 \times 2 \times 1} \left(\frac{2}{3} \times \frac{5}{1}\right)^4 \frac{4 \times \frac{1}{5}}{4 \times \frac{1}{5} - \frac{2}{3}}$$

$$= 30.8642$$

(c) $$p(0) = \frac{1}{y + z}$$

$$= 0.0213.$$

The average time spent in the system $$= \frac{\mu(\lambda/\mu)^k}{(k - 1)!(k\mu - \lambda)^2} p(0) + \frac{1}{\mu}$$

$$= \frac{\frac{1}{5}\left(\frac{10}{4}\right)^4}{(3 \times 2 \times 1)\left(\frac{4}{5} - \frac{2}{3}\right)^2} \times 0.0213 + 5$$

$$= 9.9 \text{ s}$$

The average waiting time spent in the queue = 9.9 - 5

= 4.9 s

The average queue length $$= \frac{\lambda\mu(\lambda/\mu)^k}{(k - 1)!(k\mu - \lambda)^2} p(0)$$

$$= \frac{2}{3} \times 4.9$$

= 3.3 vehicles

The reduction in the time spent in the system due to the change in the system of operation.

$$= \frac{30.0 - 9.9}{30.0}$$

$$= 67 \text{ per cent}$$

The reduction in the average waiting time spent in the queue

$$= \frac{25.0 - 4.9}{25.0}$$

$$= 80 \text{ per cent}$$

The reduction in the average queue length

$$= \frac{4.2 - 3.3}{4.2}$$

$$= 21 \text{ per cent}$$

EXAMPLE 11 Priority Intersections, Gap and Lag Acceptance

(a) Describe the operation of priority highway intersections and the function of lag and gap acceptance.

(b) The observations given below were obtained at a priority intersection, calculate the unbiased and biased mean gap and lag acceptances together with their standard deviations and the critical lag. Illustrate the relationships between these values.

1	*2*		*3*	
lag or gap class (s)	*first decisions number*		*all decisions number*	
	accepted	*rejected*	*accepted*	*rejected*
0.5-1.4	0	30	0	181
1.5-2.4	0	33	0	168
2.5-3.4	8	41	10	105
3.5-4.4	30	26	40	64
4.5-5.4	38	15	52	31
5.5-6.4	32	5	44	11
6.5-7.4	27	3	45	3
7.5-8.4	18	1	25	1
8.5-9.4	15	0	17	0
9.5-10.4	4	0	0	0

Solution

(a) The simplest form of highway intersection is where control over the conflicting traffic movements is exercised by assigning priority to a major road stream of vehicles so requiring a conflicting stream of minor road vehicles to give way. This form of control is to be found in a variety of forms ranging from the simplest Stop and Give Way controls to the regulation of the merging action at motorway intersections.

Priority intersections function because minor road vehicles are able to enter or cross the major road traffic stream using the larger headways or gaps in the major road flow. It is generally assumed that minor road drivers waiting to enter the major road make a decision whether to enter a gap in the major road on the basis of the size of the gap. If a driver arrives at a Give Way line and immediately enters the major road then the vehicle would not normally enter a complete gap between two vehicles but only a portion of a gap, usually referred to as a lag. Frequently gaps and lags are not differentiated in traffic engineering practice.

When a minor road driver waits at a stop or give way line then the driver may or may not enter a given gap or lag. If the driver enters, then the driver is said to accept the gap or lag, conversely the driver is said to reject the gap or lag if he does not enter the major road.

Driver reactions vary, some are more cautious than others and the acceleration performance of Vehicles also varies; this means that there is a wide range of minimum gaps or lags which drivers will accept. Frequently it is necessary to find the mean value of the accepted gap or lag for all drivers passing through an intersection. As well as determining the mean value, observations can also be used to find the form of the distribution of gap or lag acceptance.

When making observations to determine the mean lag or gap accepted by drivers particular care has to be taken to ensure that the results are not biased by the slower drivers, who will reject many gaps before accepting one gap, in comparison with faster drivers who will reject few gaps before accepting a gap.

Frequently to prevent this bias occurring only the decision of a minor road driver when he first arrives at the junction is recorded. A note is taken of the size of the gap in the major road traffic and whether the driver accepts or rejects this gap. As an alternative all the rejections or drivers are recorded and a mathematical relationship used to determine the unbiased value of the average gap which is accepted.

In early traffic studies of gap acceptance the critical lag was frequently used. This was defined as that lag which had a value such that the number of rejected lags greater than the critical lag is equal to the number of accepted lags less than the critical lag. The critical lag is by definition a measure of first or unbiased driver decisions and there is a mathematical connection between the critical lag and the unbiased mean value.

(b) The determination of true and biased mean gap and lag for a particular junction will now be determined for the given observations.

Initially the mean and standard deviation of the first driver decisions will be calculated, practically all of these observations will be lags. Using the method of Probit analysis (1) it is possible to obtain a linear relationship between the probit of the percentage acceptance and the mid-value of the lag class. The percentage acceptance of first decisions is calculated from column 2 divided by the sum of columns 2 and 3 and tabulated in column 6 of Table 11.1. A plot of the probit of these acceptances (obtained from tables in ref 1 and given in column 7) against the mid point of the lag class is given in Figure 11.1.

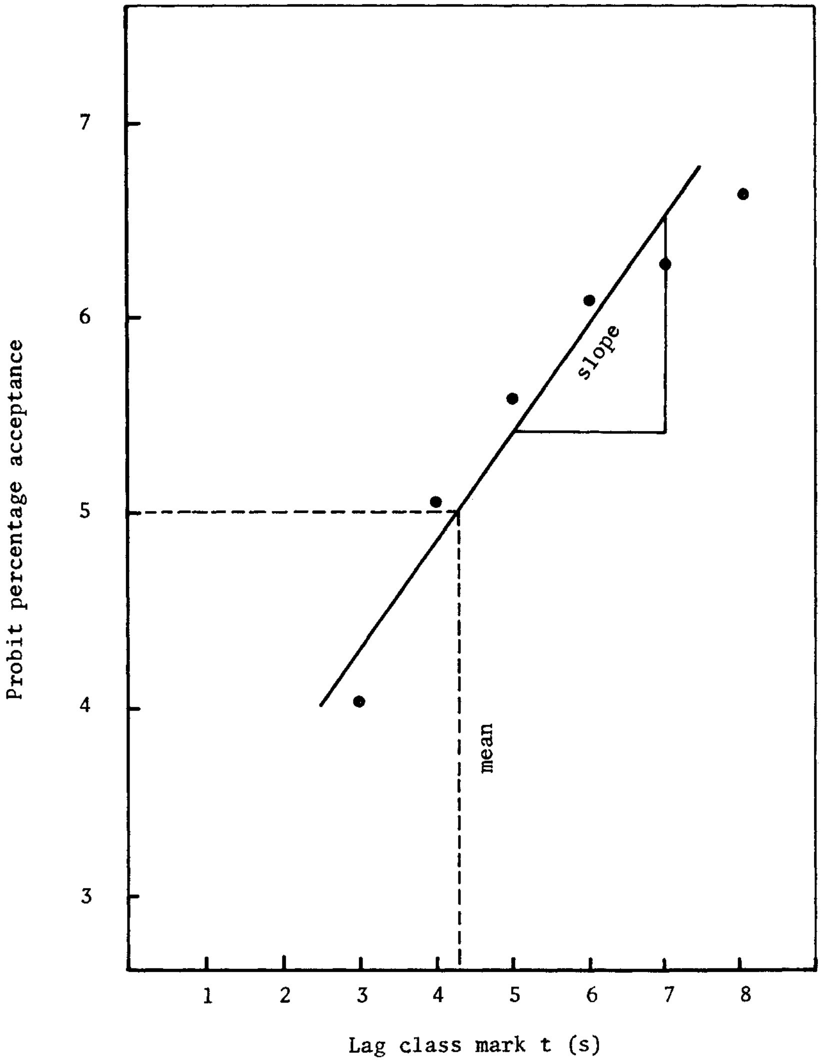


Figure 11.1

Graphical determination of unbiased mean lag

If the distribution is cumulative normal then the plotted points will lie on a straight line; inspection of Figure 11.1 indicates that this is approximately true. Linear regression can be carried out to give the following relationship

$$\text{Probit acceptance} = 0.55\ (\text{mid point lag class}) + 2.64 \qquad (11.1)$$

or less accurately a best fit straight line can be drawn on the graph and the equation of the line found.

The mean gap acceptance occurs when the probit of the acceptance is 5 and the standard deviation is the reciprocal of the slope of the regression or best fit line. From equation 11.1 the mean lag acceptance is 4.3 s and the standard deviation is 1.8 s.

Similarly the percentage acceptance for all driver decisions is calculated as before and entered in column 8. The probit of this acceptance is entered in column 9 and is plotted against the mid point of the gap class in Figure 11.2 and the best fit straight line drawn on it or the relationship found by linear regression.

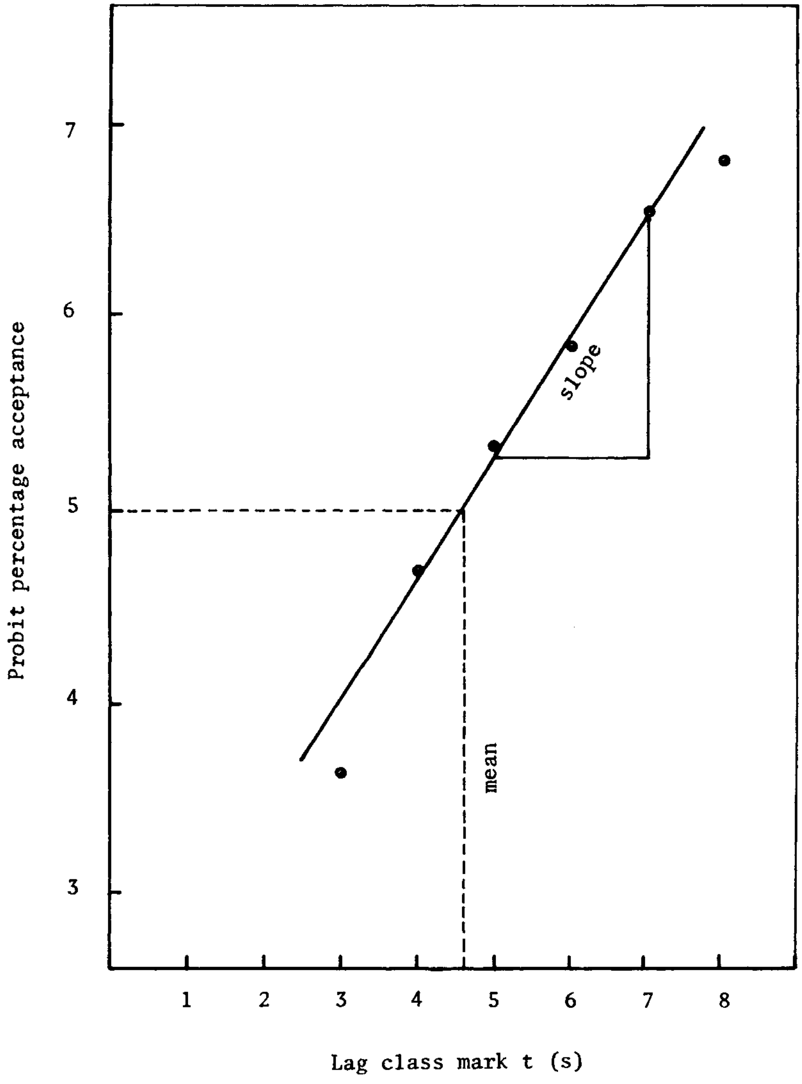


Figure 11.2

Graphical determination of biased mean gap

The relationship is

Probit acceptance = 0.63 (mid point lag class) + 1.98 (11.2)

giving a mean gap acceptance of 4.8 s and a standard deviation of 1.6 s.

The critical lag can be obtained by plotting the curves of number of rejected lags >t and the number of accepted lags <t against the lag. These values are tabulated in columns 10 and 11 respectively. The intersection of the two curves gives the value of the critical lag as 4.1 s.

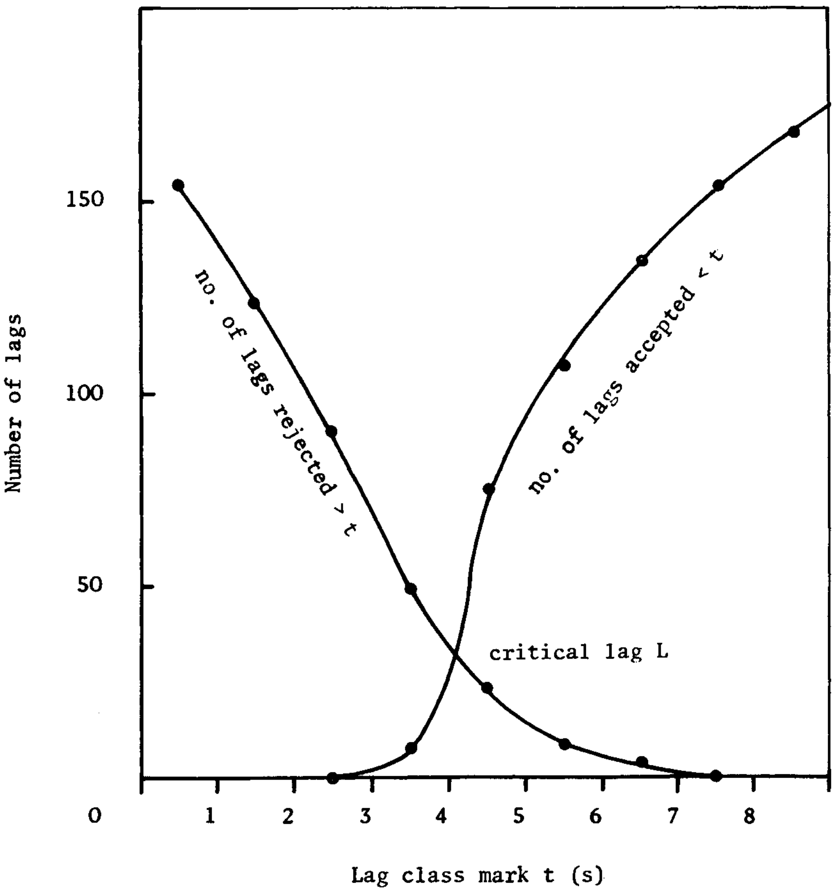


Figure 11.3

Graphical determination of the critical lag

Table 11.1 Major road flow 760 vehicles/h

6	7	8	9	10	11
first decision percentage acceptance	*probit of first decision percentage acceptance*	*all decision percentage acceptance*	*probit of all decision percentage acceptance*	*number of rejected lags > lower class limit*	*number of accepted lags < lower class limit*
0		0		154	0
0		0		124	0
16.4	4.0178	8.7	3.6405	91	8
53.4	5.0904	38.5	4.7076	50	38
71.8	5.5740	62.7	5.3239	24	76
86.5	6.1021	80.0	5.8416	9	108
90.0	6.2816	93.8	6.5382	4	135
94.7	6.6164	96.2	6.7744	1	153
100		100		0	168
100		100		0	172

It has been shown (2) that the mean lag or first decision is related to the mean gap and lag (the biased value) when both have a cumulative normal distribution. The relationship is

$$\text{true mean lag} = \text{biased mean lag and gap} - s^2q \tag{11.3}$$

where s is the standard deviation of the lag and gap acceptance distributions
q is the flow in the priority stream.

From equation 11.1 true mean = 4.3 s
standard deviation = 1.8 s

From equation 11.2 biased mean = 4.8 s
standard deviation = 1.6 s

The observations do not exactly fit the theoretical model in that the two standard deviations are not equal; as an approximation a mean value of 1.7 s will be used for s.

From equation 11.3

$$\text{true mean lag} = 4.8 - \frac{1.7^2 \times 760}{3600}$$

$$= 4.2 \text{ s}$$

This value can be compared with the observed values of 4.3 s and can be considered a good approximation.

The critical lag is also related to the mean lag by the following relationship

$$\text{critical lag} = \text{true mean} - \frac{s^2\ q}{2}$$

$$\approx 4{,}3 - \frac{(1.7)^2 \times 760}{3600 \times 2}$$

$$\approx 4{,}0\ \text{s}$$

This value can be compared with the observed value of 4.1 s and can be considered a good approximation.

References

(1) Finney, D.J., Probit analysis, A statistical treatment of the sigmoid response curve. Cambridge University Press 1947.

(2) Ashworth, R., The analysis and interpreation of gap acceptance data. Transportation Sc, 4. 270-280, 1970.

EXAMPLE 12 Delays at Priority Intersections Illustrated by an Example

(a) Describe how delays at priority intersections can be estimated.

(b) At a highway intersection minor road vehicles cut through a two way traffic stream consisting of 600 veh/h in each direction. If the average gap accepted by minor road vehicles is 6 s; the minimum headway between major road vehicles passing through the junction is 2 s and the minimum headway between vehicles emerging from the minor road is 3 s. Calculate the maximum minor road flow which can cross the major road.

Solution

(a) Determination of capacities and delays with this form of control is frequently made using a theoretical model developed by Tanner (1). In this model minor road drivers either accept or reject gaps between vehicles in the major road traffic stream. The parameters used in the model are

(1) α, the average gap or lag in the major road stream which is accepted by minor road drivers when entering or crossing the major road,

(2) q_1 and q_2, the major and minor road flows respectively,

(3) β_1, the minimum time headway between major road vehicles passing through the intersection,

(4) β_2, the minimum headway between minor road vehicles emerging from the minor road.

The parameter β_1 allows a modification to be made for the layout of the major road where vehicles may pass through the junction in one or more streams. The value of α will depend upon the movement of the minor road vehicle, left or right turning or straight ahead in a cutting action; the layout of the junction and estimated speeds at the junction.

A guide to gap acceptance values recommended for priority junctions in the United Kingdom is given by the Department of Transport (2). Values vary from 4 s for a merge or cut with a single major road stream when the design speed is equal to or less than 65 km/h to a value of 8 s when a vehicle has to cut one traffic stream and merge with another and design speed is greater than 65 km/h.

The average delay to minor road vehicles is given by

$$\frac{\frac{1}{2}E(y^2)/Y + q_2 Y \exp(-\beta_2 q_1)\left[\exp(\beta_2 q_1) - \beta_2 q_1 - 1\right]/q_1}{1 - q_2 Y\left[1 - \exp(-\beta_2 q_1)\right]} \qquad (12.1)$$

$$\text{where } E(y) = \frac{\exp\left[q_1(\alpha - \beta_1)\right]}{q_1(1 - \beta_1 q_1)} - \frac{1}{q_1}$$

$$E(y^2) = \frac{2\exp\left[q_1(\alpha - \beta_1)\right]}{q_1^{\,2}(1 - \beta_1 q_1)^2}\left\{\exp\left[q_1(\alpha - \beta_1)\right]\right.$$

$$\left. - \alpha q_1(1 - \beta_1 q_1) - 1 + \beta_1 q_1 - \beta_1^{\,2} q_1^{\,2} + \tfrac{1}{2}\beta_1^{\,2} q_1^{\,2}/(1 - \beta_1 q_1)\right\}$$

$$Y = E(y) + 1/q_1$$

The maximum discharge from a minor road is given by

$$q_2 \max = \frac{q_1(1 - \beta_1 q_1)}{\exp\left[q_1(\alpha - \beta_1)\right]\left[1 - \exp(-\beta_2 q_1)\right]} \qquad (12.2)$$

(b) A solution to this problem can be obtained by taking the sum of flows in each direction as the major road flow q_1 and the minimum headway between major road vehicles passing through the intersection in both directions as half the value in one direction, i.e. 1 s.

From equation 12.2

$$q_2 \max = \frac{q_1(1 - \beta_1 q_1)}{\exp\left[q_1(\alpha - \beta_1)\right]\left[1 - \exp(-\beta_2 q_1)\right]}$$

where $q = 1200/3600 = 0.3333$ veh/s
$\beta_1 = 1$ s
$\alpha = 6$ s
$\beta_2 = 3$ s

$$q_2 \max = \frac{0.3333\ (1 - 1 \times 0.3333)}{\exp\left[0.3333(6 - 1)\right]\left[1 - \exp(-3 \times 0.3333)\right]}$$

$$= \frac{0.2222}{5.2936 \times 0.6321}$$

$$= 0.6641$$

$$= 239 \text{ veh/h}$$

References

(1) Tanner, J.C., A theoretical analysis of delays at an uncontrolled intersection. Biometrika, 49(1), (2), 1962.

(2) Department of Transport. Technical Memorandum H11/76. The design of major/minor priority junctions. 1976.

EXAMPLE 13 The Capacity of Oversaturated Priority Intersections

(a) Describe how the capacity and delay at an oversaturated priority intersection may be estimated.

(b) An intersection is estimated to carry the hourly flows shown in Table 13.1 in the design year and has the geometric features shown in Figure 13.1. Comment on likely future traffic flow conditions.

Total major road carriageway width (w)	9.90 m
Minor road visibility to right (Vb-a)	20.00 m
Minor road visibility to left (Vb-c)	20.00 m
Major road right turn visibility (Vc-b)	90.00 m

There is no explicit provision for right turning vehicles

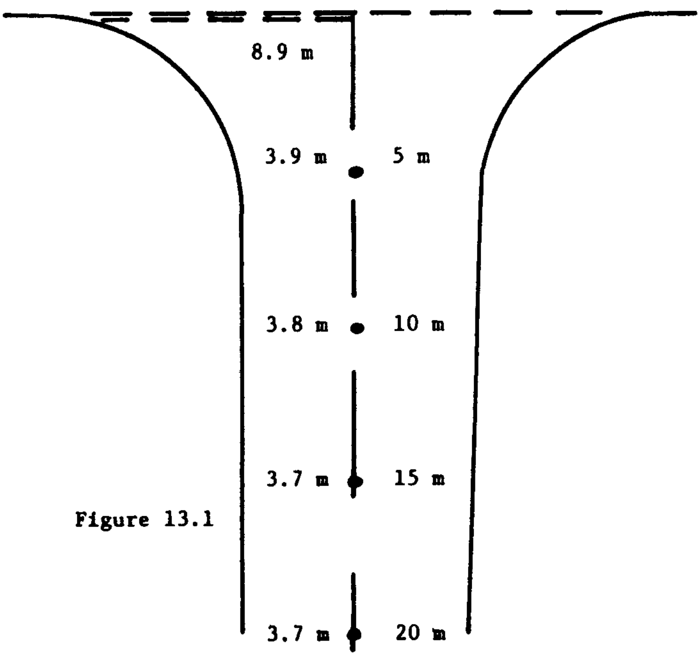


Figure 13.1

Table 13.1 Design Hour Flows

movement q	design hour flow (pcu/h)
C-A	269
A-C	231
A-B	282
B-A	275
B-C	383
C-B	318

Solution

$$Sq_{b-a} = D(627+14W_{cr}-Y[0.364q_{a-c}+0.114q_{a-b} + 0.229q_{c-a}+0.520q_{c-b}])$$

$$Sq_{b-c} = E(745-Y[0.364q_{a-c}+0.144q_{a-b}])$$

$$Sq_{c-b} = F(745-0.364Y[q_{a-c}+q_{a-b}])$$

$$Y = (1-0.0345W)$$

$$D = [1+0.094(w_{b-a}-3.65)]\,[1+0.0009(Vr_{b-a}-120)]\,[1+0.0006(V\ell_{b-a}-150)]$$

$$E = [1+0.094(w_{b-c}-3.65)][(1+0.0009(Vr_{b-a}-120)]$$

$$F = [1+0.094(w_{c-b}-3.65)][1+0.0009(Vr_{c-b}-120)]$$

w_{b-a} and w_{b-c} $= (4.45+1.95+1.9+1.85+1.85)/5$

$= 2.4$ m

D = [1+0.094(2.4-3.65)] [1+0.0009(20-120)]

[1+0.0006(20-150)]

= 0.88x0.91x0.92

= 0.74

E = [1+0.094(2.4-3.65)] [1+0.0009(20-120)]

= 0.88x0.91

= 0.80

F = [1+0.094(2.1-3.65)] [1+0.0009(20-120)]

= 0.85x0.91

= 0.78

Y = (1-0.0345x9.9) = 0.66

Traffic flow will not arrive at a junction during the design hour at a uniform rate and to allow for the peakiness of vehicle arrivals it is usual to multiply the design hour flows by a factor of 1.125. The flow rates to be considered in design are given in Table 13.2.

