Contents

Week 2

Basic traffic flow descriptors

Flow/Volume, q, [veh/h]

The number of vehicles N, that pass specific point during the time period T.

$$q = \frac{N}{T} \tag{1}$$

Headway, h, [s/veh]

The time (in $'s'$) between successive vehicles, as their front bumpers pass a specific point. We denote respectively by $h_1, h_2, h_3, \dots$ first, second, third, … headway. The total counting time T actually represents the sum of recorded headways

$$T = \sum_{i=1}^{N} h_i \tag{2}$$

Substitute Eq.(2) to Eq.(1), we get

$$q = \frac{N}{T} = \frac{N}{\sum_{i=1}^{N} h_i} = \frac{1}{\frac{1}{N}\sum_{i=1}^{N} h_i}$$

$$\Longrightarrow q = \frac{1}{\bar{h}}, \text{where } \bar{h} \text{ represent average headway}$$

Note: In CIVL6415, we use h instead of $\bar{h}$ to represent the average headway.

Speed, v

- Time-mean speed, v_t, [km/h]

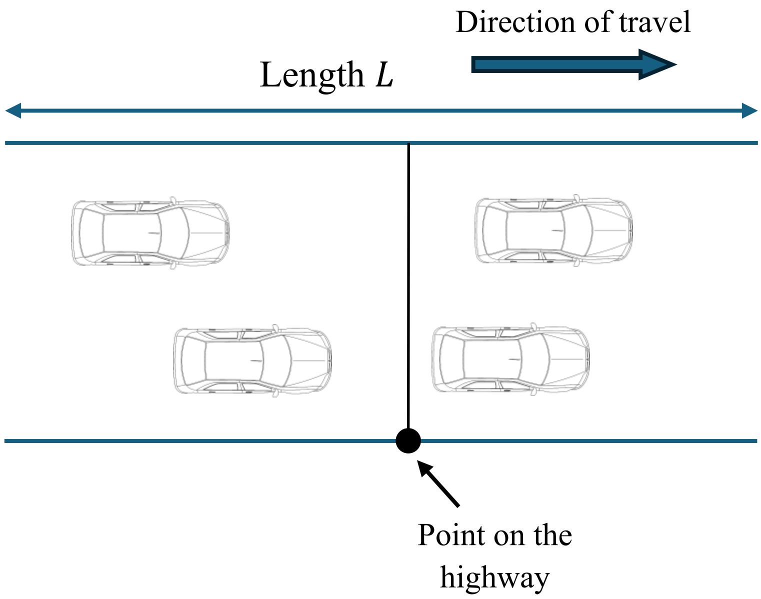
$$v_i = \frac{d_i}{T}$$

where d_i is travel distance of ith vehicle during time interval T

$$v_t = \frac{1}{N}\sum_{i=1}^{N} v_i$$

where v_i represent recorded speed of the ith vehicle

Note: Time-mean speed can be calculated by calculating the arithmetic mean speed.



- Spece-mean speed, v_s, [km/h]

Let the highway has a length of L. We denote by t_i the time needed by the ith vehicle to travel along this highway section.

$$v_s = \frac{L}{\frac{1}{N}\sum_{i=1}^{N} t_i}$$

t_i: represent travel time of the vehicles travelling along the observed highway section.

$$= \frac{L}{\frac{1}{N}\sum_{i=1}^{N} \frac{L}{v_i}}$$

L: Length of the observed highway section

$$= \frac{1}{\frac{1}{N}\sum_{i=1}^{N} \frac{1}{v_i}}$$

v_i: recorded speed of the ith vehicle

Note: Time-mean speed can be calculated by calculating the harmonic mean speed.

Example 1. Measurement points are located at the beginning and at the end of the highway section whose length equals 1 km. The recorded speeds and travel times are shown in Table below.

Vehicle Number	Speed at Point A (km/h)	Travel Time Between Point A and Point B (s)
1	80	45
2	75	50
3	62	56
4	90	39
5	70	53

Speeds of the five vehicles are recorded at the beginning of the section (point A). The vehicle appearance at points A and point B were also recorded.

Calculate the time-mean speed and the space-mean speed.

Solution.

The time-mean speed v_t at point A is

$$v_t = \frac{1}{N}\sum_{i=1}^{N} v_i = \frac{1}{5}(80 + 75 + 62 + 90 + 70) = 75.4 \text{ km/h}$$

The space-mean speed represents measure of the average traffic speed along the observed highway section.

$$\bar{t} = \frac{1}{N}\sum_{i=1}^{N} t_i = \frac{1}{5}(45 + 50 + 56 + 39 + 53) = 48.6\ s = 0.0135\ h$$

$$v_s = \frac{L}{\frac{1}{N}\sum_{i=1}^{N} t_i} = \frac{L}{\bar{t}} = \frac{1 \text{ km}}{0.0135\ h} = 74.07 \text{ km/h}$$

Exercise. The speed of five vehicles were measured with radar at the midpoint of a 0.5 km section of roadway. The speeds for vehicles 1, 2, 3, 4, and 5 were 44, 42, 51, 49, and 46 km/h, respectively. Assuming all vehicles were traveling at constant speed over this roadway section. Calculate the time-mean and space-mean speeds.

