EASC-2220
Mechanics of Materials

Project 2
Level 4

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The intention of this document is to provide the information listed below for the traffic signal support structure.

1) Reaction forces at the fixed support
2) Principle stresses on the four elements located at the base and around the vertical beam
3) Principle planes of the elements
4) Maximum shear stress on the elements
5) Shear plane on the elements

Blue cells contain **modifiable** values, while white cells are calculation outputs and **should not be changed**.

Axial Conventions										
CCW	+M		Right	+X		Up	+Y		Out of Pg	+Z
CW	-M		Left	-X		Down	-Y		Into Page	-Z
Rotational conventions are from the perspective of the positive positive side of the axis, facing the origin.										

Material, Environmental, and Structure Properties						
Material		Al T6061		Wind Speed (m/s)		Mass Per Traffic Bulb (kg)
Density	2700	kg/m^3		51		1
Tens & Comp Yield	276	MPa				
Shear Strength	207	MPa				

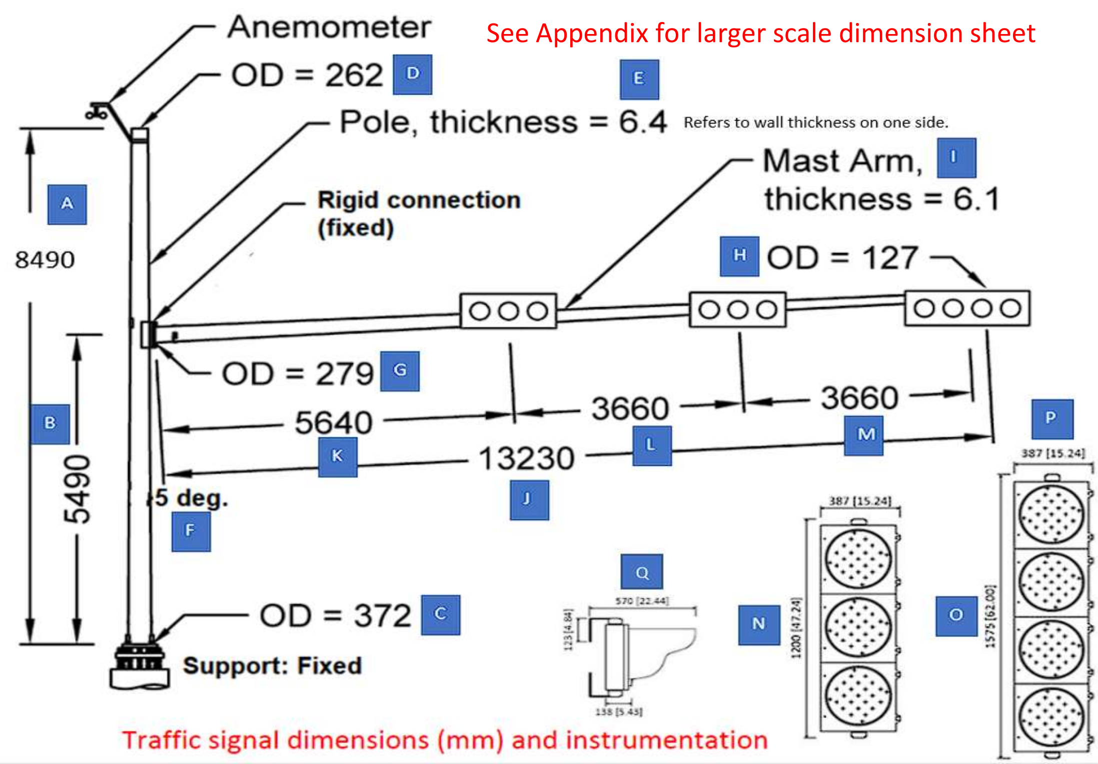
Wind Speed is calculated and has maximum effect in -Z direction

Reactions			Safety Factor
RMx	50.8	kNm	2
RMy	-40.9	kNm	
RMz	8.8	kNm	Structure Status
Rx	0.0	kN	
Ry	2.8	kN	SAFE
Rz	10.1	kN	

Stresses & Stress Planes					
	Element 1	Element 2	Element 3	Element 4	
σ_{Max}	87.5	27.6	10.9	35.4	MPa
σ_{Min}	-11.0	-41.2	-88.1	-22.5	MPa
τ_{Max}	49.2	34.4	49.5	29.0	MPa
$\theta_{principle}$	-19.5	39.3	19.4	-38.6	deg
θ_{shear}	25.5	-5.7	-25.6	6.4	deg