Table 13.2 Design Hour Flows used in Design

movement q	flow pcu/h
C-A	303
A-C	260
A-B	317
B-A	309
B-C	431
C-B	358

$$sq_{b-a} = 0.74(627+14.0-0.66[0.364x260$$

$$+ 0.114x317+0.229x303+0.520x358])$$

$$= 0.74(627-0.66[94.64+36.14+69.39+186.16])$$

$$= 0.74(627-0.66x386.33)$$

$$= 275 \text{ pcu/h}$$

$$sq_{b-c} = 0.80(745-0.66[0.364x260+0.144x317])$$

$$= 0.80(745-0.66[94.64+45.65])$$

$$= 0.80(745-92.59)$$

$$= 522 \text{ pcu/h}$$

This capacity will require modification due to the effect of the queue in the right hand lane.

The modified value is then 0.75 sq_{b-c} as the ratio of flow to capacity for flow b-a is greater than unity.

$$Sq_{b-c} = 392 \text{ pcu/h}$$

$$Sq_{c-b} = 0.78(745-0.364x0.66[260+317])$$

$$= 473 \text{ pcu/h}$$

It is now possible to tabulate non-priority flows and capacities to determine future traffic conditions as shown in Table 13.3.

Table 13.3 Ratios of Flow to Capacity

movement	flow (pcu/h) q	capacity (pcu/h) s	$\frac{\text{flow}}{\text{capacity}}$
B-A	309	275	1.12
B-C	431	392	1.10
C-B	358	473	0.76

Comparison of the values in Table 13.3 indicate that extensive queueing can be expected for movements B-A and B-C. The right turn C-B from the major road into the minor road has a flow/capacity ratio of 0.76 and this is normally considered to be satisfactory for junctions in urban areas and marginally satisfactory in rural areas.

EXAMPLE 14 Geometric Delay at an At-Grade Roundabout

(a) Describe the two sources of delay experienced by vehicles at highway intersections.

(b) Calculate the geometric delay for straight ahead movements at a conventional at-grade roundabout, shown in outline in Figure 14.1, with an inscribed circle diameter of 22m, where the angle turned through at entry is 30^{o} and at exit is 25^{o} and where the approach and exit link speeds are 50 mph.

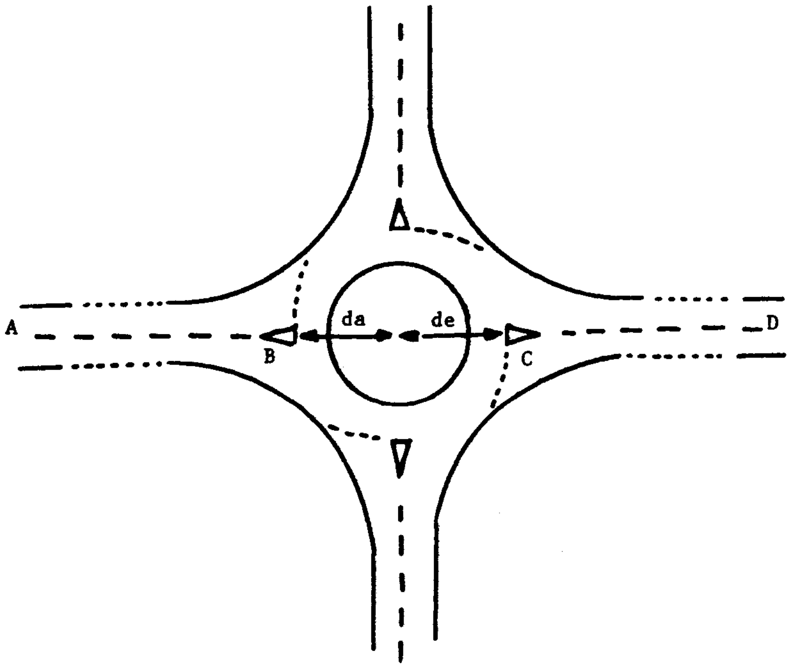


Figure 14.1

Solution

(a) Vehicles experience two types of delay at non-signalised intersections. Firstly, vehicles have frequently to slow down to negotiate the junction, even in the absence of conflicting traffic. This type of delay is associated with the size and shape of the intersection, factors referred to as its geometric characteristics, and for this reason the delay is referred to as geometric delay. Secondly, as traffic flow increases the interactions between vehicles produces queueing or congestion delay. Whilst this latter type of delay predominates at peak flow times the former type of delay is always present and forms a significant proportion of the total delay. The calculation of geometric delay at non-signalised intersections is described in reference (1).

Speed on approach and exit links (V_A,V_D) = 22.32 m/s.

Speed on entry to internal section of roundabout (V_B, V_C, V_{BC})

$$V_{BC} = 0.96 \sqrt{ICD} + 2.03$$

$$= 0.96 \sqrt{22} + 2.03$$

$$= 6.53 \text{ m/s}$$

Deceleration rate on approach (a_{AB})

$$a_{AB} = 1.06(V_A-V_B)/V_A+0.23$$

$$= 1.06 (22.32-6.53)/22.32+0.23$$

$$= 0.98 \text{ m/s}^2$$

Acceleration rate on exit (a_{CD})

$$a_{CD} = 1.11(V_D-V_C)/V_D+0.02$$

$$= 1.11(22.32-6.53)/22.32+0.02$$

$$=, 0.81 \text{ m/s}^2$$

Measured distance from entry to exit of intersection (d_{BC}) = 20m

Measured distance from entry point to centre of intersection (d_a) = 9m

Measured distance from centre of intersection to exit point (d_e) = 9m

Distance from deceleration point to entry point (d_{AB})

$$d_{AB} = (V_A^2-V_B^2)/2a_{AB}$$

$$= (22.32^2-6.53^2)/2 \times 0.98$$

$$= 232.42\text{m}$$

Time of travel from deceleration point to entry point (t_{AB})

$$t_{AB} = (V_A-V_B)/a_{AB}$$

$$= (22.32-6.53)/0.98$$

$$= 16.11\text{s}$$

Time of travel within intersection (t_{BC})

$$t_{BC} = d_{BC}/V_{BC}$$
$$= 20/6.53$$
$$= 3.06s$$

Distance from exit point to end of acceleration period (d_{CD})

$$d_{CD} = (V_D^2-V_C^2)/2a_{CD}$$
$$= (22.32^2-6.53^2)/2 \times 0.81$$
$$= 281.20m$$

Time of travel from exit point to end of acceleration point (t_{CD})

$$t_{CD} = (V_D-V_C)/a_{CD}$$
$$= (22.32-6.53)/0.81$$
$$= 19.49s$$

Overall journey time (JT)

$$JT = t_{AB}+t_{BC}+t_{CD}$$
$$= 16.11+3.06+19.49$$
$$= 38.66s$$

Overall geometric delay (g)

$$g = JT- (d_{AB}-d_a)/V_A+(d_{CD}+d_e)/V_D$$
$$= 38.66- (232.42+9)/22.32+(281.20+9)/22.32$$
$$= 38.66- 10.82+13.00$$
$$= 14.84s$$

Reference

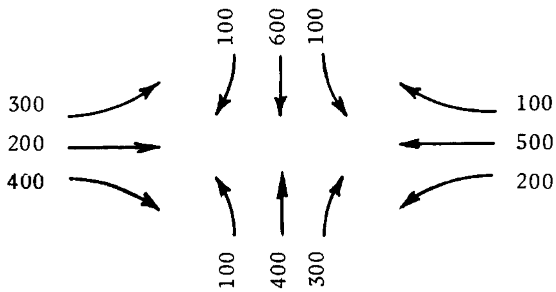
(1) McDonald, M., N.B. Hounsell, and R.M. Kimber. Geometric delay at non-signalised intersections. Transport and Road Research Laboratory Supplementary Report 810. Crowthorne, 1984.

EXAMPLE 15 Relationship between Entry and Circulating Flow at Roundabouts

The capacity of roundabouts can be predicted using relationships between the entry and the circulating flow. Use the relationship derived by the Transport and Road Research Laboratory to calculate the reserve capacity of the intersection whose geometric features are:

inscribed circle diameter, D, 40 m
entry width, e, 8m
approach half width, v, 7.3 m
circulation width, u, 8 m
effective length over which flare is developed, ℓ^1, 20 m
entry radius, r, 25 m
entry angle, ϕ, 30°.

The present day peak hour traffic flows are given below.



Design hour flows (pcu/h)

Solution

Initially the flows will be assigned to the individual circulating sections in the conventional manner and then separated into entry and circulating flows as shown in Figure 15.1.

Research by the Transport and Road Research Laboratory (1) has resulted in a unified formula for predicting the capacity of roundabout entries for both offside and conventional roundabouts. The best predictive equation was found to be

$$Q_e = k(F - f_c Q_c) \text{ when } f_c Q_c \leqslant F \quad (15.1)$$

$$= 0 \qquad \text{when } f_c Q_c > F$$

where $k = 1 - 0.00347\ (\phi - 30) - 0.978\ ((1/r) - 0.05)$,

$F = 303\ x_2$,

$f_c = 0.210\ t_D\ (1 + 0.2\ x_2)$,

$t_D = 1 + 0.5/(1 + \exp\ ((D - 60)/10))$,

$x_2 = v + (e - v)/(1 + 2s)$,

$s = 1.6\ (e - v)/\ell^1$,

and e, v, ℓ^1, D and r are in metres, ϕ in degrees and their significance is illustrated by Figure 15.2. Vehicles are classified as light or heavy according to the number of wheels, 3 or 4 and more than 4 respectively.

Substituting the geometric values given in the questions gives

$$k = 1 - 0.00347\ (30 - 30) - 0.978\ ((1/25) - 0.05) = 1.01$$

$$s = 1.6\ (8 - 7.3)/20 = 0.056$$

$$x_2 = 7.3 + (8 - 7.3)/(1 + 2 \times 0.056) = 7.929$$

$$F = 303 \times 7.929 = 2402.5$$

$$t_D = 1 + 0.5/(1 + \exp\ ((40 - 60)/10)) = 1.4404$$

$$f_c = 0.210 \times 1.4404\ (1 + 0.2 \times 7.929) = 0.7822$$

Substituting these values in equation 15.1 gives

$$Q_e = 1.01\ (2402.5 - 0.7822\ Q_c)$$

$$= 2426.5 - 0.79\ Q_c \text{ pcu/h}$$

The east approach capacity = 2426.5 - 0.79 x 1100 = 1558 pcu/h
The west approach capacity = 2426.5 - 0.79 x 800 = 1795 pcu/h
The north approach capacity = 2426.5 - 0.79 x 900 = 1716 pcu/h
The south approach capacity = 2426.5 - 0.79 x 700 = 1874 pcu/h

The reserve capacities of the approaches are

east approach = (1558 - 800)/800 = 94.8 per cent
west approach = (1795 - 900)/900 = 99.4 per cent
north approach = (1716 - 800)/800 = 114.5 per cent
south approach = (1874 - 900)/900 = 108.2 per cent.

Reference

1. Kimber, R.M. The traffic capacity of roundabouts. Transport and Road Research Laboratory Report 942. Crowthorne. 1980.

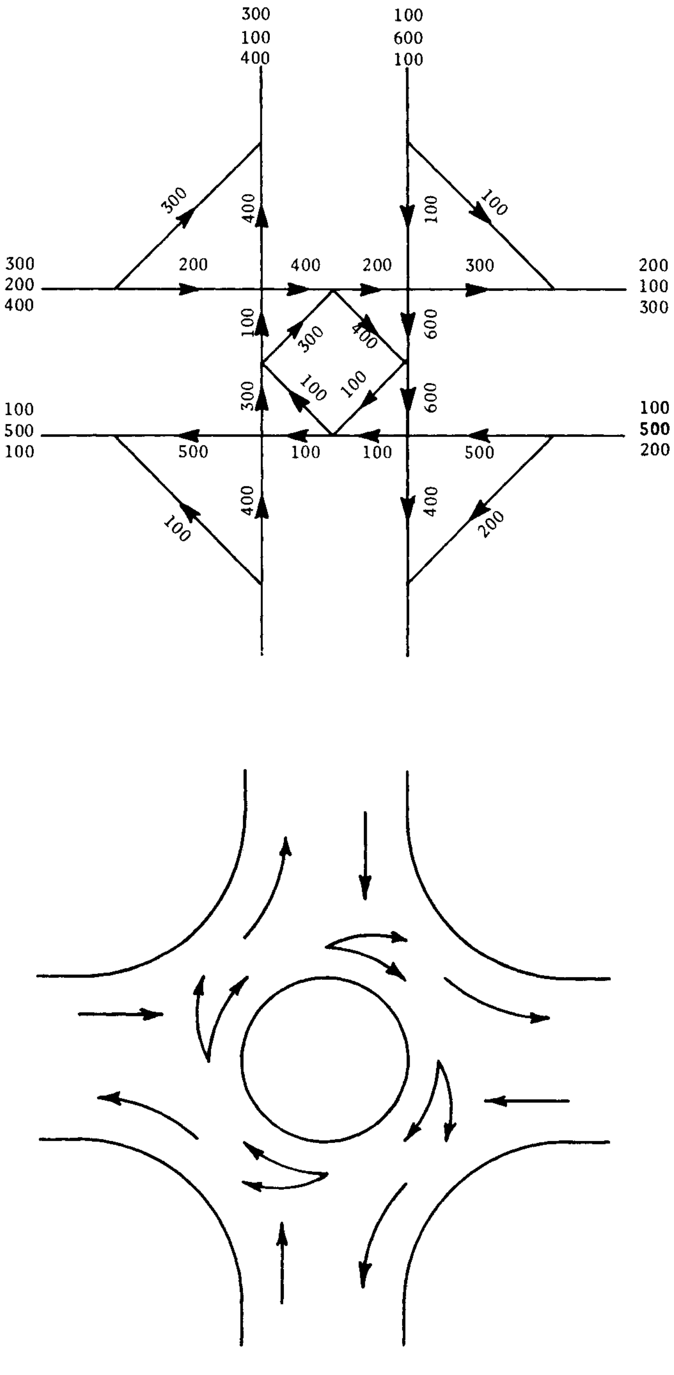

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Figure 15.1

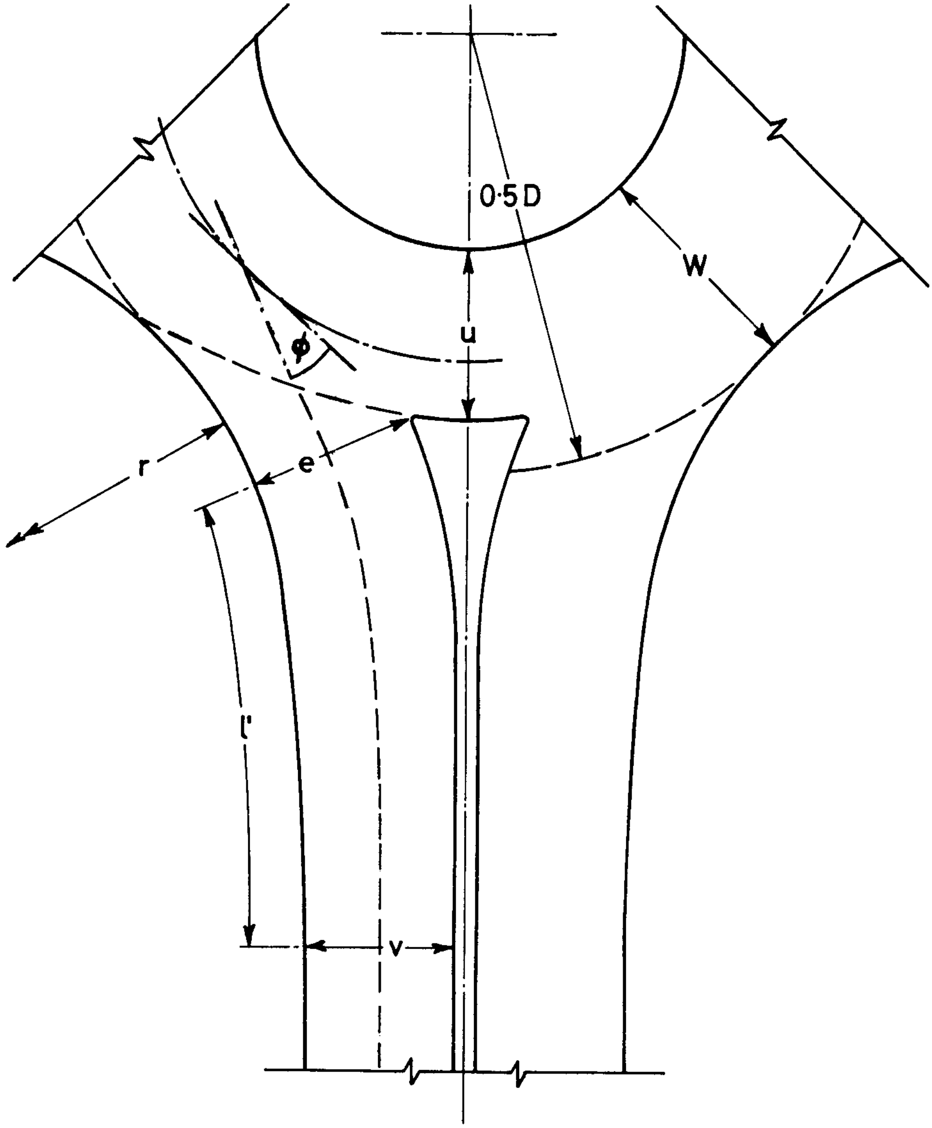


Figure 15.2

EXAMPLE 16 Determination of Roundabout Entry Width

At the entry to a roundabout intersection the entry flow is 800 pcu/h and the circulating flow is 1100 pcu/h. Given the geometric features and traffic flow relationships below, determine the required entry width for a 100 per cent reserve capacity.

inscribed circle diameter	40 m
approach half width	7.3 m
effective length over which flare is developed	20 m
entry radius	25 m
entry angle	30 degrees

Solution

$Q_e = k(F - f_c Q_c)$ when $f_c Q_c \leqslant F$

$k = 1-0.00347\ (\emptyset-30) - 0.978\ ((1/r) - 0.05)$

where $\emptyset$ is the entry angle, given as 30 degrees

r is the entry radius, given as 25 m

$k = 1-0.00347\ (30-30) - 0.978\ ((1/25)-0.05) = 1.01$

$F = 303\ x_2$

$f_c = 0.210\ t_D\ (1+0.2\ x_2)$

$t_D = 1+0.5/(1+\exp(D-60)/10)$

where D is the inscribed circle diameter, given as 40 m

$t_D = 1+0.5/(1+\exp(40-60)/10)$

$= 1.4404$

$f_c = 0.210 \times 1.4404\ (1+0.2\ x_2)$

$x_2 = v + (e-v)/(1+2s)$

where v is the approach half width, given as 7.3 m

$s = 1.6\ (e-v)/\ell^1$

where e is the entry width

and ℓ^1 is the effective length over which flare is developed, given as 20 m

$s = 1.6\ (e-7.3)/20$

$x_2 = 7.3+(e-7.3)/(1+2 \times 1.6(e-7.3)/20)$

$F = 303\ x_2$

$= 303\ (7.3+(e-7.3)/(1+2 \times 1.6(e-7.3)/20))$

The required entry capacity for a hundred per cent reserve capacity is 2 x 800 or 1600 pcu/hour when the circulating flow is 1100 pcu/hour.

$Q_e = k(F-f_cQ_c)$

$1600 = 1.1(303\ x_2-(0.210 \times 1.4404(1+0.2\ x_2)1100))$

$= 1.1(303\ x_2-332.73-66.55\ x_2)$

$= 333.30\ x_2-366.00-73.21\ x_2$

$1966 = 260.09\ x_2$

$x_2 = 7.56$

and $x_2 = v+(e-v)/(1+2s)$

$7.56 = 7.3+(e-7.3)/(1+2 \times 1.6(e-7.3)/20)$

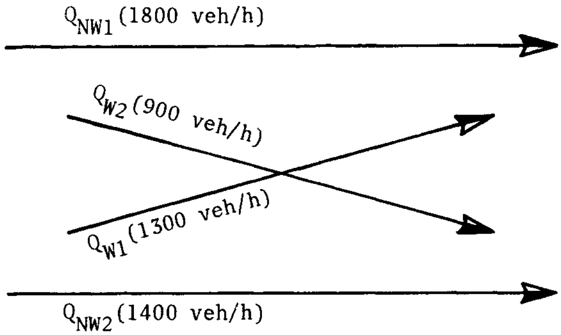
$e = 7.56$ m

The required entry width is 7.6 m.

EXAMPLE 17 Design of Weaving Sections

(a) Describe the procedure used in Great Britain for the design of highway weaving sections.

(b) The predicted future design hour traffic flows through a weaving section between interchanges on an urban all-purpose 7.3m dual carriageway which serves as a primary distributor are shown below. The weaving section is straight with a length of 500m and has an overall gradient over a length of 0.5km upstream and downstream of the weaving section of less than 2 per cent. The maximum hourly flow for the design of the weaving section is 1600 vehicles per hour per lane and the design speed for the highways upstream of the weaving section is 80 kilometres per hour. The percentage of heavy goods vehicles in the flow is 10 per cent. Calculate the number of lanes required in the weaving section.



Solution

(a) Weaving takes place in many situations on highways where traffic streams merge, diverge and cross whilst travelling in the same general direction. This form of weaving is to be found on main carriageways between intersections and is caused by conflicts in the paths of entering, leaving and straight through vehicles. It is also to be found on link roads within free flow intersections and in the area of some junctions where entering and leaving vehicles conflict.

To design the weaving section it is necessary to predict the future design hour flows. For Main Urban road types this is the 30th highest hourly flow and for Inter-Urban and Recreational road types the 50th and 200th highest hourly flows respectively are used. The definitions of road types are given in the Department of Transport Traffic Appraisal Manual (1). The required design hour flows are those predicted to occur in the fifteenth year after opening to traffic. A correction must be made to the design hour traffic flows for heavy vehicle content and highway gradient. These corrections are given in reference (2).

It is also necessary to know the design speed of the mainline carriageway upstream of the weaving area, the mainline is normally taken as the one carrying the major flow. The maximum allowable hourly flows per lane used in the design of weaving sections are given in reference (2) and vary with carriageway width and road type.

The design of a weaving section makes use of two graphs which are given in reference (2). The length of the weaving section is given by the larger graph of Figure 17.1, which relates the minimum length of the weaving section to the total weaving flow for differing ratios of the maximum allowable hourly flow per lane to the upstream design speed. The smaller graph of Figure 17.1 relates the length of the weaving section to the design speed, the greater of the two weaving lengths is then used for design. On rural motorways the desirable minimum weaving length is recommended as 2 kilometres but in extreme cases when the predicted traffic flows are at the lower end of the range given for the carriageway width being considered then an absolute minimum length of 1 kilometre may be considered.

