

CIVL6415

# TRAFFIC ANALYSIS AND SIMULATION

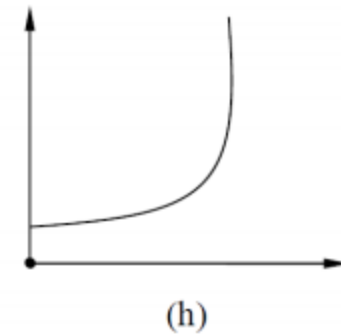
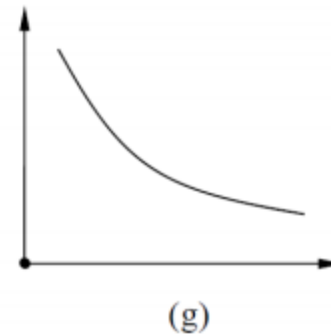
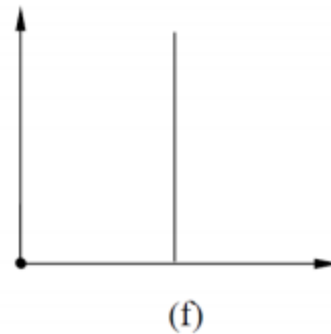
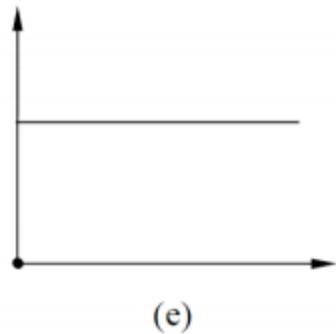
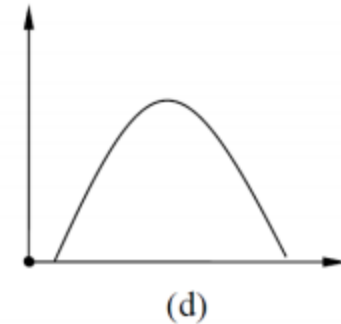
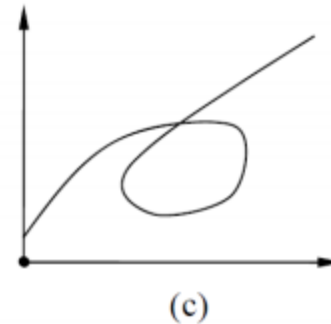
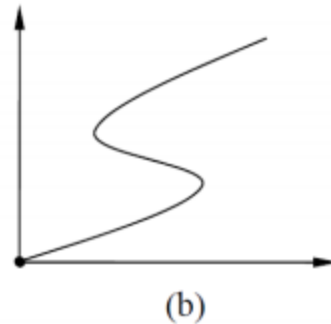
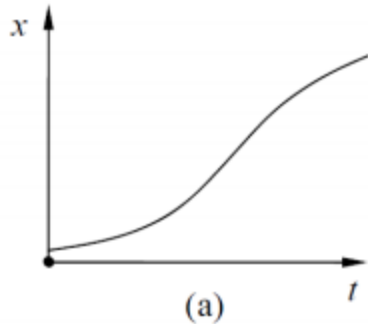
## MODULE 1

### Traffic flow fundamentals

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The University of Queensland

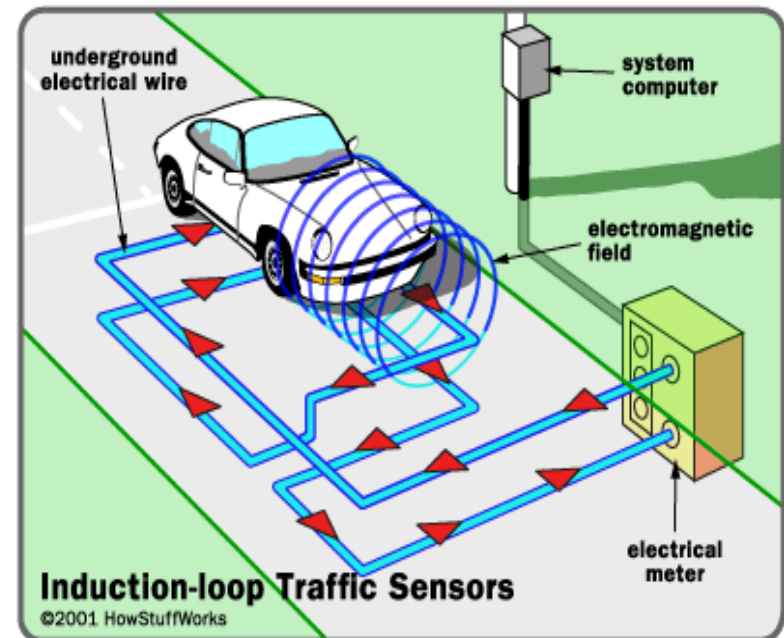
- **Basic Descriptors of Traffic Flow**
  - Flow, Density (Occupancy), Speed
  - Headway, Spacing
- **Relationships between Traffic Flow Parameters**
  - Flow, Density, and Speed
  - Flow vs. Headway
  - Density vs. Spacing
  - Space mean speed vs. Time mean speed
- **Graphical Relationships for Uninterrupted Flow**
  - Speed vs. Density
  - Flow vs. Density
  - Speed vs. Flow
- **Generalized Definitions of Flow, Density and Speed**

# Space-time diagram

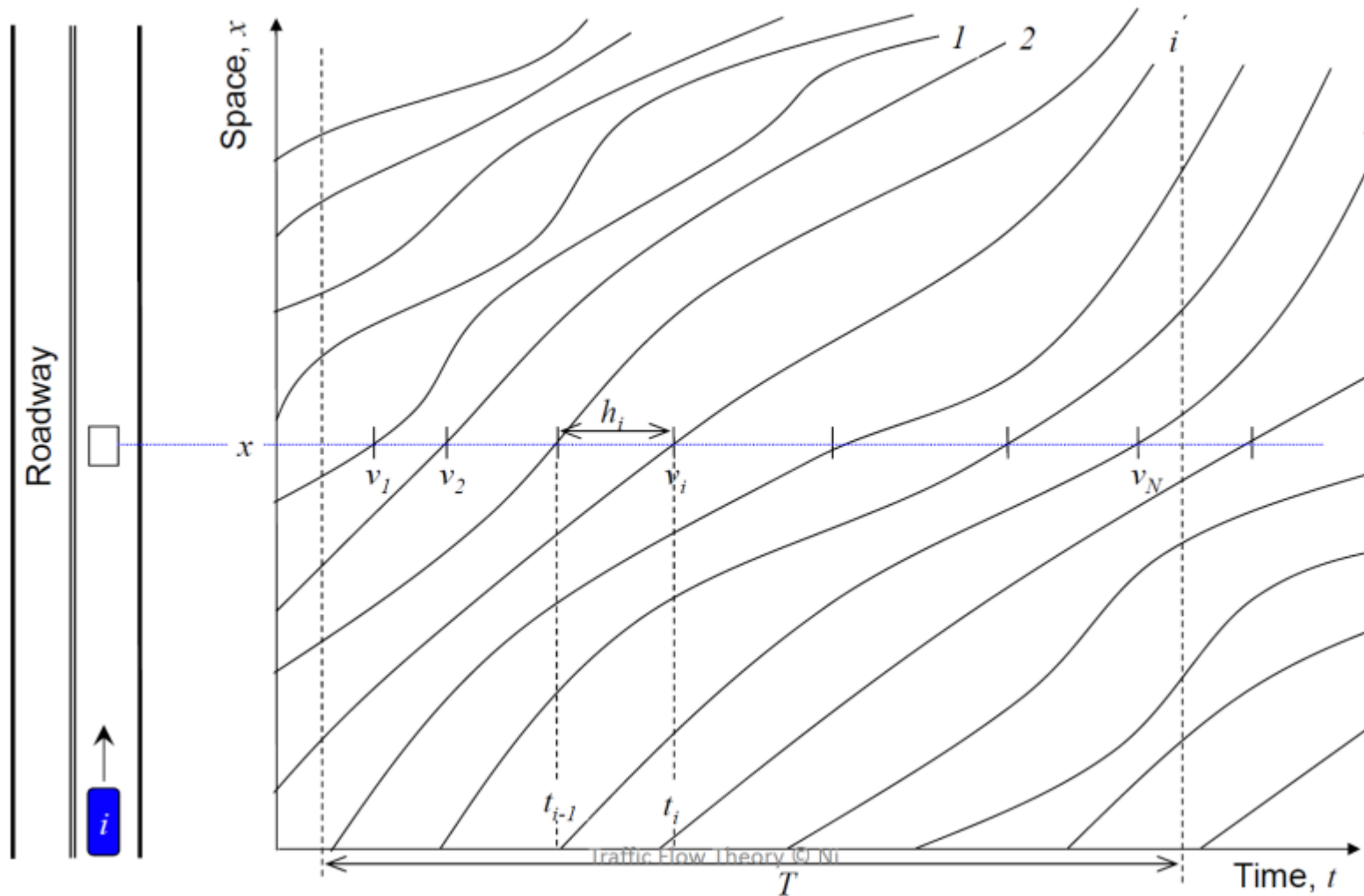


# Point sensors

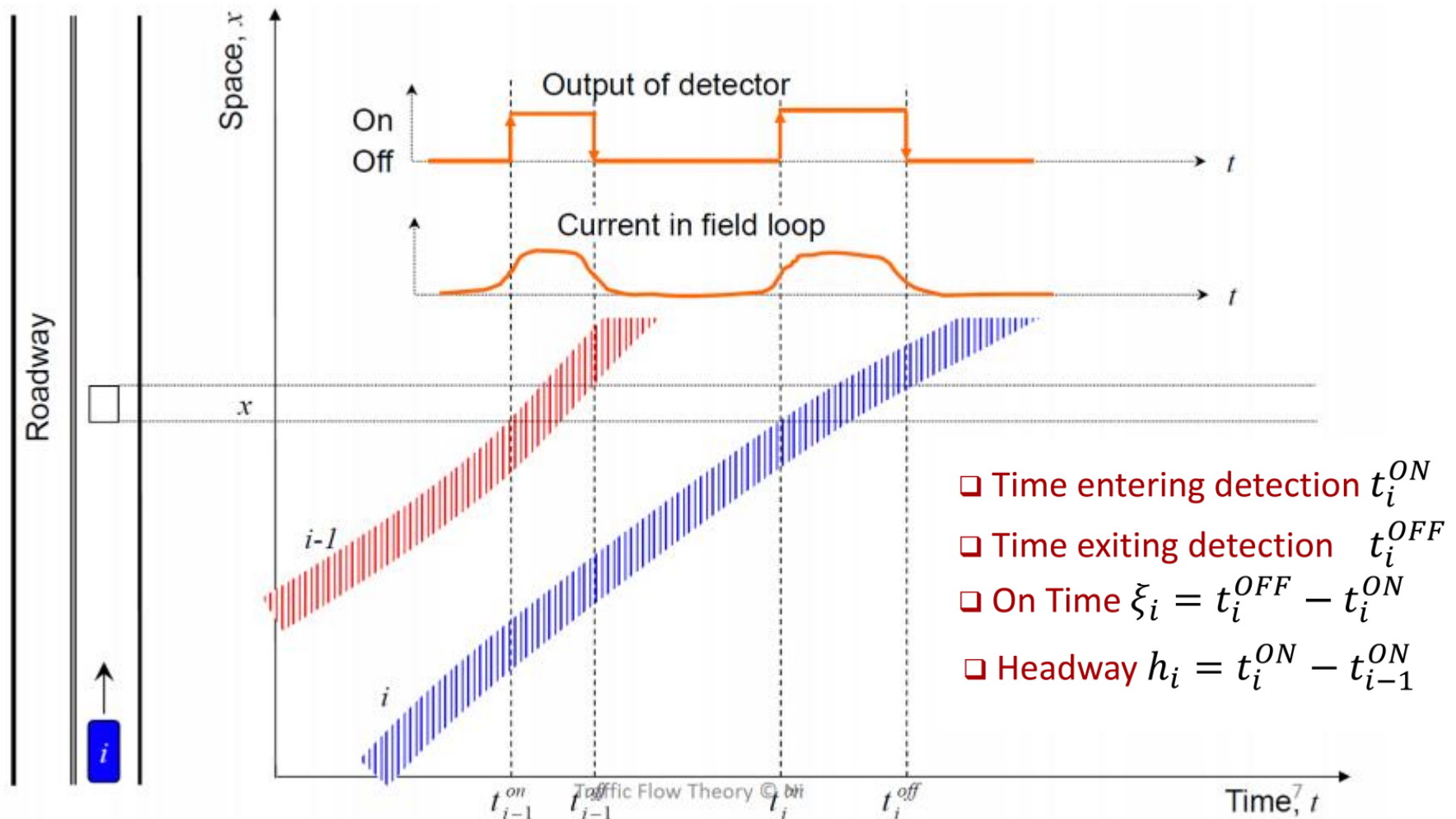
- Inductive loops
  - Coil of wire in pavement
  - Energised by AC
  - Detector senses the 'presence' of vehicle
  - Single loop: volume & occupancy
  - Dual loops: vol, occ & speed
  - typically spaced at 500 m on motorways
  - actuated signals
  - dominant technology (low cost)
- CCTV cameras



# Point sensor data



# Point sensor data





# Basic Descriptors of Traffic Flow

## ■ Three Principal Traffic Flow Descriptors

- Volume ( $q$ )
  - Density ( $k$ )
  - Speed ( $v$ )
  - Headway ( $h$ )
  - Spacing ( $s$ )
- + Occupancy (Occ)



## ▪ Flow ( $q$ ) (also called 'Volume' or 'Flow rate')

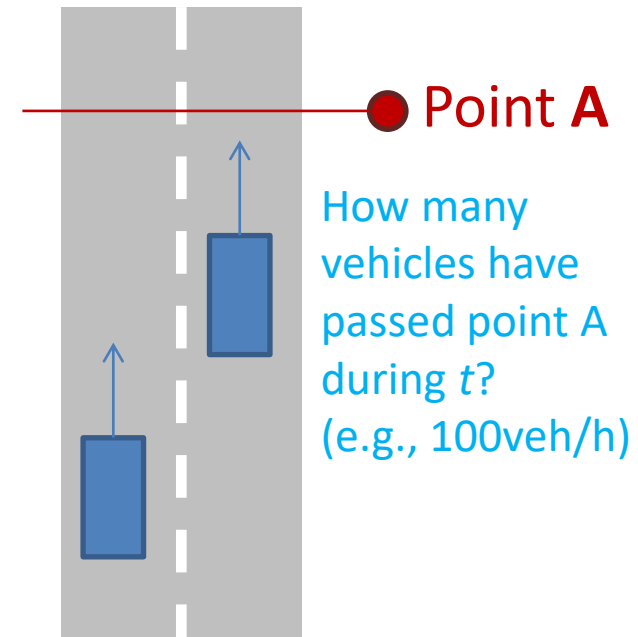
- Number of vehicles per unit time passing a given point on a road
- *vehicles per second (veh/s), vehicles per hour (veh/h), or vehicles per day (veh/d).*

Flow

$$q = \frac{N}{T}$$

where:

- $N$  = number of vehicles passing some designated roadway point during time  $t$ , and
- $T$  = duration of time interval.



