

Essential Maths for DTC DPhil Students

Michaelmas Term 2020

Problem Sheet 8: complex numbers

Introductory problems

1. Simplify:

a) $2 - 2i + 3 + i$

b) $4 - 6i + 19 + 4i$

c) $(1 - i)^2$

d) $(2 + i)^2$

e) $(4 - 3i)/(2 + 6i)$

f) $(1 + 2i)/(1 - 3i)$

g) $(2 - i)^{-2} + (2 + i)^{-2}$

h) $(5 - i)^{-2} - (5 + i)^{-2}$

2. Find the sum, difference, product and quotient of the complex numbers $z_1 = 5 + 3i$ and $z_2 = -4 + 2i$.

Main problems

1. Solve the following equations for z :

a) $(7 + i)z - 3i = 6$

b) $\frac{(z - i)}{(z + i)} = \frac{2}{3}$

c) $z^2 + (1 + 4i)z + (15 + 27i) = 0$

2. Represent the complex numbers $z_1 = 5 - 2i$ and $z_2 = -2 + 4i$ on a sketch/plot of the complex plane (an Argand diagram). Write down z_1 and z_2 in polar form (either give r and θ for each, or write them as exponentials). What is the product of z_1 and z_2 ?

3. Find the sum, difference, product and quotient of the complex numbers $z_1 = 5e^{4i}$ and $z_2 = 3e^{-2i}$. Can you represent the results on an Argand diagram?

4. What are the (x, y) coordinates of the complex number $5e^{4i}$?

5. If $z = 1 + i$, mark on an Argand diagram the four points A, B, C and D representing z, z^2, z^3 and z^4 respectively. Find by calculation or from your diagram, the moduli and arguments of the complex numbers $z^2 - 1$ and $z + z^4$.

6. Find the complex numbers represented by the vertices of a square if one vertex represents $3 + 3i$ and the centre of the square represents $1 + 2i$.

Extension problems

1. Experiment with using Python to solve the problems and confirm your pen & paper solutions.