Modifiable Dimension Entry		
A	8.4900	m
B	5.4900	m
C (Z)	0.3720	m
D	0.2620	m
E	0.0064	m
F	0.0873	rad
G	0.2790	m
H	0.1270	m
I	0.0061	m
J	13.2300	m
K	5.6400	m
L	3.6600	m
M	3.6600	m
N	1.2000	m
O	1.5750	m
P	0.3870	m
Q	0.5700	m

See Appendix for larger scale dimension sheet



Traffic signal dimensions (mm) and instrumentation

Calculations Page 1

Wind Pressure	1594.413	N/m^2

Vertical Beam		
Height	8.490	m
OD Base	0.372	m
OD Top	0.262	m
Taper Loss/Meter	0.013	m dia/m
Surface Area	2.691	m^2
Local Windforce	4291.092	N
Cntrd & COG dx, dz	0.000	m
Center of Grav. dy	3.994	m
Centroid dy	3.999	m
I_x	59.054	m^4
I_y	0.023	m^4
Volume	0.053	m^3
Mass	143.154	kg
Weight	1404.338	N

3 Segment Traffic Light		
Height	0.387	m
Length	1.200	m
Surface Area	0.464	m^2
Local Windforce	740.445	N
Centroid dx	5.769	m
Centroid dx_2	9.415	m
Centroid $dy_{1,2}$	5.684	m
Centroid dz_1	0.499	m
Centroid dz_2	0.457	m
I_x	15.007	m^4
I_y	15.511	m^4
I_{x2}	15.007	m^4
I_{y2}	41.222	m^4
Weight	29.430	N

4 Segment Traffic Light		
Height	0.387	m
Length	1.575	m
Surface Area	0.610	m^2
Local Windforce	971.835	N
Centroid dx	13.061	m
Centroid dy	5.684	m
Centroid dz	0.415	m
I_x	19.697	m^4
I_y	104.1	m^4
Weight	39.240	N

Note that letters refer to labels in dimension diagram

Assumptions

1) Wind is linear and blows only towards -Z axis
2) Anemometer is massless and dimensionless
3) Tapered beams have continuous wall thickness
4) Center of gravity of the light fixtures are located at Q/2
5) Bottom edge of all light fixtures align with the top of dimension B

--General Equations--

$$\text{Wind Pressure} = 0.613 * \text{Velocity}^2$$

$$\text{Taper Loss Per Meter (TLoss)} = \frac{(\text{Root Diameter} - \text{Tip Diameter})}{\text{Length}}$$

$$\text{Dia. of Tapered Beam at Length} = \text{Root Dia} - (\text{TLoss} * \text{Length})$$

-- Vertical Beam Calculations --

$$\text{Vertical Beam (Trapezoid) Surface Area} = \text{Height} * \frac{\text{Base} + \text{Top}}{2}$$

$$\text{Local Windforce} = \text{Wind Force} * \text{Surface Area}$$

Vertical Beam Center of Gravity dy - See Appendix

$$\text{Vertical Beam Centroid dy} = \frac{\text{Height}}{3}\left(\frac{2\text{Top} + \text{Base}}{\text{Top} + \text{Base}}\right)$$

Volume - See Appendix

-- 3 Segment Traffic Light Calculations --

$$\text{Centroid dx} = \text{Radius of Vertical Beam} + (K)\cos(F)$$

$$\text{Centroid dx}_2 = \text{Radius of Vertical Beam} + (K + L)\cos(F)$$

$$\text{Centroid dy}_{1,2} = B + (P/2)$$

$$\text{Centroid dz}_{1,2} = \text{Radius of Mast} + (Q/2)$$

-- 4 Segment Traffic Light Calculations --

$$\text{Centroid dx} = \text{Radius of Vertical Beam} + (K + L + M)\cos(F)$$

$$\text{Centroid dy} = B + (P/2)$$

$$\text{Centroid dz} = \text{Radius of Mast} + (Q/2)$$

Note that the radius of the mast and the vertical beam differ over the span

Calculations Page 2

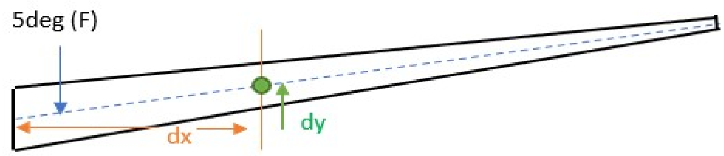
Mast (Whole)		
Length	13.230	m
OD Root	0.279	m
OD Tip	0.127	m
Taper Loss/Meter	0.011	m dia/m
Surface Area	2.686	m^2
Local Windforce	4282.099	N
Center of Grav. dx	5.892	m
Center of Grav. dy	6.006	m
I_x	96.873	m^4
I_y	130.592	m^4
Volume	0.050	m^3
Mass	134.787	kg
Weight	1322.264	N

-- Mast (Whole) Calculations --

$$\text{Mast (Trapezoid) Surface Area} = \text{Length} * \frac{\text{Root} + \text{Tip}}{2}$$