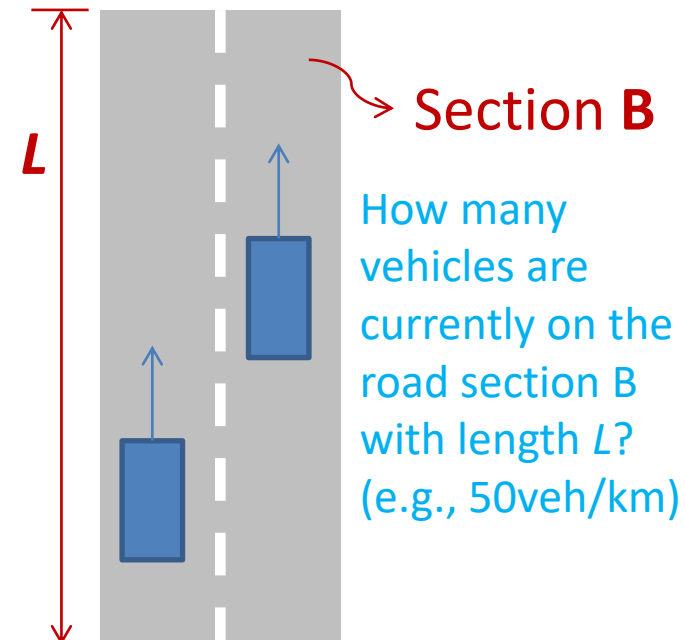
## ■ Density ( $k$ ) (also known as 'Concentration')

- Number of vehicles present within a unit length of lane or road at a given instant of time
- *vehicles per kilometre (veh/km) or vehicles per metre (veh/m)*

$$\text{Density } k = \frac{N}{L}$$

where:

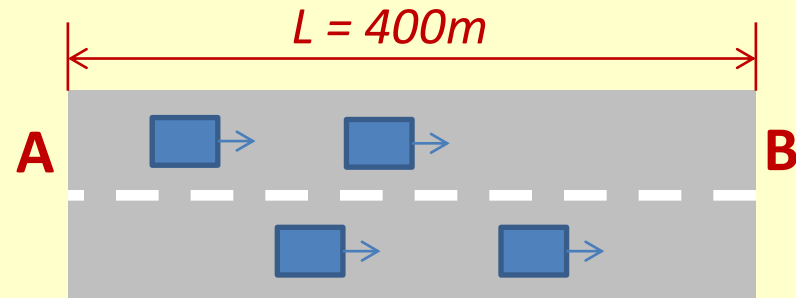
- $N$  = number of vehicles occupying some length of roadway at some specified time, and
- $L$  = length of roadway.



## Question 1

# Flow and Density

- An observer located at point A observes four vehicles passing point A during 30 sec. The figure shows their positions at an instant of time by photography. Calculate flow (veh/h) and density (veh/km).



- A.  $q = 0.13 \text{ veh/h}$ ,  $k = 0.01 \text{ veh/km}$
- B.  $q = 8 \text{ veh/h}$ ,  $k = 1 \text{ veh/km}$
- C.  $q = 240 \text{ veh/h}$ ,  $k = 10 \text{ veh/km}$
- D.  $q = 480 \text{ veh/h}$ ,  $k = 10 \text{ veh/km}$
- E. None of the above

## ▪ Headway (h)

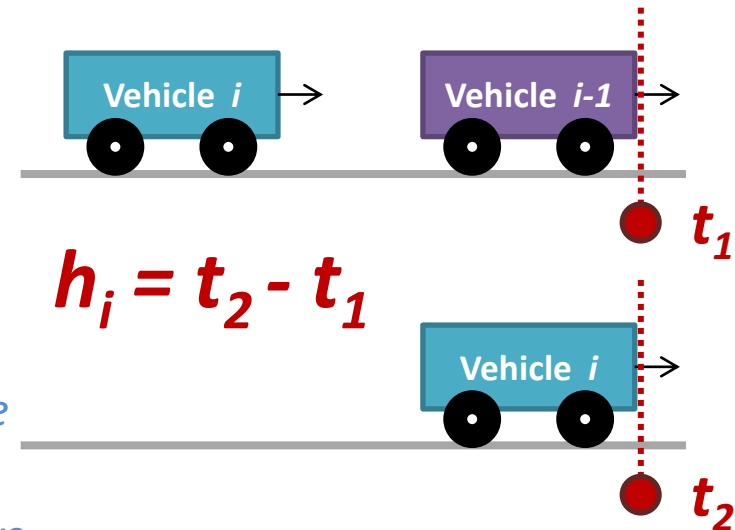
- Time interval separating the passing of a fixed point by two **consecutive vehicles** in a traffic stream, expressed as *seconds per vehicle (s/veh)*
- The average headway (h) of the stream over a given time interval is **the arithmetic mean of the series of headways** occurring over that interval

Average  
headway

$$h = \frac{1}{N} \sum_{i=1}^N h_i$$

where:

- $h_i$  = time headway of vehicle  $i$  (the elapsed time between the arrivals of vehicles  $i$  and  $i-1$ )
- $N$  = number of measured vehicle time headways



## ■ Spacing (s)

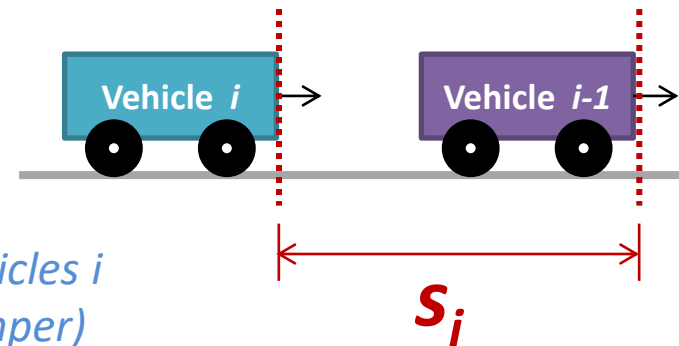
- Distance between the fronts of two consecutive vehicles in a traffic stream at a given instant of time, expressed as *metres per vehicle* (m/veh)
- The average spacing (s) of the stream over a given length of lane or carriageway is the arithmetic mean of the individual spacings occurring over that length at that instant of time

Average  
spacing

$$s = \frac{1}{N} \sum_{i=1}^N s_i$$

where:

- $s_i$  = spacing of vehicle  $i$  (the distance between vehicles  $i$  and  $i-1$ , measured from front bumper to front bumper)
- $N$  = number of measured vehicle spacings



## ■ Speed ( $v$ )

- Distance travelled by a vehicle during a unit of time
- Travel distance / travel time
- *metres per second* (m/s) or *kilometres per hour* (km/h or kph)

## ■ Two Ways in which average speeds can be obtained:

### – Time Mean Speed ( $v_t$ ):

- The average speed for vehicles that travel an equal amount of time
- Average speed over time = Average travel distance / Fixed time

### – Space Mean Speed ( $v_s$ ):

- The average speed for vehicles that travel an equal amount of distance
- Average speed over space = Fixed distance / Average travel time

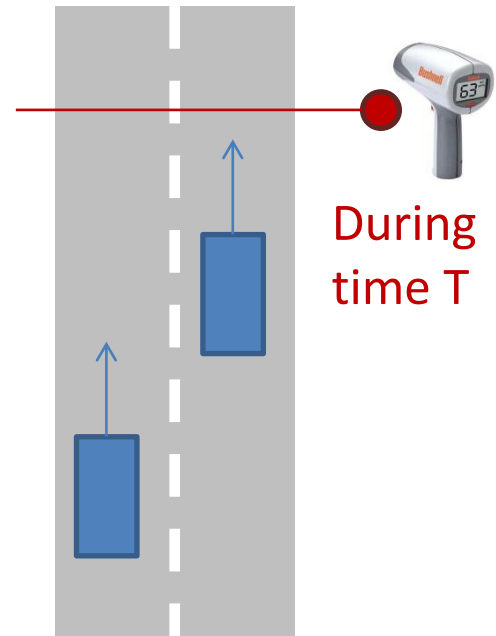
## ■ Time Mean Speed ( $v_t$ )

- Arithmetic mean of the measured speeds of all vehicles passing a given point during a given time interval
- Measured by taking a reference point on the roadway over a fixed period of time  $T$
- Average travel distance / Fixed time  $T$ :

Time mean speed  $v_t = \frac{\frac{1}{N} \sum_{i=1}^N d_i}{T} = \frac{\frac{1}{N} \sum_{i=1}^N T v_i}{T} = \frac{1}{N} \sum_{i=1}^N v_i$

where

- $d_i$  = travel distance of vehicle  $i$  during  $T$
- $v_i$  = individual speed (or 'spot speed') of vehicle  $i$
- $N$  = number of vehicles during  $T$



## ■ Time Mean Speed ( $v_t$ )

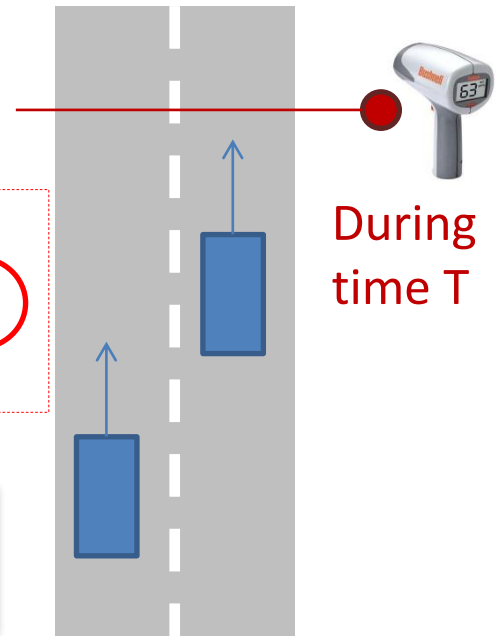
- Arithmetic mean of the measured speeds of all vehicles passing a given point during a given time interval
- Measured by taking a reference point on the roadway over a fixed period of time  $T$
- Average travel distance / Fixed time  $T$ :

Time mean speed  $v_t = \frac{\frac{1}{N} \sum_{i=1}^N d_i}{T} = \frac{\frac{1}{N} \sum_{i=1}^N T v_i}{T} = \frac{1}{N} \sum_{i=1}^N v_i$

where

- $d_i$  = travel distance
- $v_i$  = individual speed
- $N$  = number of vehicles during  $T$

**Arithmetic mean of spot speeds**





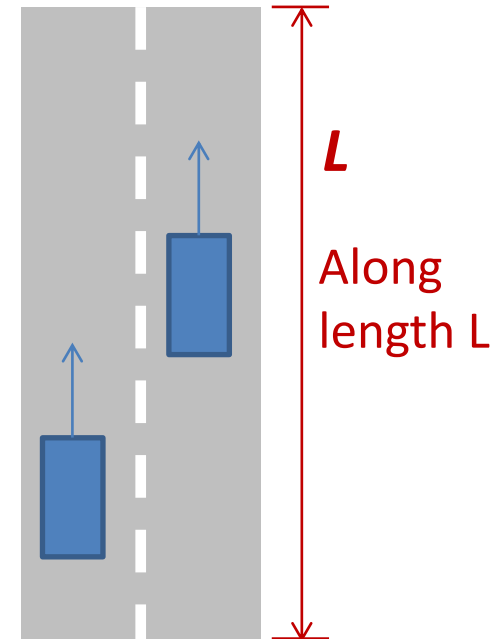
## ■ Space Mean Speed ( $v_s$ )

- Arithmetic mean of the measured speeds of all vehicles **within a given length of lane or carriageway**, at a given instant of time
- Measured by taking the whole roadway segment into account
- Fixed length  $L$  / Average travel time:

Space mean speed  $v_s = \frac{L}{\frac{1}{N} \sum_{i=1}^N t_i} = \frac{L}{\frac{1}{N} \sum_{i=1}^N \frac{L}{v_i}} = \frac{1}{\frac{1}{N} \sum_{i=1}^N \frac{1}{v_i}}$

where:

- $t_i$  = travel time of vehicle  $i$  to traverse length  $L$
- $v_i$  = individual speed (or 'spot speed') of vehicle  $i$
- $N$  = number of vehicles along  $L$  at a given time



## ■ Space Mean Speed ( $v_s$ )

- Arithmetic mean of the measured speeds of all vehicles **within a given length of lane or carriageway**, at a given instant of time
- Measured by taking the whole roadway segment into account
- Fixed length  $L$  / Average travel time:

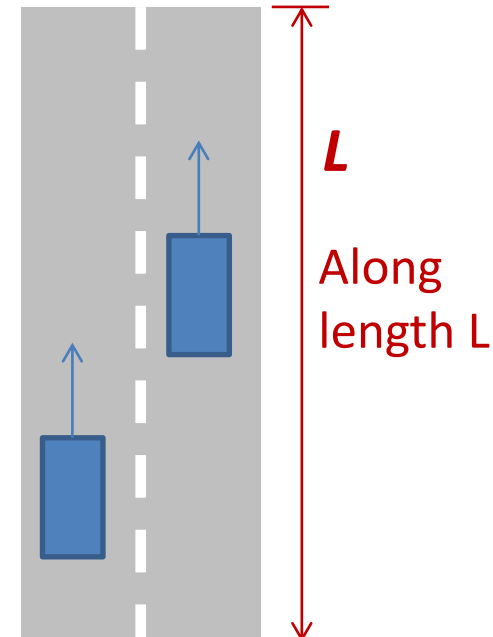
Space mean speed

$$v_s = \frac{L}{\frac{1}{N} \sum_{i=1}^N t_i} = \frac{L}{\frac{1}{N} \sum_{i=1}^N \frac{L}{v_i}} = \frac{1}{\frac{1}{N} \sum_{i=1}^N \frac{1}{v_i}}$$

where:

- $t_i$  = travel time of vehicle  $i$  to traverse length  $L$
- $v_i$  = individual speed
- $N$  = number of vehicles

**Harmonic mean of spot speeds**



## Question 2

# Speed

- Three vehicles traverse a 1 km segment of freeway in 1.2, 1.5, and 1.0 minutes respectively. What are the time-mean and space-mean speeds (km/h) ? (Assume that the speeds are constant along the segment).
  