To calculate the required width of the weaving section the minor weaving flow is multiplied by a weighting factor which takes into account the reduction in traffic flow caused by weaving. This factor depends upon the ratio of the minimum length to the actual length of the weaving section.

The number of lanes required is given by

$$N = \frac{Q_{nw} + Q_{w1}}{D} + \left(\frac{2 \times L_{min}}{L_{act}} + 1\right) \frac{Q_{w2}}{D} \qquad (17.1)$$

where

N = required number of traffic lanes

Q_{nw} = total non-weaving flow (vph)

Q_{w1} = major weaving flow (vph)

Q_{w2} = minor weaving flow (vph)

D = maximum allowable mainline flow (vph/lane)

L_{min} = minimum weaving length from Figure 17.1 (m)

L_{act} = actual weaving length available (m)

It can be seen that the maximum value of the weighting factor is 3 when the actual length of the weaving section is equal to the minimum length as given by Figure 17.1. The Department of Transport gives the following advice when the value of N is not an integer (3). Obviously if the junction can be moved then the actual weaving length will change and the value of N can approximate to a whole number of lanes. If this is not possible then if the size of the fractional part is small and the weaving flow is low then rounding down is possible, conversely a high fractional part and a high weaving flow will favour rounding up to an additional lane. Consideration should be given to the uncertain nature of future traffic predictions, the difficulty of obtaining land in urban areas and the difference between urban commuter and recreational flows.

(b) Traffic flow must first be corrected for traffic composition and highway gradient, for 10 per cent heavy goods vehicles in the flow and a gradient of less than 2 per cent no correction is necessary

Q_{nw} = 1800 + 1400 = 3200 vph

Q_{w1} = 1300 vph

Q_{w2} = 900 vph

For a maximum hourly flow of 1600 vph and a design speed of 80 kph, D/V is 20 and from Figure 17.1 the minimum weaving length for a total weaving flow of 2200 vph is 300m. (The absolute minimum weaving length is 220m for a design speed of 80 kph).

The number of lanes required within the weaving section is given by

$$N = \frac{Q_{nw} + Q_{w1}}{D} + \left(\frac{2 \times L_{min}}{L_{act}} + 1\right) \frac{Q_{w2}}{D}$$

$$= \frac{3200 + 1300}{1600} + \left(\frac{2 \times 300}{500} + 1\right) \frac{900}{1600}$$

$$= 2.81 + 1.24$$

$$= 4.05 \text{ lanes}$$

A decision has to be made whether to provide 4 or 5 lanes and it can be shown that if it is possible to increase the

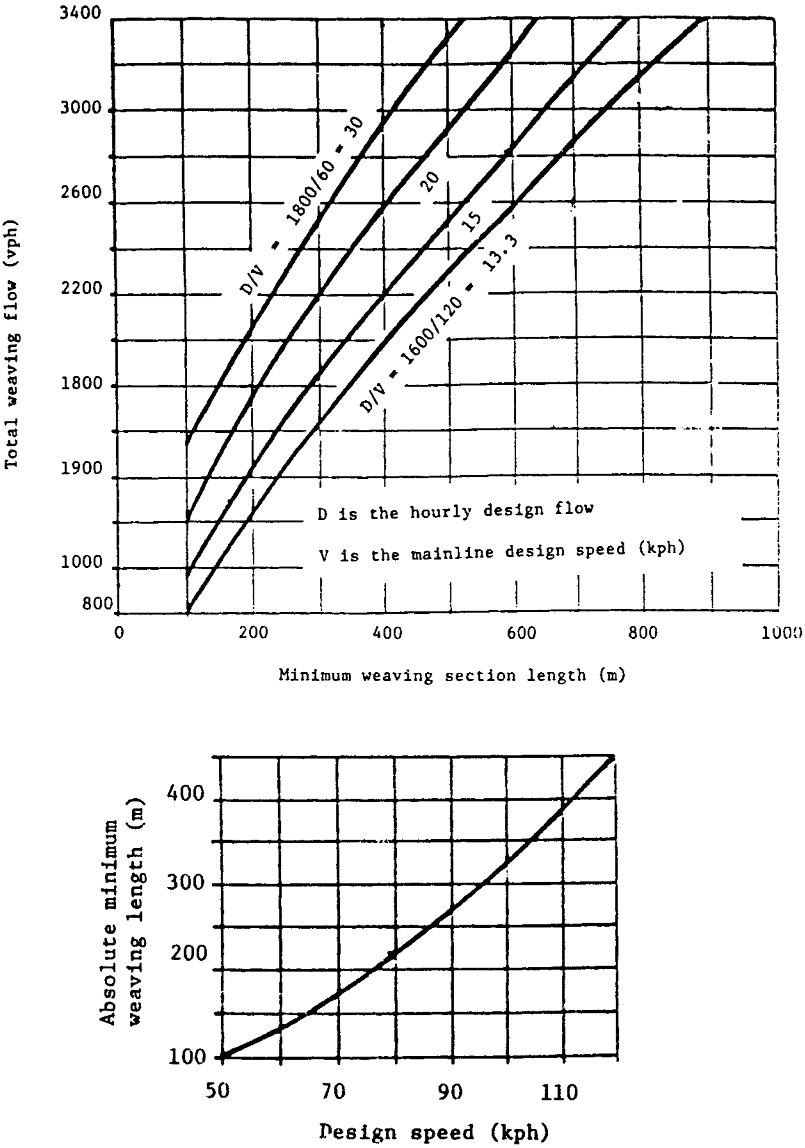


Figure 17.1 Weaving section lengths

actual length of the weaving section to 550 m then only four lanes will be required. The fractional part of a lane required is, however, very small and in an urban situation it is likely that only 4 lanes would be provided regardless of the possibility of increasing the weaving length.

References

(1) Department of Transport. Traffic Appraisal Manual, 1982.

(2) Department of Transport. Layout of Grade Separated Junctions. Departmental Standard TD 22/86. London 1986.

(3) Department of Transport. Layout of Grade Separated Junctions. Departmental Advice Note TA 48/86. London 1986.

EXAMPLE 18 Merging on to High Speed Roads

Describe how the capacity of merging links to high speed roads may be estimated. A single lane link merges with a two lane highway and the merging relationship between entering link flow and upstream main carriageway flow is

Link entry flow = 1600-0.44 upstream flow

Develop a diagram illustrating the variation of link entry capacity with mainline flow.

Solution

The capacity of merging and diverging links or ramps onto high speed highways was initially examined in detail in the United States and is reported in the Highway Capacity Manual (1).

There are three aspects of the capacity of a link or connector which must be considered. These are, the capacities at either end of the link and the capacity of the link itself. Usually the link has one termination with an all-purpose highway and the capacity at the junction can be determined by the methods used for priority junctions and roundabouts. It is the merging and diverging operation with the high speed road which presents the greatest difficulty and which will be considered later in Example 19.

Maximum hourly flows used in the design of merging areas are given in reference (2). These values differ according to road class, all-purpose or motorway and are given in Table 18.1.

Table 18.1 Maximum Flows Used in Merging Design

road class	maximum hourly flow vph/lane	
	main line, two lane links and slip roads $\geqslant$ 6m	single lane links and slip roads $\leqslant$ 5m
all-purpose	1600	1200
motorway	1800	1350

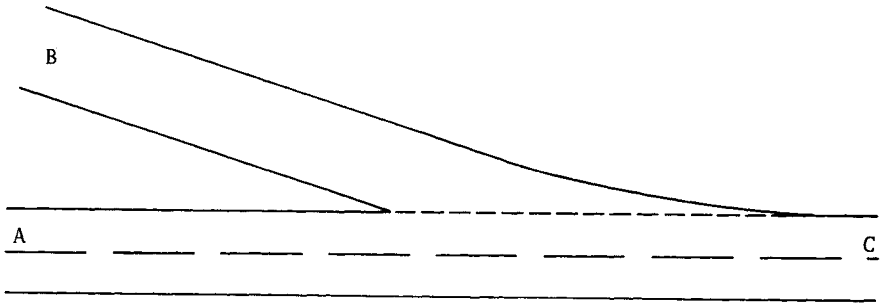


Figure 18.1

When designing the merging intersection shown in Figure 18.1 there are five factors to be considered.

(1) The limiting value of the flow upstream of the merging area at "A". From Table 19.1 for a two lane all-purpose main line this value is 3200 vph, on Figure 18.2 this value is represented as line (a).

(2) The limiting value of the entry link flow at "B". From Table 19.1 for a single lane all-purpose link this value is 1200 vph, on Figure 18.2 this value is represented as line (b).

(3) The limiting value of the merged flow downstream of the junction at "C". The sum of flow "A" and "B" cannot exceed the maximum flow for two lanes of a main line which is 3200 vph. On Figure 18.2 this condition is superimposed on the diagram as line (c).

(4) The condition that the entry flow should not exceed the upstream flow, this is shown on Figure 18.2 as line (d).

(5) The limitation due to the merging process itself. Using the relationship given it is possible to represent the merging criteria for a single lane entry link onto a two lane main line carriageway. This is shown as line (e) on Figure 18.2.

In Figure 18.2 the shaded area represents the permissible range of entry and upstream main line flows. This approach may be extended to other geometric arrangements at junctions as illustrated in Example 19.

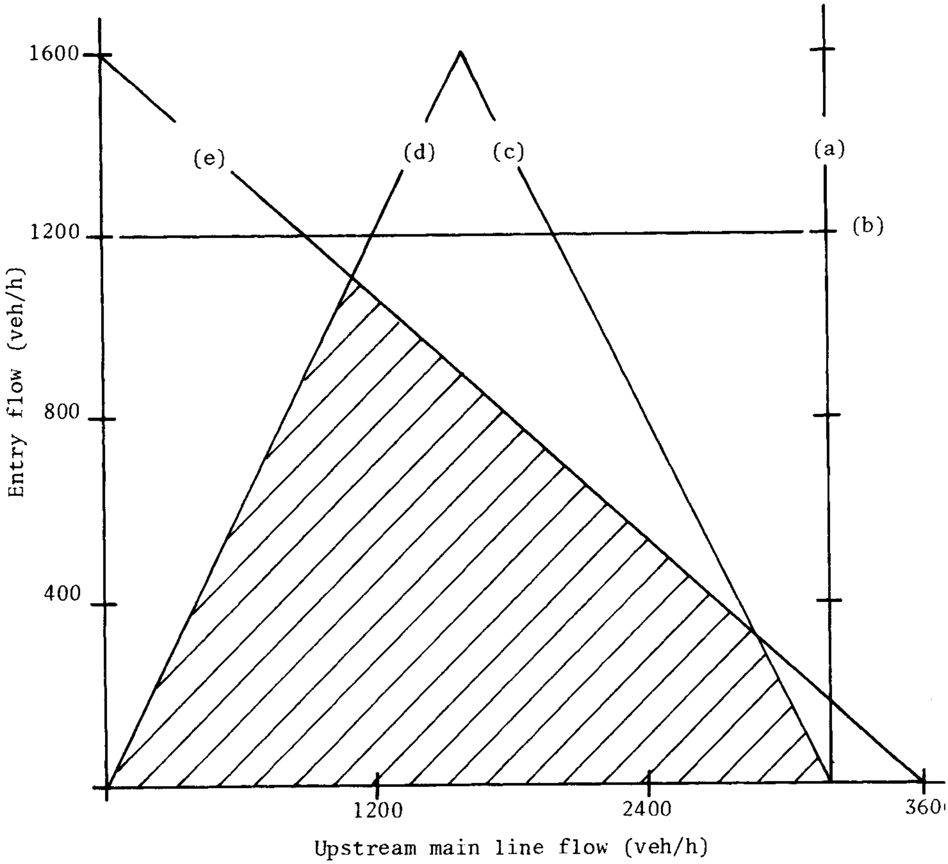


Figure 18.2

References

(1) Highway Capacity Manual. Transportation Research Board. Washington, 1985.

(2) Department of Transport. Layout of Grade Separated Junctions. Departmental Standard TD22/86. London 1986.

EXAMPLE 19 Design of Merging and Diverging Lanes at Grade Separated Junctions

Briefly describe the design methods used for merging and diverging links or connectors in the United Kingdom and check the design in the following situations.

(a) The design year traffic flows at a merge at a rural motorway junction are

upstream main line	2000 vph	(15% heavy vehicles)
entry link or connector	800 vph	(10% heavy vehicles)

These two design hour flows coincide in time and the average gradient through the junction is 3% uphill. Preliminary studies indicate a two lane link and a two lane main line upstream carriageway.

(b) The design combination of hourly flows on a proposed rural motorway junction is

upstream main line	2400 vph	(15% heavy vehicles)
entry link or connector	1600 vph	(15% heavy vehicles)

The gradient on the main line is 1.5% uphill and on the entry link is 5% uphill. Preliminary studies indicate a two lane link and a two lane main line upstream carriageway.

(c) The combination of design hour traffic flows on a proposed rural motorway junction is

downstream main line	2000 vph	(20% heavy vehicles)
diverge link or connector	1500 vph	(15% heavy vehicles)

The main line and the link have zero gradient. Preliminary studies indicate three lanes on the upstream main line carriageway, two lanes on the downstream main line carriageway and a single lane link.

(d) The combination of design hour traffic flows on a proposed all-purpose junction is

downstream main line	2000 vph	(20% heavy vehicles)
diverge link or connector	1500 vph	(15% heavy vehicles)

The gradient on the main line and on the diverge links is 3% uphill. Preliminary studies indicate three lanes on the upstream main line carriageway, two lanes on the downstream main line carriageway and a two lane link.

Solution

Factors considered in the design of merging and diverging areas at grade separated junctions are

(a) the number of lanes and the design hour flows upstream and downstream of the merging or diverging areas on the main line carriageway,

(b) the number of lanes and the design hour flow on the merging or diverging link or connector,

(c) the traffic composition and the gradient of the main line carriageway and the merging or diverging links, details are given in Table 19.1,

(d) maximum hourly flows per lane based on experience of United Kingdom highway flow and which differ for all-purpose roads and for motorways, details are given in Table 19.2.

Table 19.1 Percentage Correction Factors for Gradient (based on reference (1))

percentage heavy goods vehicles	mainline gradient		merge connector gradient		
	<2%	>2%	< 2%	> 2%< 4%	> 4%
5	-	+10	-	+15	+30
10	-	+15	-	+20	+35
15	-	+20	+5	+25	+40
20	+5	+25	+10	+30	+45

Table 19.2 Maximum Hourly Flows for Merge, Diverge and Weave Sections (based on reference (1))

road class	maximum hourly flow vph/lane	
	main line, two lane links and slip roads $\geqslant$6m	single lane links and slip roads $\leqslant$ 5m
all-purpose	1600	1200
motorway	1800	1350

Using the principles developed in Example 18 to produce Figure 18.2 the Department of Transport have developed flow region diagrams for merging and diverging area design and Figure 19.1 and 19.2 are based on these recommendations. Because of the differing maximum flow levels for motorways and all-purpose roads the flow values given in these figures must be multiplied by 1.125 when motorway merges and diverges are being considered.

When selecting hourly flow levels to be used in design it is recommended that for highways of the Main Urban road type as defined in reference (2) the 30th highest hourly flow should be used whilst for Inter-Urban and Recreational road types the 50th and 200th highest hourly flows respectively should be used.

Using the worst combination of corrected flows predicted for the 15th year after opening of the highway the appropriate flow region is used with Figure 19.1 or 19.2 to obtain the appropriate flow region. Tables 19.3 and 19.4 then allow a suitable merging or diverging layout to be selected.

The use of these design recommendations will now be illustrated by consideration of examples (a), (b), (c) and (d).

Example (a)

Correct for traffic composition and gradient (Table 19.1).

Upstream main line flow = 2000 x 1.20

= 2400 vph

Entry link flow = 800 x 1.20

= 960 vph

As a motorway merge is being considered the above flows must be divided by 1.125 for entry into Figure 19.1. The resulting

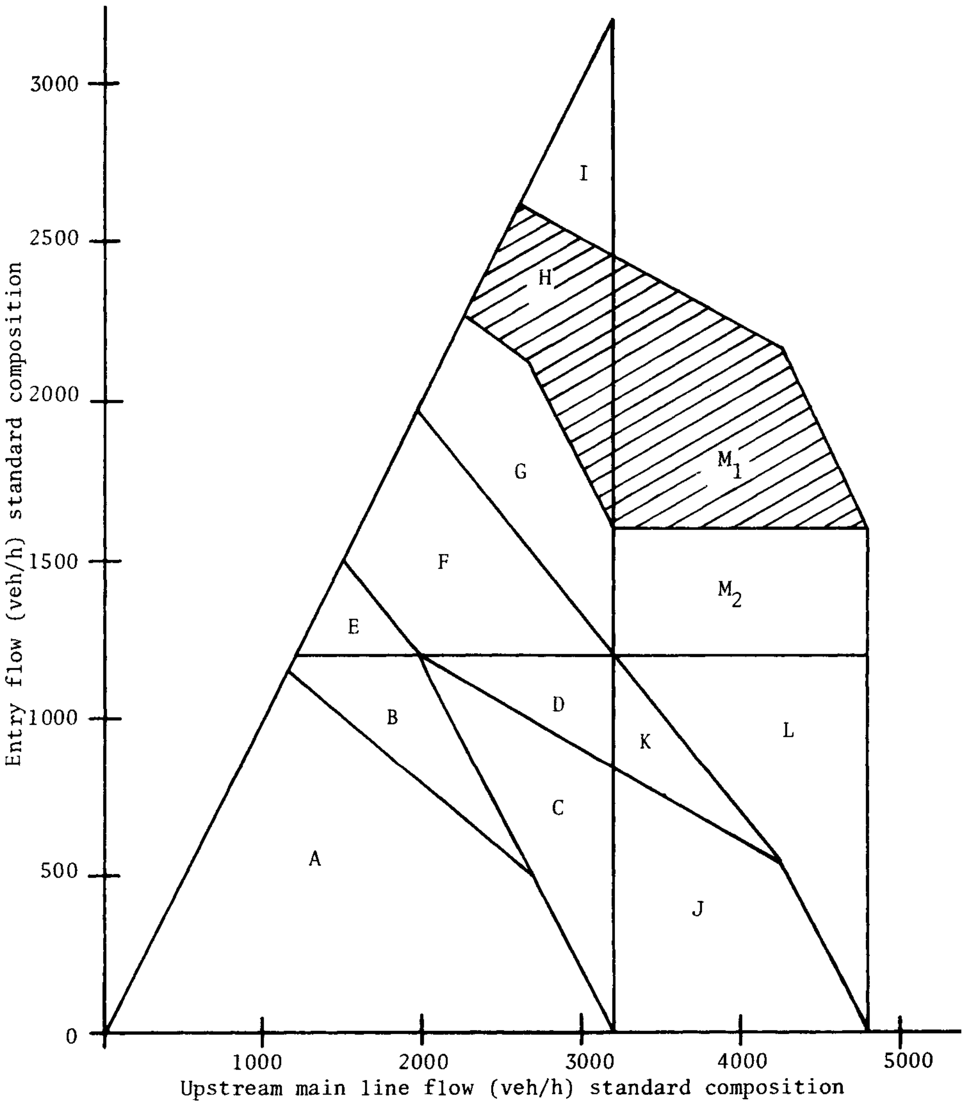


Figure 19.1

(Based on Tech. Mem. H18/75)

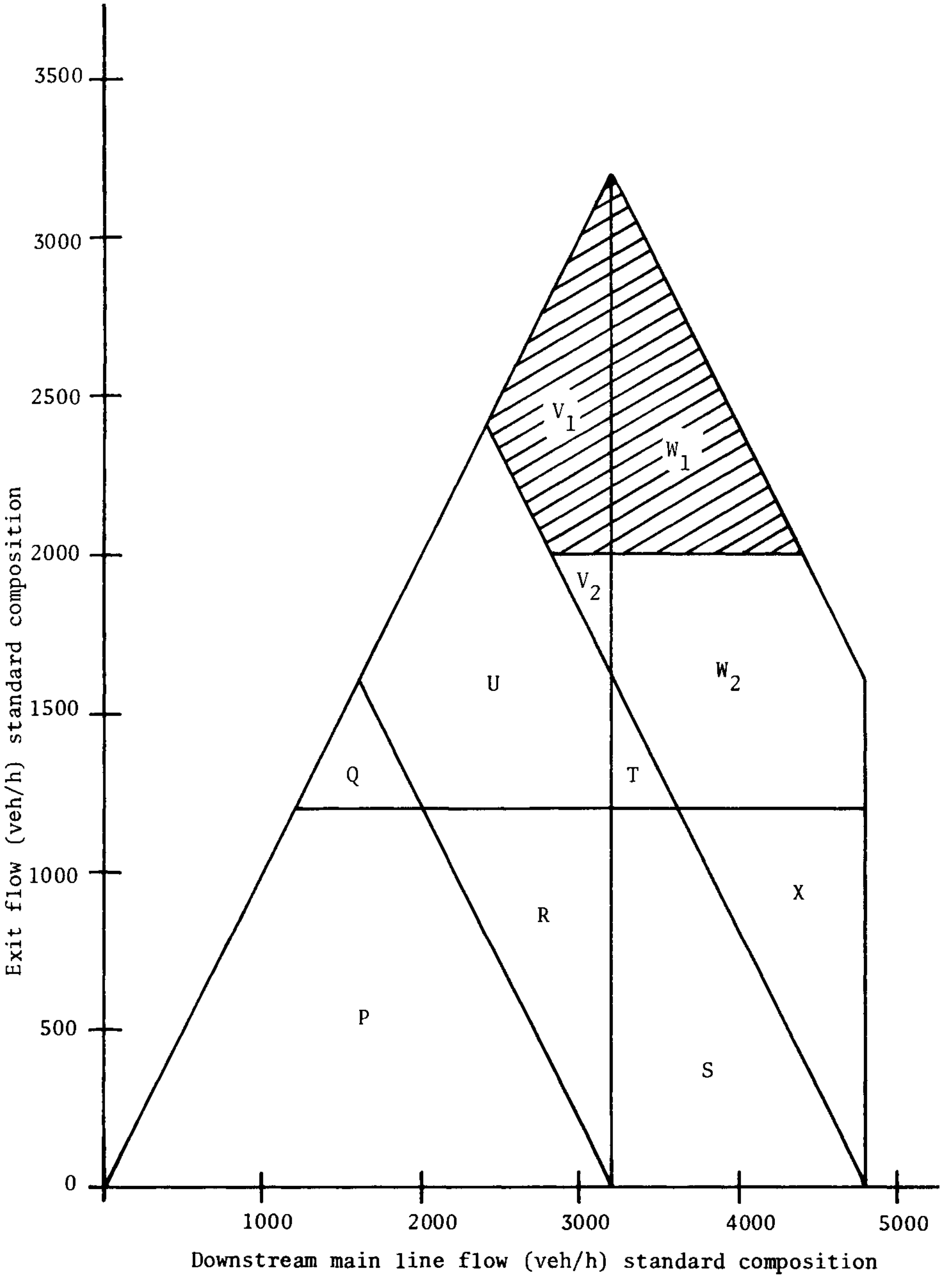


Figure 19.2

(Based on Tech. Mem. H18/75)

Table 19.3 Merging lane types for flow regions in Figure 19.1

		1	2	3	4	5	6	7	8	9	10	11
	merging lane type	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*
number of lanes	*upstream mainline*	*2*	*2*	*2*	*2*	*3*	*3*	*3*	*3*	*2*	*2*	*3*
	link entry	*1 (1)*	*1 (2)*	*2 (2)*	*2 (2)*	*1 (1)*	*1 (2)*	*2 (2)*	*2 (2)*	*1 (1)*	*2 (2)*	*1 (1)*
	downstream mainline	*2*	*2*	*2*	*3*	*3*	*3*	*3*	*4*	*3*	*4*	*4*
	A	√	√	√	√	√	√	√	√	√	√	√
	B	X	√	√	√	√	√	√	√	√	√	√
	C	X	X	X	√	√	√	√	√	√	√	√
	D	X	X	X	√	X	√	√	√	√	√	√
	E	X	X	√	√	X	X	√	√	X	√	X
	F	X	X	X	√	X	X	√	√	X	√	X
	G	X	X	X	√	X	X	X	√	X	√	X
	H	X	X	X	X	X	X	X	√	X	√	X
	I	X	X	X	X	X	X	X	X	X	√	X
	J	X	X	X	X	√	√	√	√	X	X	√
	K	X	X	X	X	X	√	√	√	X	X	√
	L	X	X	X	X	X	X	X	√	X	X	√
	M	X	X	X	X	X	X	X	√	X	X	X

√ indicates merging lane type is suitable for particular flow region, X otherwise.