Center of Gravity dx - See Appendix

$$\text{Center of Gravity dy} = B + (\text{dx} * \tan(F))$$



Volume - See Appendix

Mast Section 1		
Length	5.040	m
OD Root	0.279	m
OD Tip	0.221	m
Surface Area	1.260	m^2
Local Windforce	2009.343	N
Centroid dx	2.564	m
Centroid dy	5.714	m
I_x	41.158	m^4
I_y	10.940	m^4
Mast Section 2		
Length	2.460	m
OD Root	0.207	m
OD Tip	0.179	m
Surface Area	0.475	m^2
Local Windforce	757.689	N
Centroid dx	7.562	m
Centroid dy	6.152	m
I_x	17.985	m^4
I_y	27.415	m^4
Mast Section 3		
Length	2.273	m
OD Root	0.165	m
OD Tip	0.139	m
Surface Area	0.346	m^2
Local Windforce	551.482	N
Centroid dx	11.112	m
Centroid dy	6.462	m
I_x	14.445	m^4
I_y	42.860	m^4

-- Mast Section 1 Calculations --

$$\text{Length} = K - N/2$$

$$\text{Mast (Trapezoid) Surface Area} = \text{Length} * \frac{\text{Root} + \text{Tip}}{2}$$

$$\text{Centroid dx} = \frac{h}{3}\left(\frac{Root+2Tip}{Root+Tip}\right)\cos(F) + \text{Radius of Vertical Beam}$$

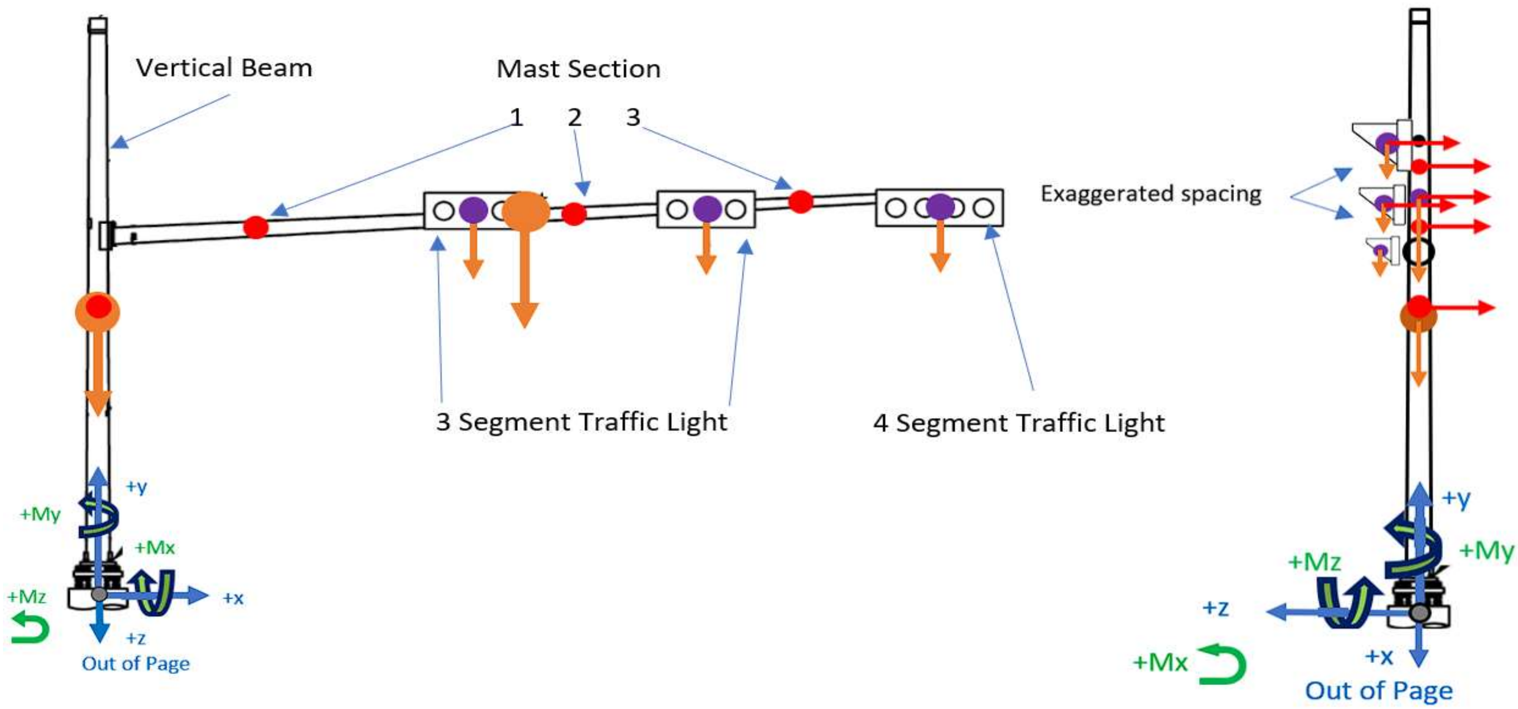
$$\text{Centroid dy} = B + (\text{dx} * \tan(F))$$

