Control in Robotics
Part 3: Force Control

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Introduction

- In this chapter we deal with the motion control problem for situations in which the robot manipulator end effector is in contact with the environment.
- Many robotic tasks involve intentional interaction between the manipulator and the environment.
- Usually, the end effector is required to follow in a stable way the edge or the surface of a workpiece while applying prescribed forces and torques.
- The specific feature of robotic problems such as polishing, deburring, or assembly, demands control also of the exchanged forces at the contact.
- These forces may be explicitly set under control or just kept limited in a indirect way, by controlling the end-effector position.

Introduction

- In any case, force specification is often complemented with a requirement concerning the end-effector motion, so that the control problem has in general hybrid (mixed) objectives.
- Three control strategies are discussed; namely, impedance control, parallel control, and hybrid force/motion control.
- Impedance control tries to assign desired dynamic characteristics to the interaction with rather unmodelled objects in the workspace. Parallel control provides the additional feature of regulating the contact force to a desired value. Hybrid force/motion control exploits the partition of the task space into directions of admissible motion and of reaction forces, both arising from the existence of a rigid constraint.

Impedance control

- The underlying idea of impedance control is to assign a prescribed dynamic behaviour for a robot manipulator while its end effector is interacting with the environment.
- The desired performance is specified by a generalized dynamic impedance, i.e., by a complete set of linear or nonlinear second-order differential equations representing a mass-spring-damper system.
- As an example, consider automated fish skinning operatio which is one of the most complex parts for an automation system.
- In order to visualize the difficulties, consider how this operation will be accomplished manually.

Impedance control

- While the fish fillet is placed on the cutting board on its skinned side, a human shall angle sharp knife very slightly down the flesh such that the edge are toward the skin.
- By this means the operator will ensure that the skin is cut away and only little flesh will be removed by the skin.
- In general, impedance control is suitable for those tasks where contact forces must be kept small, typically to avoid jamming among parts in assembly or insertion operations, while accurate regulation of forces is not required.
- In fact, an explicit force error loop is absent in this approach, so that it is often stated that "force is controlled by controlling position".

Impedance control

- In the following, we will first introduce the convenient format of the manipulator dynamic model in task space coordinates.
- The imposition of a desired impedance at the end-effector level will then be obtained by an inverse dynamics scheme designed in the proper task space coordinates.
- Simplified control laws are then recovered from this general scheme.

Task space dynamic model

- When additional external forces act from the environment on the manipulator end effector, the dynamic model becomes

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u + J^T(q)f \tag{1}$$

- Notably $J(q) \in \mathbb{R}^{m\times n}$ is the geometric Jacobian of the manipulator relating end-effector velocity to joint velocity as

$$v = \begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(q)\dot{q}$$

- We will consider only the case m = n and, without loss of generality, n = 6.
- With reference to the analytical Jacobian, the differential kinematics equation can be written in the form

$$\dot{x} = J_a(q)\dot{q} = T_a^{-1}J(q)\dot{q} \tag{2}$$

Task space dynamic model

- The two sets of generalized forces f and fa performing work on v and $\dot{x}$, respectively, are related by the virtual work principle, i.e.,

$$J^T(q)f = J_a^T(q)f_a$$

- Then, the model (1) can be rewritten as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = u + J_a^T(q)f_a \qquad (3)$$

- By further differentiation of (2) we have

$$\ddot{x} = J_a(q)\ddot{q} + \dot{J}_a(q)\dot{q}$$

Task space dynamic model

- Then, it is easy to see that the dynamic model in the task space becomes

$$M_x(q)\ddot{x} + C_x(q,\dot{q})\dot{x} + g_x(q) = J_a^{-T}(q)u + f_a \tag{4}$$

where

$$\begin{aligned} M_x(q) &= J_a^{-T}(q)M(q)J_a^{-1}(q) \\ C_x(q,\dot{q}) &= J_a^{-T}(q)C(q,\dot{q})J_a^{-1}(q) - M_x(q)\dot{J}_a(q)J_a^{-1}(q) \\ g_x(q) &= J_a^{-T}(q)g(q) \end{aligned}$$

- It is easy to conclude that the Skew Symmetry Property and Boundedness of the Inertia Matrix property hold for $M_x(q)$ and $C_x(q,\dot{q})$.

