Essential Maths for DTC DPhil Students

Michaelmas Term 2020

Problem Sheet 8: complex numbers

Introductory problems

1. Simplify:
   a) $2-2i \quad + \quad 3+i$
   b) $4-6i \quad + \quad 19+4i$
   c) $(1-i)^2$
   d) $(2+i)^2$
   e) $(4-3i)/(2+6i)$
   f) $(1+2i)/(1-3i)$
   g) $(2-i)^{-2}+(2+i)^{-2}$
   h) $(5-i)^{-2}-(5+i)^{-2}$

2. Find the sum, difference, product and quotient of the complex numbers $z_1 = 5+3i$ and $z_2 = -4+2i$.

Main problems

1. Solve the following equations for z:
   a) $(7+i)z - 3i = 6$
   b) $\dfrac{(z-i)}{(z+i)} = \dfrac{2}{3}$
   c) $z^2 + (1+4i)z + (15+27i) = 0$

2. Represent the complex numbers $z_1 = 5-2i$ and $z_2 = -2+4i$ on a sketch/plot of the complex plane (an Argand diagram). Write down z_1 and z_2 in polar form (either give r and θ for each, or write them as exponentials). What is the product of z_1 and z_2?
3. Find the sum, difference, product and quotient of the complex numbers $z_1 = 5e^{4i}$ and $z_2 = 3e^{-2i}$. Can you represent the results on an Argand diagram?
4. What are the (x, y) coordinates of the complex number $5e^{4i}$?
5. If $z = 1+i$, mark on an Argand diagram the four points A, B, C and D representing z, z^2, z^3 and z^4 respectively. Find by calculation or from your diagram, the moduli and arguments of the complex numbers $z^2 - 1$ and $z + z^4$.
6. Find the complex numbers represented by the vertices of a square if one vertex represents $3+3i$ and the centre of the square represents $1+2i$.

Extension problems

1. Experiment with using Python to solve the problems and confirm your pen & paper solutions.