Ans. $v_t = 46.4 \text{ km/h}, \quad v_s = 46.17 \text{ km/h}$

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- Space-mean speed (v_s) is ALWAYS LESS THAN OR EQUAL TO time-mean speed (v_t)

$$\boldsymbol{v_s \leq v_t}$$

Slower vehicle occupy any given segment of road for a longer period of time than faster vehicles and therefore receive a greater weighting in the calculation of space-mean speed than time-mean speed.

Approximate relationship between v_t and v_s:

$$v_t = v_s + \frac{\sigma_s^2}{v_s}, \qquad \text{where } \sigma_s^2 = \frac{\sum_{i=1}^{N}(v_i - v_s)^2}{N-1} \text{ is the variance of space mean speed}$$

Density, k, [veh/km]

The number of vehicles occupying a unit length of road/lane at a specific time point. In another word, traffic density in vehicles per unit distance.

$$k = \frac{N}{L}$$

N: Number of vehicles occupying some length of roadway at some specified time

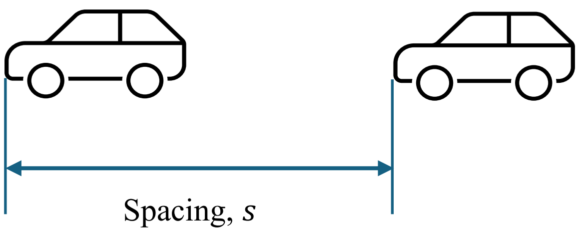
L: Length of roadway

Spacing, s

Individual spacing between successive vehicles (measured from front bumper to front bumper). The roadway length, L can be defined as

$$L = \sum_{i=1}^{N} s_i$$

s_i: spacing of the ith vehicle (the distance between vehicles i and $i-1$)



$$k = \frac{N}{L} = \frac{N}{\sum_{i=1}^{N} s_i} = \frac{1}{\frac{1}{N}\sum_{i=1}^{N} s_i} = \frac{1}{\bar{s}}$$

$\bar{s}$: Average spacing ($\sum \frac{s_i}{n}$) in unit distance per vehicle.

In CIVL6415, we use s instead of $\bar{s}$ to represent average spacing

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Example 2. An observer located at point A observes four vehicles passing point A during 30 sec. The Figure shows their positions at an instant of time by photography. Calculate flow and density. Length of highway is 400 m.

Solution.

$$q = \frac{N}{T} = \frac{4}{30} = 0.13\ veh/s$$

$$k = \frac{N}{L} = \frac{4}{400} = 0.01\ veh/m = 10\ veh/km$$

Occupancy, Occ (%)

$$Occ = \frac{\sum_{i=1}^{N} t_{o,i}}{T}$$

$t_{o,i}$: Duration of the period when the detection zone is occupied by vehicle i

N: Number of vehicles detected during the time period T

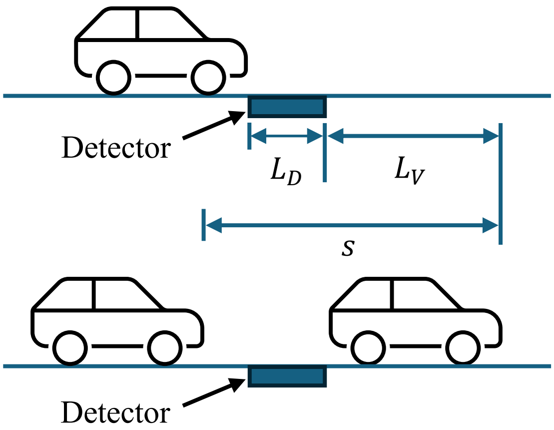
T: Observation time period

$$Occ = \frac{\sum_{i=1}^{N} t_{o,i}}{T} = \frac{L_V + L_D}{s}$$

where L_V is average length of a vehicle (m)

L_D is effective length of detector (m)

s is average spacing of vehicles (m/veh)



Obtain density from occupancy:

$$Occ = \frac{L_V + L_D}{s} \tag{O.1}$$

And we know

$$s\left[\frac{\text{m}}{\text{veh}}\right] = \frac{1}{k\left[\frac{veh}{km}\right] \times \frac{1}{1000}\left[\frac{km}{m}\right]} = \frac{1000}{k} \tag{O.2}$$

From Eq. $(O.1)$ and $(O.2)$,

$$k = \frac{1000}{s} = \frac{1000 \cdot Occ}{L_V + L_D}$$
$$= \frac{10 \cdot (\%Occ)}{L_V + L_D}$$

$\%Occ$: Occupancy expressed as a percentage

L_V: Vehicle length (m)

L_D: Detector length (m)

Example 3. A detector records a %occupancy of 20% for a 15-minute analysis period. If the average vehicle length is 9 m and the detector is 1 m long, what is the density?

