MATH3301 Assignment 1 - Design Theory

See the Course Profile for the due date. This is not the assignment for MATH7331 students.

(1) **(4 marks)** For each $m \in \{5, 6, 7, 8\}$, construct a C_m-decomposition of K_{2m+1}.

(2) **(12 marks)** Determine congruence conditions on v that are equivalent to the obvious necessary conditions for the existence of a $(v, 15, 2)$-design, and use theorems from class to investigate existence of $(v, 15, 2)$-designs for $v \leq 140$.

(3) **(10 marks)** A *subsystem* of a Steiner triple system $(V, \mathcal{B})$ is a Steiner triple system $(U, \mathcal{A})$ such that $U \subseteq V$ and $\mathcal{A} \subseteq \mathcal{B}$. Construct a Steiner triple system of order 19 with a subsystem of order 7.

(4) **(6 marks)** Consider Theorem 4.2.8 in the typed notes. Extend this theorem by considering congruence classes of m modulo 60. To do this, you should state and prove a theorem that guarantees existence of transversal designs with block size larger than those given by Theorem 4.2.8 for certain congruence classes of m modulo 60.

(5) **(8 marks)** Prove that there exist 6 mutually orthogonal Latin squares of order n for each $n \in \{70, 80, 84, 85, 86\}$.