- A.  $v_t = 41.8$  km/h,  $v_s = 40.9$  km/h
- B.  $v_t = 40.9$  km/h,  $v_s = 41.8$  km/h
- C.  $v_t = 50.0$  km/h,  $v_s = 48.7$  km/h
- D.  $v_t = 48.7$  km/h,  $v_s = 50.0$  km/h
- E. None of the above

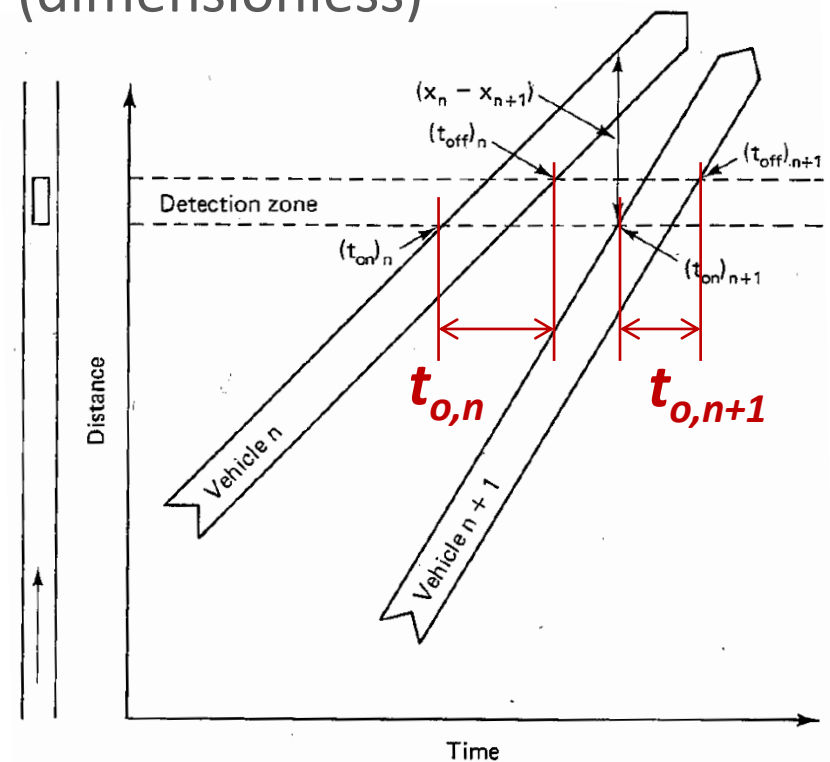
## ■ Occupancy (Occ)

- The proportion of time for which a detector is “occupied” or covered by a vehicle at a given location over a defined period of time  $T$ , expressed as a proportion (dimensionless)

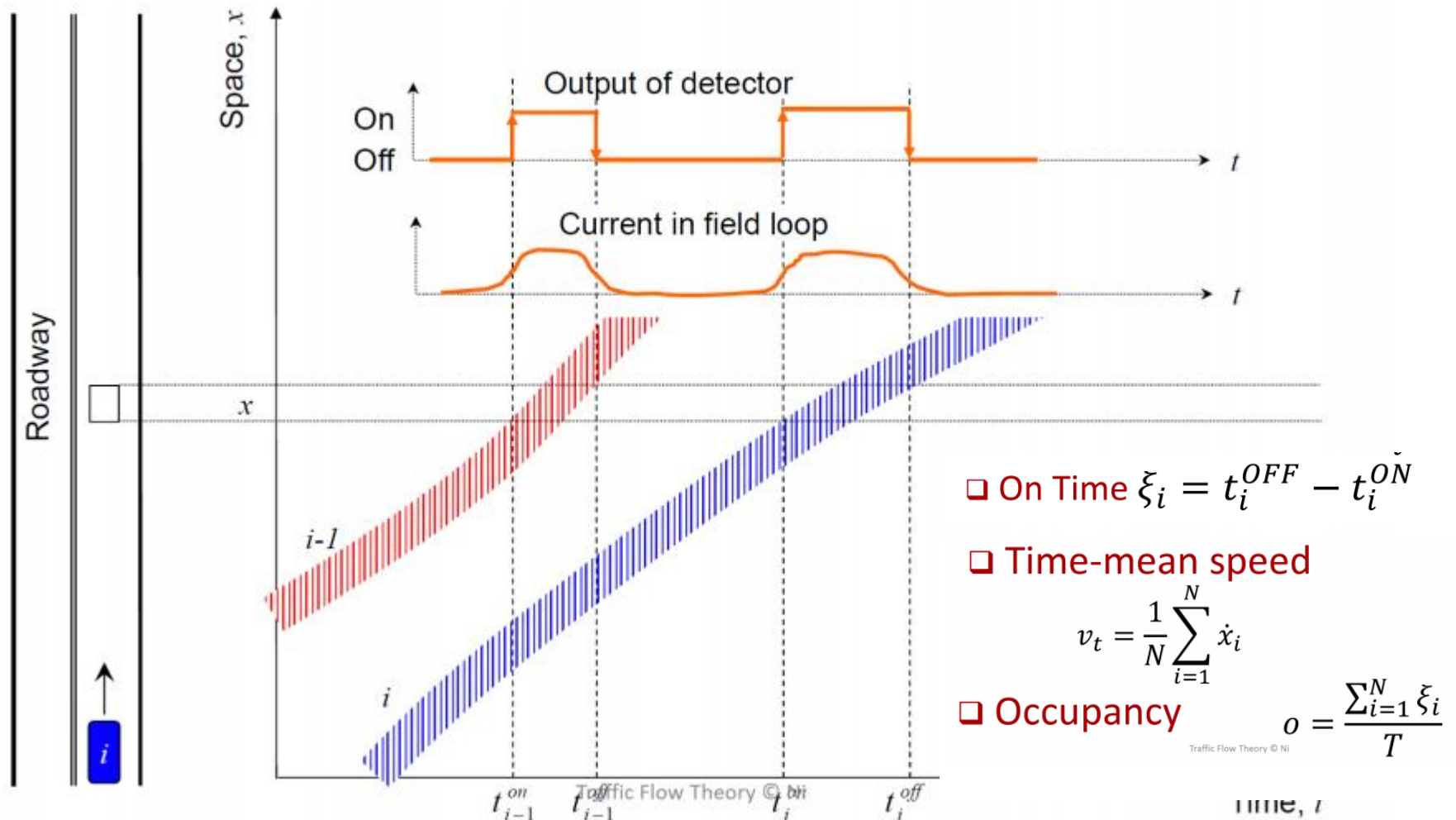
$$\text{Occupancy } Occ = \frac{\sum_{i=1}^N t_{o,i}}{T}$$

where:

- $t_{o,i}$  = duration of the period when the detection zone is occupied by vehicle  $i$
- $N$  = number of vehicles detected during time period  $T$
- $T$  = observation time period



# Point sensor data



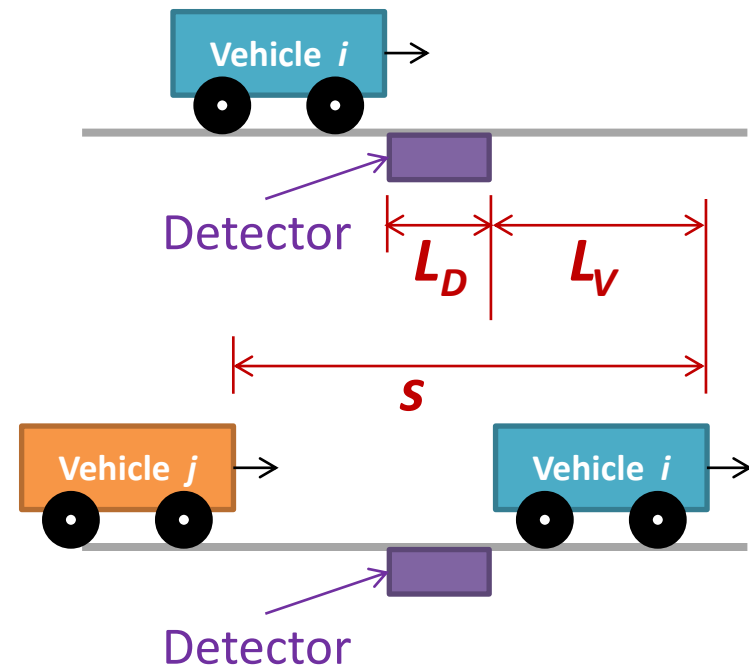
## ■ Occupancy (Occ)

– Related to the average spacing ( $s$ ) as follows:

$$\text{Occupancy } Occ = \frac{\sum_{i=1}^N t_{o,i}}{T} = \frac{L_V + L_D}{s}$$

where:

- $t_{o,i}$  = duration of the period when the detection zone is occupied by vehicle  $i$
- $L_V$  = average length of a vehicle (m)
- $L_D$  = effective length of detector (m)
- $s$  = average spacing of vehicles (m/veh)



## ■ Density and Occupancy

- Occupancy is often used as a ‘surrogate’ for density ( $k$ ) because density is difficult to measure directly (e.g., need aerial photography)
- Obtain Density from Occupancy:

$$Occ = \frac{L_V + L_D}{s} \quad \leftarrow s[\text{m/veh}] = \frac{1000}{k[\text{veh/km}]}$$

$$k = \frac{1000}{s} = \frac{1000 \cdot Occ}{L_V + L_D} = \frac{10 \cdot (\%Occ)}{L_V + L_D}$$

where:

- $\%Occ$  = occupancy expressed as a percentage
- $L_V, L_D$  = vehicle length and detector length, respectively (m)

# Density and Occupancy

- A detector records a %occupancy of 20% for a 15-minute analysis period. If the average vehicle length is 9m and the detector is 1m long, what is the density?

A. 0.2 veh/km

B. 4 veh/km

C. 10 veh/km

D. 20 veh/km

E. None of the above

$$Occ = \frac{\sum_{i=1}^N t_{o,i}}{T}$$

$$k = \frac{1000 \cdot Occ}{L_V + L_D} = \frac{10 \cdot (\% Occ)}{L_V + L_D}$$





# Relationships between Traffic Flow Parameters

# Macroscopic vs. Microscopic

## ■ Macroscopic Parameters

- Describing the entire traffic stream

- Flow ( $q$ )
- Density ( $k$ )
- Speed ( $v_s, v_t$ )

## ■ Microscopic Parameters

- Describing individual vehicles

- Headway ( $h_i$ )
- Spacing ( $s_i$ )
- Speed ( $v_i$ )

# Fundamental Relationships

- Flow is the product of Density and (Space Mean) Speed

$$q = k \times v_s$$

$$\left[ \frac{\text{veh}}{\text{h}} \right] = \left[ \frac{\text{veh}}{\text{km}} \right] \times \left[ \frac{\text{km}}{\text{h}} \right]$$

- Flow is the inverse of Average Headway

$$q = 1/h$$

$$q = \frac{3600}{h} \left[ \frac{\text{veh}}{\text{h}} \right] = 1 / \left[ \frac{\text{s}}{\text{veh}} \times \frac{1\text{h}}{3600\text{s}} \right]$$

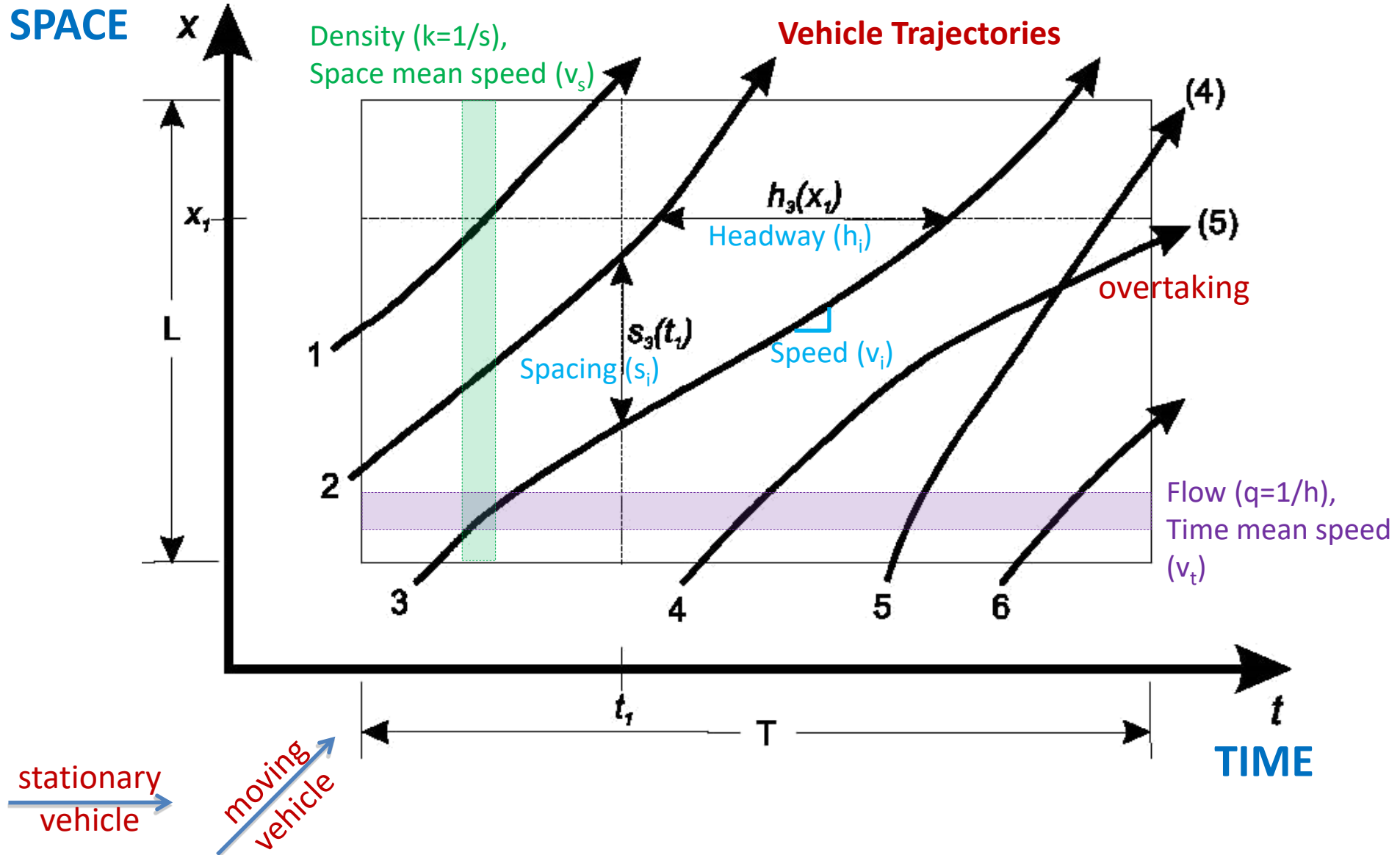
- Density is the inverse of Average Spacing

$$k = 1/s$$

$$k = \frac{1000}{s} \left[ \frac{\text{veh}}{\text{km}} \right] = 1 / \left[ \frac{\text{m}}{\text{veh}} \times \frac{1\text{km}}{1000\text{m}} \right]$$