Solution.

$$\%Occ = 20, \qquad t = 15\ min = 0.25\ h, \qquad L_V = 9\ m, \qquad L_D = 1\ m$$

$$k = \frac{10 \cdot (\%Occ)}{L_V + L_D} = \frac{10 \times 20}{9 + 1} = 20\ veh/m = 0.02\ veh/km$$

Macroscopic vs. Microscopic

- Macroscopic: Describing the entire traffic stream

  Flow (q), Density (k), Speed (v_s, v_t)
- Microscopic: Describing individual vehicles

  Headway (h_i), Spacing (s_i), Speed (v_i)

Fundamental Relationships (Be careful with units)

$$q = k \times v_s, \qquad \left[\frac{\text{veh}}{\text{h}}\right] = \left[\frac{\text{veh}}{\text{km}}\right] \times \left[\frac{\text{km}}{\text{h}}\right]$$

Flow (q) is the inverse of Average Headway (h)

$$q = \frac{1}{h}, \qquad \left[\frac{\text{veh}}{\text{h}}\right] = \frac{1}{\left[\frac{\text{s}}{\text{veh}} \cdot \frac{1\ \text{h}}{3600\ \text{s}}\right]}$$

Density (k) is the inverse of Average Spacing (s)

$$k = \frac{1}{s}, \qquad \left[\frac{\text{veh}}{\text{km}}\right] = \frac{1}{\left[\frac{\text{m}}{\text{veh}} \cdot \frac{1\ \text{km}}{1000\ \text{m}}\right]}$$

Other derived relations:

$$h = \frac{s}{v}, \qquad s = h \cdot v$$

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Example 4. Traffic flow on a one lane ramp is 360 veh/h, what is the average headway?

A. 0.1 s
B. 10 s
C. 36 s
D. 60 s
E. None of the above

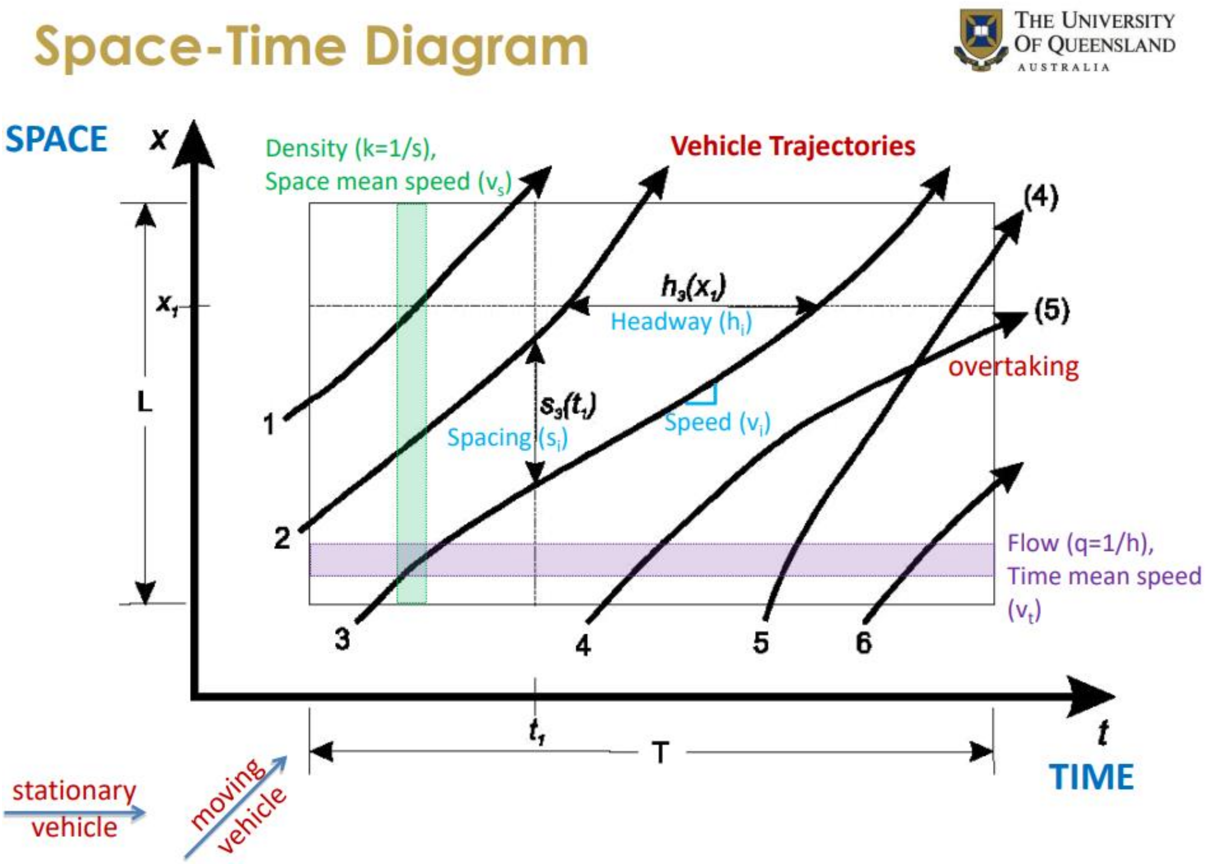
Answer.