Table 19.4 Diverging lane types for flow regions in Figure 19.2

	diverging lane type	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
number of lanes	*upstream mainline*	*2*	*2*	*3*	*3*	*3*	*4*	*3*	*4*	*4*
	link	*1*	*2*	*1*	*2*	*2*	*2*	*1*	*1*	*2*
	downstream mainline	*2*	*2*	*3*	*2*	*3*	*3*	*2*	*3*	*2*
	P	√	√	√	√	√	√	√	√	√
	Q	X	√	X	√	√	√	X	X	√
	R	X	X	√	√	√	√	√	√	√
	S	X	X	√	X	√	√	X	√	X
	T	X	X	X	X	√	√	X	X	X
	U	X	X	X	√	√	√	X	X	√
	V	X	X	X	X	X	√	X	X	√
	W	X	X	X	X	X	√	X	X	X
	X	X	X	X	X	X	√	X	√	X

√ indicates diverging lane type is suitable for particular flow region. X otherwise.

values are 2133 and 853 vph, giving flow region B which by reference to Table 19.3 allows for a 2 lane upstream main line and a two lane link or connector, a 2 lane downstream main line and a merging lane type 2 with a two lane entry with or without a ghost island. Other alternative layouts may be used.

Example (b)

Correct for traffic composition and gradient (Table 19.1).

Upstream main line flow = 2400 x 1.0

= 2400 vph

Entry link flow = 1600 x 1.40

= 2240 vph

As in Example (a) divide the above flows by 1.125 for entry into Figure 19.1, giving 2133 vph and 1991 vph. The required flow region is G. For a two lane link or connector and a two lane upstream main line flow region G a suitable layout is type 4 with a three lane downstream main line and a two lane entry with or without a ghost island.

Example (c)

Correct for traffic composition and gradient (Table 19.1).

Downstream main line flow = 2000 x 1.05

= 2100 vph

Diverge link flow = 1500 x 1.05

= 1575 vph

As in Examples (a) and (b) the above flows must be divided by 1.125 for entry into Figure 19.2, giving 1867 vph and 1400 vph. The given flow region is U which by reference to Table 19.4 suggests one suitable layout is type 4. For a 3 lane upstream main line and a 2 lane downstream main line, a two lane link or connector is required as opposed to the proposed single lane link or connector.

Example (d)

Correct for traffic composition and gradient (Table 19.1).

Downstream main line flow = 2000 x 1.25

= 2500 vph

Diverge link flow = 1500 x 1.25

= 1875 vph

From Figure 19.2 the required flow group is as in Example (c) group U. A suitable layout which is suggested in Table 19.4 is as before diverging lane type 4.

References

(1) Department of Transport. Layout of Grade Separated Junctions. Departmental Standard TD 22/86. London 1986.

(2) Department of Transport. Traffic Appraisal Manual. London 1982.

EXAMPLE 20 Introduction to Traffic Signal Control

(a) Describe the traffic and site conditions which justify the installation of signal control at a priority intersection.

(b) Discuss the factors which affect the maximum number of vehicles which are discharged per cycle over a traffic signal stopline.

(c) A traffic signal approach has a width of 7.3 m, an overall uphill gradient of 4 per cent. There is one 25s effective green period per 60s cycle. Traffic flow consists of 85 per cent light vehicles and 15 per cent medium goods vehicles. Turning radius for all vehicles is 15 m. Calculate the maximum possible number of vehicles discharged per cycle from the approach when,

(i) the approach is divided into two lanes, one of which is designated for 15 per cent left turn and 85 per cent straight ahead vehicles and the other lane is for opposed right turn vehicles as shown in figure 20.1. The degree of saturation of the opposing flow is 0.5.

(ii) the right turn flow is unopposed.

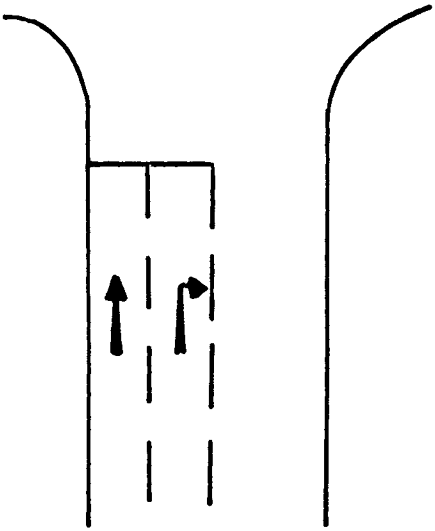


Figure 20.1

Solution

(a) The provision of traffic signal control may be justified by considering one or more of the following factors:

(i) the reduction in delay to traffic or pedestrians passing through the intersection

(ii) the reduction in accidents due to signal control

(iii) the ability to impose traffic management policies

(iv) the need to optimise traffic flow on an area-wide basis

Considering each of these factors in turn.

The great advantage of the priority intersection is that the major road traffic streams are only impeded to the extent that they may be prevented from overtaking in the vicinity of the intersection. For this reason the overall delay to traffic passing through the intersection may be low when expressed as delay per vehicle and yet delays to minor road vehicles may be high. The use of signal control may reduce delays to these minor road drivers and yet at the same time impose a small delay on a large number of major road drivers who previously were unimpeded at the intersection. In some traffic flow situations this can result in an increase in overall delay.

A precise warrant for the establishment of signal control on delay grounds cannot be given for the general case for it will depend on the distribution of flows and turning movements at the junction and particularly the number of vehicles turning right from the minor road.

In the United Kingdom a decision on the installation of traffic signal control is assisted by an economic assessment of the reduction in delay, expected changes in accident characteristics, capital and operating costs and also by a general policy of using signal control in urban traffic control schemes.

Because of the more positive control which is achieved with traffic signals it is possible to facilitate the movement of pedestrians through intersections using special pedestrian phases. If traffic flows do not warrant the provision of signal control then Pelican pedestrian signals may be installed away from the junction with guard rails to limit pedestrian movement.

If it is not possible to site the Pelican crossing away from the junction then signals may be installed at the junction and special pedestrian phases or walk with traffic facilities incorporated.

The provision of pedestrian facilities will almost invariably result in a decrease in the overall traffic capacity of the junction. If this is unacceptable then the signal phasing may, in certain circumstances, be modified.

The provision of such pedestrian facilities will certainly reduce the probability of pedestrian accidents but if the existing priority junction has a past accident record of at least 5 injuries to persons per year then the provision of signal control will be justified on these grounds alone.

One of the advantages of traffic signal control is that it is possible to positively control the movement of traffic into and through an area. A well known example of this approach to traffic management was the Nottingham zone and collar experiment. Here traffic signal control was used to limit the number of vehicles entering the major radial roads from the adjacent residential zones and at the same time priority was given to buses entering the centre of the city. A similar system is still in operation in Southampton where buses are given a direct entry onto a major road leading to the centre of the City whilst non-bus vehicles have either limited entry in controlled queues or may only proceed away from the congested central area.

In recent years the extension of signal linking from the optimisation of traffic flow along a major route to minimising journey times throughout an entire area has led to the widespread establishment of Area or Urban Traffic Control Schemes. The positive optimising control which can be exercised at a central computer over traffic signal controlled intersections is an added incentive to their establishment in preference to priority controlled systems such as roundabouts.

(b) The factors which affect the maximum rate of discharge of vehicles across the stop line of a traffic signal approach may initially be summarised as follows:

(i) the width of the approach

(ii) the geometric design of the intersection

(iii) the composition of the traffic flow

(iv) the length of the green period per cycle, and

(v) the lost time during the green period.

Dealing with each of these factors in greater detail.

For unopposed traffic streams in individual lanes, the saturation flow S_1 is given by,

$$S_1 = (S_o - 140\ d_n)/(1+1.5f/r) \text{ pcu/h}$$

where,

$$S_o = 2080-42\ d_G G + 100\ (w-3.25)\ \text{pcu/h}$$

where,

d_n and d_G are nearside lane and gradient dummy variables (0 for downhill gradients and 1 for uphill gradients, 0 for non-nearside lanes and 1 for nearside lane or single lane approaches respectively)

f is the proportion of turning vehicles in a lane

r is the radius of curvature (m) of the vehicle path

For streams containing opposed right turning vehicles in individual lanes, the saturation flow S_2 is given by,

$$S_2 = S_g + S_c\ \text{pcu/h}$$

where

$$S_g = (S_o-230)/(1+(T-1)f)$$

and $T = 1 + 1.5/r + t_1/t_2$

$$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$$

$$t_2 = 1-(fX_o)^2$$

and $S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$$X_o = q_o/\lambda n_\ell s_o$$

where,

S_g and S_c are the saturation flows in lanes of opposed mixed turning traffic during and after the effective green period respectively,

X_o is the degree of saturation on the opposing arm,

N_s is the number of storage spaces available inside the intersection which right turners can use without blocking following straight ahead vehicles,

s_o and n_ℓ are the saturation flow and the number of lanes of the opposing entry respectively,

P is the ratio of passenger car units to vehicles for the stream being considered.

The composition of the traffic is taken into account when calculating capacity by the use of passenger car equivalent units.

The following units are used in the United Kingdom.

Light vehicles (3 or 4 wheeled vehicles)	are	1 p.c.u.
Medium goods vehicles (vehicles with 2 axles but more than 4 wheels)	are	1.5 p.c.u.
Heavy goods vehicles (vehicles with more than 2 axles)	are	2.3 p.c.u.
Buses and coaches	are	2.0 p.c.u.
Motor cycles	are	0.4 p.c.u.
Pedal cycles	are	0.2 p.c.u.

As the maximum discharge or saturation flow of an approach is expressed in terms of passenger car units per hour of green the length of time during which the signal is effectively green during a cycle determines the total flow which is discharged. The effective green time is considered to be the actual green time on the signal indication plus the amber period which terminates the green minus the starting and end lost times due to acceleration and deceleration. The sum of these lost times has previously been taken as 2s but more recent work has indicated a wide range of values with an average starting lost time of 1.35s and an end lost time of 0.13s.

(c) In case (1) the nearside lane saturation flow may be calculated from,

$$S_1 = (S_o-140dn)/(1+1.5f/r)$$

$$S_o = 2080-42d_GG+100(w-3.25)$$

$$= 2080-42 \times 1 \times 4 + 100\ (3.65-3.25)$$

$$= 2208 \text{ pcu/h}$$

$$S_1 = (2208-140)/(1+1.5 \times 0.15/15)$$

$$= 2037 \text{ pcu/h.}$$

The farside lane saturation flow must take into account the effect of the opposing flow. Assume N_s is equal to 2.

$$S_2 = S_g + S_c$$

$$S_g = (S_o-230)/(1+(T-1)f)$$

$$S_o = 2080 - 42 \times 1 \times 4 + 100\ (3.65-3.25)$$

$$= 2208 \text{ pcu/h.}$$

$$T = 1 + 1.5/r + t_1/t_2$$

$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$

$= 12 \times 0.5^2/(1+0.6(1-1)N_s)$

$= 3$

$t_2 = 1-(fX_o)^2$

$1-(1 \times 0.5)^2$

0.75

$T = 1 + 1.5/15 + 3/0.75$

$= 5.1$

$S_g = (2208-230)/(1 + (5.1-1)1)$

$= 388$ pcu/h.

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$P = 0.85 + 0.15 \times 1.5$

$= 1.08$

$S_c = 1.08(1+2)(1 \times 0.5)^{0.2}\ 3600/25$

$= 406$ pcu/h.

$S_2 = 388 + 406$ pcu/h.

$= 794$ pcu/h.

The maximum discharge per cycle is the saturation flow multiplied by the effective green time per cycle.

For the nearside lane containing straight ahead and left turning vehicles the maximum discharge per cycle is,

$2037 \times 25/3600$

$= 14$ pcu/cycle

For the farside lane containing opposed right turning vehicles the maximum discharge per cycle is,

$794 \times 25/3600$

$= 6$ pcu/cycle.

In case (ii) the maximum discharge from the nearside lane is the same as in case (i). The maximum discharge or saturation flow from the farside lane which is now unopposed is given by,

$$S_1 = (S_o - 140d_n)/(1+1.5f/r) \text{ pcu/h}$$

$$S_o = 2080 - 42d_G G + 100(w-3.25)$$

as before,

$$S_o = 2208 \text{ pcu/h}$$

$$S_1 = 2208/(1+1.5/15)$$

$$= 2007 \text{ pcu/h}$$

The maximum number of vehicles discharged per cycle is,

$$2007 \times 25/3600$$

$$= 14 \text{ pcu/cycle}$$

EXAMPLE 21 Traffic Signal Cycle Times

(a) Describe how the calculated cycle time for fixed time operation is divided into green and red signal indications.

(b) At a highway intersection traffic conflicts are to be controlled by a 3-phase signal system. The design hour traffic flows are given in Table 21.1.

Table 21.1 Design Hour Traffic Flows

approach		right turn	straight ahead	left turn
north	light vehicles	400	100	30
	medium goods	60	60	10
	heavy goods	25	50	5
	buses	10	10	0
	motor cycles	10	5	5
south	light vehicles	300	100	30
	medium goods	40	20	10
	heavy goods	30	30	5
	buses	0	10	10
	motor cycles	5	10	5
west	light vehicles	15	300	20
	medium goods	10	30	5
	heavy goods	10	30	5
	buses	0	0	0
	motor cycles	5	5	5
east	light vehicles	10	200	40
	medium goods	5	100	20
	heavy goods	5	30	10
	buses	0	0	0
	motor cycles	5	5	5

All approaches have two 3.25m wide lanes, start plus end lost times are 1.3s per green period, the intergreen period is 4s, left turn radius 15m, right turn radius 25m, level approaches, number of storage spaces within the intersection for right turn vehicles 2.

Determine the optimum cycle times and minimum green times for fixed time operation.

Solution

(a) With the cycle time determined the total lost time per cycle is deducted from the cycle time to give the effective green time available for traffic flow during each cycle. The effective green time is then divided between the phases in proportion to the maximum ratio of flow to saturation flow for each phase. The actual green time is then the effective green time plus the lost time due to starting delays minus the 3s amber period.

(b) The first step is to convert the traffic flow into passenger car units, using the following equivalent values. Converted flows are given in Table 21.2 and summarised in Table 21.3.

Table 21.2 Design Flows Expressed in Passenger Car Units

approach		right turn	straight ahead	left turn
north	light vehicles	400	100	30
	medium goods	90	90	15
	heavy goods	58	115	12
	buses	20	20	0
	motor cycles	4	2	2
south	light vehicles	300	100	30
	medium goods	60	30	15
	heavy goods	69	69	12
	buses	0	20	20
	motor cycles	2	4	2
west	light vehicles	15	300	20
	medium goods	15	45	8
	heavy goods	23	69	12
	buses	0	0	0
	motor cycles	2	2	2
east	light vehicles	10	200	40
	medium goods	8	150	30
	heavy goods	12	69	23
	buses	0	0	0
	motor cycles	2	2	2

light vehicles (3 or 4 wheeled vehicles)	1.0 pcu
medium goods vehicles (vehicles with 2 axles but more than 4 wheels)	1.5 pcu
heavy goods vehicles (vehicles with more than 2 axles)	2.3 pcu
buses and coaches	2.0 pcu
motor cycles	0.4 pcu

Table 21.3 Summarised values of design hour flows

approach	right turn	straight ahead	left turn
north	572	327	59
south	431	223	79
west	55	416	42
east	32	421	95

The possible stages which can be used to control the traffic conflicts are shown in Figure 21.1.

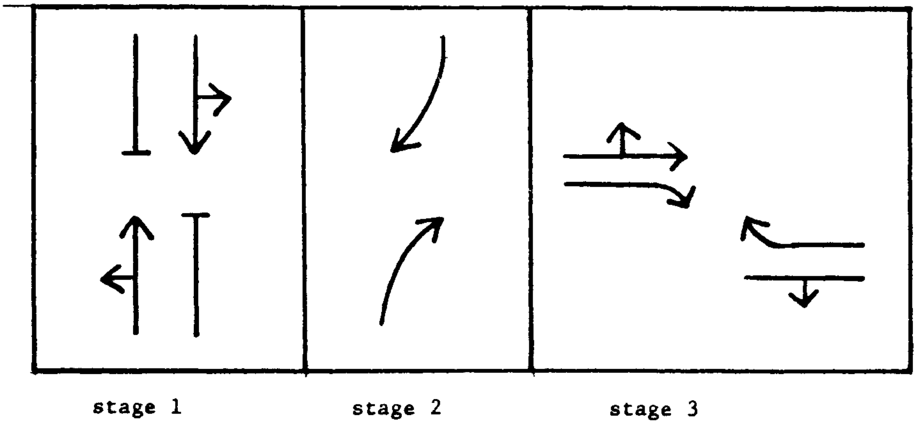


Figure 21.1

Saturation flows for stage 1

$$S_1 = \frac{2080-140\ d_n-42d_gG+100(w-3.25)}{1+1.5\ f/r}\ \text{pcu/h}$$

where S_1 is the saturation flow for lane 1

d_n is 1 and 0 for nearside and non-nearside lanes respectively

d_g is 1 and 0 for uphill and downhill gradients respectively

G is the percentage gradient (per cent)

w is the lane width (m)

f is the proportion of turning vehicles

r is the radius of curvature of the vehicle path

For the north approach the proportion of left turn vehicles
= 59/386
= 0.15

For the south approach the proportion of left turn vehicles
= 79/302
= 0.26

d_n = 1 for nearside lanes
G = 0 for a level approach
w = 3.5 m

For the north approach

$$S_{1N} = \frac{2080-140x1}{1+1.5x0.15/15}$$

$$= 1911\ \text{pcu/h}$$

For the south approach

$$S_{1S} = \frac{2080-140x1}{1+1.5x0.26/15}$$

$$= 1891\ \text{pcu/h}$$

Saturation flows for stage 2

For the north approach

$$S_{2N} = \frac{2080}{1+1.5x1/25}$$

$$= 1962\ \text{pcu/h}$$

For the south approach

$$S_{2S} = 1962 \text{ pcu/h}$$

Saturation flows for stage 3 for the unopposed straight ahead flow.

For the west approach the proportion of left turn vehicles
$= 42/458$
$= 0.09$

For the east approach the proportion of left turn vehicles
$= 95/516$
$= 0.18$

$$S_{3W(\text{unopposed})} = \frac{2080-140}{1+1.5\text{x}0.09/15}$$

$$= 1923 \text{ pcu/h}$$

$$S_{3E(\text{unopposed})} = \frac{2080-140}{1+1.5\text{x}0.18/15}$$

$$= 1906 \text{ pcu/h}$$

Saturation flows for stage 3 for the opposed right turn flow.

X_o, the degree of saturation of the opposing lane depends upon the green and cycle times and as these are unknown at this stage the assumption is made that for both west and east opposing flows the degree of saturation is 0.5,

P a conversion factor from veh/h to pcu/h is,

$$1 + \sum_i (a_i-1)p_i \text{ where}$$

a_i is the pcu value of vehicle type i

p_i is the proportion of vehicles of type i in the stream.

For the west approach

$$P = 1 + \Big[(1-1)15/40+(1.5-1)10/40+(2.3-1)10/40 + (0.4-1)5/40\Big]$$

$$= 1 + \Big[0+0.13+0.33-0.08\Big]$$

$$= 1.38$$

(this is the same result as would be obtained by a comparison of the flow in vehicles and in pcus)

For the east approach

$$P = 1.28$$

For both approaches N_s is given as 2 vehicle spaces.

For the west approach

$$S_{3W(opposed)} = S_{gw}+S_{cw}$$

where S_{gw} is the saturation flow during the effective green period and S_{cw} is the saturation flow after the effective green period.

$$S_{gw} = (S_o-230)/(1+(T-1)f)$$

$$S_o = 2080-42\ d_gG+100(w-3.25)$$

$$= 2080 \text{ pcu/h}$$

$$T = 1+1.5/r+t_1/t_2$$

$$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$$

$$= 12x0.5^2/(1+0.6(1-1)2)$$

$$= 3$$

$$t_2 = 1-(fX_o)^2$$

$$= 1-(1x0.5)^2$$

$$= 0.75$$

$$T = 1+1.5/25+3/0.75$$

$$= 1+0.06+4$$

$$= 5.06$$

$$S_{gw} = (S_o-230)/(1+(T-1)f)$$

$$= (2080-230)/(1+(5.06-1)1)$$

$$= 366 \text{ pcu/h}$$

$$S_{cw} = P(1+N_s)fX_o)^{0.2}\ 3600/\lambda c$$

$$S_{cw} = 1.38(1+2)(1x0.5)^{0.2}\ 3600/0.3x60$$

(as λ and c are not known the assumption is made from an examination of demand flow that λ, the ratio of effective green to cycle time is 0.3 and that c, the cycle time, is 60s, these assumptions will be examined at a later stage).

$$S_{cw} = 723 \text{ pcu/h}$$

$$S_{3W(opposed)} = S_{gw}+S_{cw}$$
$$= 366+723$$
$$= 1089 \text{ pcu/h}$$

For the east approach

S_{ge} = 366 pcu/h (as for the west approach)

$$s_{ce} = P(1+N_s)(fX_0)^{0.2}\ 3600/\lambda c$$
$$= 1.28\ (1+2)(1x0.5)^{0.2}\ 3600/0.3x60$$

(assumptions for λ and c as for west approach)

$$= 672 \text{ pcu/h}$$

$$S_{3E(opposed)} = S_{gE} + S_{CE}$$
$$= 366+672$$
$$= 1038 \text{ pcu/h}$$

The y values (demand flow/saturation flow) can now be calculated and are given in Table 21.4.

Table 21.4 Values of demand and saturation flows

Stage	approach	demand flow (pcu/h)	saturation flow (pcu/h)	y
1	north (straight and left turn)	386	1911	0.20*
	south (straight and left turn)	302	1891	0.16
2	north (right turn)	572	1962	0.29*
	south (right turn)	431	1962	0.22
3	west (nearside lane)	458	1923	0.24
	west (farside lane)	55	1089	0.05
	east (nearside lane)	516	1906	0.27*
	east (farside lane)	32	1038	0.03

The optimum cycle time C_o may be calculated from,

$$C_o = \frac{1.5L + 5}{1-\Sigma y_{max}}$$

The total lost time per cycle is the sum over the three stages of a 1s intergreen lost time and a 1.3s start and end lost time and is equal to 6.9s per cycle.

The y_{max} values are indicated by * in Table 21.4.

$$C_o = \frac{1.5x6.9+5}{1-0.76}$$

$$= 63.96s$$

Effective green time per cycle is 63.96-6.9=57.1s

Effective green time stage 1 = 57.1x0.20/0.76

= 15.0s

Effective green time stage 2 = 57.1x0.29/0.-6

= 21.8s

Effective green time stage 3 = 57.1x0.27/0.76

= 20.3s

Some of the initial assumptions made in the analysis will now be examined and their effects assessed.