- Other derived relations:  $h = s/v$      $s = h \cdot v$

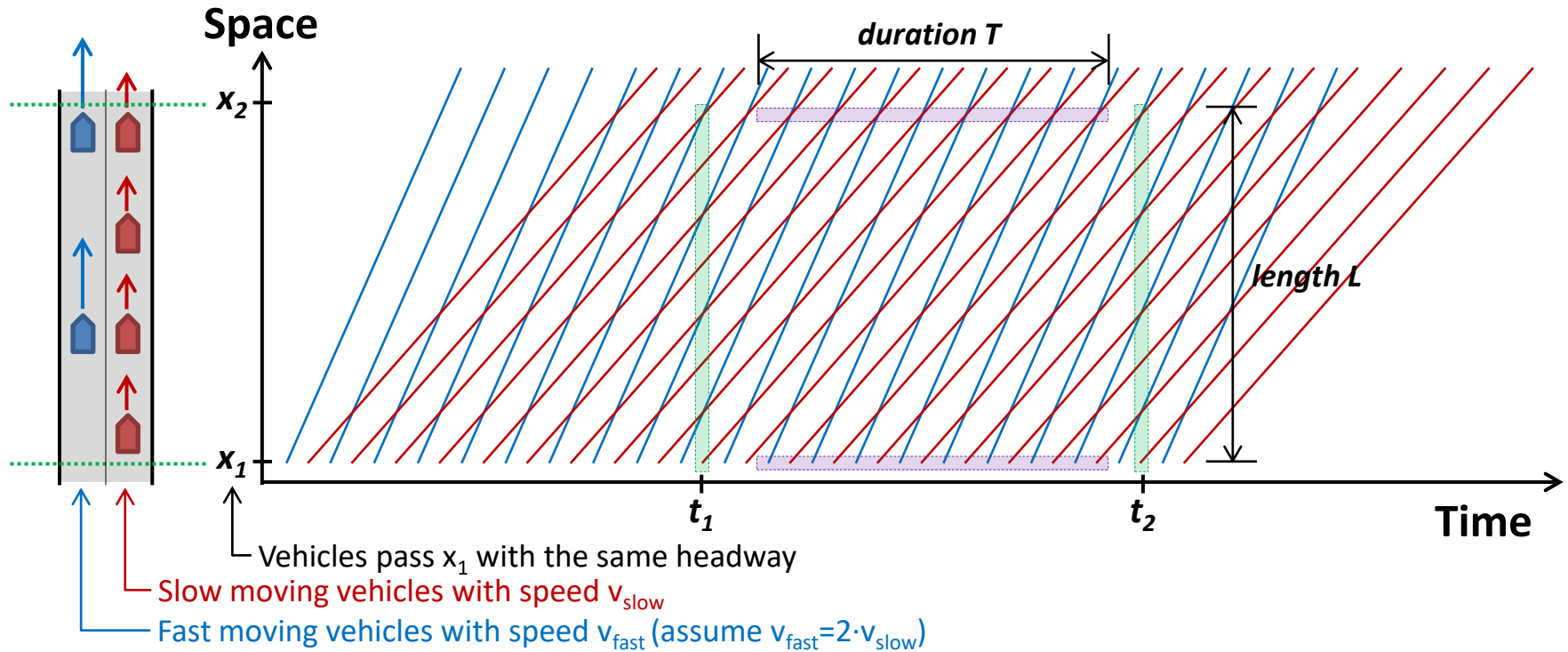
# Space-Time Diagram



# Headway and Flow

- Traffic flow on a one lane ramp is 360 veh/h, what is the average headway (seconds)?
  
- A. 0.1 s
- B. 10 s
- C. 36 s
- D. 60 s
- E. None of the above

# Why $V_s \leq V_t$ ??



**Slower vehicles occupy any given segment of road for a longer period of time than faster vehicles, and therefore receive a greater weighting in the calculation of space mean speed than they do in the calculation of time mean speed.**

[Time Mean Speed] During  $T$  at any given location  $x$ , the sample contains an equal number of slow and fast vehicles  
 $\rightarrow v_t = \frac{1}{2} v_{slow} + \frac{1}{2} v_{fast}$

[Space Mean Speed] Over  $L$  at any given time  $t$ , the sample contains twice as many slow vehicles as fast vehicles  
 $\rightarrow v_s = \frac{2}{3} v_{slow} + \frac{1}{3} v_{fast}$

- The Approximate Relationship between Time Mean Speed and Space Mean Speed

$$v_t = v_s + \frac{\sigma_s^2}{v_s}$$

$\sigma_s^2$  = the variance of space mean speed,  
i.e.,  $\sigma_s^2 = \frac{\sum_{i=1}^N (v_i - v_s)^2}{N-1}$

- Space mean speed is always less than or equal to time mean speed ( $V_s \leq V_t$ )

When all vehicles have exactly the same speed, the two mean speeds are equal ( $V_s = V_t$ ).



# Graphical Relationships for Uninterrupted Flow



## ■ Uninterrupted Flow Vs. Interrupted Flow

- **Uninterrupted flow** occurs in a traffic stream that is not delayed or interfered with by factors external to the traffic stream itself
- **Uninterrupted flow facilities**
  - No intersections at grade, traffic signals, STOP or GIVE WAY signs, direct property access (e.g., controlled-access roads such as **motorways**)
  - **The characteristics of the traffic stream** are based solely on the interactions among vehicles and with the roadway and the general environment.

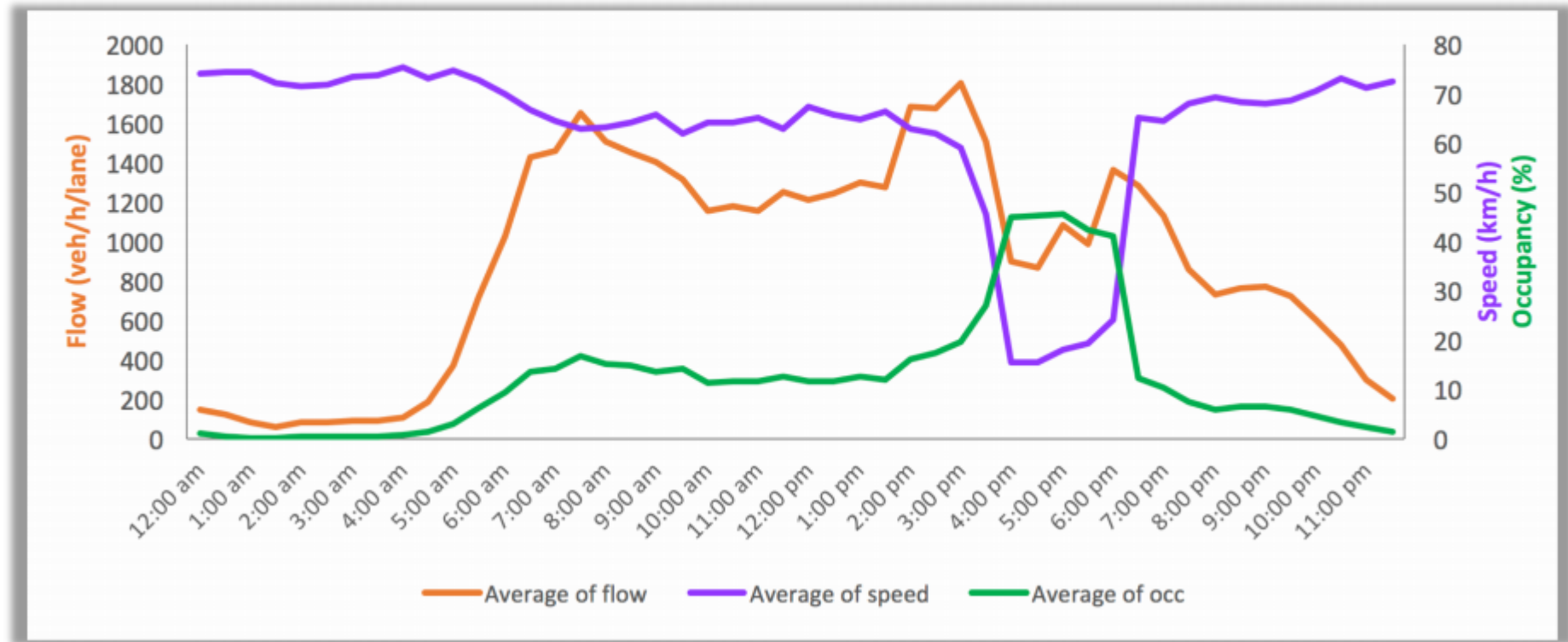


## ■ Uninterrupted Flow Vs. Interrupted Flow

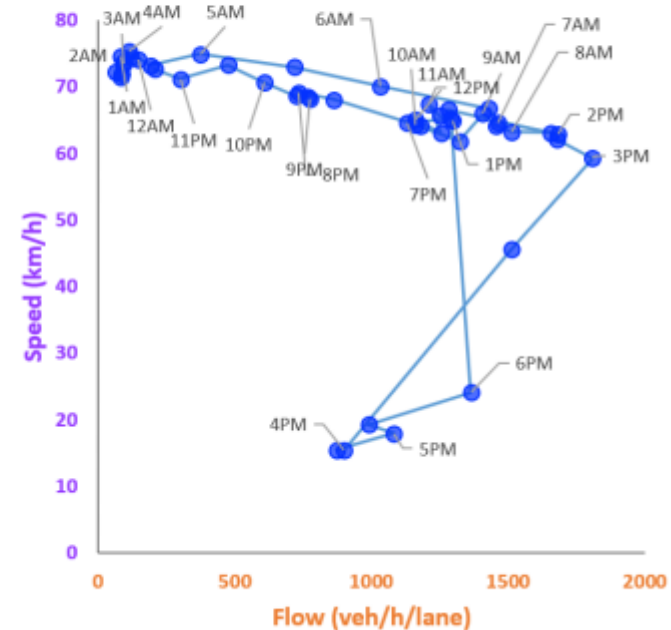
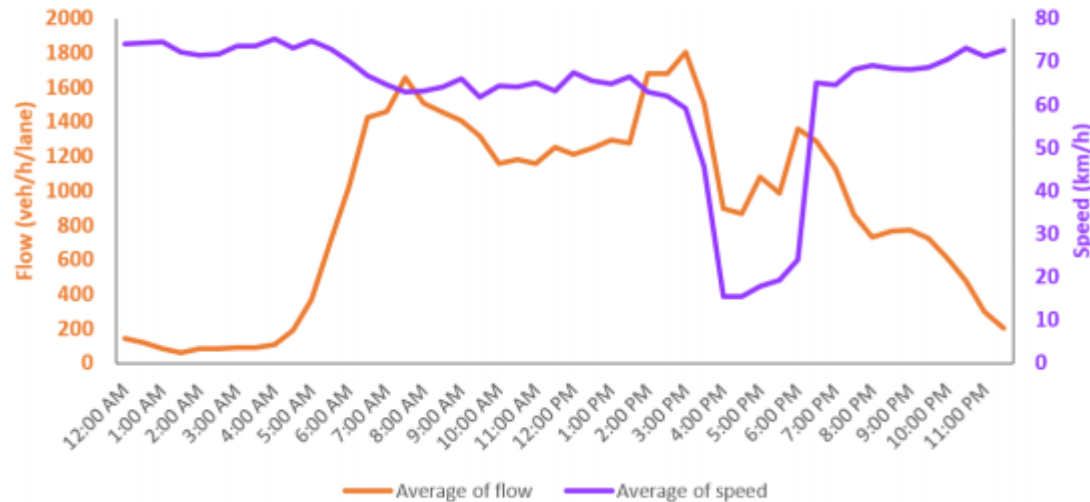
- **Interrupted flow** occurs when external factors have significant effects on the traffic flow.
- **Interrupted flow facilities**
  - Incorporate fixed external interruptions into their design and operation: traffic signal, STOP or GIVE WAY signs, unsignalized at-grade intersections, land access operations (e.g., **arterials** and other urban streets)
  - **The characteristics of the traffic stream** are primarily based on **external interruptions** (e.g., “**red**” signals stop traffic periodically and create a platoons of vehicles); vehicle-vehicle interactions and vehicle-roadway interactions play a secondary role in defining the traffic flow.