$$q = 360\frac{\text{veh}}{\text{h}}$$

Fundamental relationship used: $q = \frac{1}{h} \Longrightarrow h = \frac{1}{q}$

$$h = \frac{1}{360\left[\frac{\text{veh}}{h}\right]} = \frac{1}{360}\left[\frac{h}{\text{veh}}\right] \times \frac{3600}{1}\left[\frac{s}{h}\right] = 10\ s/\text{veh}$$

Option B is correct.



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Types of traffic flow

- Uninterrupted Flow

Occurs in a traffic stream that is not delayed or interfered with by factors external to the traffic stream itself. (i.e. No traffic signals, STOP or GIVE WAY signs)

The characteristic of the traffic stream are based solely on the intersections among vehicles and with the roadway and the general environment.

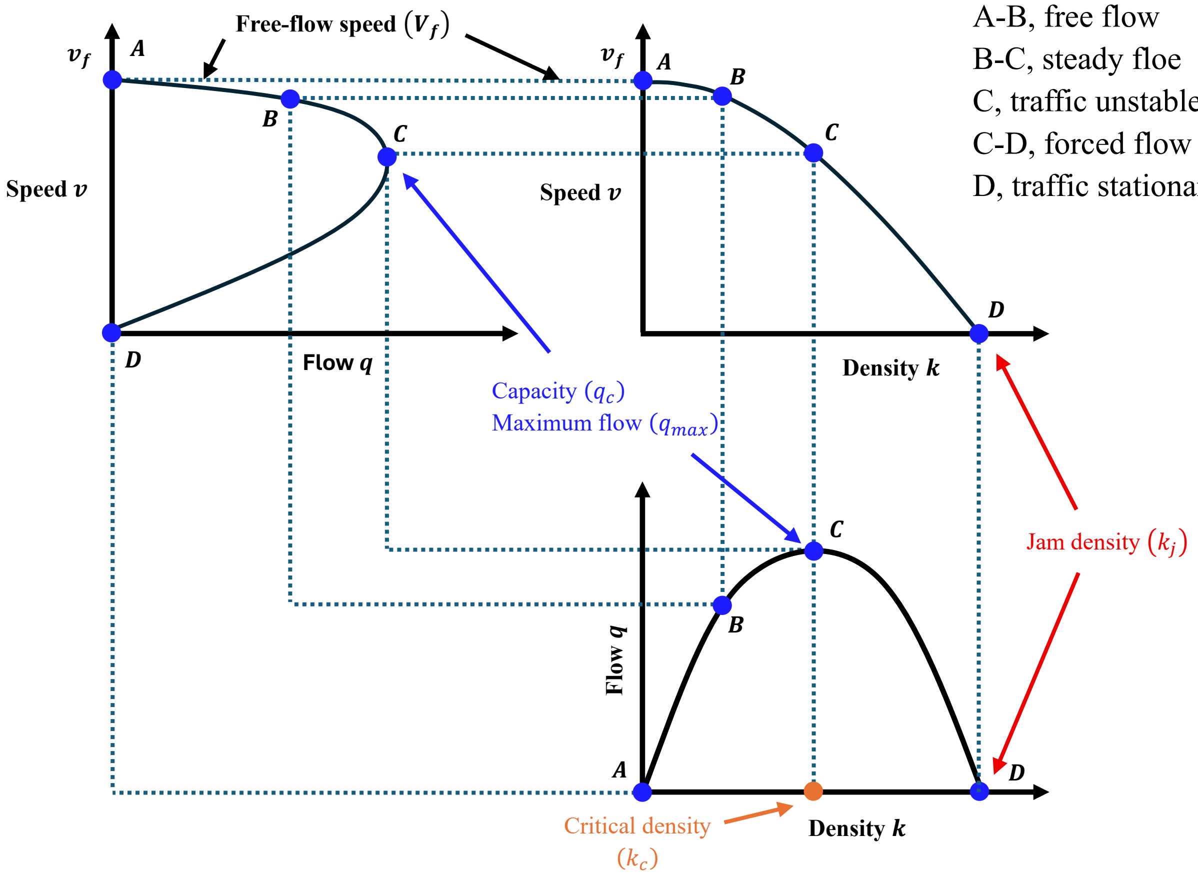
- Interrupted Flow

Occurs when external factors have significant effects on the traffic flow. (e.g. Traffic signal)

The characteristic of the traffic stream are primarily based on external interruptions (e.g. red lights); vehicle-vehicle interactions and vehicle-roadway interactions play a secondary role in defining the traffic flow.