The ratios of effective green to cycle time for west and east approaches are 20.3/64.0 equals 0.32 and this can be compared with the initial assumption of 0.3.

The degree of saturation for the opposing flow on the west approach is 458x63.96/(1921x20.3) equals 0.75 and this can be compared with the initial assumption of 0.5.

The degree of saturation for the opposing flow on the east approach is 516x63.96/(1896x20.3) equals 0.86 and this can be compared with the initial assumption of 0.5.

The effect of the error in the initial assumption of the degree of saturation on the opposed saturation flow for the west approach will be investigated initially.

$$T = 1+1.5/r+t_1/t_2$$

$t_1 = 12X_o^2\ (1+0.6(1-f)N_s)$ and $t_2 = 1-(fX_o)^2$

X_o previously assumed as 0.5 is now 0.86

t_1 now becomes 8.88

t_2 now becomes 0.26

$T = 1+1.5/25+8.88/0.26$

T now becomes 35.21

$S_{gw} = (S_o-230)/(1+(T-1)f)$

$= (2080-230)/(1+(35.21-1)1)$

$= \frac{1850}{35.21}$

$= 53$ pcu/h

$S_{cw} = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$= 1.38(1+2)(1x0.86)^{0.2}\ 3600/0.32x64$

S_{cw} now becomes 706 pcu/h

$S_{3W(opposed)} = S_{gw}+S_{cw}$

$S_{3W(opposed)}$ now becomes 53+706 pcu/h

759 pcu/h

Similarly the effect of the change in the degree of saturation on the west approach on the opposed saturation flow for the east approach will also be investigated.

$t_1 = 12X_o^2\ (1+0.6(1-f)N_s)$

X_o previously assumed as 0.5 is now 0.75

t_1 now becomes 6.75

$t_2 = 1-(fX_o)^2$

t_2 now becomes 0.44

$T = 1+1.5/r+t_1/t_2$

T now becomes 16.40

$S_{ge} = (S_o-230)/(1-(T-1)f)$

$= (2080-230)/(1+(16.40-1)1)$

$= 113$ pcu/h

$S_{ce} = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$= 1.28(1+2)(1x0.86)^{0.2}\ 3600/0.32x64$

S_{ce} now becomes 655 pcu/h

$$S_{3E(opposed)} = S_{gE} + S_{cE}$$

$S_{3E(opposed)}$ now becomes 113+655 pcu/h

= 768 pcu/h

Using these revised values of saturation flow it is now possible to revise Table 21.4, as shown in Table 21.5.

Table 21.4 Revised values of demand and saturation flows

stage	approach	demand flow (pcu/h)	saturation flow (pcu/h)	y
1	north (straight & left turn)	386	1906	0.20
	south (straight & left turn)	302	1865	0.16
2	north (right turn)	572	1962	0.29
	south (right turn)	431	1962	0.22
3	west (nearside lane)	458	1921	0.24
	west (farside lane)	55	759	0.07
	east (nearside lane)	516	1896	0.27
	east (farside lane)	32	768	0.04

The calculation of optimum cycle time and green times is unaltered by the revisions in Table 21.5.

EXAMPLE 22 Right Turning Flows at Traffic Signals Illustrated by an Example

(a) Describe how the problem of a single large right turning movement at a signal controlled intersection can be dealt with using late start or early cut-off facilities.

(b) At an intersection to be controlled by traffic signals the design hour traffic flows are given below.

		right turning	straight ahead	left turning
north approach 3.5 m wide	light vehicles	12	120	10
	medium goods	6	15	2
	buses	0	6	0
south approach 3.5 m wide	light vehicles	20	440	25
	medium goods	15	10	10
	buses	0	6	0
west approach 7.0 m wide	light vehicles	30	920	40
	medium goods	12	30	10
	buses	0	0	0
east approach 7.0 m wide	light vehicles	330	310	60
	medium goods	20	10	15
	buses	0	0	0

The intersection approaches are level, 4s intergreen periods, total start and end lost times are 2s per green period, left and right turn radii are 25 m and 2 right turn vehicles may wait in the intersection without obstructing the straight ahead flow. Using an early cut-off to facilitate right turning movement calculate the optimum cycle time and green times.

Solution

(a) Large right turning flows at signal controlled intersections present a problem for not only are right turning vehicles delayed by the straight ahead flow in the opposite direction but they also delay following vehicles. Where the right turning movement is balanced by a similar one in the opposite direction then a frequently employed solution is the addition of a third phase which allows uninterrupted flow for the two right turning movements. When, however, the right turning flow exists on only one approach, then it is frequently better from the point of view of overall delay, to allow the right turning flow to take place without obstruction at either the beginning or the end of the green period. These stages in the flow are shown in Figure 22.1.

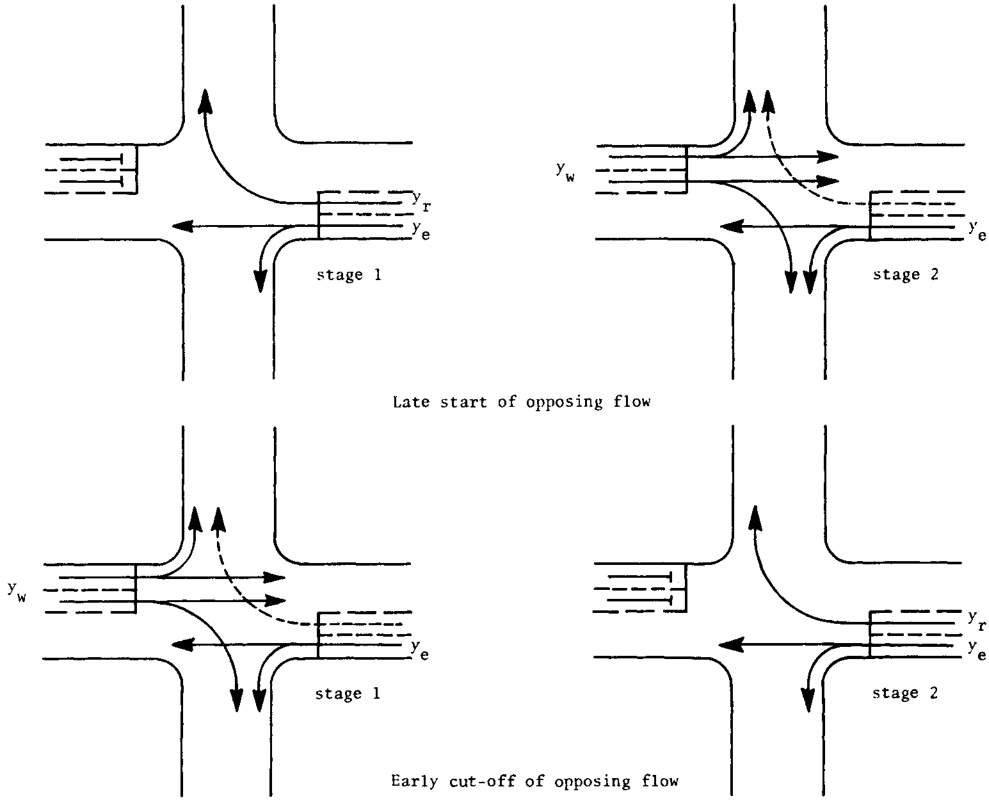


Figure 22.1

For the design of the signal settings the procedure used for normal signal setting calculations is used. The only difference is in the determination of the y_{max} value for stages 1 and 2. For the examples given in Figure 22.1 the green time which is available for west/east movements is fully used by the straight ahead and left turning vehicles on the east approach. The straight ahead and turning movements on the west approach, however, share the available west/east green time with the right turning vehicles on the east approach, thus

y_{max} west/east is the greater of y_e or $y_r + y_w$

If y_e is the greater, then the green time which the flow from the east requires should be divided in the proportion of y_r and y_w for the right turning and west approach streams. Should $y_r + y_w$ be greater then the y value for the first stage will be y_r and for the second stage will be y_w.

The choice of an early cut-off or a late release will depend to some extent upon the layout of the junction. If right turning vehicles block the movement of straight ahead and left turning vehicles on the same approach then there are advantages in allowing them to clear the intersection at the commencement of the green period by means of a late release of the opposing flow. A disadvantage of this technique, however, is that traffic flow is uncertain when the opposing flow is allowed to proceed. Will the turning movement, once having become established, continue, or will the vehicles in the opposing flow establish their movement immediately?

Where it is possible for right turning vehicles to wait without causing an obstruction to other vehicle movements then the early cut-off of the opposing flow is to be preferred. If a filter signal is used this is followed by an amber signal and the length of the right turning period can be adjusted to the traffic flow by vehicle detection.

(b) An inspection of the design hour traffic flows reveals a large right turning flow on the east approach which would have difficulty in passing through the straight ahead flow on the west approach. For this reason there will be an early cut-off of the west approach flow during which the right turning flow on the east approach will be permitted to flow without obstruction.

The design hour flows must first be converted into passenger car units and these values are tabulated in Table 22.1.

Table 22.1 Design hour flows expressed in passenger car units

approach	movement (pcu/h)		
	right turning	straight ahead	left turning
north	21	155	13
south	43	467	40
west	48	965	55
east	360	325	83

The next step is the calculation of saturation flows. Both north and south approaches consist of single lanes with mixed left, right and straight ahead movements. The saturation flow is obtained by separately calculating the saturation flows for left (S_L), right (S_R) and straight ahead vehicles (S_A) and combining in their respective proportions f_L, f_R and f_A using the relationship,

$$S_{combined} = \frac{S_L\ S_R\ S_A}{f_L\ S_R\ S_A + f_R\ S_L\ S_A + f_A\ S_L\ S_R}$$

$$S_{L(north,south)} = \frac{2080-140d_n-42d_GG+100(w-3.25)}{1+1.5/25}$$

$$= \frac{2080-140+100(3.5-3.25)}{1+1.5/25}$$

$$= 1807 \text{ pcu/h}$$

$S_R = S_g + S_c$

$S_g = (S_o-230)/(1+(T-1)f)$

$S_o = 2080-42\ d_GG+100(w_\ell-3.25)$

$T = 1+1.5/r+t_1/t_2$

$t_1 = 12X_o^2/(1+0.6(1-f)N_s)$

$t_2 = 1-(fX_o)^2$

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

X_o is the degree of saturation of the opposing flow.

For the north approach, the proportion of right turning vehicles (f) is,

21/189

= 0.11

For the south approach, the proportion of right turning vehicles (f) is,

43/550

= 0.08

For the north approach the degree of saturation of the south approach must be considered, an initial assumption of 0.5 for both north and south approaches will be made.

The conversion factor (P) between pcu's and vehicles can be calculated from the traffic data.

For right turning vehicles on the north approach P is 21/18 or 1.17.
For right turn vehicles on the south approach P is 43/35 or 1.23.

For the north approach

$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$

$= 12 \times 0.5^2/(1+0.6(1-0.11)2)$

$= 1.45$

$t_2 = 1-(fX_o)^2$

$= 1-(0.11 \times 0.5)^2$

$= 1.00$

$T = 1-1.5/r+t_1/t_2$

$1+1.5/25+1.45/1.00$

$= 2.51$

$S_o = 2080-42\ d_G G+100\ (w_e-3.25)$

$= 2080+100(3.5-3.25)$

$= 2105$ pcu/h

$S_g = (S_o-230)/(1+(T-1)f)$

$= (2105-230)/(1+(2.51-1)0.11)$

$= 1608$ pcu/h

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$= 1.17(1+2)(0.11 \times 0.5)^{0.2}\ 3600/(0.25 \times 50)$

(initial assumptions of 0.25 for λ and 50s for the cycle time are made).

$S_c = 566$ pcu/h

$S_R = S_g+S_c$

$= 1608+566$ pcu/h

$S_{R(north)} = 2174$ pcu/h

For the south approach

$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$

$= 12 \times 0.5^2/(1+0.6(1-0.08)2)$

$= 1.43$

$t_2 = 1-(fX_o)^2$

$= 1-(0.08 \times 0.5)^2$

$= 1.00$

$T = 1+1.5/r+t_1/t_2$

$= 1+1.5/25+1.43/1.00$

$= 2.49$

$S_o = 2105$ pcu/h (as for north approach)

$S_g = (S_o-230)/(1+(T-1)f)$

$= (2105-230)/(1+(2.49-1)0.08)$

$= 1675$ pcu/h

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$S_c = 1.23(1+2)(0.08 \times 0.5)^{0.2}\ 3600/(0.25 \times 50)$

initial assumptions of 0.25 for λ and 50s for the cycle time are made.

$S_c = 558$ pcu/h

$S_R = S_g+S_c$

$= 1675+558$

$S_{R(south)} = 2233$ pcu/h

$S_A = 2080-140d_n-42d_G G+100(w-3.25)$pcu/h

$= 2080-140+100(3.5-3.25)$

$S_{A(north,south)} = 1965$ pcu/h

$$S_{combined} = \frac{S_L\ S_R\ S_A}{f_L\ S_R\ S_A + f_R\ S_L\ S_A + f_A\ S_L\ S_R}$$

For the north approach (pcu/h)

$S_L = 1807$, $S_R = 2174$, $S_A = 1965$

$f_L = 0.07$, $f_R = 0.11$, $f_A = 0.82$

$$S_{combined(north)} = \frac{1807x2174x1965}{0.07x2174x1965+0.11x1807x1965+0.82x1807x2174}$$

$$= 1978 \text{ pcu/h}$$

For the south approach (pcu/h)

$S_L = 1807$, $S_R = 2233$, $S_A = 1965$

$f_L = 0.07$, $f_R = 0.08$, $f_A = 0.85$

$$S_{combined(south)} = \frac{1807x2233x1965}{0.07x2233x1965+0.08x1807x1965+0.85x1807x2233}$$

$$= 1972 \text{ pcu/h}$$

For the east approach, the nearside lane carries straight ahead and left turning vehicles and the farside lane carries only unopposed right turning vehicles.

For the nearside lane.

$$S_{nearside(east)} = \frac{2080-140\ d_n-42\ d_G G + 100(w-3.25)}{1 + 1.5\ f/r} \text{pcu/h}$$

$f = 83/408 = 0.20$

$$S_{nearside(east)} = \frac{2080-140+25}{1+1.5x0.20/25}$$

$$= 1942 \text{ pcu/h}$$

For the farside lane

$f = 1$

$$S_{farside(east)} = \frac{2080+25}{1+1.5\ x\ 1/25}$$

$$= 1986 \text{ pcu/h}$$

For the west approach, straight ahead vehicles may select either the nearside or the farside lane and the principle of lane groups will be used to determine the equivalent flow and saturation flow for this approach.

If q_S is the total straight ahead flow

q_L is the left turning flow

q_R is the right turning flow

S_{S1} and S_{S2} are the straight ahead saturation flows for the nearside and farside lanes respectively

S_{L1} is the saturation flow for left turn movement on the nearside lane

S_{R2} is the saturation flow for the opposed right turn movement

$$S_{S1} = 2080\text{-}140\ d_n - 42\ d_G G + 100\ (w-3.25)\ \text{pcu/h}$$

$$= 2080\text{-}140\text{+}25\ \text{pcu/h}$$

$$= 1965\ \text{pcu/h}$$

$$S_{S2} = 2080\text{+}25\ \text{pcu/h}$$

$$= 2105\ \text{pcu/h}$$

$$S_{L1} = S_{S1}/(1+1.5/r)$$

$$= 1965/(1+1.5/25)$$

$$= 1854\ \text{pcu/h}$$

$$S_{R2} = S_g + S_c$$

$$S_g = (S_o - 230)/(1+(T-1)f)$$

and

$$S_o = S_{S2}$$

f may be assumed to be 1.

$$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$$

$$= 12\ X_o^2$$

X_o will be assumed to be 0.7.

t_1 = 5.88

$t_2 = 1-(fX_o)^2$

$= 1-(1 \times 0.7)^2$

$= 0.51$

$T = 1+1.5/25+5.88/0.51$

$= 12.59$

$S_g = (2105-230)/(1+(12.59-1)1)$ pcu/h

= 149 pcu/h

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$P = 360/350$

$= 1.03$

$S_c = 1.03(1+2)(1 \times 0.7)^{0.2}\ 3600/(0.3 \times 50)$ pcu/h

The assumption is made that λ is 0.3 and that the cycle time is 50s.

S_c = 691 pcu/h

$S_{R2} = S_g + S_c$

= 149+691 pcu/h

= 840 pcu/h

The equivalent flow on the west approach is,

$q_L\ (S_{S1}/S_{L1}) + q_S + q_R\ (S_{S2}/S_{R2})$

= 55(1965/1854)+505+48(2105/840)

= 58+505+120

= 683 pcu/h

The equivalent saturation flow on the west approach is

$S_{S1} + S_{S2}$

= 1965 + 2105 pcu/h

= 4070 pcu/h

The flows and saturation flows for each approach are given in Table 22.2.

Table 22.2 Demand and Saturation Flows

approach	demand flow (pcu/h)	saturation flow (pcu/h)	y
north	189	1978	0.10
south	550	1972	0.28
west	679	4070	0.17
east nearside farside	 408 360	 1942 1986	 0.21 0.18

y_{max} for north/south phase is 0.28

y_{max} for west/east phase is the maximum of either y_{west}+ $y_{east(farside)}$ or $y_{east(nearside)}$

i.e. 0.17+0.18 or 0.21

$y_{max\ west/east} = 0.35$

$$C_o = \frac{1.5L + 5}{1 - \Sigma y_{max}}$$

The lost time L consists of 2s start and end lost times and a 1s intergreen lost time per stage.

$$C_o = \frac{1.5 \times 9 + 5}{1-(0.28+0.35)}$$

$$= 50s$$

Available effective green time per cycle is 50-9 equals 41s. This can be divided between the north/south and the east/west movements in proportion to their y values,

effective green north/south = 41x0.28/(0.28+0.35)=18.2s

effective green east/west = 41x0.35/(0.28+0.35)=22.8s

actual green north/south = 18.2-1=17.2s

actual green east/west = 22.8-1=21.8s

actual green right turn movement from east

$= 21.8 \times 0.18/(0.18+0.17)=11.2s$

actual green straight ahead movement from west

$= 21.8 \times 0.17/(0.18+0.17)=10.6s$

In the practical situation these green and cycle times would be rounded to integer values.

EXAMPLE 23 Variation of Delay with Cycle Time at Traffic Signals Illustrated by an Example

(a) Discuss the variations of overall average delay to vehicles which occur at fixed-time signal controlled intersections with variations in the cycle time.

(b) At an intersection controlled by 2 phase signal operation set at the optimum cycle time of 30 s, saturation flow on all approaches is equal and the flows used for design purposes are in the ratio of 2:1 for phase 1 and phase 2. A 4 s intergreen period is employed. The traffic flow on phase 2 increases by 100 per cent and a 15 s intergreen period is introduced for pedestrian use on the change of green from phase 1 to phase 2. Starting delays may be taken as 4 s per cycle in both cases. Show that the optimum cycle time increases by approximately 250 per cent.

Solution

(a) As the cycle time varies at an intersection, all other factors remaining constant then the variation in average delay to vehicles passing through the intersection shown in Figure 23.1 occur.

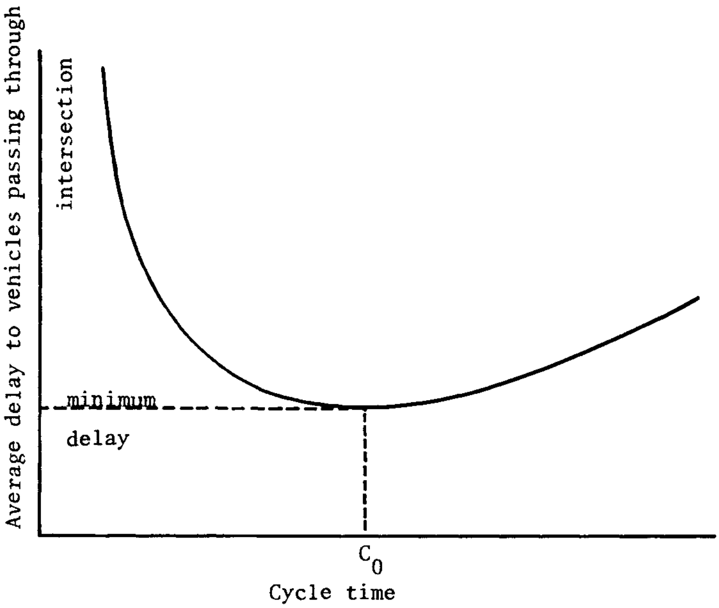


Figure 23.1

At very short cycle times delay to vehicles is high, as the cycle time decreases then the average delay decreases, until a value of cycle time is reached at which delay is a minimum, this is referred to as the optimum cycle time C_0. Any increase in cycle time beyond this value results in increased delays.

To understand this variation it is necessary to consider the manner in which traffic flows during the cycle. When there are queues on the traffic signal approaches then traffic will cross the stop lines at the saturation flow rate during the effective green time but there will be periods when vehicles do not cross the stop line, referred to as lost time due to starting delays. Numerically this period is calculated from the sum of the actual green period plus the 3 s amber period at the end of the green minus the effective green period.

There is also another period of time which is lost to traffic flow which occurs during the period from the end of green on one phase to the start of green on the next phase, referred to as the intergreen period. Within this period traffic will not be crossing the stopline for a small period of time which depends on the length of the intergreen period. As the intergreen period contains a 3 s period of amber which terminates every green period in the United Kingdom and is normally included as running time, subject to a correction for deceleration which is included in the starting delay described above, the lost time in the intergreen period is taken as the intergreen period minus 3 s. During the cycle there will be one intergreen period for each phase.

Thus there are two elements of lost time to traffic flow which occur during the signal cycle, lost time due to starting delays and lost time during the intergreen period. As these periods are independent of cycle time they comprise a considerable period of the cycle time when the cycle time is short. Conversely as the cycle time increases operation of the signals becomes more efficient as shown in Figure 23.1.

As the cycle time continues to increase however saturated flow across the stop time is not maintained for the whole of the longer green periods. This means that vehicles on some phases are delayed whilst those on other phases cross the stop line at widely spaced intervals. For this reason another factor in addition to lost time is used in the determination of the value of the optimum cycle time, this is the intensity of trafficking on the most heavily loaded approach for each phase. It is defined by the ratio of flow to saturation flow or the value for the most heavily loaded approach of each phase.

Using lost time and the y_{max} values the approximate optimum cycle time C_o is given by

$$C_o = \frac{1.5L + 5}{1 - \Sigma y_{max}} \text{ s}$$

where L is the total lost time per cycle (s)
and Σy_{max} is the sum of the y_{max} values for each phase.

(b) Under initial conditions

$$1\text{st } C_o = \frac{1.5L + 5}{1 - \Sigma y_{max}} \text{ s}$$

In this case the total lost time per cycle (L) is composed of 2 s starting delays per green period and 1 s per intergreen period

hence $L = 2(2 + 1)$ s

as there are two phases per cycle.

And $\Sigma y_{max} = y_{max}$ for phase 1 + y_{max} for phase 2

$$= 2y_2 + y_2 = 3y_2$$

$$1\text{st } C_o = \frac{1.5 \times 6 + 5}{1 - 3y_2} = 30 \text{ s}$$

giving $y = 0.18$

Under final conditions the total lost time per cycle (L) is composed of

2 s starting delays per green period,
12 s in the intergreen period between phase 1 and 2, plus
1 s in the intergreen period between phase 2 and 1.

And $\Sigma y_{max} = 2y_2 + 2y_2 = 4y_2 = 0.72$

$$2\text{nd } C_o = \frac{1.5 \times 17 + 5}{1 - 0.72} = 105 \text{ s}$$

Percentage increase in optimum cycle time $= \frac{75}{30} \times 100 = 250$ per cent.