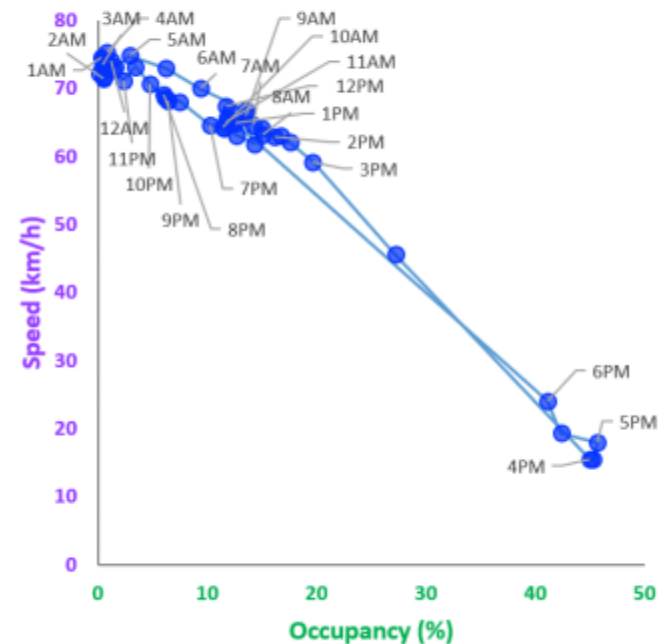
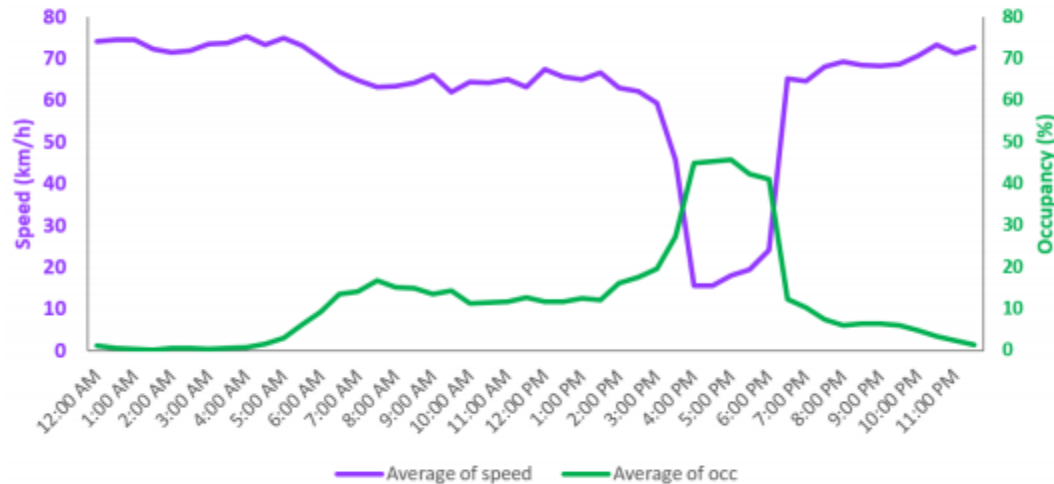
## ■ Flow, Density, and Speed Relationship



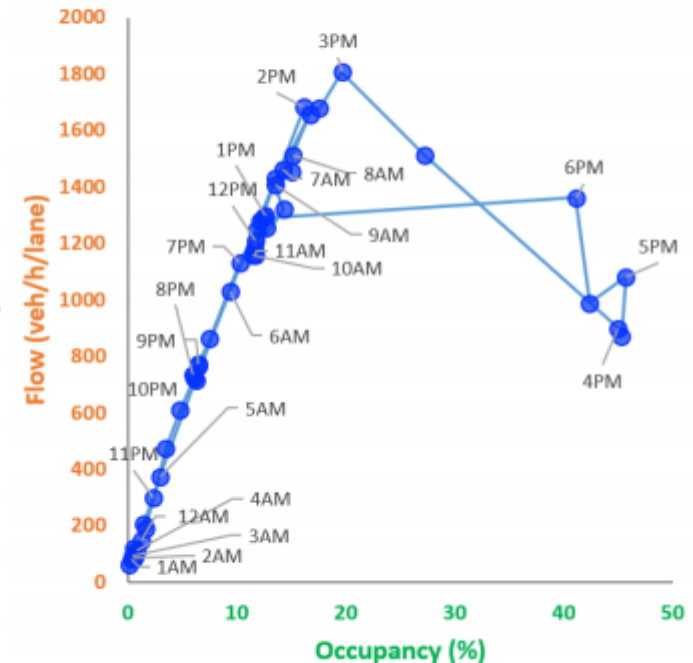
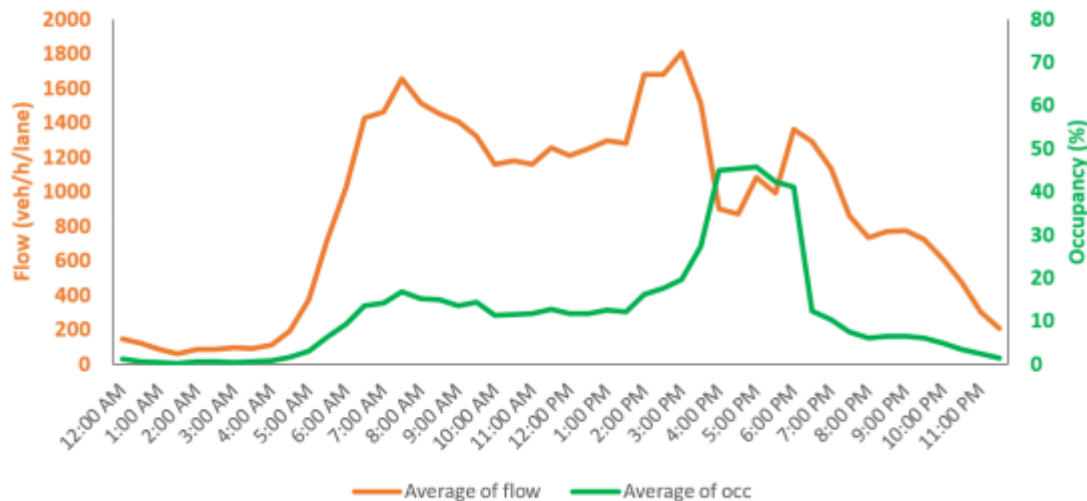
## Speed (v) – Flow (q) Plot



## ■ Speed (v) – Density (k) Plot

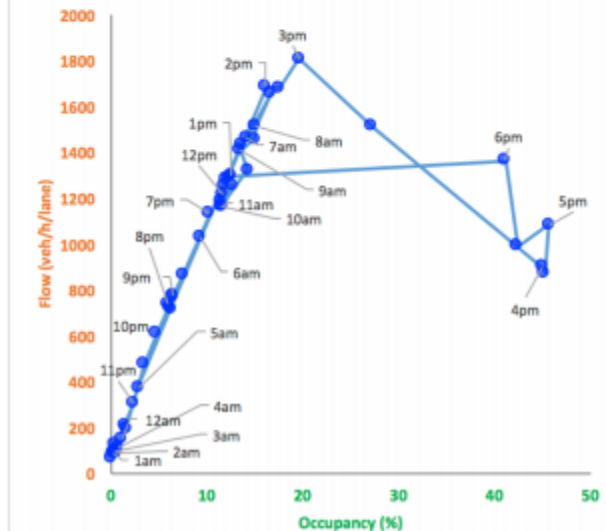
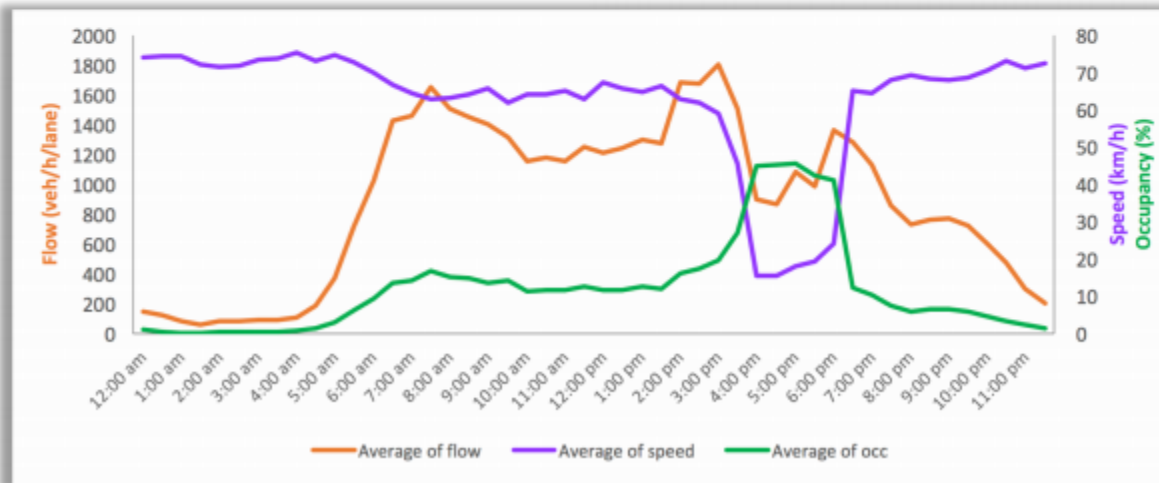
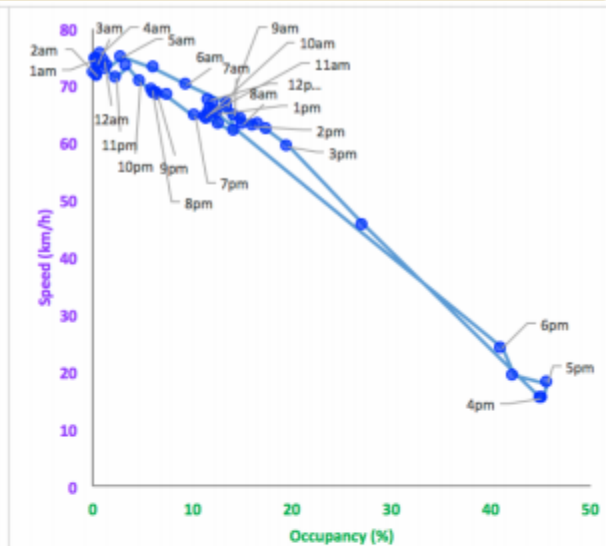
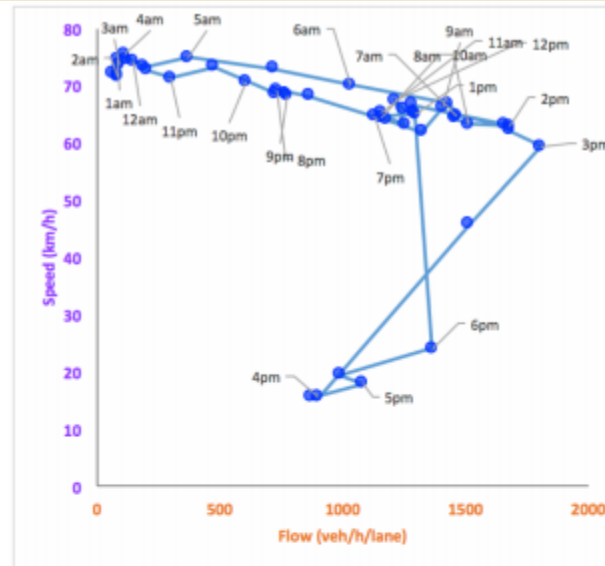