Fundamental diagrams (FD) of uninterrupted traffic flow

A, no interaction
A-B, free flow
B-C, steady floe
C, traffic unstable
C-D, forced flow
D, traffic stationary



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Traffic Stream Model

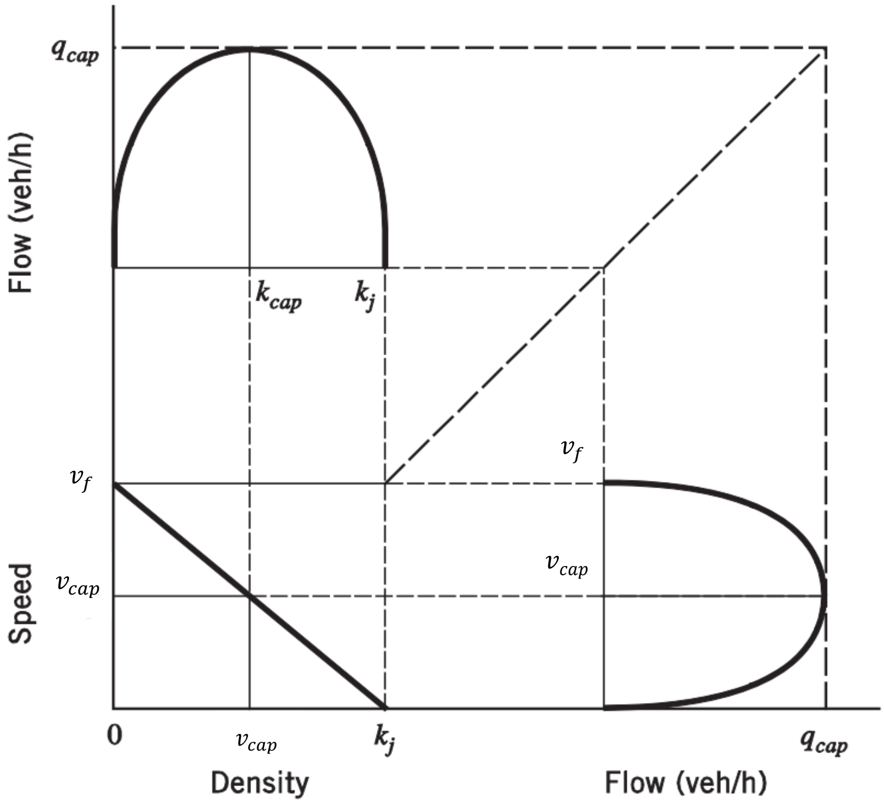
Speed-Flow-Density Relationships

$$q = v_f\left(k - \frac{k^2}{k_j}\right)$$

$$k_{cap} = k_c = \frac{1}{2}k_j$$

$$q_{cap} = q_c = \frac{1}{4}v_f k_j$$

v_f: Free-flow speed
k_c, k_{cap}: Critical density
k_j: Jam density
q_c, q_{cap}: Capacity



$$v = v_f\left(1 - \frac{k}{k_j}\right)$$

$$v_{cap} = \frac{1}{2}v_f$$

v: Space-mean speed (km/h)
v_f: Free-flow speed (km/h)
k_j: Jam density (veh/km)
v_{cap}: Speed at capacity or maximum flow

$$q = k_j\left(v - \frac{v^2}{v_f}\right), \qquad v_{cap} = \frac{1}{2}v_f$$

$$q_{cap} = q_c = \frac{1}{4}v_f k_j$$

q: Flow rate or volume (veh/h)
k_j: Jam density (veh/km)
v: Space-mean speed (km/h)
v_f: Free-flow speed (km/h)
v_{cap}: Speed at capacity or maximum flow
q_{cap}: Capacity or maximum flow

$$q: \left[\frac{\text{veh}}{h}\right]$$

$$k: \left[\frac{\text{veh}}{km}\right]$$

$$v: \left[\frac{km}{h}\right]$$

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Example. A section of highway is known to have a free-flow speed of 88 km/h and a capacity of 3300 veh/h. In a given hour, 2100 vehicles were counted at a specified point along this highway section. If the linear speed-density relationship applies, what would you estimate the space-mean speed of these 2100 vehicles to be?

Solution.

$$v_f = 88\frac{\text{km}}{h}, \qquad q_{cap} = 3300\frac{\text{veh}}{h}, \qquad q = 2100\frac{veh}{h}$$

$v = ?$

$$q_{cap} = \frac{1}{4}v_f k_j \Longrightarrow 3300\left[\frac{\text{veh}}{h}\right] = \frac{1}{4}\cdot 88\left[\frac{\text{km}}{h}\right]\cdot k_j \Longrightarrow k_j = 150\ \text{veh}/km$$

$$q = k_j\left(v - \frac{v^2}{v_f}\right) \Longrightarrow 2100 = 150\cdot\left(v - \frac{v^2}{88}\right)$$

$$2100 = 150\cdot\left(v - \frac{v^2}{88}\right)$$

$$14 = v - \frac{v^2}{88}$$

$$-v^2 + 88v - 14\times 88 = 0$$

$$\Longrightarrow v = 17.47\ \text{km/h or } 70.53\ \text{km/h}$$

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Generalised Traffic Flow Variables

Edie (1965) proposed generalised definitions of traffic flow (q), density (k), and speed (v). Let us explain this generalised definitions in the case of the region A (Fig. 4.24). Fig 4.24 shows the trajectory of six vehicles passing through the region A. In the considered case, the region A is in the form of a rectangle with sides are L and T. The thin horizontal rectangle and thin vertical rectangle are also shown in Fig 4.24 (these rectangles are shaded).