EXAMPLE 24 Delay at Traffic Signals Illustrated by an Example

Explain how delay at traffic signal controlled intersections may be estimated. For a signal approach with a saturation flow of 3672 pcu/h and a demand flow of 720 pcu/h calculate the average delay on the approach when the cycle time is 40s, 60s and 80s. Given the total lost time per cycle is 12s and the effective green time for the approach is 0.3 of the available effective green time, comment on the calculated results.

Solution

The delay to vehicles on a traffic signal approach is estimated in United Kingdom practice using a combination of queueing theory and experimental results.

Vehicles arriving on a traffic signal approach are a combination of vehicles which are arriving at random and those which are arriving at regular intervals.

For those vehicles arriving at regular intervals it has been shown (1) that the average delay can be determined from,

$$\frac{C(1-\lambda)^2}{2(1-\lambda x)}$$

and for those arriving at random from

$$\frac{x^2}{2q(1-x)}$$

a third term is required to accurately model results obtained from simulation, thus

$$-0.65\left(\frac{C}{q^2}\right)^{\frac{1}{3}} x^{(2+5\lambda)}$$

These can be combined to give the average delay per vehicle (d) where

C = cycle time

λ = proportion of the cycle that is effectively green for the phase under consideration, i.e. effective green divided by cycle time

q = flow

s = saturation flow

x = degree of saturation

The calculation of delay on the approach for varying cycle time is given in Table 24.1. It can be seen that the delay per passenger car unit (d) is lowest for a 60s cycle time indicating that delay/cycle time relationship for the approach is likely to be of the form previously shown in Figure 23.1.

Table 24.1 Delay Calculations

cycle time	40	60	80
g_{eff}	8.40	14.40	20.40
λ	0.21	0.24	0.26
q pcu/s	0.20	0.20	0.20
s pcu/s	1.02	1.02	1.02
x	0.93	0.82	0.75
$\frac{C(1-\lambda)^2}{2(1-\lambda x)}$	15.51	21.57	27.21
$\frac{x^2}{2q(1-x)}$	30.89	9.34	5.63
$0.65\ (\frac{C}{q^2})^{\frac{1}{3}}\ x^{(2+5\lambda)}$	5.21	3.94	4.26
d/pcu (s)	41.19	26.97	28.58

Reference

(1) Webster, F.V., Road Research Technical Paper No. 39, Ministry of Transport. Road Research Laboratory. H.M.S.O. London.

EXAMPLE 25 Design of a Traffic Signal Controlled Intersection

The highway intersection and traffic flows shown in Figure 25.1 are to be controlled by the traffic signal installation and the traffic management measures shown in Figure 25.2. All traffic lanes have a width of 3.25 m, turning vehicle radii 20 m, waiting spaces for right turn vehicles 2 and the average passenger car unit/vehicle ratio is 1.2. Intergreen periods are 4s and 10s per cycle, starting and end lost times total 2s per phase. Two phase operation is to be used. Calculate the optimum cycle time and the actual green times.

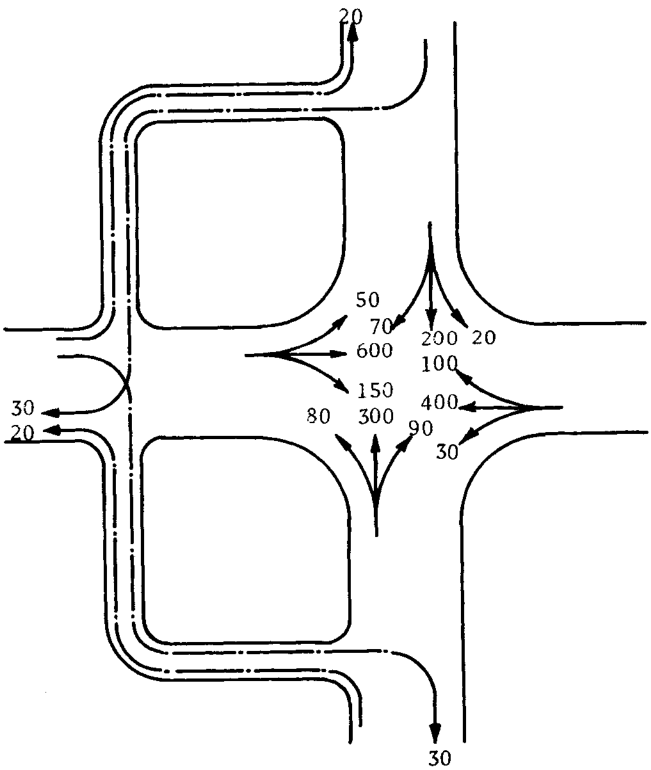


Figure 25.1

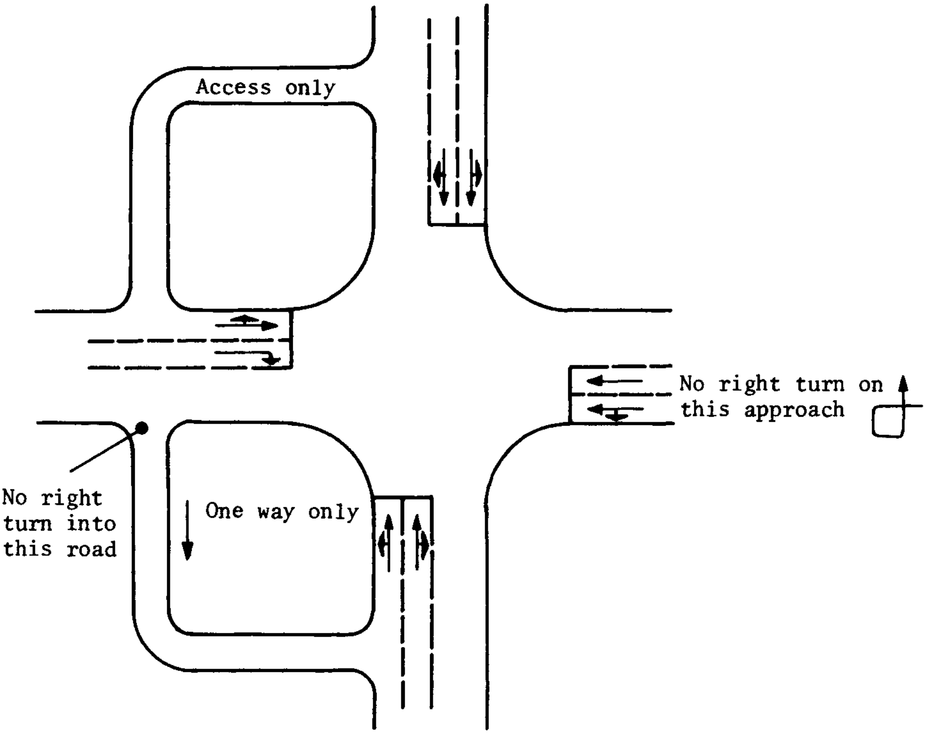


Figure 25.2

Traffic management proposals

Solution

As a result of the traffic management measures the flows at the junctions and the turning movements must be re-assigned. The revised design hour flows are given in Table 25.1.

The saturation flows for the approaches will now be calculated.

Nearside lane of the west approach.

$$S_{w(nearside)} = (S_o - 140d_n)/(1+1.5f/r) \text{ pcu/h}$$

and $S_o = 2080-42d_G G+100(w-3.25)$

$$S_o = 2080$$

$$S_{w(nearside)} = (2080-140)/(1+1.5 \times 0.1/2.0)$$

$$= 1926 \text{ pcu/h}$$

Table 25.1 Re-assigned traffic flows after traffic management

approach	movement	flow (pcu/h)
north	left turn	20
	straight ahead	200
	right turn	100
south	left turn	100
	straight ahead	400
	right turn	90
west	left turn	70
	straight ahead	600
	right turn	180
east	left turn	30
	straight ahead	500
	right turn	0

Right turn lane of the west approach. This flow is opposed by the flow on the east approach and it is necessary at this stage to make assumptions regarding the degree of saturation of the opposing flow from the east and also regarding the effective green time for the west approach. It will be assumed that the degree of saturation of the east approach is 0.6 and that the effective green time for the west approach is 25s.

$$S_{w(offside)} = S_g + S_c$$

$$S_g = (S_o - 230)/(1 + (T-1)f)$$

$$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$$

$$= 12 \times 0.6^2$$

$$= 4.32$$

$$t_2 = 1-(fX_o)^2$$

$$= 0.64$$

$T = 1+1.5/r + t_1/t_2$

$= 1+1.5/20+4.32/0.64$

$= 7.83$

$S_o = 2080-42d_G G+100(w-3.25)$

$= 2080$ pcu/h

$S_g = (2080-230)/(1+(7.83-1)1)$

$= 236$ pcu/h

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$p = 1.2$

$S_c = 1.2(1+2)(0.6)^{0.2}\ 3600/25$

$= 466$ pcu/h

$S_{w(offside)} = 702$ pcu/h

The saturation flow for the east approach can be calculated using the concept of lane grouping, as shown in Figure 25.3, where,

$q = q_s + q_L\ (S_{S1}/S_{L1})$

and $S_E = S_{S1} + S_{S2}$

$S_{S1} = (S_o-140d_n)$

$S_o = 2080-42d_G G+100(w-3.25)$

$= 2080$ pcu/h

$S_{S1} = 2080-140$

$= 1940$ pcu/h

$S_{S2} = 2080$ pcu/h

$S_{L1} = S_{S1}/(1+1.5/r)$

$= 1940/(1+1.5/20)$

$= 1805$ pcu/h

$q = 500 + 30\ (1940/1805)$

$= 532$ pcu/h

$S = 1940 + 2080$

$= 4020$ pcu/h

Considering the north and south approaches, the highest ratio of flow to saturation flow will occur on the south approach and the saturation flow for this approach will be calculated as it will be used in the calculation of the cycle time and green times.

When calculating the saturation flow of the south approach, the concept of lane grouping will be used and in addition assumptions are necessary for the degree of saturation of the north approach (0.6) and for the effective green time for the south approach (15s).

The equivalent flow (q) on the south approach as shown in Figure 25.4 is,

$$q_L\ (S_{S1}/S_{L1}) + q_S + q_R\ (S_{S2}/S_{R2})$$

$S_{S1} = 1940$ pcu/h

$S_{L1} = 1805$ pcu/h

$S_{S2} = 2080$ pcu/h

$S_{R2} = S_g + S_c$

$$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$$
$$= 12 \times 0.6^2$$
$$= 4.32$$

$$t_2 = 1-(fX_o)^2$$
$$= 1-0.6^2$$
$$= 0.64$$

$$T = 1+1.5/r + t_1/t_2$$
$$= 1+1.5/20 + 4.32/0.64$$
$$= 7.83$$

$$S_g = (S_o-230)/(1+(T-1)f)$$
$$= (2080-230)/(1+(7.83-1)1)$$
$$= 236 \text{ pcu/h}$$

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$P = 1.2$

$S_c = 1.2(1+2)(0.6)^{0.2} \times 3600/15$

$= 788$ pcu/h

$S_{R2} = 236+788$

$= 1024$ pcu/h

$q = 100\ (1940/1805) + 400 + 90\ (2080/1084)$

$= 107 + 400 + 173$

$= 680$ pcu/h

$S = S_{S1} + S_{S2}$

$= 1940 + 2080$

$= 4020$ pcu/h.

The design values of flow and saturation flow are given in Table 25.2.

Table 25.2 Design values of flow and saturation flow

approach	flow (pcu/h)	saturation flow (pcu/h)	y
west nearside	675	1926	0.35*
west offside	180	702	0.26
east	532	4020	0.13
south	680	4020	0.17*

*values used for design

$$C_o = \frac{1.5L + 5}{1 - \Sigma y_{max}}$$

$$= \frac{1.5 \times 12 + 5}{1 - (0.35 + 0.17)}$$

$$= 48s$$

The available effective green time is the cycle time minus the lost time is 48-12 = 36s.

This available effective green time is divided between the phases in proportion to the y_{max} values.

The effective green for the north/south phase is,

$0.17 \times 36/(0.35 + 0.17)$

$= 11.8s$

The effective green for the west/east phase is,

$0.35 \times 36/(0.35 + 0.17)$

$= 24.2s$

A check will now be made on the previous assumptions.

Effective green, north/south approaches, calculated 12s, assumed 15s

Effective green, west/east approaches, calculated 24s, assumed 25s

Degree of saturation, north approach, calculated 0.4, assumed 0.6

Degree of saturation, east approach, calculated 0.3 assumed 0.6

It can be seen that the assumed values of effective green times are similar to the resulting calculated values but the assumed degrees of saturation differ from those which were subsequently calculated.

The right turn saturation flow of the offside lane of the west approach assumed a degree of saturation of 0.6, a revised value of 0.3 will now be used.

$S_{w(offside)} = S_g + S_c$

$S_g = (S_o-230)/(1+(T-1)f)$

$t_1 = 12\ X_o^2/(1+0.6(1-f)N_s)$

$= 12 \times 0.3^2$

$= 1.08$

$t_2 = 1-(fX_o)^2$

$= 0.91$

$T = 1+1.5/r+t_1/t_2$

$= 1+1.5/20+1.08/0.91$

$= 2.26$

$S_o = 2080$ pcu/h

$S_g = (2080-230)/(1+(2.26-1)1)$

$= 819$ pcu/h

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$S_c = 1.2(1+2)(0.3)^{0.2}\ 3600/25$

$= 407$ pcu/h

$S_{w(offside)} = 1226$ pcu/h

When calculating the saturation flow of the south approach the degree of saturation of the north approach was assumed to be 0.6 and now it will be assumed to be 0.3.

Referring to the previous calculation.

$S_{R2} = S_g + S_c$

$t_1 = 12X_o^2/(1+0.6(1-f)N_s)$

$= 1.08$

$t_2 = 1-(fX_o)^2$

$= 0.91$

$T = 1+1.5/r+t_1/t_2$

$= 2.26$

$S_o = 2080$ pcu/h

$S_g = 819$ pcu/h

$S_c = P(1+N_s)(fX_o)^{0.2}\ 3600/\lambda c$

$= 1.2(1+2)(0.3)^{0.2}\ 3600/15$

$= 679$ pcu/h

S_{R2} = 819+679

= 1498 pcu/h

q = 100 (1940/1805)+400+90 (2080/1498)

= 107+400+125

= 632 pcu/h

Revised values of flow and saturation flow are given in Table 25.3.

Table 25.3 Revised values of flow and saturation flow

approach	flow (pcu/h)	saturation flow (pcu/h)	y
west nearside	675	1926	0.35*
west offside	180	1226	0.15
east	532	4020	0.13
south	632	4020	0.16*

*values used for design

$$C_o = \frac{1.5L + 5}{1-\Sigma y_{max}}$$

$$= \frac{1.5 \times 12 + 5}{1 - (0.35+0.16)}$$

$$= 47s$$

The revised value of the cycle time is similar to the first approximation and adjustment of effective green times is not required.

EXAMPLE 26 Design of a Three Phase Traffic Controlled Intersection

The design hour traffic flows at an intersection to be controlled by three phase traffic signals are given below.

approach	movement	design flows (vehicles per hour)		
		light vehicles	medium goods	heavy goods
north	left turn	40	10	5
	straight ahead	300	50	20
	right turn	200	80	10
south	left turn	50	40	5
	straight ahead	400	40	10
	right turn	280	70	20
west	left turn	80	5	0
	straight ahead	350	20	5
	right turn	60	10	0
east	left turn	90	10	0
	straight ahead	420	30	10
	right turn	30	5	0

The north approach has an uphill gradient of 5 per cent. All approaches comprise 2 lanes, the widths on the north/south approaches are 3.5 m and on the east/west approaches are 3.0m. A three phase system is to be employed, intergreen times are to be 5s and start and end lost times total 1.5s per phase. Turning radii are 15 m for left turn and 25 m for right turn movements. Calculate the optimum cycle time and the maximum green times.

Solution

Inspection of the design hour traffic flows indicates heavy right turn movements on the north and south approaches and a separate phase will be provided to allow unobstructed flow for these movements as illustrated in Figure 26.1.

It will first be necessary to convert the flows to passenger car units using the equivalent values of 1.0 for light vehicles, 1.5 for medium goods vehicles and 2.3 for heavy goods vehicles. Design flows are given in Table 26.1.

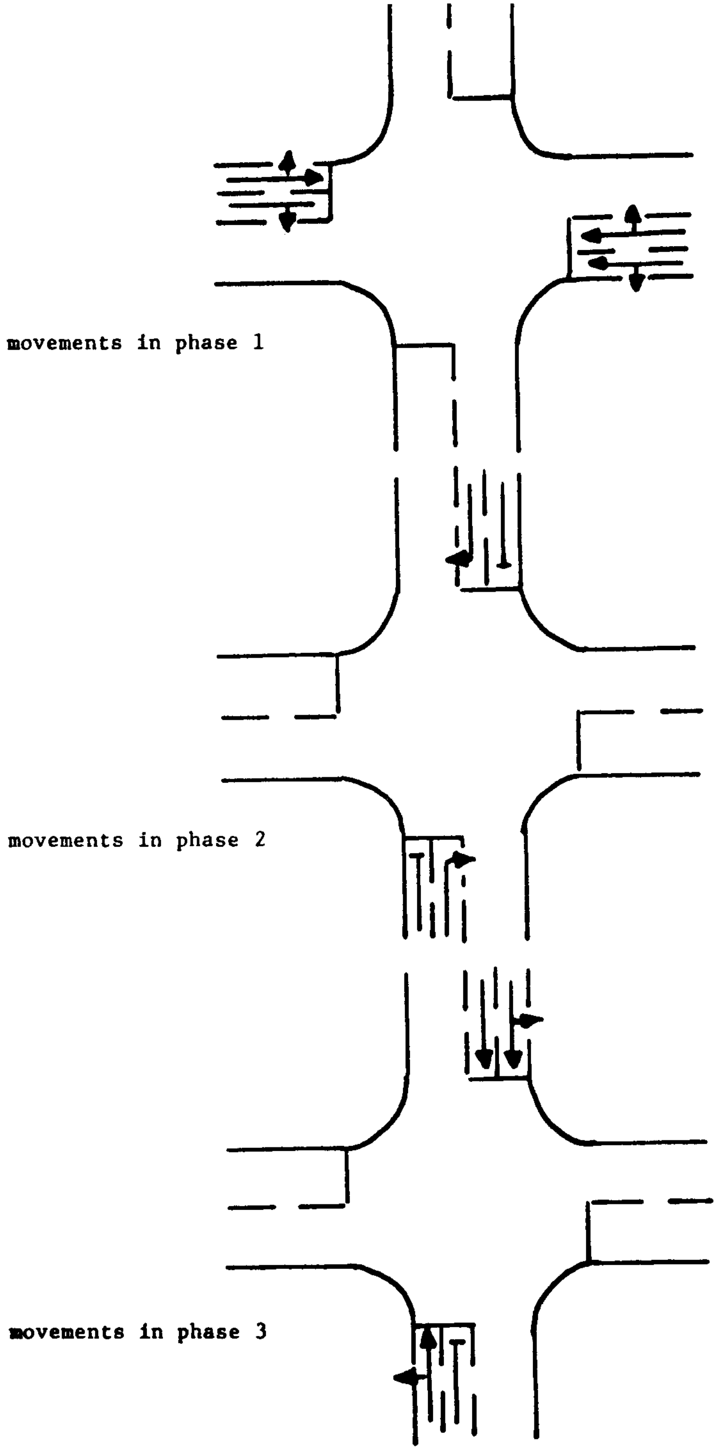

movements in phase 1
movements in phase 2
movements in phase 3

Figure 26.1

Table 26.1 Design Flows in Passenger Car Units

approach	movement	design flow (pcu/h)
north	left turn	67
	straight ahead	421
	right turn	343
south	left turn	122
	straight ahead	483
	right turn	431
west	left turn	88
	straight ahead	392
	right turn	75
east	left turn	105
	straight ahead	488
	right turn	38

The saturation flows for the approaches used in the calculation of the optimum cycle time and the maximum green times will now be calculated.

For the movements taking place in phase 1 (i.e. the west/east movements) the highest ratio of demand flow to saturation flow will occur on the east approach.

The flow group for the east approach can be identified and the equivalent flow q_E is,

$$q_E = q_L\ (S_{S1}/S_{L1}) + q_S + q_R\ (S_{S2}/S_{R2})$$

where L, S and R refer to left, straight ahead and right turn movements respectively and 1 and 2 refer to the nearside and farside lanes respectively.

$$S_{S1} = (S_o\text{-}140\ dn)/(1+1.5\ f/r)$$

$$= S_o\text{-}140\ pcu/h$$

$$S_o = 2080+100(3.5\text{-}3.25)$$

$$= 2105\ pcu/h$$

$$S_{S1} = 1965\ pcu/h$$

$$S_{L1} = (2105\text{-}140)/(1+1.5/15)$$

$$= 1786\ pcu/h$$

$$S_{S2} = 2105\ pcu/h$$

In previous examples an attempt was made to obtain a more exact solution by the use of the saturation flow expression for an opposed right turn flow, in this example an approximate solution will be obtained by the use of the unopposed right turn saturation flow expression.

$$S_{R2} = 2105/(1+1.5/25)$$

$$= 1986\ pcu/h$$

$$S_{R2} = 2105/(1+1.5/25)$$

$$= 1986\ pcu/h$$

$$q_E = 105(1965/1786)+488+38(2105/1986)$$

$$= 116+488+40$$

$$= 644\ pcu/h$$

The equivalent saturation flows for the east approach is given by,

$$S = S_{S1} + S_{S2}$$

$$= 1915+2055$$

$$= 3970\ pcu/h$$

For movements taking place in phase 2 (i.e. the north/south right turning movements) the highest ratio of demand flow to saturation flow will occur on the south approach. Saturation flow for the right turn lane on the south approach S_{SR} is given by:

$S_{SR} = (S_o - 140dn)/(1+1.5f/r)$

$S_o = 2080+100(3.5-3.25)$

$= 2105$ pcu/h

$S_{SR} = (2105)/(1+1.5/25)$

$= 1906$ pcu/h

For movements taking place in phase 3 (i.e. north/south straight ahead and left turning movements) the highest ratio of demand flow to saturation flow will occur on the south approach.

$S_s = (S_o - 140dn)/(1+1.5f/r)$

$S_o = 2080+100(3.5-3.25)$

$= 2105$ pcu/h

$f = 122/(122+483)$

$= 0.20$

$S_s = (2105-140)/(1+1.5x0.2/25)$

$= 1942$ pcu/h

Table 26.2 Demand and Saturation Flows

phase	movement	demand flow (pcu/h)	saturation flow (pcu/h)	y
1	all, east approach	644	3970	0.16
2	right turn, south approach	431	1906	0.23
3	straight ahead and left turn, south approach	605	1942	0.31

$$C_o = \frac{1.5L+5}{1-\Sigma y_{max}}$$

$$L = 3(2+1.5)s$$

$$= 10.5s$$

$$C_o == \frac{1.5x10.5+5}{1-(0.16+0.23+0.31)}$$

$$= \frac{20.75}{0.3}$$

$$= 69s$$

Effective green time/cycle = 69-10.5

= 58.5s

Effective green time phase 1 = 0.16x58.5/0.7 = 13.4s

phase 2 = 0.23x58.5/0.7 = 19.2s

phase 3 = 0.31x58.5/0.7 = 25.9s

Actual green time = effective green time + start and end lost time - 3s amber

Actual green time phase 1 = 13.4-1.5 = 11.9s

phase 2 = 19.2-1.5 = 17.7s

phase 3 = 25.9-1.5 = 24.4s

EXAMPLE 27 Platoon Dispersion between Signal Controlled Intersections

A highway link connects two traffic signal controlled intersections A and B. The average travel time of vehicle platoons between the two intersections is 30 s. Details of the characteristics of intersections A and B are given below.

intersection	*cycle time (s)*	*green time (s)*	*flow (pcu/h)*	*saturation flow (pcu/h)*
A	120	60	720	2400
B	120	60	720	1800

Using the standard platoon dispersion relationship employed by Transyt and dividing the cycle time into 6 s units calculate the average delay to vehicles travelling from A to B when the offset of B relative to A is zero. Ignore the effect of turning vehicles and assume that the flow crosses stopline A with a uniformly distributed platoon.