## ■ Flow (q) – Density (k) Plot



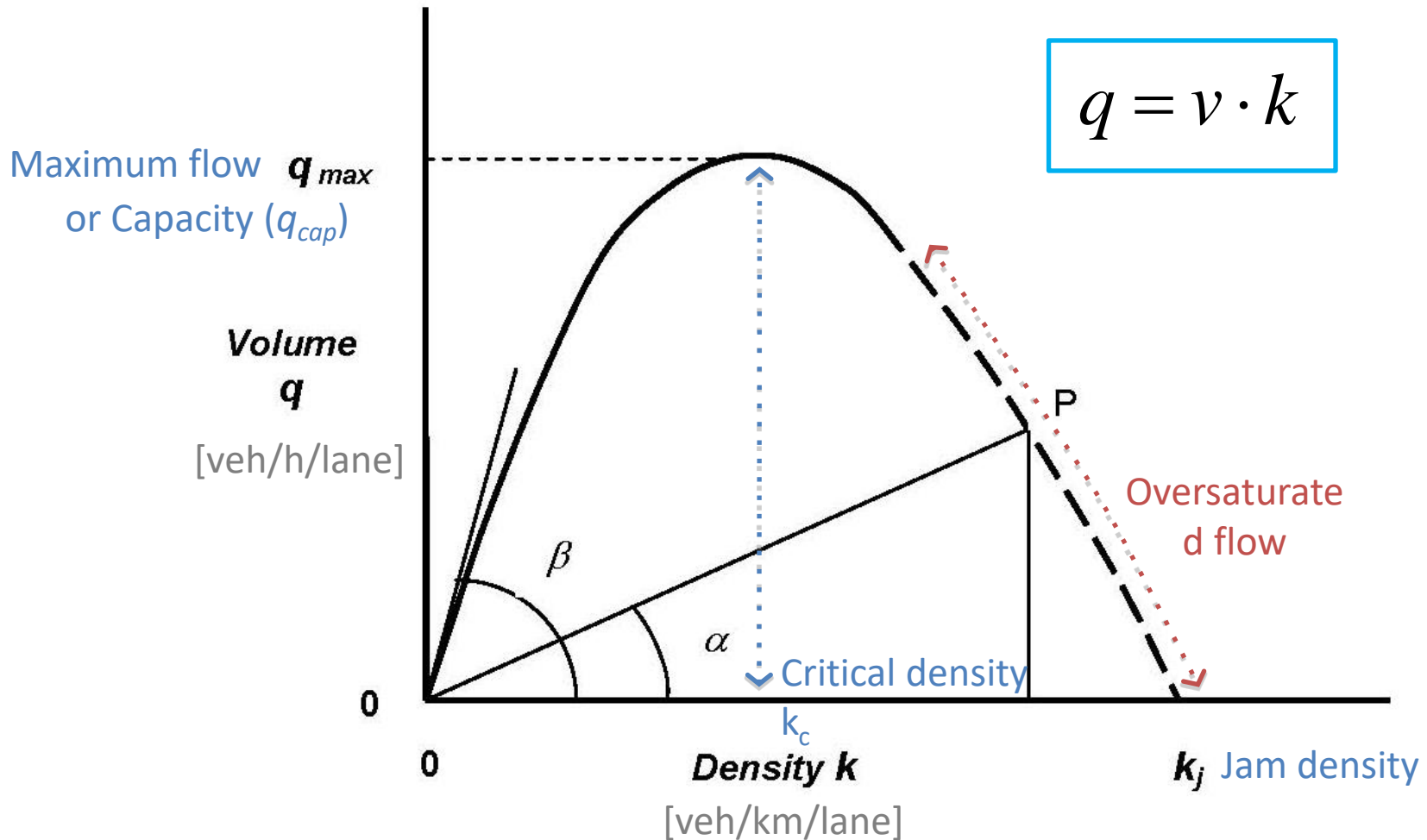
# Traffic Characteristics

## ■ Fundamental relationships of 'uninterrupted' traffic flow



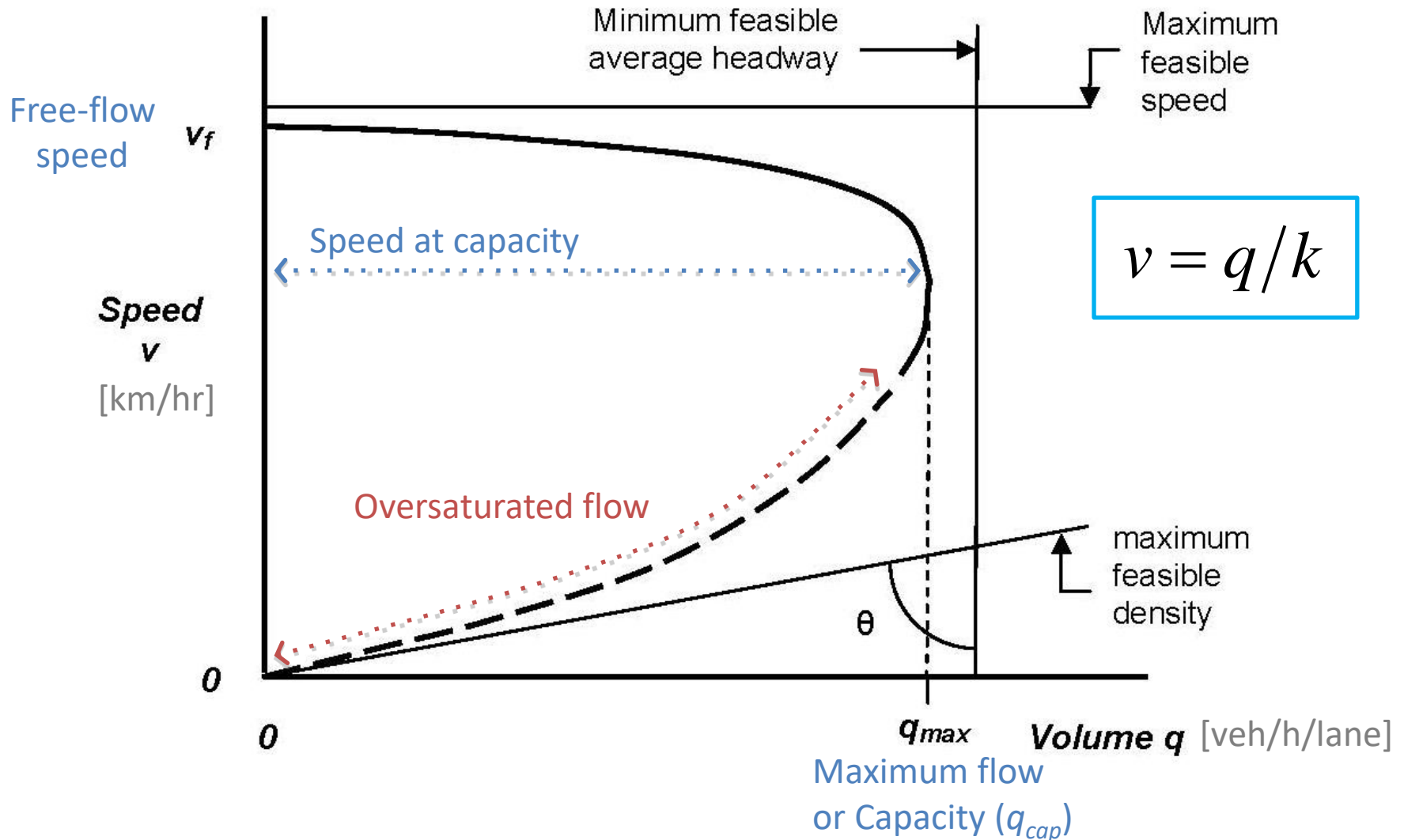


# Flow-Density Relationship

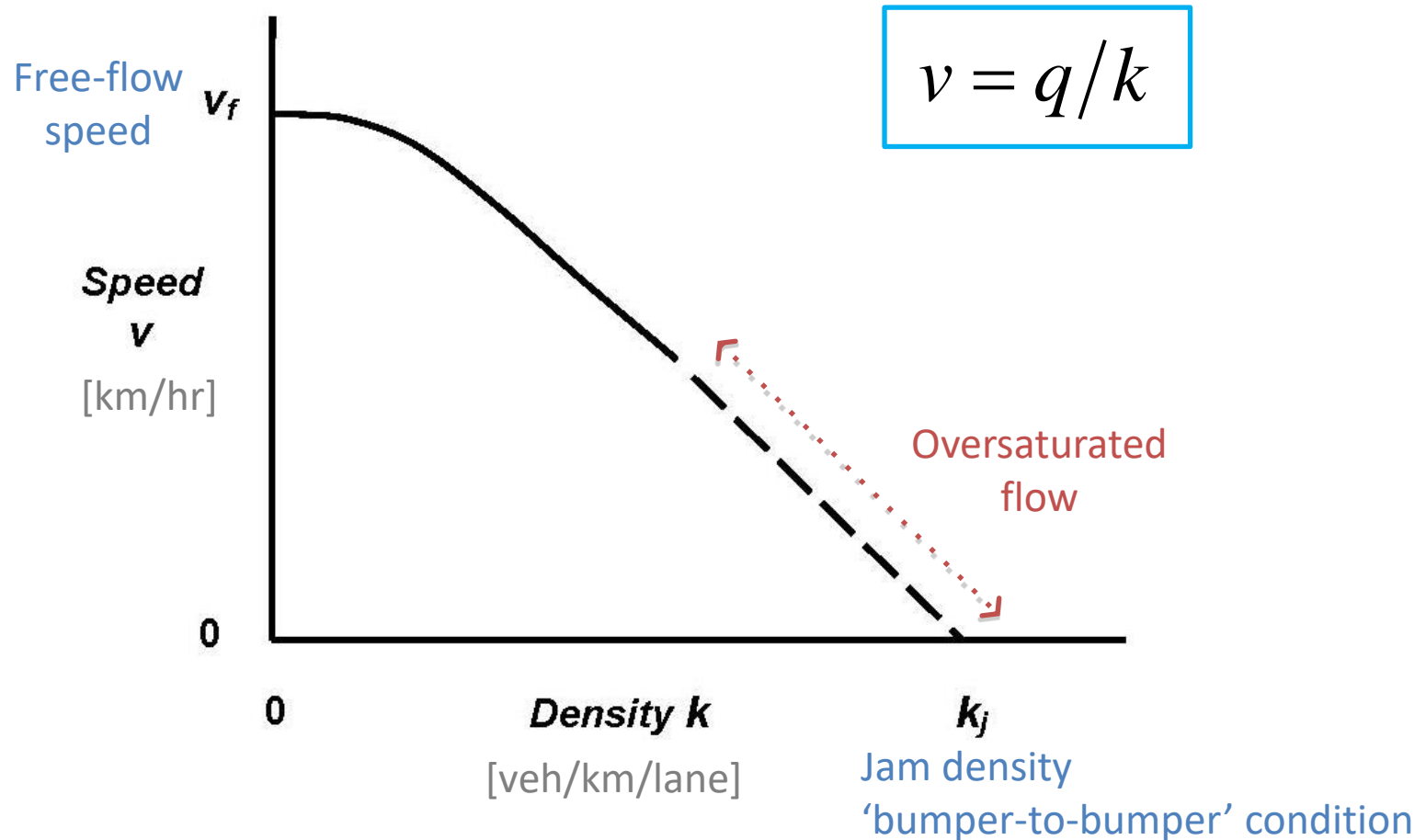




# Speed-Flow Relationship



# Speed-Density Relationship

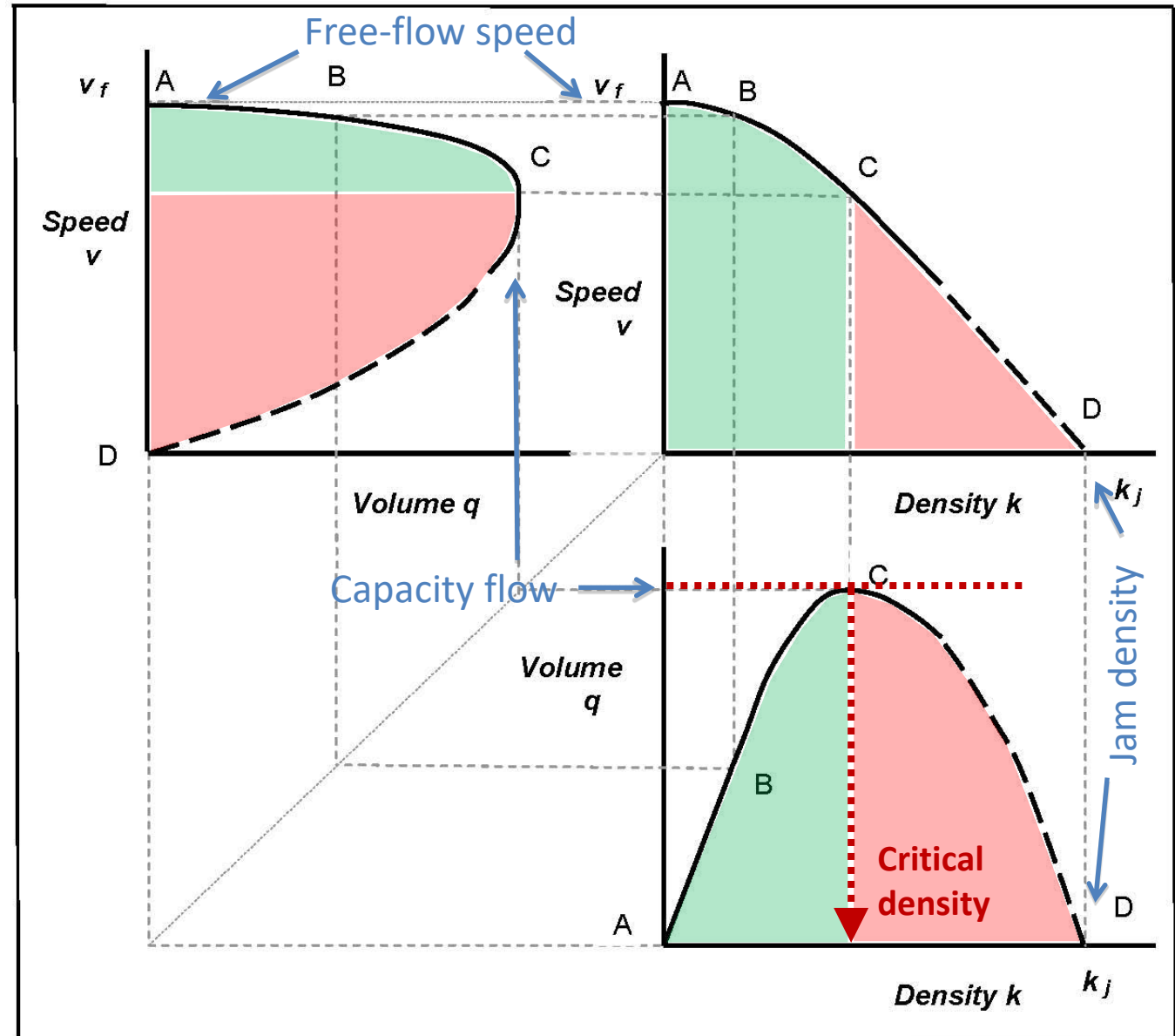


# Flow-Density-Speed Relationships

## Fundamental Diagrams (FD)

‘Fundamental relationships’ of ‘uninterrupted’ traffic flow

At **A**, no interaction  
**A-B**, free flow  
**B-C**, steady flow  
 At **C**, traffic unstable  
**C-D**, forced flow  
 At **D**, traffic stationary



## ■ Linear Speed-Density Relationship

- The first traffic stream model, developed by **Greenshields** (known as 'Greenshields model')
- Assumes that the space mean **speed** of the traffic would **decrease linearly** from the mean free speed,  $v_f$ , at zero density, to zero speed at the jam density,  $k_j$ .

$$v = v_f \left( 1 - \frac{k}{k_j} \right) \dots (1)$$

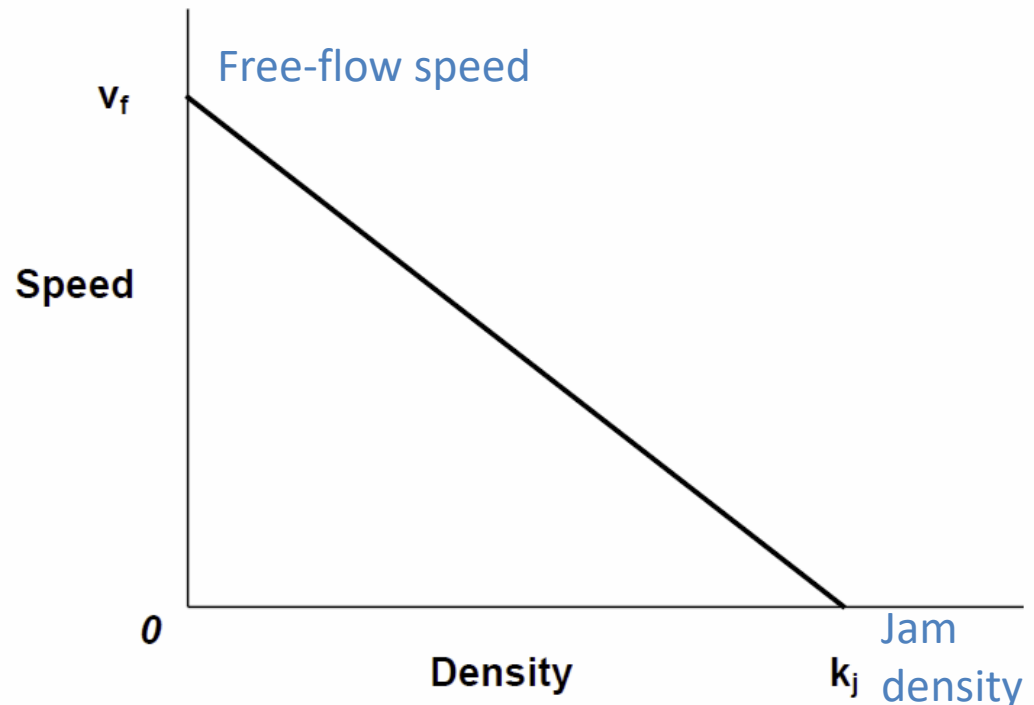
where:

$v$  = **space-mean speed** (km/h)

$v_f$  = **free-flow speed** (km/h)

$k$  = **density** (veh/km)

$k_j$  = **jam density** (veh/km)



## ■ Parabolic Flow-Density Relationship

–  $q = k \cdot v \leftarrow \text{Eq. (1)}$

$$q = v_f \left( k - \frac{k^2}{k_j} \right) \dots (2)$$

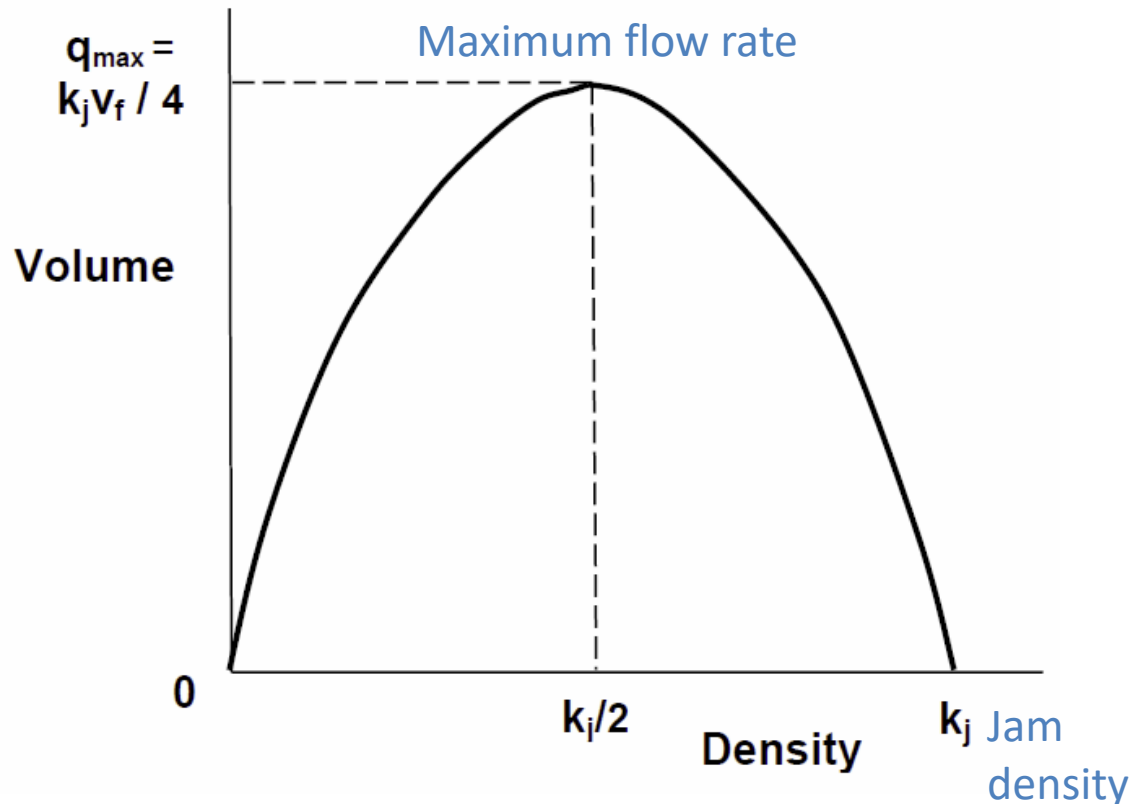
where:

$q$  = flow rate or volume (veh/h)

$v_f$  = free-flow speed (km/h)

$k$  = density (veh/km)

$k_j$  = jam density (veh/km)



## ■ *Parabolic Speed-Flow Relationship*

- Rearranging Eq.(1)  $\rightarrow k = k_j \left(1 - v/v_f\right)$
- $q = k \cdot v \leftarrow k$

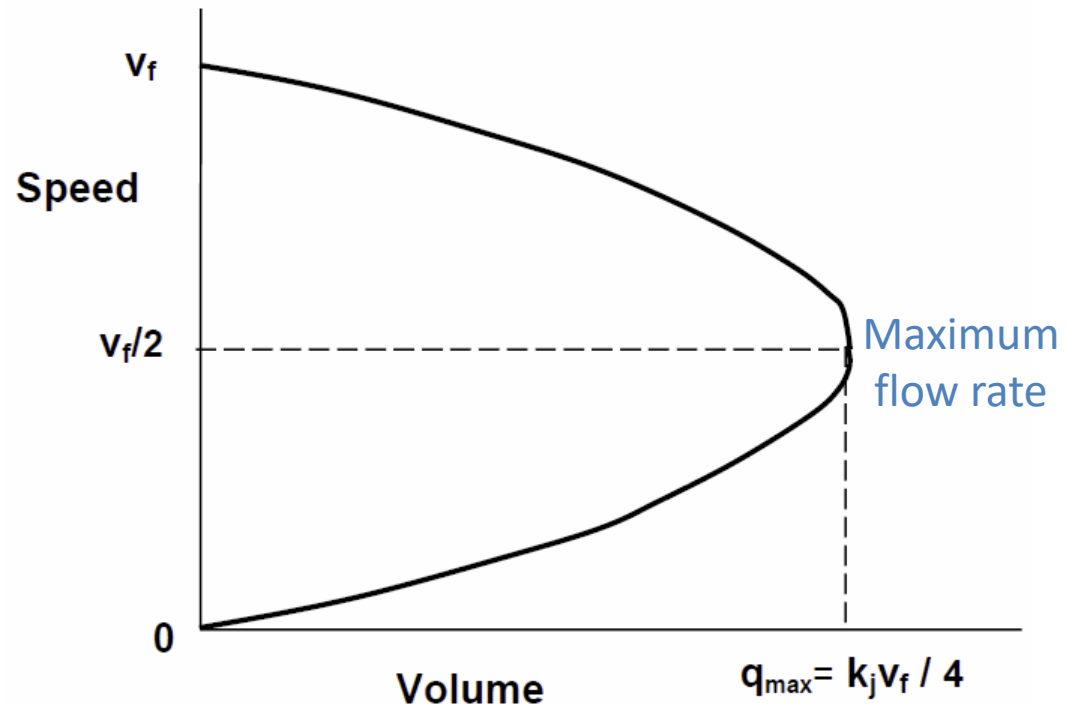
$$q = k_j \left( v - \frac{v^2}{v_f} \right) \dots (3)$$

where:

$q$  = flow rate or volume (veh/h)

$v$  = **space-mean** speed (km/h)

$v_f$  = free-flow speed (km/h)



## ■ Parabolic Speed-Flow Relationship

- Rearranging Eq.(1)  $\rightarrow k = k_j \left(1 - v/v_f\right)$
- $q = k \cdot v \leftarrow k$

$$q = k_j \left( v - \frac{v^2}{v_f} \right) \dots (3)$$

$$\frac{dq}{dv} = 0 \Rightarrow v_{cap} = \frac{v_f}{2}$$

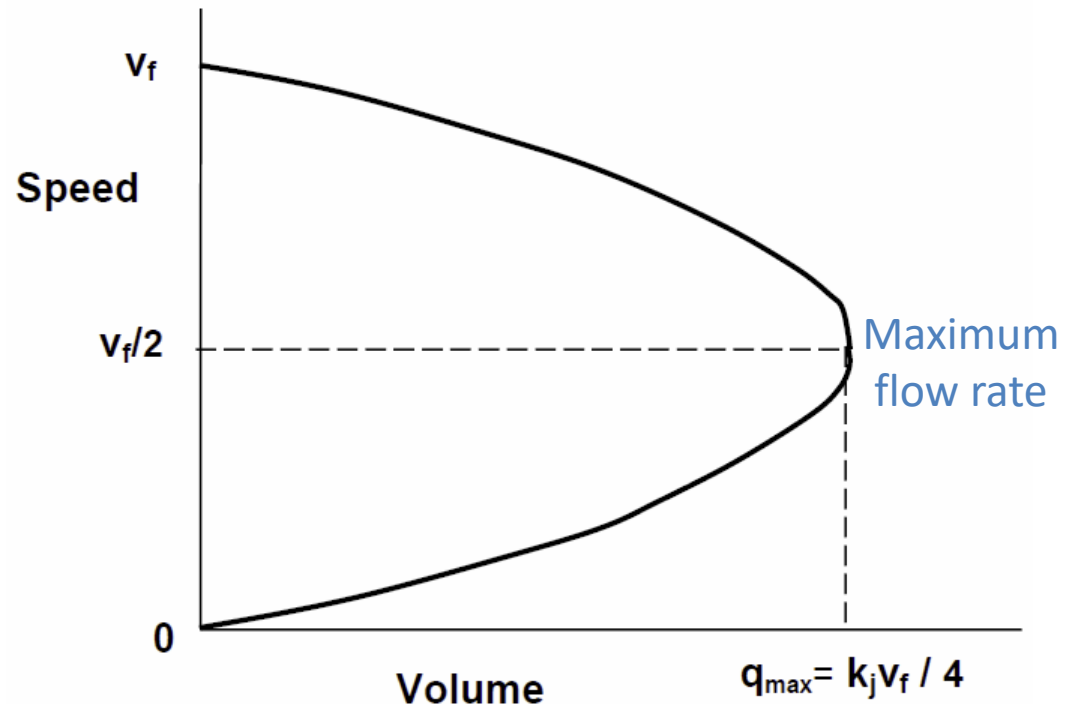
where:

$q$  = flow rate or volume (veh/h)

$v$  = **space-mean** speed (km/h)

$v_f$  = free-flow speed (km/h)

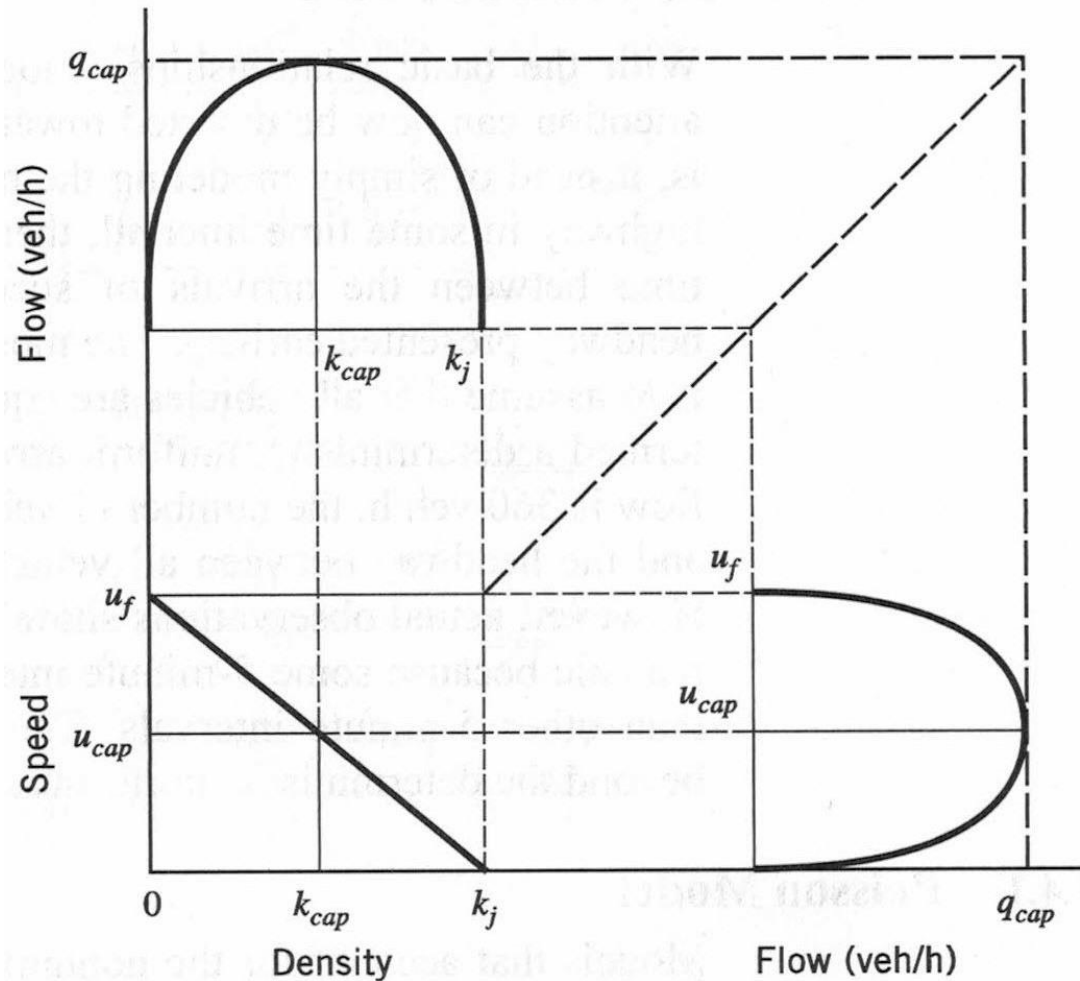
$v_{cap}$  = speed at capacity or maximum flow