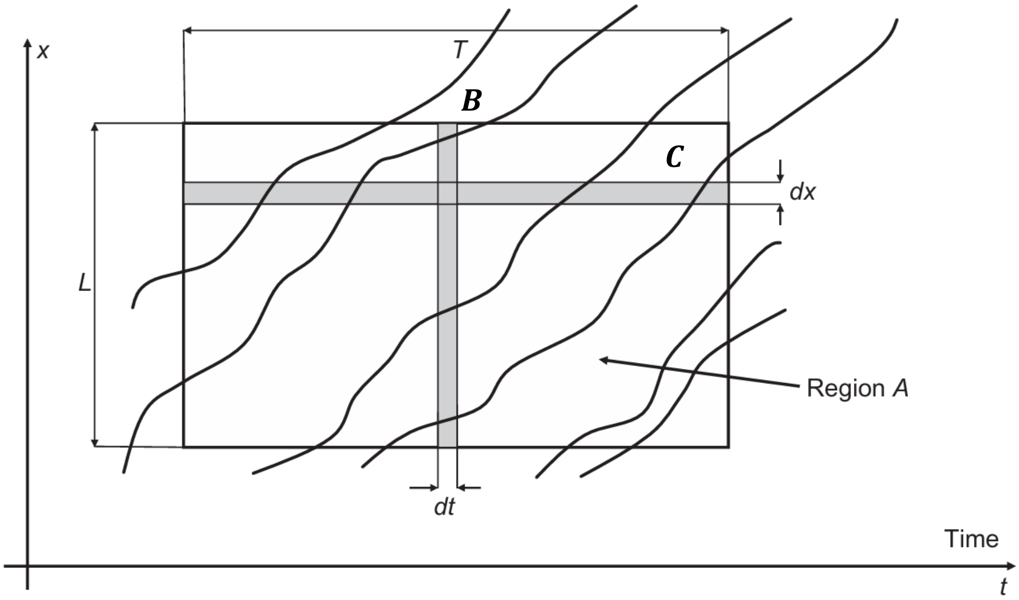

FIG. 4.24

Trajectories of six vehicles.

The thin, horizontal rectangle is related to a fixed surveillance point at the highway. The flow at the surveillance point dx, is equal to $\frac{N}{T}$, where N is the number of vehicles. In the case shown in Fig 4.24, $N = 4$. By multiplying both nominator and denominator by dx, the flow could be expressed as

$$\frac{N \cdot dx}{T \cdot dx}$$

The denominator $T \cdot dx$ represents the area of thin horizontal rectangle. The denominator is expressed in units of distance × time.

The nominator $N \cdot dx$ represents the total distance travelled by all vehicles in this thin horizontal rectangle. The flow represents the ratio of distance travelled by vehicles in a region to region's area. Any time-space region is composed of thin, elementary rectangles. We conclude that the flow $q(C)$ in ***any region*** C represents the distance $d(C)$ travelled by vehicles in a region C to the region's area $|C|$, for dx arbitrary small, each vehicle has same value of dx

$$q(C) = \frac{N \cdot dx}{T \cdot dx} = \frac{d(C)}{|C|}$$

$d(C)$: distance travelled by vehicles in a region C

$|C|$: any regions area C (area of horizontal rectangle)

Note: If dx gets larger, each vehicle's dx is different, in that case, $d(C) = dx_1 + dx_2 + dx_3 + \cdots + dx_n$