Impedance Control Structure

- To achieve a desired dynamic characteristic for the interaction between the manipulator and the environment at the contact, the design of the control input u is carried out in two steps.
- The first step decouples and linearizes the closed-loop dynamics in task space coordinates so as to obtain

$$\ddot{x} = u_0$$

where u_0 is an external auxiliary input still available for control. This is achieved by designing an inverse control law in task space as

$$u = J_a^T(q)(M_x(q)u_0 + C_x(q,\dot{q})\dot{x} + g_x(q) - f_a) \qquad (5)$$

Impedance Control Structure

- In the second step, the desired impedance model that dynamically balances contact forces f_a at the manipulator end effector is chosen as a linear second-order mechanical system described by

$$M_d(\ddot{x} - \ddot{x}_d) + B_d(\dot{x} - \dot{x}_d) + K_d(x - x_d) = f_a$$

  where M_d, B_d, K_d, are 6×6 matrices specifying desired inertia, damping, and stiffness, respectively.
- Then u_0 is chosen as

$$u_0 = \ddot{x}_d + M_d^{-1}(B_d(\dot{x}_d - \dot{x}) + K_d(x_d - x) + f_a)$$

- The implementation of the above controller requires feedback from the manipulator state $(q, \dot{q})$ and measure of the contact force f_a. However, no explicit force loop is imposed in this control law.

Impedance Control Structure

- The overall impedance control law is expressed as

$$\begin{aligned} u = & J_a^T(q)\Big(M_x(q)\ddot{x}_d + C_x(q,\dot{q})\dot{x} + g_x(q) \\ & + M_x(q)M_d^{-1}(B_d(\dot{x}_d - \dot{x}) + K_d(x_d - x)) \\ & + (M_x(q)M_d^{-1} - I)f_a\Big) \end{aligned} \tag{6}$$

- A relevant simplification occurs in the above control law when a nonlinear impedance is prescribed. Replacing formally $M_d = M_x(q)$ in (6) leads to

$$u = J_a^T(q)\Big(M_x(q)\ddot{x}+C_x(q,\dot{q})\dot{x}+g_x(q)+B_d(\dot{x}_d-\dot{x})+K_d(x_d-x)\Big) \tag{7}$$

- Remarkably, no force feedback is required in this case. The price to pay is that the effective inertia of the manipulator will be the natural one in the task space.

PD Control and Stiffness Control

- Provided that quasi-static assumptions are made ($\dot{x}_d = 0$ and $\dot{q} \approx 0$ in the nonlinear dynamic terms), the following controller is obtained from (7):

$$u = J_a^T(q)(K_d(x_d - x) - B_d\dot{x}) + g(q) \tag{8}$$

- It can be recognized that this is nothing but the task space PD control law, with added gravity compensation, that was considered in the previous chapter.
- If gravity terms are mechanically balanced,(g(q) $=$ 0), and no additional damping is included ($B_d = 0$), then (8) reduces to the so-called stiffness control:

$$u = J_a^T(q)K_d(x_d - x) \tag{9}$$

Parallel control

- The above impedance control schemes perform only indirect force control, through a closed-loop position controller, without explicit closure of a force feedback loop.
- In other words, it is not possible to specify a desired amount of contact force with an impedance controller, but only a satisfactory dynamic behaviour between end-effector force and displacement at the contact.
- An effective strategy to embed the possibility of force regulation is given by the so-called parallel control.
- The key concept is to provide an impedance controller with the ability of controlling both position and force.
- This is obtained by closing an outer force control loop around the inner position control loop, which is typically available in a robot manipulator.