Solution

The standard platoon dispersion relationship employed in Transyt is

$$q_2(i + t) = F\,q_1(i) + (1 - F)q_2(i + t - 1)$$

and F = 1/1(1 + 0.4 average journey time)

where t is 0.8 times the average journey time of the platoon,
q_1 is the discharge rate of vehicles per time interval from intersection A,
q_2 is the arrival rate of dispersed vehicles per time interval at intersection B.

The cycle time is divided into 20 number 6 s intervals which are referred to as time intervals.

The discharge rate per time interval of 6 s at intersection A is obtained from the given saturation flow.

$$q_1 = \frac{2400 \times 6}{3600} = 4 \text{ pcu/time interval}$$

During each cycle 720 x 120/3600 = 24 pcu are discharged from signal A. The discharge of vehicles per time interval at intersection B is also obtained from the given saturation flow and is

$$\frac{1800 \times 6}{3600} = 3 \text{ pcu/time interval}$$

The smoothing coefficient F is given by

F = 1/(1 + 0.4 x 5 time intervals) = 0.33

Table 27.1

1	*2*	*3*	*4*	*5*	*6*	*7*
time interval	*signal indication* *A*	*B*	*vehicle discharge at intersection A* *pcu*	*vehicle arrival at intersection B* *pcu*	*vehicle departure at intersection B* *pcu*	*queue at stopline B* *pcu*
1	G	G	4		0	0
2	G	G	4		0	0
3	G	G	4		0	0
4	G	G	4		0	0
5	G	G	4	1.32	1.32	0
6	G	G	4	2.20	2.20	0
7	G	G		2.79	2.79	0
8	G	G		3.19	3.00	0.19
9	G	G		3.46	3.00	0.65
10	G	G		3.64	3.00	1.29
11	R	R		2.44	0	3.73
12	R	R		1.63	0	5.36
13	R	R		1.09	0	6.45
14	R	R		0.73	0	7.18
15	R	R		0.49	0	7.67
16	R	R		0.33	0	8.00
17	R	R		0.22	0	8.22
18	R	R		0.15	0	8.37
19	R	R		0.10	0	8.47
20	R	R		0.07	0	8.54
21	G	G	4		3.00	5.54
22	G	G	4		3.00	2.54
23	G	G	4		2.54	0
24	G	G	4		0	0
25	G	G	4	1.32	1.32	0
26	G	G	4	2.20	2.20	0
27	G	G		2.79	2.79	0
28	G	G		3.19	3.00	0.19
29	G	G		3.46	3.00	0.65
30	G	G		3.64	3.00	1.29
31	R	R		2.44	0	3.73
32	R	R		1.63	0	5.36
33	R	R		1.09	0	6.45
34	R	R		0.73		7.18
35	R	R		0.49		7.67
36	R	R		0.33		8.00

one cycle

The shape of the dispersed platoon is now calculated in Table 27.1 using the platoon dispersion relationship and the assumption that the platoon leader has a travel time of 0.8 of the average time of travel.

Column 1 lists the time intervals, each interval being equal to 6 s. The effective red and green indications at intersections A and B are indicated in columns 2 and 3. The discharge of vehicles at intersection A is shown in column 4 and occurs during the first 6 time intervals of the green period. The calculated dispersed platoon arrival rate at intersection B is shown in column 5. Column 6 is the discharge rate at intersection B, calculated from the saturation flow, and the queue at the stopline which is the cumulative difference between vehicle arrivals and departures is shown in column 7.

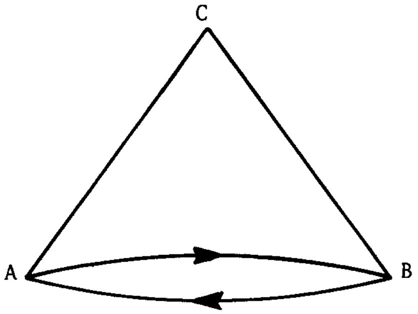
A typical cycle is selected from Table 27.1 and the average delay during this cycle calculated from the sum of the queue lengths multiplied by the real time value of the intervals and divided by the total number of vehicles which passed through the intersection during the cycle.

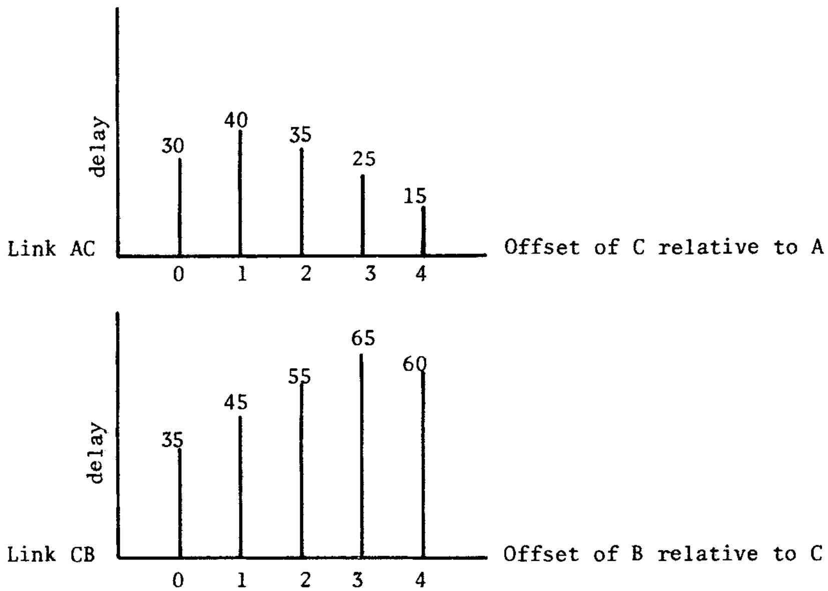
Considering one cycle

$$\text{average delay} = \frac{\Sigma\text{queue length x 6 s}}{\text{no. of vehicles}} = \frac{82.2 \times 6}{24} = 20.6 \text{ s}$$

EXAMPLE 28 Combination Method of Minimising Delays in Traffic Signal Controlled Networks

State the assumptions made in the Combination Method of optimising offsets in a network of traffic signal controlled intersections. For the network of highway links and the delay offset relationships shown in Figure 28.1 determine the offsets which minimise delays throughout the network.





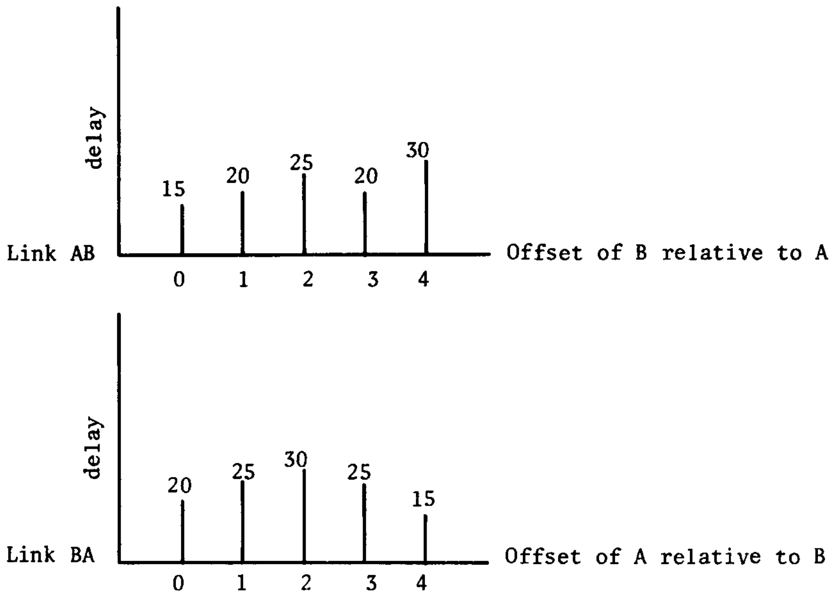


Figure 28.1

Solution

The assumptions made by the Combination Method are

(1) the settings of the signals do not affect the traffic flow or the assignment of traffic to the individual links,

(2) all signals have a common cycle time, or some sub-multiple of the common cycle,

(3) at each intersection the distribution of green times among the stages is known,

(4) the delay to vehicles on any link depends solely on the difference between the settings of the signals at each end of the link.

First step in the Combination Method is the parallel combination of the delays on the link AB, carried out in Table 28.1.

The second step in the Combination Method is the series combination of the links AC and CB into one link, this is carried out in Table 28.2.

Table 28.1 Parallel combination for link AB

offset of B rel. to A	*offset of A rel. to B*	*delay*
0	0	15 + 20 = 35
1	4	20 + 15 = 35
2	3	25 + 25 = 50
3	2	20 + 30 = 50
4	1	30 + 25 = 55

Table 28.2 Series combination of links AC and CB

offset of C rel. to A	*offset of B rel to C*	*delay*	
0	0	30 + 35 = 65	offset of B rel to A zero
1	4	40 + 60 = 100	
2	3	35 + 65 = 100	
3	2	25 + 55 = 80	
4	1	15 + 45 = 60*	
0	1	30 + 45 = 75	offset of B rel to A one
1	0	40 + 35 = 75	
2	4	35 + 60 = 95	
3	3	25 + 65 = 90	
4	2	15 + 55 = 70*	
0	2	30 + 55 = 85	offset of B rel to A two
1	1	40 + 45 = 85	
2	0	35 + 35 = 70*	
3	4	25 + 60 = 85	
4	3	15 + 65 = 80	
0	3	30 + 65 = 95	offset of B rel to A three
1	2	40 + 55 = 95	
2	1	35 + 45 = 80	
3	0	25 + 35 = 60*	
4	4	15 + 60 = 75	
0	4	30 + 60 = 90	offset of B rel to A four
1	3	40 + 65 = 105	
2	2	35 + 55 = 90	
3	1	25 + 45 = 70	
4	0	15 + 35 = 50*	

The combination of links AC and CB which produce minimum delays are taken from Table 28.2 and given in Table 28.3.

Table 28.3 Delay offset relationship for combined links AC and CB

offset of B rel. to A	*delay*
0	65
1	70
2	70
3	60
4	50

The combination of links AC and CB and the link AB can now be combined by addition giving the delay/offset relationship for all links combined shown in Table 28.4.

Table 28.4 Delay offset relationship for the network

offset of B rel. to A	*combined delay*
0	65 + 35 = 100
1	70 + 35 = 105
2	70 + 50 = 120
3	60 + 50 = 110
4	50 + 55 = 105

From Table 28.4 it can be seen that minimum delay occurs when the offset of B relative to A is 0. From Table 28.2 it can be seen that when the offset of B relative to A is 0 then minimum delay occurs when the offset of C relative to A is 4.

EXAMPLE 29 Costs and Benefits of Highway Traffic Flow

The following information relates to traffic flow on a highway

Speed/flow relationship $v = 54 - \frac{14(Q - 300)}{1000}$ km/h

marginal resource cost of travel $= 5 + \frac{40}{v} + 0.0001v^2$ p/km/veh

value of vehicle occupant's time 200 p/vehicle/h
benefit of trip 12 p/vehicle/km

Determine:

(a) the approximate highway flow which maximises the net benefit from the operation of the highway.

(b) the maximum highway flow when the costs and benefits of individual trips are considered.

Illustrate the relationships between individual costs and benefits and net benefits from the operation of the highway.

Solution

The calculation will be carried out in a tabular manner as shown in Table 29.1 where costs and benefits for a 1 km length of highway are calculated.

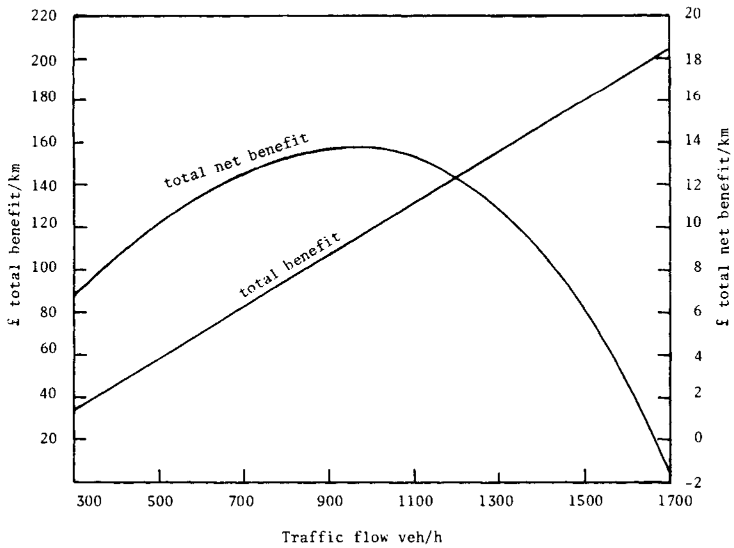


Figure 29.1

Table 29.1

1	2	3	4	5	6	7	8
flow	*speed*	*marginal resource cost*	*travel time cost*	*total cost*	*total cost*	*benefit of trips*	*net benefit of trips*
veh/h	*km/h*	*p/km/veh*	*p/km/veh*	*p/km/veh*	*£/km*	*£/km*	*£/km*
300	54.2	6.03	3.70	9.73	29.19	36	6.81
400	52.6	6.04	3.80	9.84	39.36	48	8.64
500	51.2	6.04	3.01	9.95	49.75	60	10.25
600	49.8	6.05	4.02	10.07	60.42	72	11.58
700	48.4	6.06	4.13	10.19	71.33	84	12.67
800	47.0	6.07	4.26	10.33	82.64	96	13.36
900	45.6	6.09	4.39	10.48	94.32	108	13.68
1000	44.2	6.10	4.52	10.62	106.20	120	13.80
1100	42.8	6.12	4.67	10.79	118.69	132	13.31
1200	41.4	6.14	4.83	10.97	131.64	144	12.26
1300	40.00	6.16	5.10	11.66	145.08	156	10.92
1400	38.6	6.19	5.18	11.37	159.18	168	8.82
1500	37.2	6.21	5.38	11.59	173.85	180	6.15
1600	35.8	6.25	5.59	11.84	189.44	192	2.56
1700	34.4	6.28	5.81	12.09	205.53	204	-1.53

Column 2 is calculated from the speed/flow relationship
Column 3 is calculated from the marginal resource cost of travel
Column 4 is calculated from the value of vehicle occupant's time divided by column 2
Column 5 is the sum of columns 3 and 4
Column 6 is the product of columns 1 and 5
Column 7 is the product of column 1 and the benefit of a trip/vehicle/km
Column 8 is column 7 minus column 6

It can be seen from Table 29.1 that when the net benefit of trips per km are considered the flow which maximises net benefits is approximately 1000 vehicles per hour. Traffic flows greater than this could be expected because at a flow of 1000 vehicles per hour the total cost of a trip (column 5) is 10.62 pence per kilometre per vehicle and the benefit of a trip is 12 pence per kilometre per vehicle.

When only the costs and benefits of individual trips on the highway are considered then the resulting maximum flow is approximately 1700 vehicles per hour. At this flow the total cost of a trip (column 5) is 12.09 pence per kilometre and the benefit of a trip 12 pence per kilometre.

EXAMPLE 30 Economic Assessment of Highway Improvements

It is proposed to improve an existing highway by realignment, the provision of dual carriageways and junction improvements. The scheme is expected to take 2 years to complete with the major part of the construction operation being completed in the first year; traffic can be expected to benefit from the improvement in the second year of construction.

Assuming that the items of benefit are time and accident cost savings, and the items of cost are constructional costs and the increased maintenance costs calculate for 10 years of operation,

(1) the Net Present Value of the scheme
(2) the NPV/C ratio
(3) the First Year Rate of Return.

Given:

Accident rates: existing road $0.85/10^6$ vehicle km.
improved road $0.25/10^6$ vehicle km.

Average cost of an accident £5410.
Average value of vehicle time £1.50 per hour.
Length of existing road 10 km.
Length of improved road 11 km.
Average speed on existing road 40 km/h.
Average speed on improved road 80 km/h.
Discount rate 7 per cent.
Average vehicle operating cost = $2 + 30/v + 0.00006\ v^2$ p/km.

year	*predicted flow*		*construction cost* £	*maintenance cost* £
	unimproved million v.km/year	*improved million v.km/year*		
0			750000	
1	20	25	100000	
2	21	26		10000
3	22	27		10000
4	23	28		10000
5	24	29		10000
6	25	30		10000
7	26	31		50000
8	27	32		10000
9	28	33		10000
10	29	34		10000

Solution

The reduction in accident costs per year due to the improved road can be calculated from

accident cost (0.85 flow per year on unimproved road - 0.25 flow per year on improved road)

the resulting accident cost saving is given in column 2 of Table 30.1.

The reduction in vehicle operating cost due to the road improvement is now calculated.

Vehicle operating cost on unimproved road $= 2 + 30/40 + 0.00006 \times 40^2$
$= 2.846$ p/km

Vehicle operating cost on improved road $= 2 + 30/80 + 0.00006 \times 80^2$
$= 2.759$ p/km

Table 30.1 Calculation of total discounted benefits

1	*2*	*3*	*4*	*5*	*6*
Year	*accident cost saving*	*travel time saving*	*operating cost saving*	*total benefits*	*discounted benefits*
	£	£	£	£	£
0					
1	58 157.50	281 250.00	-120 550.00	218 857.50	204 539.71
2	61 403.50	300 000.00	-119 680.00	241 723.50	211 130.66
3	64 649.50	318 750.00	-118 810.00	264 589.50	215 983.84
4	67 895.50	337 500.00	-117 940.00	287 455.50	219 298.42
5	71 141.50	356 625.00	-117 070.00	310 696.50	221 522.31
6	74 387.50	375 00.00	-116 200.00	333 187.50	222 016.89
7	77 633.50	393 750.00	-115 330.00	356 053.50	221 732.22
8	80 879.50	412 500.00	-114 460.00	378 919.50	220 534.59
9	84 125.50	431 250.00	-113 590.00	401 785.50	218 544.69
10	87 371.50	450 000.00	-112 720.00	424 651.50	215 871.28
				Σ2	171 174.61

The difference in vehicle operating cost due to the improved road can now be calculated from the given predicted flows and the above operating costs. It should be noted that although travel time and accident savings are normally positive, vehicle operating cost savings may be either positive or negative. This is due to changes in speed which may change the speed on the improved road nearer or further from the operating cost optimum. In addition the improved road may be longer and generate more traffic than the unimproved road. The resulting increase in operating cost is given in column 4 of Table 30.1.

The sum of columns 2, 3 and 4 is the net benefit which is entered in column 5. This net benefit is discounted by multiplying the values in column 5 by

$$\text{net benefit discounted to year } n = \frac{\text{net benefit}}{1.07^n}$$

these discounted values are given in column 6.

These changes in cost due to the improvement of the highway are now calculated in a tabular manner in Table 30.1.

The costs of improving the road are the actual construction costs together with the increased maintenance costs and these costs are also discounted to the year when construction commences. This is now calculated in a tabular manner in Table 30.2.

Table 30.2 Calculation of total discounted costs

year	*construction and maintenance costs*	*discounted costs*
0	750 000	750 000
1	100 000	93 458
2	10 000	8 734
3	10 000	8 163
4	10 000	7 629
5	10 000	7 130
6	10 000	6 663
7	50 000	31 135
8	10 000	5 820
9	10 000	5 439
10	10 000	5 083
		Σ929 254

The Net Present Value of the discounted costs and benefits due to the improvement of the road is given by the subtraction of the sum of the discounted costs from the sum of the discounted benefits, that is, £2 171 174.61 minus £929 254 or £1 241 920.61. As the Net Present Value is positive the scheme is worth carrying out on economic grounds.

To allow the economic advantages of schemes of different sizes to be compared the Net Present Value divided by the present value of the cost is calculated

$$\text{i.e., } \frac{£1\ 241\ 920.61}{£929\ 254} = 1.34$$

A simple method of calculating the economic benefits of construction schemes is the first year rate of return. This is

$$\frac{\text{the net benefits during the first year of operation}}{\text{the capital cost of the scheme}}$$

$$= \frac{204\ 539.71}{850\ 000}$$

$$= 24.06 \text{ per cent}$$

This rate of return indicates that the scheme can be justified on economic grounds because the rate of return is greater than the discount rate.

EXAMPLE 31 Calculation of Highway Operational Costs

The following average hourly traffic flows were predicted for the design year on a single two-way two lane carriageway in a semi-urban area.

cars	light goods	other goods	buses	percentage other goods	percentage buses
692	52	34	18	4.3	2.3

The highway has the following characteristics; 0.5 major intersections/km; 27.5 minor intersections and drives/km; 50 per cent of roadside is developed; length of link 15 km. Using the following information calculate the yearly highway operational costs.

$$V = V_{co} + S\left(\frac{Q - 300}{1000}\right) \text{ km/h} \tag{31.1}$$

$$V_o = 50 - 10(I - 0.8) - 3(A - 27.5)/20 \text{ km/h} \tag{31.2}$$

$$S = -25 - 4(V_o - 50)/3 - 30(I - 0.8) - 2(B - 65)/5 \text{ km/h} \tag{31.3}$$

$$\frac{V_o}{V_{co}} = \frac{100}{102 - 2(P_h + P_b)/15} \tag{31.4}$$

Table 31.1 Proportions of the mean hourly flow level for four vehicle classes assumed within the four flow groups

flow group	*number of hours represented*	*vehicle classes*			
		cars	*van light*	*other goods*	*buses*
1	3800	0.25	0.25	0.25	0.25
2	3420	1.213	1.575	1.575	1.575
3	1160	2.135	1.575	1.575	1.575
4	380	3.120	1.575	1.575	1.575

The value of occupants time per vehicle is given by

average car	131.3 pence/h
light goods vehicle	205.4 pence/h
other goods vehicle	213.6 pence/h
public service vehicle	898.8 pence/h.

The combined marginal resource costs for the operation of vehicles is given by

$$C = a + \frac{b}{V} + cV^2 \tag{31.5}$$

where C is the cost in pence/km
V is the average link speed (km/h).

The values of a, b and c for the four vehicle classes are

Table 31.2 Vehicle operating cost parameters at 1976 prices

	a	*b*	*c*
car	1.81	25.08	0.000056
light goods	3.32	33.61	0.000074
other goods	7.80	45.82	0.000127
buses	10.71	71.62	0.000149

Solution

From equation 31.2 where I is the density of major intersections and A is the density of minor intersections

$$V_o = 50 - 10(0.5 - 0.8) - 3(27.5 - 27.5)/20 = 53 \text{ km/h}$$

From equation 31.3 where B is the percentage of roadside which is developed

$$S = - 25 - 4(53 - 50)/3 - 30(0.5 - 0.8) - 2(50 - 65)/5 = -14\text{km/h}$$

When considering the traffic flow on a highway considerable variations from the average hourly flow occur throughout the year. In the Department of Transport method using the COBA program the total number of hours in the year are divided into 4 groups with varying proportions of the vehicle classes as shown in Table 31.1.

Using the flows for the design year, the hourly flow levels for the vehicle classes within the flow groups are calculated and given in Table 31.3.