Mast Sections 2 & 3 follow similarly to Section 1

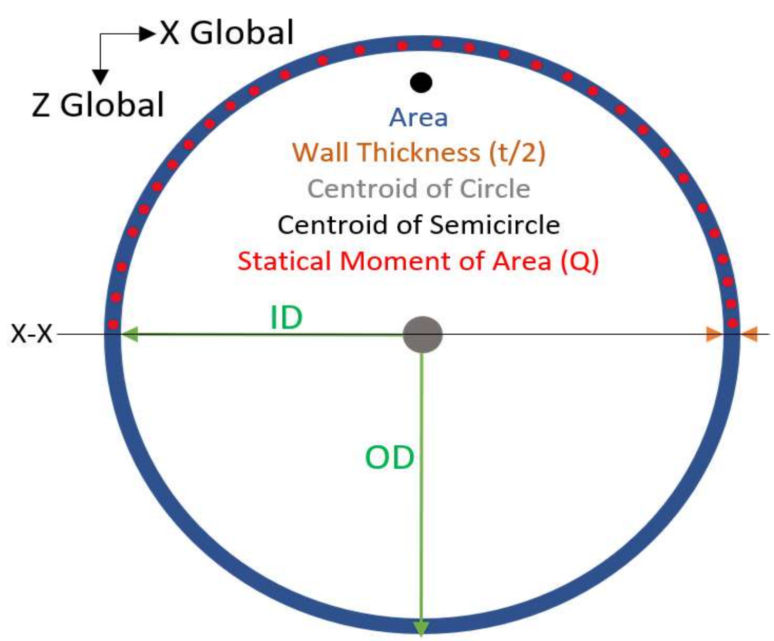
ΣXY Inertias (m^4)	
ΣI_x	182.352
ΣI_y	242.078

Calculations Page 3		
Reactions		
RMx	50764.6	Nm
RMy	-40946.0	Nm
RMz	8750.7	Nm
Rx	0.0	N
Ry	2824.7	N
Rz	10062.3	N

Linear Axis' (X,Y,Z)
Rotational Axis' (Mx, My, Mz)
Origin (0,0,0)
Center of Gravity & Weight Force
Centroid & Wind Force
COG = Centroid for Wind Force & Weight Force



Fixed Base Support Cross Section		
Outer Diameter (OD)	0.372	m
Inner Diameter (ID)	0.359	m
Area	0.007	m^2
J (Polar Moment of Inertia)	2.46E-04	m^4
$I_x = I_y = J/2$	1.23E-04	m^4
Centroid of Semicircle (y)	0.116	m
Statical Moment of Area (Q)	4.28E-04	m^3



The directions of the reactions are flipped to observe the effect of the external forces on the material.

Mohrs Circle Coordinates

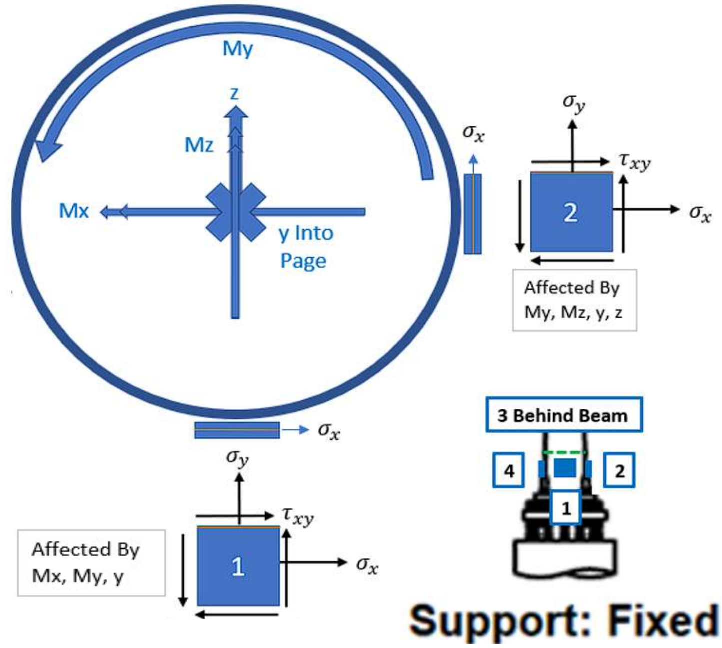
$$Pt1 = (\sigma_x, -\tau_{xy})$$
$$Pt2 = (\sigma_y, \tau_{xy})$$

Stress Element Diagram

Visible orange edges are kept on top of the element diagrams as though viewed from the outside of the pole.