## Speed-Flow-Density Relationships

$$q = v_f \left( k - \frac{k^2}{k_j} \right) \quad \dots (2)$$

$$v = v_f \left( 1 - \frac{k}{k_j} \right) \quad \dots (1)$$



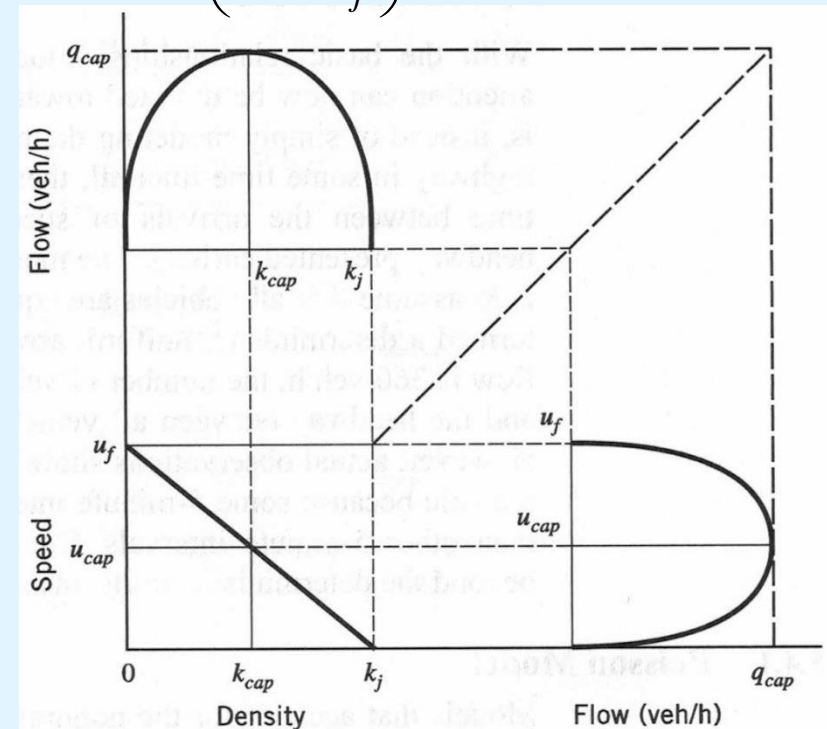
$$q = k_j \left( v - \frac{v^2}{v_f} \right) \quad \dots (3)$$



# Exercise: Traffic Stream Model

- A section of highway is known to have a free-flow speed of 88 km/h and a capacity of 3300 veh/h. In a given hour, 2100 vehicles were counted at a specified point along this highway section. If the linear speed-density relationship in Eq. (1) applies, what would you estimate the space-mean speed of these 2100 vehicles to be?

$$q = v_f \left( k - \frac{k^2}{k_j} \right) \dots (2)$$



$$v = v_f \left( 1 - \frac{k}{k_j} \right) \dots (1)$$

$$q = k_j \left( v - \frac{v^2}{v_f} \right) \dots (3)$$

# Exercise: Traffic Stream Model

- A section of highway is known to have a free-flow speed of 88 km/h and a capacity of 3300 veh/h. In a given hour, 2100 vehicles were counted at a specified point along this highway section. If the linear speed-density relationship in Eq. (1) applies, what would you estimate the space-mean speed of these 2100 vehicles to be?

$$q_{cap} = \frac{v_f k_j}{4} \Rightarrow k_j = \frac{4q_{cap}}{v_f} = \frac{4 \times 3300}{88}$$
$$\Rightarrow k_j = 150 \text{ veh/km}$$

$$\begin{cases} q = 2100 \text{ veh/h} \\ k_j = 150 \text{ veh/km} \end{cases} \Rightarrow v = ?$$

Rearranging Eq. (3):

$$q = k_j \left( v - \frac{v^2}{v_f} \right) \Rightarrow \frac{k_j}{v_f} v^2 - k_j v + q = 0$$
$$\Rightarrow \frac{150}{88} v^2 - 150v + 2100 = 0$$
$$\Rightarrow v = 70.53 \text{ km/h or } 17.47 \text{ km/h}$$



# Generalized Definitions of Flow, Density, and Speed

## ■ A More Generalized Definition of Flow and Density:

$$q = \frac{N}{T} = \frac{N \times dx}{T \times dx} = \frac{d(B)}{|B|}$$

$dx$  : infinitesimal distance

$d(B)$  : sum of distances travelled by all vehicles during  $T$

$|B|$  : area of time-space rectangle  $B$  bounded by  $T$  and  $dx$

**Flow ( $q$ ) : total distance travelled by all vehicles within  $B$  divided by the area of  $B$**

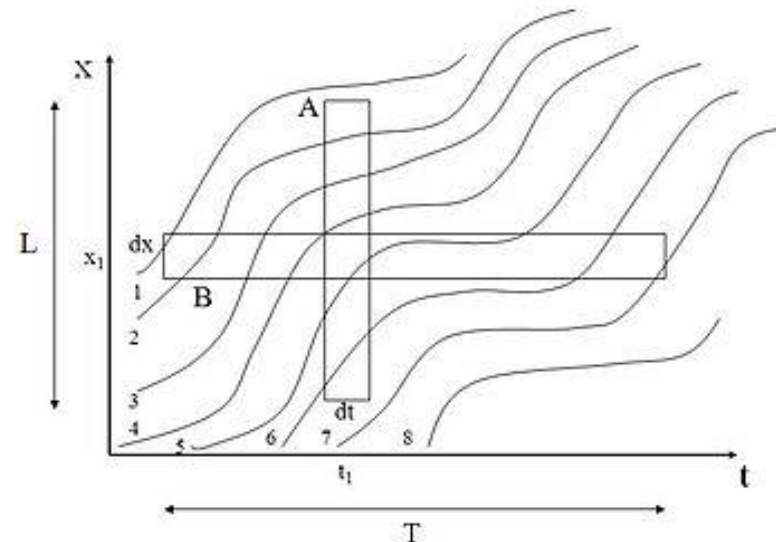
$$k = \frac{N}{L} = \frac{N \times dt}{L \times dt} = \frac{t(A)}{|A|}$$

$dt$  : infinitesimal duration

$t(A)$  : sum of times spent by all vehicles within  $A$

$|A|$  : area of time-space rectangle  $A$  bounded by  $L$  and  $dt$

**Density ( $k$ ) : total travel time spent by all vehicles within  $A$  divided by the area of  $A$**



## ■ Unifying the Generalized Definition of $q$ , $k$ , and $v$ :

– “Edie’s generalized definition” (Edie, 1965)

$$q(C) = \frac{d(C)}{|C|}$$

Total travel distance  
(veh·km)

Total space-time  
region (km·hrs)

$$k(C) = \frac{t(C)}{|C|}$$

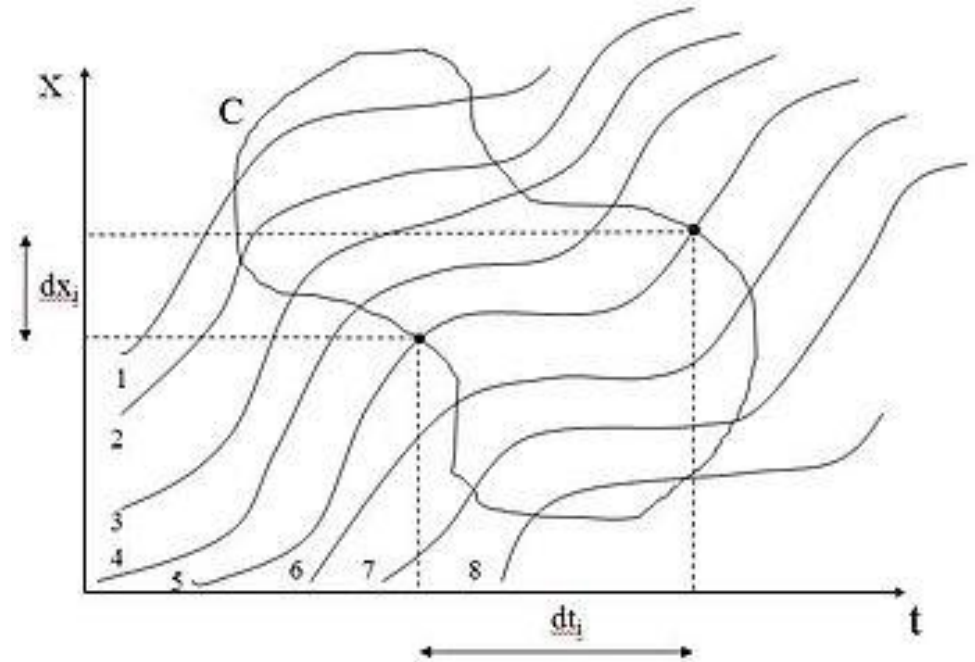
Total travel time  
(veh·hrs)

Total space-time  
region (km·hrs)

$$v(C) = \frac{d(C)}{t(C)}$$

Total travel distance  
(veh·km)

Total travel time  
(veh·hrs)

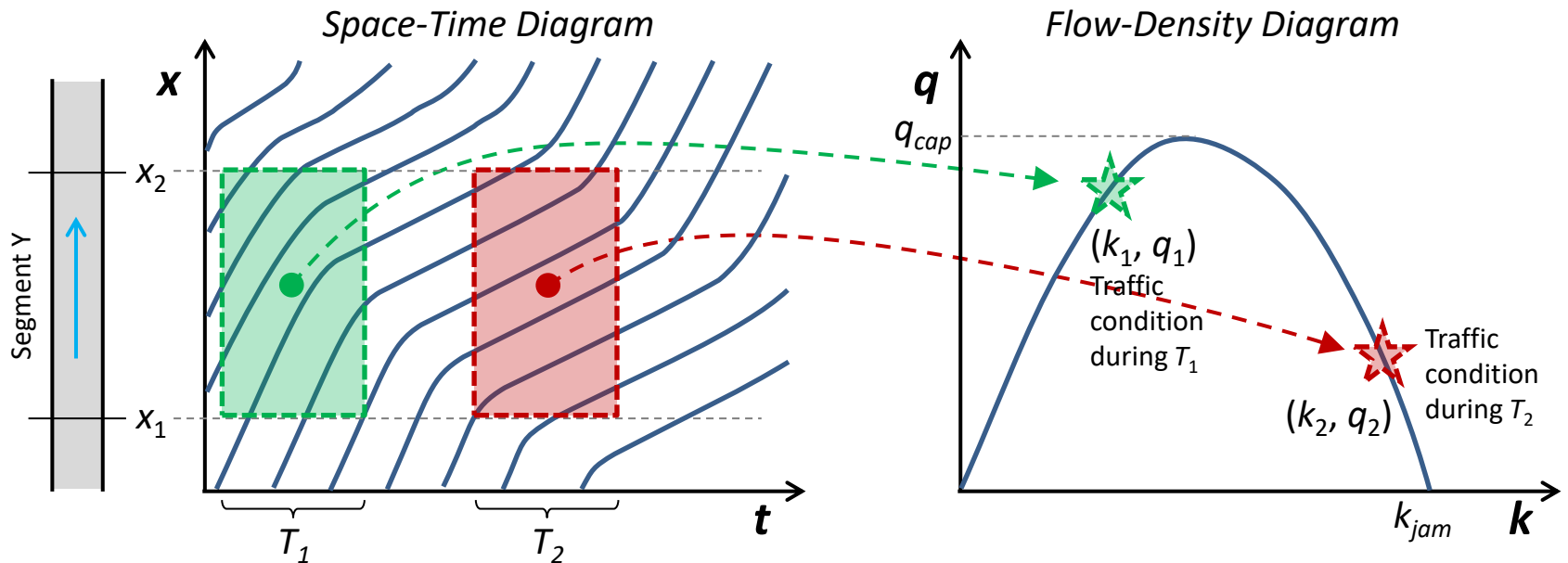


where

$$d(C) = \sum_{i=1}^N dx_i \quad t(C) = \sum_{i=1}^N dt_i$$

# Space-Time Diagram (x-t plot) vs. Fundamental Diagram (q-k plot)

## ■ Relation between Space-Time Diagram and Flow-Density Diagram



Points on the fundamental diagram (FD) describe all possible traffic conditions on the road segment Y.  
→ **FD is a (steady-state) property of the road.**

# Edie's Generalized Definition

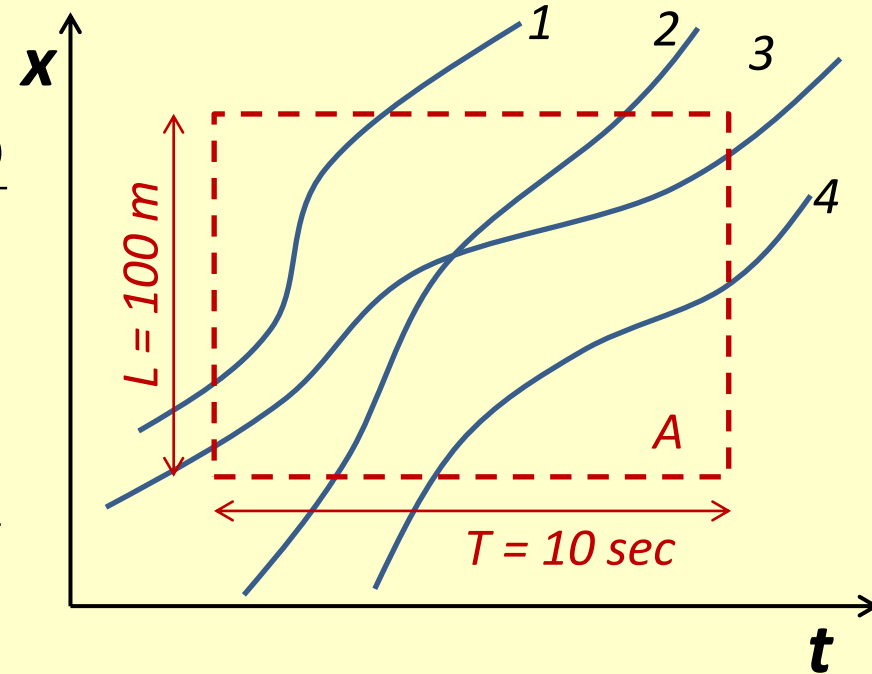
- Calculate flow  $q$  in region A using Edie's generalized definition.

Veh i	$dt_i$	$dx_i$
1	3 sec	70 m
2	6 sec	100 m
3	10 sec	80 m
4	5 sec	50 m

$$q(A) = \frac{d(A)}{|A|}$$

$$k(A) = \frac{t(A)}{|A|}$$

$$v(A) = \frac{d(A)}{t(A)}$$



- A. 1080 veh/h
- B. 1440 veh/h
- C. 1880 veh/h
- D. 2400 veh/h
- E. None of the above

# Next

- Tutorial on Wednesday
- Please don't forget to request the Aimsun License!