Table 31.3 Traffic flow levels during the design year by vehicle class

flow group	number of hours represented	cars	light goods	other goods	buses	total flow	percentage other goods	percentage buses
1	3800	173.0	13.0	8.5	4.5	199	4.3	2.3
2	3420	839.4	81.9	53.6	28.4	1003	5.4	2.8
3	1160	1477.4	81.9	53.6	28.4	1641	4.6	2.4
4	380	2159.0	81.9	53.6	28.4	380	2.3	1.2

Before the journey speed on the road can be predicted using equation 31.1 it is necessary to correct the value of V_o, the speed at a flow of 300 vehicle/hour/standard lane for the non-standard composition of traffic. This is carried out using the relationship given by equation 31.4, i.e.

$$V_{co} = \frac{102 - 2(P_h + P_b)/15}{100} \times 53 \text{ km/h}$$

and the calculated values entered in Table 31.4.

Table 31.4 Corrected free speeds

flow group	V_{co} km/h
1	53.6
2	53.5
3	53.6
4	53.8

The journey speed can now be calculated using equation 31.1 and assuming equal flows in each direction so that Q is half the total flow, these journey speeds are calculated and entered in Table 31.5.

Table 31.5 Journey speeds for design year

flow group	journey speed V_{co} km/h
1	53.6
2	50.7
3	46.3
4	41.7

The costs of vehicles travelling on the highway for a period of one year is composed of the cost of vehicle occupant's time and the marginal resource cost of fuel, oil, tyres, maintenance and depreciation.

The occupant's time cost for the 15 km section of highway being considered is calculated for each vehicle class in flow group 1 of Table 31.1.

The total time costs for flow group 1 during the design year, based on the given prices is

$$\text{hours represented} \ \frac{\text{length of highway}}{\text{journey speed}} \ \Sigma \ \begin{array}{l}\text{hourly flow x time cost}\\ \text{per vehicle}\end{array}$$

$$3800 \ \frac{15}{53.6} \ (173.0 \times 131.3 + 13.0 \times 205.4 + 8.5 \times 213.6 + 4.5$$

$$\times \ 898.8)\text{p}$$

$$= £332\ 272.77$$

Similarly for flow group 2

$$3420 \ \frac{15}{50.7} \ (839.4 \times 131.3 + 81.9 \times 205.4 + 53.6 \times 213.6 + 28.4$$

$$\times \ 898.8)\text{p}$$

$$= £1\ 659\ 513.10$$

Similarly for flow group 3

$$1160 \ \frac{15}{46.3} \ (1477.4 \times 131.3 + 81.9 \times 205.4 + 53.6 \times 213.6 +$$

$$28.4 \times 898.8)\text{p}$$

$$= £931\ 180.95$$

Similarly for flow group 4

$$380 \ \frac{15}{41.7} \ (2159.0 \times 131.3 + 81.9 \times 205.4 + 53.6 \times 213.6 +$$

$$28.4 \times 898.8)\text{p}$$

$$= £461\ 021.79$$

Total operating costs for the 15 km length of highway for the design year is £3 383 988.61.

The marginal resource costs for the operation of the highway for the design year at the given prices will now be calculated for each of the vehicle classes.

From equation 31.5 and Table 31.2 the resource costs of car operation in flow group 1 is

$$1.81 + 25.08/5.36 + 0.000056 \times 53.6^2 \text{ or } 2.44 \text{ pence/km.}$$

Similarly for flow groups 2, 3 and 4 the resource costs of car operation are 2.45, 2.47 and 2.51 pence/km respectively.

The resource cost for flow group 1 of light goods vehicle operation is

$$3.32 + 33.61/53.6 + 0.000074 \times 53.6^2 \text{ or } 4.16 \text{ pence/km.}$$

Similarly for flow groups 2, 3 and 4 the resource costs of light goods vehicle operation are

4.17, 4.20, 4.25 pence/km respectively.

The resource cost for flow group 1 of other goods vehicle operation is

$$7.80 + 45.82/53.6 + 0.000127 \times 53.6^2 \text{ or } 9.02 \text{ pence/km.}$$

Similarly for flow groups 2, 3 and 4 the resource costs of other goods vehicle operation are

9.03, 9.06 and 9.12 pence/km respectively.

The resource cost for flow group 1 of bus operation is

$$10.71 + 71.62/53.6 + 0.000149 \times 53.6^2 \text{ or } 12.47 \text{ pence/km.}$$

Similarly for flow groups 2, 3 and 4 the resource costs of bus operation are

12.51, 12.64 and 12.69 pence/km.

For each flow group the resource cost of vehicles travelling on the highway is

$$\text{hours represented} \times \text{length of highway} \sum_{\text{vehicle classes}} \text{hourly flow} \times \text{resource cost/km}$$

For flow group 1
Resource cost of vehicle operation is

$$3800 \times 15(173.0 \times 2.44 + 13.0 \times 4.16 + 8.5 \times 9.02 + 4.5 \times 12.47)\text{p}$$

$$= £347\ 121.45$$

For flow group 2

$$3420 \times 15(839.4 \times 2.45 + 81.9 \times 4.17 + 53.6 \times 9.03 + 28.4 \times$$

12.51)p

= £1 660 757.90

For flow group 3

1160 x 15(1477.4 x 2.47 + 81.9 x 4.20 + 53.6 x 9.06 + 28.4 x

12.64)p

= £757 271.31

For flow group 4

380 x 15(2159.0 x 2.51 + 81.9 x 4.25 + 53.6 x 9.12 + 28.4 x

12.69)p

= £357 294.12

Total resource cost of all vehicle operation during the design year on a 15 km length of highway is £3 122 444.78.

Total resource cost and time costs for the 15 km length of highway during the design year at the given prices is £6 506 433.39.

EXAMPLE 32 Simulation of Highway Headway Distributions

In the computer simulation of highway traffic flow it is desired to simulate the time intervals between vehicle arrivals at a point on the highway. Assume the traffic flow is a single one-way stream of vehicles and that the highway conditions are

(a) a rural situation, flow 300 veh/h, traffic control devices are not present for at least 1 km upstream or downstream of the point on the highway being considered

(b) an urban situation, flow 800 veh/h, traffic control devices are present upstream of the point on the highway being considered.

Solution

(a) In a rural situation such as described and with a low traffic flow it can be expected that the vehicles will arrive at the point on the highway in a random manner. When this occurs then the intervals between vehicle arrivals or the time headways may be represented by the negative exponential distribution (see Example 6), then

$$\text{Probability of a headway} > t = \exp -(q\ t) \qquad (33.1)$$

where q is the vehicle arrival rate.

These headways may be simulated by the use of a random number technique in which the computer generates pseudorandom numbers between 0 and 1 which are inserted in equation 33.1. The repeated solution of this equation gives values of headways which form a negative exponential distribution.

This operation may be very simply programmed in Basic and the following program can be run on a Hewlett-Packard Personal Computer. Rearranging equation 33.1 and substituting for q the vehicle arrival rate in vehicles per second

```
X = RND
T = LOG(X)*3600/300
```

The function RND generates a random number with equal probability between 0 and 1. The function may be specified by other names in other types of computer but most computers provide this facility.

When incorporated into a simulation program the statements will generate a series of headways to a negative exponential distribution. Whenever the program is run the same series of random numbers will be generated and this can produce an artificial situation which is not found on highways when separate observations of headways at the same traffic volume are made. It is possible to avoid this difficulty by incorporating the statement RANDOMIZE into the program. Thus generalising the traffic flow

```
DISP"TRAFFIC VOLUME VEH/HOUR";
INPUT Q
RANDOMIZE
X=RND
T=-LOG(X)*3600?Q
PRINT"HEADWAY EQUALS" T
END
```

would generate a new series of pseudorandom numbers and headways every time the program is run.

(b) In an urban situation with a high traffic flow then congested traffic conditions can be expected with bunched vehicles. In such a situation the double exponential distribution can be expected to model observed headways. This distribution is simple to simulate and has the added advantage that the degree of congestion may be varied (see Example 7). Using this distribution then

$$\text{Probability of a headway} > t = r \exp - \left[\frac{t-c}{t_1-c}\right] + (1-r) \exp - \left[\frac{t}{t_2}\right] \quad (33.2)$$

$t > c$

$$= r + (1-r) \exp - \left[\frac{t}{t_2}\right]$$

$t < c$

where r is the proportion of restrained vehicles, t_1 and t_2 are the mean headways of restrained and unrestrained vehicles and c is the minimum headway between following vehicles.

The first term represents the headways between those vehicles which are following the preceding vehicle, i.e. restrained vehicles whilst the second term represents the free flowing vehicles, i.e. those unrestrained by the presence of the preceding vehicle either because of distance, speed differences or by the act of overtaking.

The simulation of headways to this distribution may be achieved by simulating the restrained and unrestrained vehicles separately. The proportion of restrained vehicles is r and initially a pseudo-random number is generated, if this number is less than or equal to r then a restrained headway is generated. If the pseudo-random number is greater than r then an unrestrained headway is generated. The following statements would generate headways as required, using upper case characters for the appropriate lower case ones in equation 33.2.

```
10   DISP"PROPORTION OF RESTRAINED VEHICLES";
20   INPUT R
30   DISP"TRAFFIC VOLUME VEH/HOUR";
40   INPUT Q
50   DISP"MEAN HEADWAY RESTRAINED VEHICLES";
60   INPUT T2
70   T1=(3600/Q-(1-R)*T2)/R
80   RANDOMIZE
90   Y=RND
100  IF Y<=R THEN 140
110  X=RND
120  T=T2*LOG(X)
130  GOTO 160
140  Z=RND
150  T=C-(T1-C)*LOG(Z)
160  PRINT"HEADWAY EQUALS"T
170  END
```

The program has been generalised to allow the generation of headways for a range of traffic volumes. In the absence of any further information regarding the traffic flow characteristics a value of 0.75 would be input for R as congestion and bunching is present. Experience also indicates that a value of 2.5 s would be appropriate for T2. With Q, R and T2 determined then T1 is obtained from the connection between traffic volume and mean headway

```
Q=3600/(R*T1+(1-R)*T2)
```

EXAMPLE 33 Simulation of Delay at Highway Priority Intersections

Write a program in Basic to simulate the traffic flow and determine the average delay to minor road vehicles at a priority intersection with the following highway and traffic conditions.

(a) The major road flow is in a single stream and may vary from low flow non-congested to highflow congested conditions

(b) The minor road flow is in a single stream, all vehicles turn left from the minor into the major road, the flow may vary from low flow non-congested to high flow congested conditions.

(c) The lag and lag acceptance of minor road drivers may be represented by a uniform distribution with a variation of plus or minus half the mean value.

(d) The minor road traffic flow rises instantaneously from zero to a maximum peak value which is maintained until the end of the peak period T2 when it falls instantaneously to zero.

The variations in minor road flow described above are illustrated in Figures 34.1 and 34.2.

Solution

The program given below has been written in Basic for the Hewlett-Packard personal computer. It consists of a series of instructions for vehicle movements which are updated by the successive repetition of the program at 0.5s intervals of real time.

For ease of explanation the program will be divided into sections and each section followed by an explanation.

(a) Input and record details

```
10    DISP "PRIORITY JUNCTION PROGRAM"
20    PRINT "PRIORITY JUNCTION PROGRAM"
30    WAIT 3000
40    DISP "PEAK TIME EQUALS";
50    INPUT T2
60    PRINT PEAK TIME EQUALS" T2
70    DISP "MAJOR ROAD T1 EQUALS";
80    INPUT M1
90    PRINT MAJOR ROAD T1 EQUALS" M1
100   DISP "MAJOR ROAD T2 EQUALS";
110   INPUT M2
```

```
120     PRINT "MAJOR ROAD T2 EQUALS" M2
130     DISP "MINOR ROAD T1 EQUALS";
140     INPUT S1
150     PRINT "MINOR ROAD T1 EQUALS" S1
160     DISP "MINOR ROAD T2 EQUALS";
170     INPUT S2
180     PRINT "MINOR ROAD T2 EQUALS" S2
190     DISP "RESTRAINED PROPORTION MAJOR ROAD EQUALS";
200     INPUT R1
210     PRINT "RESTRAINED PROPORTION MAJOR ROAD EQUALS" R1
220     DISP "RESTRAINED PROPORTION MINOR ROAD EQUALS;
230     INPUT R2
240     PRINT "RESTRAINED PROPORTION MINOR ROAD EQUALS" R2
250     DISP "MINIMUM EXIT TIME EQUALS";
260     INPUT E
270     PRINT "MINIMUM EXIT TIME EQUALS" E
280     DISP "MEAN ACCEPTABLE GAP";
290     INPUT A1
300     PRINT "MEAN ACCEPTABLE GAP EQUALS" A1
310     DISP "MAJOR MIN HEADWAY"
320     INPUT D1
330     PRINT "MAJOR MIN HEADWAYS EQUALS" D1
340     DISP "MINOR MIN HEADWAY";
350     INPUT D2
360     PRINT "MINOR MIN HEADWAY EQUALS" D2
```

These statements input the basic information required to generate major and minor road headways to a double exponential distribution in lines 70 to 240 and 310 to 360. The length of the peak period is input as line 50. A further parameter is input, lines 250 to 270, this is the minimum time interval between minor road vehicles entering the major road when the major road flow is zero, corresponding to the parameter $\beta 2$ in Example 12.

(b) Setting of variable parameters to zero.

```
370     Q=0
380     T=0
390     C1=0
400     C2=0
410     X=0
420     Y=0
430     D3=0
440     M=0
```

The parameter Q is the queue length of the minor road which is initially assumed to be zero, T is the realtime which is incremented by 0.5s every time the traffic situation is updated, C1 and C2 are the cumulative sum of the generated headways on the major and minor roads, X and Y are the cumulative totals of vehicles generated on the major and minor roads, D3 is the cumulative delay to minor road vehicles. M is either 0 or 1, when M is 0 it allows the generation of an acceptable gap for a vehicle when it arrives at the Give Way line. Its function is to prevent the successive generation, at each program updating, of a variety of acceptable gaps for a vehicle waiting at the Give Way line.

(c) Program incremental instructions

```
450    D3=D3+Q
460    T=T+0.5
```

Each time the program repeats itself the cumulative delay in terms of the sum of the queue lengths is calculated by statement 450. Every time the program repeats itself the time is incremented by 0.5s.

(d) Printing instructions

```
470    IF T/50-INT(T/50)>0.99 OR T/50-INT(T/50)<0.01 THEN 490
480    GOTO 310
490    PRINT "TIME="T
500    PRINT "MAJOR ROAD FLOW'"X
510    PRINT "MINOR ROAD FLOW-"Y
520    PRINT "QUEUE="Q
530    PRINT "DELAY="D3*0.5/Y
```

Every 50s of real time (statement 470) the program prints out the real time T (statement 490), the number of vehicles which have arrived on the major and minor roads (statements 500 and 510), the queue length at time T (statement 520) and the average delay per vehicle which is the sum of the queue lengths observed at 0.5s intervals multiplied by the time interval between observations and divided by the number of minor road vehicles arriving at the intersection (statement 530). If T is not 0s, 50s ... etc. then the program proceeds to statements 310 (statement 480).

(e) Generation of headways

```
540    IF T>T2 and W<0.001 THEN 910
550    IF C1-T  <=0 THEN 580
560    IF C2-T <=0 THEN 412
570    GOTO 810
580    P=RND
590    IF P <=R1 THEN 610
600    GOTO 640
610    P=RND
620    H1=S-(M1-S)*LOG(P)
630    GOTO 660
640    P=RND
650    H1=M2*LOG(P)
660    C1=C1+H1
670    X=X+1
680    GOTO 560
690    IF T  >=T2 THEN 810
700    P=-RND
710    IF P <=R2 THEN 443
720    GOTO 760
```

```
730    P=RND
740    H2=1-(S1-1)*LOG(P)
750    GOTO 780
760    P=RND
770    H2=-S2*LOG(P)
780    C2=C2+H2
790    Y=Y+1
800    Q=Q+1
```

The major and minor road headways are to be generated to a distribution which by the variation of the input parameters can represent a variety of traffic conditions, the double exponential distribution is therefore employed.

The function RND used in the program generates a random number with equal probability between 0 and 1. Most types of personal computer provide this facility but the function may have a different name.

When the real time exceeds the peak time of T2 and the queue Q on the minor road has reached zero, point P on Figure 34.2, then the simulation ends and final print statements are executed (statements 540 and 910). The cumulative major and minor road headways C1 and C2 generated previously are compared with the cumulative real time T (statements 550 and 560) and if a cumulative headway is less than or equal to the real time then a further major road headway (statements 580 to 680) or a minor road headway (statements 700 to 800) is generated. From Figure 34.1 it can be seen that when T is equal to or greater than T2, the flow of minor road vehicles ceases (statements 690). The generation of a headway indicates the arrival of a vehicle and these are totalled by X and Y (statements 670 and 790). The arrival of a minor road vehicle also increases the queue length (statement 800).

(f) The gap acceptance process

```
810    If Q<0.001 THEN 450
820    IF Q-INT(Q)<0.01 or Q-INT(Q)>0.99 THEN 850
830    Q=Q-0.5/E
840    GOTO 450
850    IF M>0.5 THEN 880
860    A=A1/2+RND*A1
870    M=1
880    IF C1-T<A THEN 450
890    Q=Q-0.5/E
900    GOTO 440
```

The gap acceptance procedure only operates when there is a minor road vehicle waiting to enter the major road (statement 810). Minor road vehicles can only enter the major road at intervals of E, the exit time. The program repeats its operation at real time intervals of 0.5s and the queue of minor road vehicles reduces by 0.5/E at each program iteration. If a queue exists then there are two possibilities, firstly that a minor road vehicle is waiting at the stop or give way line or that a vehicle be waiting at the stop line then the vehicle may previously have been assigned an acceptance gap which has been greater than the available headways on the major road or alternatively a minor road vehicle may just have arrived at the stop line and require the generation of an acceptance gap.

Statement 820 determines if a vehicle is waiting or entering the major road, if a vehicle is already entering the major road then the queue is decreased by 0.5/E (statement 830) and the program advances the real time by 0.5s (statement 450). If a vehicle is waiting and has not been assigned an acceptance gap, M equals 0, then an acceptance gap is generated (statement 860). The acceptance gap is then compared with the available gap on the major road (statement 880). If the gap is equal to or greater than the required acceptance gap the minor road vehicle commences to exit, the queue is decreased by 0.5/F (statement 890) and the program advances the real time by 0.5s (statements 900 and 440). If the gap is less than the required acceptance gap then the program returns directly to statement 450. Note that when the program returns to statement 440, M is made equal to 0 so allowing a new acceptance gap to be generated when the queue next has an integer value or when the next minor road vehicle arrives at the stop line.

(g) Final print details

```
910   D3=D3*0.5/Y
920   PRINT "MAJOR ROAD FLOW=" X*3600/T"VEH/H"
930   PRINT "MINOR ROAD FLOW=" Y*3600?T2"VEH/H"
940   PRINT "AVERAGE DELAY=" D3"SEC"
950   PRINT "TOTAL TIME=" T"SEC"
960   END
```

Final print details give the average delay to minor road vehicles which is obtained from the cumulative queue length multiplied by the time period of 0.5s between each observation of the queue length and divided by the number of minor road vehicles generated (statement 910). The major and minor road flow rates are printed (statements 920 and 930) together with the total time required for the queue to become zero (statement 950).

A print out from the running of this program is now given.

```
PRIORITY JUNCTION PROGRAM
PEAK TIME EQUALS 900
MAJOR ROAD T1 EQUALS 0
MAJOR ROAD T2 EQUALS 4
MINOR ROAD T1 EQUALS 0
MINOR ROAD T5 EQUALS 4.5
RESTRAINED PROPORTION MAJOR ROAD EQUALS 0
RESTRAINED PROPORTION MINOR ROAD EQUALS 0
MINIMUM EXIT TIME EQUALS 3
```

```
MEAN ACCEPTABLE GAP EQUALS 4
MAJOR MIN HEADWAY EQUALS 2
MINOR MIN HEADWAY EQUALS 1
TIME= 50
MAJOR ROAD FLOW= 17
MINOR ROAD FLOW= 5
QUEUE= 1
DELAY= 5.6
TIME = 100
MAJOR ROAD FLOW= 35
MINOR ROAD FLOW= 12
QUEUE= 2
DELAY= 8.875
TIME= 150
MAJOR ROAD FLOW= 44
MINOR ROAD FLOW= 22
QUEUE= 4.8
DELAY= 10.15454545
TIME= 200
MAJOR ROAD FLOW= 57
MINOR ROAD FLOW= 34
QUEUE= 9
DELAY= 16.72058824
TIME= 250
MAJOR ROAD FLOW= 69
MINOR ROAD FLOW= 45
QUEUE= 13
DELAY= 23.37777778
TIME= 300
MAJOR ROAD FLOW= 83
MINOR ROAD FLOW= 56
QUEUE= 14
DELAY= 30.71428571
TIME= 350
MAJOR ROAD FLOW= 102
MINOR ROAD FLOW= 69
QUEUE= 20
DELAY= 37.42753623
TIME= 400
MAJOR ROAD FLOW= 114
MINOR ROAD FLOW= 76
QUEUE= 19
DELAY= 47.22368421
TIME= 450
MAJOR ROAD FLOW= 126
MINOR ROAD FLOW= 92
QUEUE= 27
DELAY= 51.34782609
TIME= 500
MAJOR ROAD FLOW= 138
MINOR ROAD FLOW= 107
QUEUE= 35
DELAY= 58.02336449
TIME= 500
```

```
MAJOR ROAD FLOW= 144
MINOR ROAD FLOW= 125
QUEUE= 40
DELAY= 64.896
TIME= 600
MAJOR ROAD FLOW= 155
MINOR ROAD FLOW= 138
QUEUE= 44
DELAY= 74.11231884
TIME= 650
MAJOR ROAD FLOW= 170
MINOR ROAD FLOW= 152
QUEUE= 52
DELAY= 82.97039474
TIME= 700
MAJOR ROAD FLOW= 188
MINOR ROAD FLOW= 162
QUEUE= 53
DELAY= 93.97839506
TIME= 750
MAJOR ROAD FLOW= 197
MINOR ROAD FLOW= 174
QUEUE= 56
DELAY= 103.7643678
TIME= 800
MAJOR ROAD FLOW= 210
MINOR ROAD FLOW= 189
DELAY= 110.4465608
TIME= 850
MAJOR ROAD FLOW= 218
MINOR ROAD FLOW= 197
QUEUE= 54
DELAY= 119.8680203
TIME= 900
MAJOR ROAD FLOW= 227
MINOR ROAD FLOW= 204
QUEUE= 51.2
DELAY= 128.7083333
TIME= 950
MAJOR ROAD FLOW= 240
MINOR ROAD FLOW= 204
QUEUE= 44
DELAY= 140.5465686
TIME= 1000
MAJOR ROAD FLOW= 258
MINOR ROAD FLOW= 204
QUEUE= 36
DELAY= 150.370098
TIME= 1050
MAJOR ROAD FLOW= 273
MINOR ROAD FLOW= 204
QUEUE= 27.4
DELAY= 158.4921569
TIME= 1100
```

```
MAJOR ROAD FLOW= 286
MINOR ROAD FLOW= 204
QUEUE= 21
DELAY= 164.2328431
TIME= 1150
MAJOR ROAD FLOW= 297
MINOR ROAD FLOW= 204
QUEUE= 11
DELAY= 167.7647059
TIME= 1200
MAJOR ROAD FLOW= 306
MINOR ROAD FLOW= 204
QUEUE= 3
DELAY= 169.7573529
MAJOR ROAD FLOW= 930.9328969 VEH/H
MINOR ROAD FLOW= 816 VEH/H
AVERAGE DELAY= 170.0220588 SEC
TOTAL TIME= 1222 SEC
```