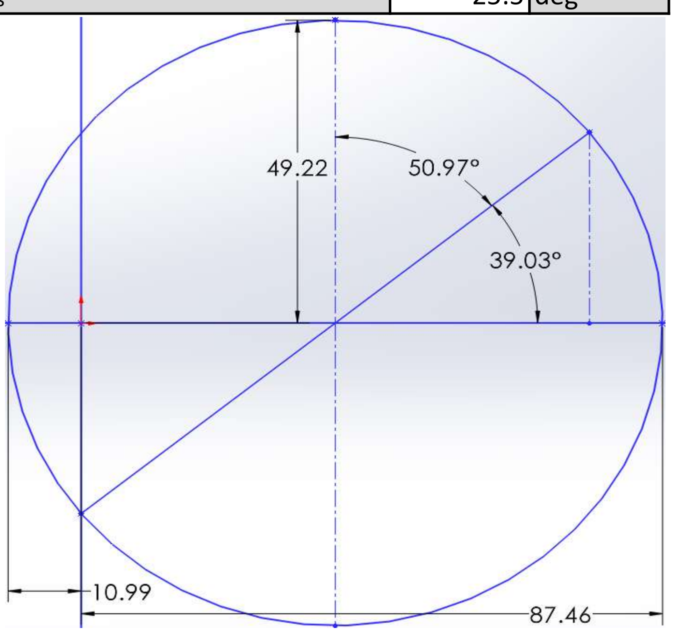
Forces from the central diagram are translated to X, Y, and XY axis' on the element diagrams

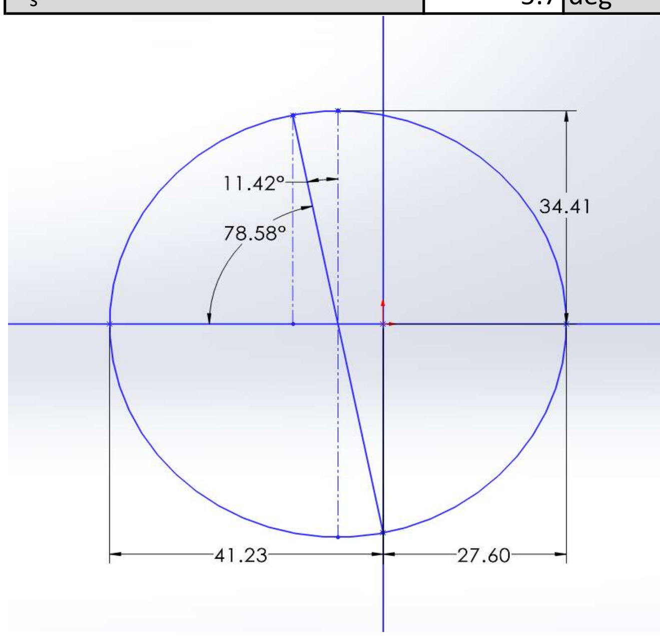
See appendix for explanation of stress axis assignment and formulas



Element 1		
Effect of Mx (Tens Y+)	76856.9	kPa
Effect of My (Shear XY+)	30995.8	kPa
Effect of Mz (N/A)	0.0	kPa
Effect of x (Shear XY + or -)	0.0	kPa
Effect of y (Comp Y-)	-384.3	kPa
Effect of z (N/A)	0.0	kPa
σ_x	0.0	kPa
σ_y	76472.6	kPa
τ_{xy}	30995.8	kPa
σ_{Min}	-10985.2	kPa
σ_{Max}	87457.8	kPa
τ_{Max}	49221.5	kPa
Θ_p	-19.5	deg
Θ_s	25.5	deg

Element 2		
Effect of Mx (N/A)	0.0	kPa
Effect of My (Shear XY+)	30995.8	kPa
Effect of Mz (Comp Y-)	-13248.4	kPa
Effect of x (N/A)	0.0	kPa
Effect of y (Comp Y-)	-384.3	kPa
Effect of z (Shear XY+)	2737.2	kPa
σ_x	0.0	kPa
σ_y	-13632.7	kPa
τ_{xy}	33733.0	kPa
σ_{Min}	-41231.2	kPa
σ_{Max}	27598.5	kPa
τ_{Max}	34414.8	kPa
Θ_p	39.3	deg
Θ_s	-5.7	deg

