MATH3405 – Differential Geometry
Semester 2, 2025
Assignment 1

To be submitted before Friday 15 August, 2025, at 5pm.

Total number of marks = 50.

Question 1 [**25 marks**] Consider the circular helix given by

$$\alpha(s) = (a\cos(s/c), a\sin(s/c), bs/c), \quad c^2 = a^2 + b^2, \quad a > 0.$$

(a) Show that α is a regular curve, and that it is parameterised by arc-length parameter.

(b) Compute the curvature, the torsion and the Frenet frame of α at s.

(c) Determine equations for the osculating plane and the normal plane of α at $s = c\pi$.

(d) How does the curvature of a circular helix change if it is compressed or dilated in the direction of the z axis? What if it is compressed or dilated in radial directions orthogonal to the z axis (that is, if the radius of the cylinder in which it lives changes)?

Question 2 [**25 marks**] Let $\alpha : I \to \mathbb{R}^3$ be a regular curve parameterised by arc-length, and assume that $\kappa(s) \neq 0$ for all $s \in I$. Prove that the trace of α is contained in a plane if and only if $\tau(s) \equiv 0$, $\forall s \in I$.

Hint 1: Show that $\tau \equiv 0$ iff $b(s)$ is constant.

Hint 2: Recall that the equation for a plane in $\mathbb{R}^3$ passing through $p \in \mathbb{R}^3$ with normal vector $\nu \in \mathbb{R}^3$ is

$$\langle q - p, \nu \rangle = 0, \qquad q \in \mathbb{R}^3.$$