Parallel control

- By a suitable design of the force control action (typically an integral term), it is possible to regulate the contact force to a desired value.
- In order to provide motion control along the feasible task space directions, also a desired position is input to the inner loop.
- The result is two control actions, working in parallel; namely, a force control action and a position control action.
- With reference to the task space dynamic model (4), the design of the control input u can be carried out as for the above inverse dynamics controller in (5), where force measurements of f_a are assumed to be available.

Parallel control

- According to the parallel control approach, the new control input u_0 can be designed as the sum of a position control action and a force control action; namely, as

$$u_0 = u_{0x} + u_{0f}$$

with

$$u_{0x} = \ddot{x}_d + M_d^{-1}\Big(B_d(\dot{x}_d - \dot{x}) + K_d(x_d - x)\Big)$$
$$u_{0f} = -M_d^{-1}\Big(K_f(f_{ad} - f_a) + K_i \int_0^t (f_{ad} - f_a)d\tau\Big)$$

- K_f, K_i are suitable force feedback matrix gains characterizing a PI action on the force error.

Parallel control

- Substituting the above controller in dynamic model (4) yields

$$\begin{aligned} M_d(\ddot{x} - \ddot{x}_d) + & B_d(\dot{x} - \dot{x}_d) + K_d(x - x_d) \\ & = K_f(f_a - f_{ad}) + K_i \int_0^t (f_a - f_{ad}) d\tau \end{aligned}$$

  which reveals that, thanks to the integral action, the force error $(f_{ad} - f_a)$ is allowed to prevail over the position error $(x_d - x)$ at steady state.

Partially Constrained Tasks

- The presented approach is based on a framework for control in situations in which motion of the manipulator is partially constrained by contact with one or more surfaces.
- We are interested in describing contact and freedoms, so we consider only the forces due to contact. Thus, other forces such as friction components and gravity are ignored.
- Every manipulation task can be broken down into subtasks that are defined by a particular contact situation occurring between the manipulator end-effector (or tool) and the work environment.
- With each such subtask, we can associate a set of constraints, called the natural constraints, that result from the particular mechanical and geometric characteristics of the task configuration.

Partially Constrained Tasks

- For instance, a hand in contact with a stationary, rigid surface is not free to move through that surface; hence, a natural position constraint exists.
- If the surface is frictionless, the hand is not free to apply arbitrary forces tangent to the surface; thus, a natural force constraint exists.
- These two types of constraint, force and position, partition the degrees of freedom of possible end-effector motions into two orthogonal sets that must be controlled according to different criteria.
- Additional constraints, called artificial constraints, specify desired motion or force. That is, each time the user specifies a desired trajectory in either position or force, an artificial constraint is defined.

Partially Constrained Tasks

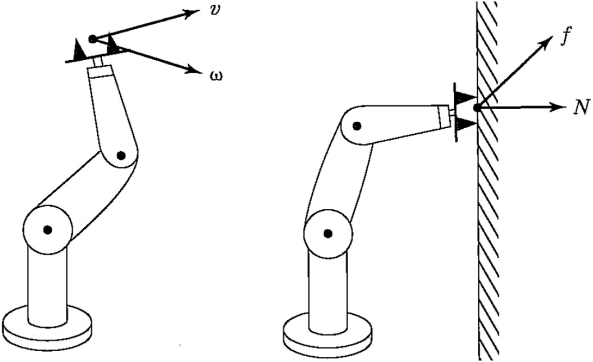


Figure : The two extremes of contacting situations. The manipulator on the left is moving in free space where no reaction surface exits. The manipulator on the right is glued to the wall so that no free motion is possible.