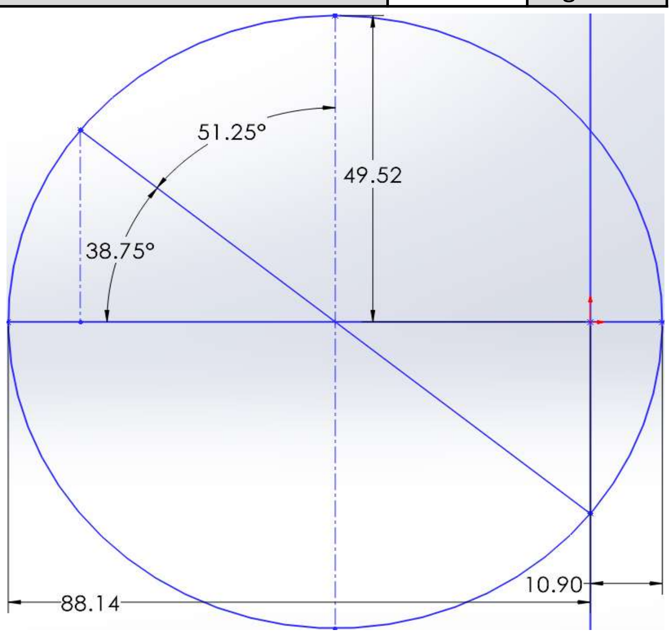


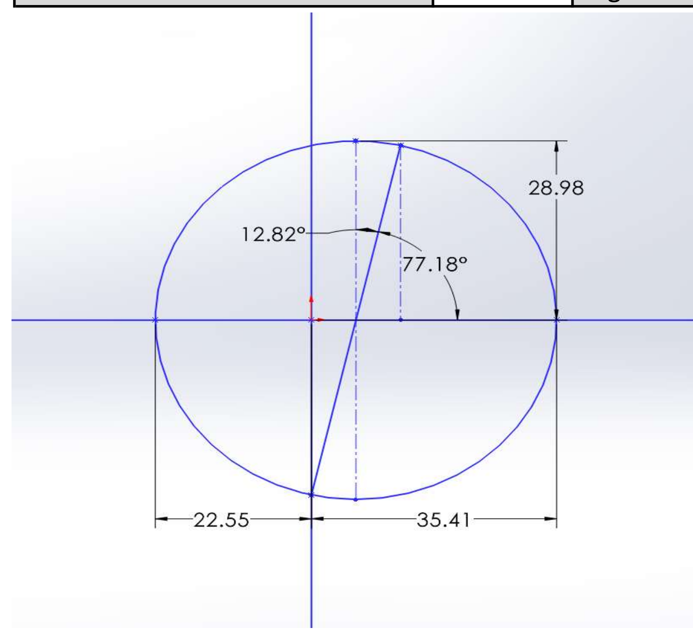


Calculations Page 4

Element 3		
Effect of Mx (Comp Y-)	-76856.9	kPa
Effect of My (Shear XY+)	30995.8	kPa
Effect of Mz (N/A)	0.0	kPa
Effect of x (Shear XY - or +)	0.0	kPa
Effect of y (Comp Y-)	-384.3	kPa
Effect of z (N/A)	0.0	kPa
σ_x	0.0	kPa
σ_y	-77241.2	kPa
τ_{xy}	30995.8	kPa
σ_{Min}	-88141.2	kPa
σ_{Max}	10900.0	kPa
τ_{Max}	49520.6	kPa
Θ_p	19.4	deg
Θ_s	-25.6	deg

Element 4		
Effect of Mx (N/A)	0.0	kPa
Effect of My (Shear XY+)	30995.8	kPa
Effect of Mz (Tens Y+)	13248.4	kPa
Effect of x (N/A)	0.0	kPa
Effect of y (Comp Y-)	-384.3	kPa
Effect of z (Shear XY-)	-2737.2	kPa
σ_x	0.0	kPa
σ_y	12864.2	kPa
τ_{xy}	28258.6	kPa
σ_{Min}	-22549.3	kPa
σ_{Max}	35413.5	kPa
τ_{Max}	28981.4	kPa
Θ_p	-38.6	deg
Θ_s	6.4	deg