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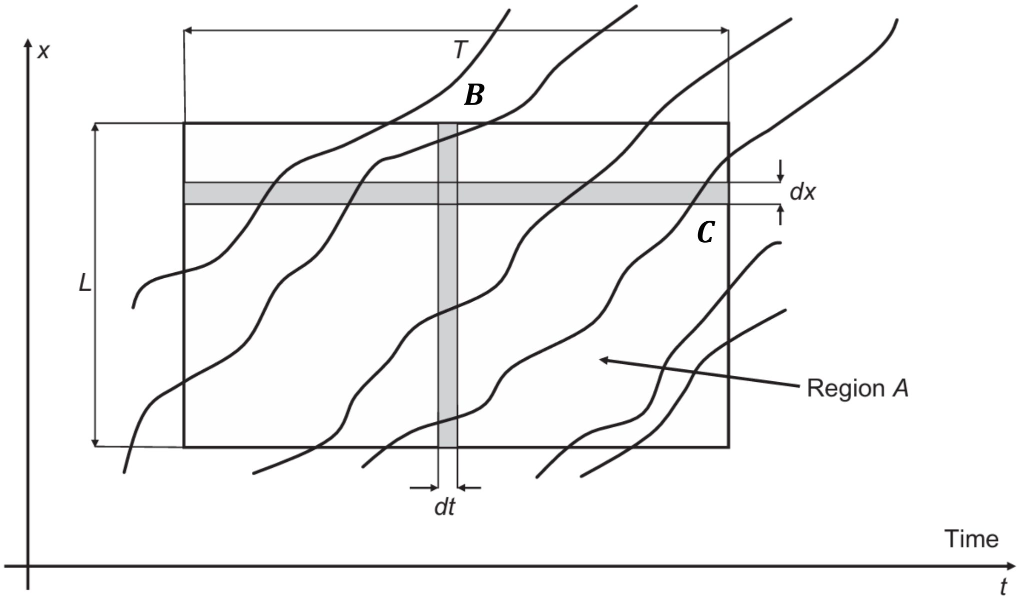

FIG. 4.24

Trajectories of six vehicles.

The thin, vertical rectangle shown in Fig 4.24 is related to an instant in time, dt. The density is equal to $\frac{N}{L}$, where N is the number of vehicles. In the case shown in Fig 4.24, $N = 3$. By multiplying both nominator and denominator by dt, the density could be expressed as

$$\frac{N \cdot dt}{L \cdot dt}$$

The nominator $N \cdot dt$ represents the total time spent by all vehicles in this thin vertical rectangle, while the denominator $L \cdot dt$ represents the area of the thin vertical rectangle.

We conclude that the density $k(A)$ in ***any region*** B represents the time $t(B)$ spent by vehicles in a region B to region's area $|B|$, for dt arbitrary small, each vehicle have same value of dt,

$$k(B) = \frac{N \cdot dt}{L \cdot dt} = \frac{t(B)}{|B|}$$

$t(B)$: time spent by vehicles in a region B

$|B|$: any regions area B (area of horizontal rectangle)

Note: If dt gets larger, each vehicle has different d_t, then $t(B) = dt_1 + dt_2 + \cdots + dt_n$

The speed $v(X)$ in ***any region*** X is equal

$$v(A) = \frac{q(X)}{k(X)} = \frac{\frac{d(X)}{|X|}}{\frac{t(X)}{|X|}} = \frac{d(X)}{t(X)}$$

$$= \frac{dx_1 + dx_2 + \cdots + dx_n}{dt_1 + dt_2 + \cdots + dt_n}$$

$d(X)$: total time spent by vehicles in a region X
$|X|$: any regions area X (area of whole rectangle)

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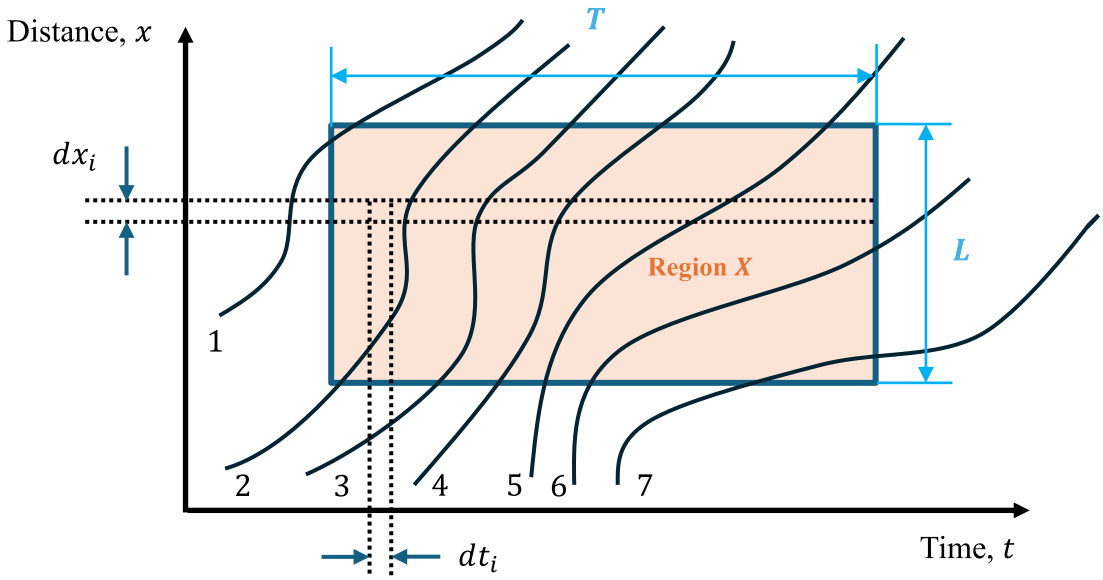
Overall, for any region C large enough,