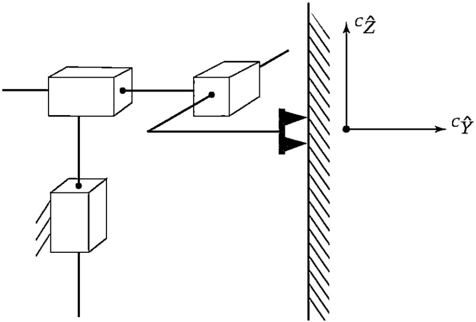
The Hybrid Position/Force Control Problem

The hybrid position/force controller must solve three problems:

- Position control of a manipulator along directions in which a natural force constraint exists.
- Force control of a manipulator along directions in which a natural position constraint exists.
- A scheme to implement the arbitrary mixing of these modes along orthogonal degrees of freedom of an arbitrary frame

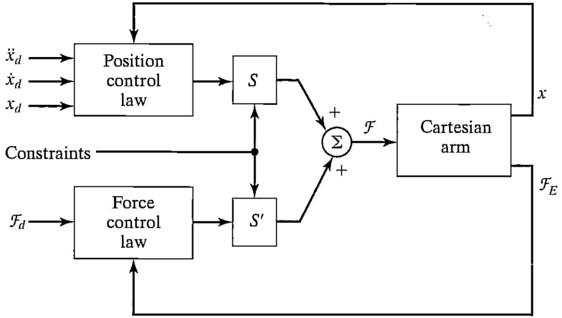
The Hybrid Position/Force Control Scheme

- We will consider a 3-DOF Cartesian manipulator with prismatic joints acting in the $\hat{Z}$, $\hat{Y}$, and $\hat{X}$ directions.
- The end-effector is in contact with a surface of stiffness k_e. Hence, force control is required in that direction, but position control in the $\hat{Z}$ and $\hat{X}$ directions.



The Hybrid Position/Force Control Scheme

- Here, we indicate the control of all three joints of our simple Cartesian arm in a single diagram by showing both the position controller and the force controller.
- The matrices S and S' determine which mode (position or force) is used to control each joint of the Cartesian arm.



The Hybrid Position/Force Control Scheme

- The S matrix is diagonal, with ones and zeros on the diagonal.
- Where a one is present in S, a zero is present in S' and position control is in effect.
- Where a zero is present in S, a one is present in S' and force control is in effect.
- Hence the matrices S and S' are simply switches that set the mode of control to be used with each degree of freedom
- For the presented problem of Cartesian arm:

$$S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad S' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The Hybrid Position/Force Control Scheme

- Generalizing the hybrid controller presented for Cartesian arm so that a general manipulator may be used is straightforward with the concept of Cartesian-based control.
- The major idea is that, through use of a dynamic model written in Cartesian space, it is possible to control so that the combined system of the actual manipulator and computed model appear as a set of independent, uncoupled unit masses.
- Because we have designed the hybrid controller for a Cartesian manipulator aligned with the constraint frame, and because the Cartesian decoupling scheme provides us with a system with the same inputoutput properties, we need only combine the two to generate the generalized hybrid position/force controller.

The Hybrid Position/Force Control Scheme

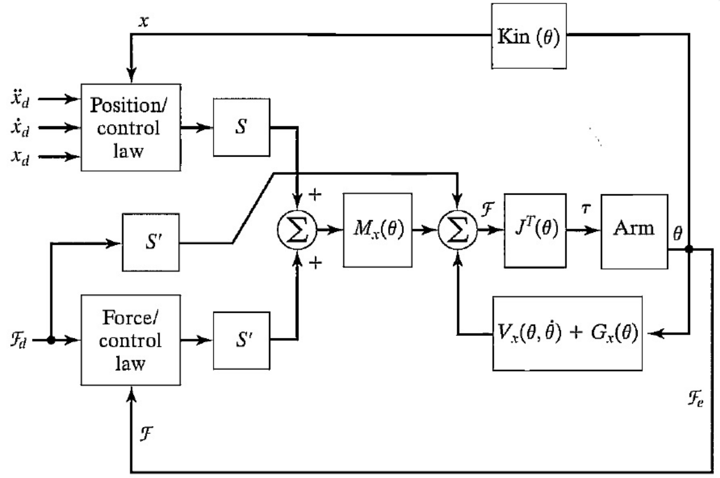


Figure : The hybrid position/force controller for a general manipulator.