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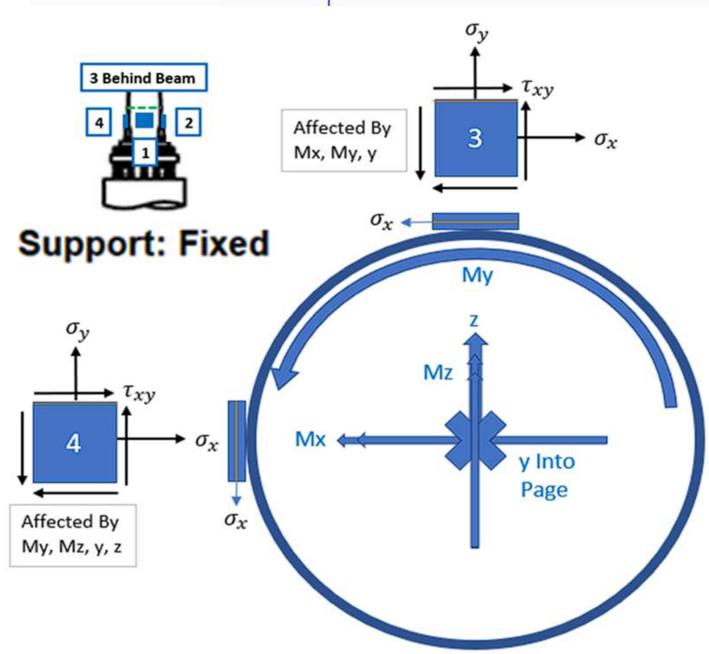
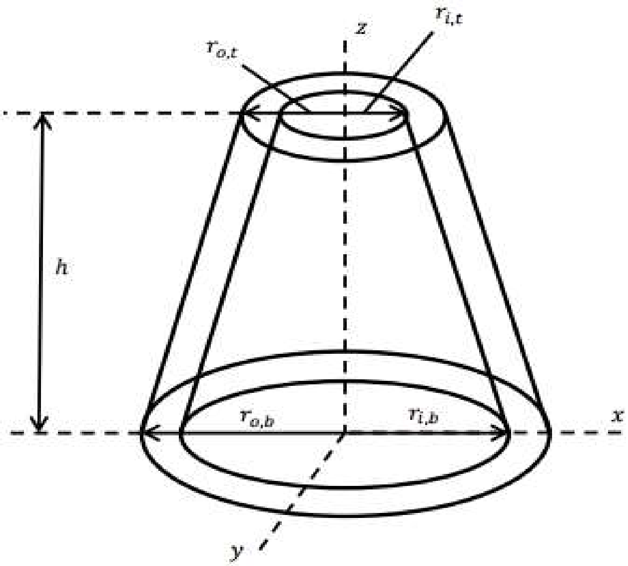


Appendix

Vertical Beam Center of Gravity dy & Volume



$$A = \left({r_{o,b}}^2 + 2r_{o,b}r_{o,t} + 3{r_{o,t}}^2\right) - \left({r_{i,b}}^2 + 2r_{i,b}r_{i,t} + 3{r_{i,t}}^2\right)$$

$$B = \left({r_{o,b}}^2 + r_{o,b}r_{o,t} + {r_{o,t}}^2\right) - \left({r_{i,b}}^2 + r_{i,b}r_{i,t} + {r_{i,t}}^2\right)$$

$$\text{Center of Gravity dx, dy} = 0\,, \frac{\mathrm{hA}}{4B}$$

$$\text{Volume Whole} = \frac{\mathrm{h}\pi}{3}\left(\mathrm{r}_{\mathrm{o,b}}^2 + \mathrm{r}_{\mathrm{o,b}}\mathrm{r}_{\mathrm{o,t}} + \mathrm{r}_{\mathrm{o,t}}^2\right)$$

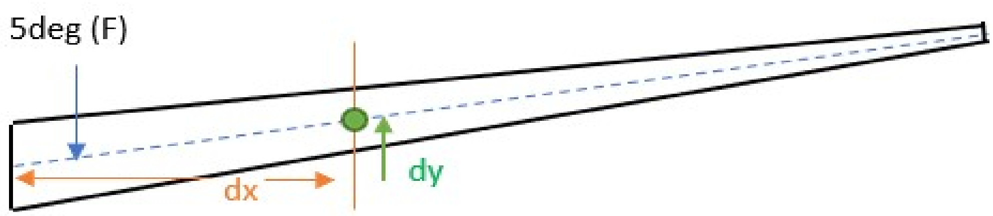
$$\text{Volume Inside} = \frac{\mathrm{h}\pi}{3}\left(\mathrm{r}_{\mathrm{i,b}}^2 + \mathrm{r}_{\mathrm{i,b}}\mathrm{r}_{\mathrm{i,t}} + \mathrm{r}_{\mathrm{i,t}}^2\right)$$