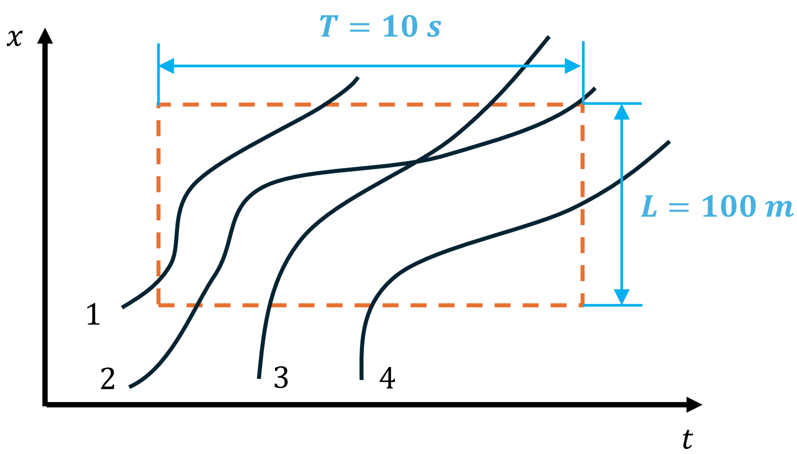
$$\text{Flow}, q(X) = \frac{d(X)}{|X|} = \frac{dx_1 + dx_2 + \cdots + dx_N}{L \times T} = \frac{\sum_{i=1}^{N} dx_i}{L \times T} \quad \left[\frac{\text{veh}}{h}\right]$$

$$\text{Density}, k(X) = \frac{t(X)}{|X|} = \frac{dt_1 + dt_2 + \cdots + dt_N}{L \times T} = \frac{\sum_{i=1}^{N} dt_i}{L \times T} \quad \left[\frac{\text{veh}}{km}\right]$$

$$\text{Speed}, v(X) = \frac{d(X)}{t(X)} = \frac{dx_1 + dx_2 + dx_N}{dt_1 + dt_2 + \cdots + dt_N} = \frac{\sum_{i=1}^{N} dx_i}{\sum_{i=1}^{N} dt_i} \quad \left[\frac{km}{h}\right]$$

Example. Calculate flow q in region A using Edie's generalised definition.

Veh i	dt_i	dx_i
1	$3\ s$	$70\ m$
2	$6\ s$	$100\ m$
3	$10\ s$	$80\ m$
4	$5\ s$	$50\ m$



Solution.

$$q(A) = \frac{d(A)}{|A|} = \frac{\sum_{i=1}^{4} dx_i}{L \times T} = \frac{dx_1 + dx_2 + dx_3 + dx_4}{L \times T} = \frac{70 + 100 + 80 + 50}{10 \times 100} = 0.3\frac{\text{veh}}{s}$$

$$= 1080\ \text{veh}/h$$

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Week 2 Exercise

1. Assume you are observing traffic in a single lane of a highway at a specific location. You measure the average headway and average spacing of passing vehicles as 3.2 s and 165 m, respectively. Calculate the flow and density of the traffic stream in this lane.

2. Given that 40 vehicles pass a given point in 1 minute and traverse a length of 1 kilometre, what is the flow, density, and time headway?

3. Given five observed velocities (60 km/hr, 35 km/hr, 45 km/hr, 20 km/hr, and 50 km/hr), what is the time-mean speed and space-mean speed?

4. Four race cars are travelling on a 2.5-mile tri-oval track. The four cars are travelling at constant speeds of 195 mi/h, 190 mi/h, 185 mi/h, and 180 mi/h, respectively. Assume you are an observer standing at a point on the track for a period of 30 minutes and are recording the instantaneous speed of each vehicle as it crosses your point. What is the time-mean speed and space-mean speed for these vehicles for this times period?

5. Four vehicles are traveling at constant speeds between sections X and Y (280 metres apart) with their positions and speeds observed at an instant in time. An observer at point X observes the four vehicles passing point X during a period of 15 seconds. The speeds of the vehicles are measured as 88, 80, 90, and 72 km/hr, respectively. Calculate the flow, density, time mean speed, and space mean speed of the vehicles.

6. (2024 Q1 MCQ) A traffic stream on a freeway has an average spacing of 50m and a flow of 1000 veh/h. What is the space mean speed for this traffic stream?

7. (2024 Q2 MCQ)A section of highway has the following flow-speed relationship, where density (k) is in veh/km and flow (q) is in veh/h.

$$q = 300v - 10v^2$$

What is the maximum speed?

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8. (2024 Q3 MCQ) Which one of the statements below about the fundamental traffic flow variables is TRUE?

a) Flow is the product of density and time mean speed.
b) Space mean speed is the arithmetic mean of spot speeds over a certain period of time.
c) Space mean speed is the harmonic mean of vehicle speeds at a given instant of time.
d) Time mean speed is always greater than space mean speed.
e) The conversion from occupancy to density requires average vehicle length and detector length.

9. (2024 Q4 MCQ) Three cars are travelling on a 5 km circular track at constant speeds of 90 km/h, 120 km/h and 150 km/h, respectively. What is the time-mean and space-mean speed?