$$\text{Volume} = \text{Volume Whole} - \text{Volume Inside}$$

Mast Center of Gravity dx, dy & Volume

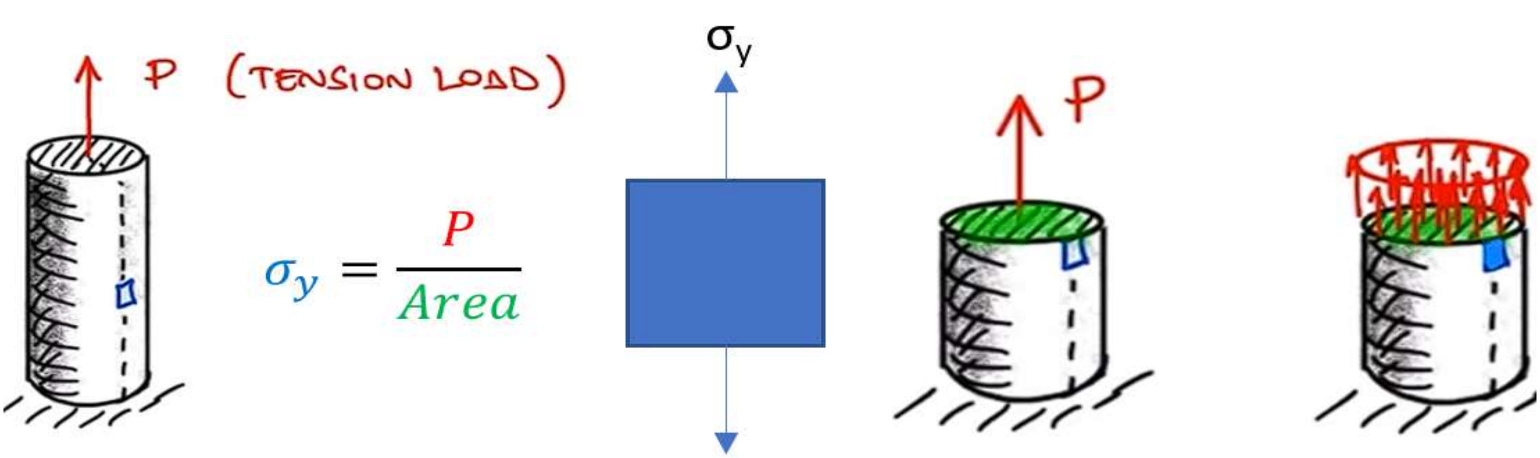
$$\text{Center of Gravity dx} = \frac{\mathrm{hA}}{4\mathrm{B}} * \cos(\mathrm{F})$$

$$\text{Center of Gravity dy} = \mathrm{B} + \left(\mathrm{dx} * \tan(\mathrm{F})\right)$$



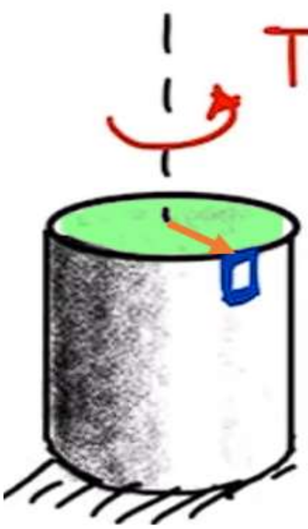
Volume calculations are identical to the vertical beam using root radius and tip radius instead of bottom and top.

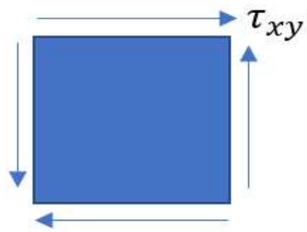
Axial Loading



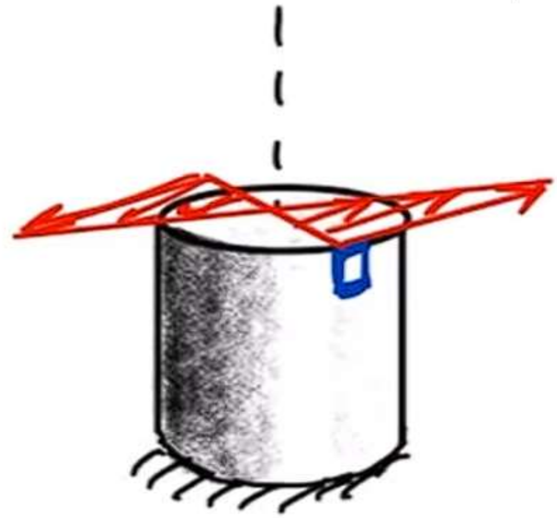
Appendix Continued

Torsion

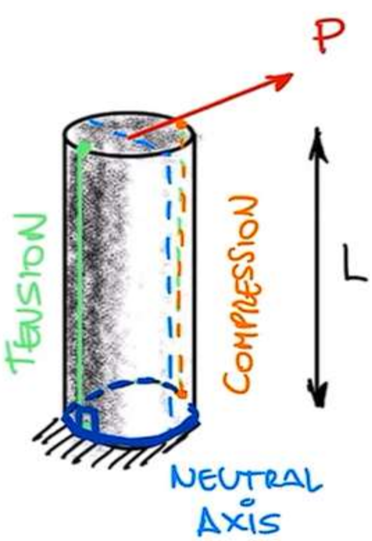


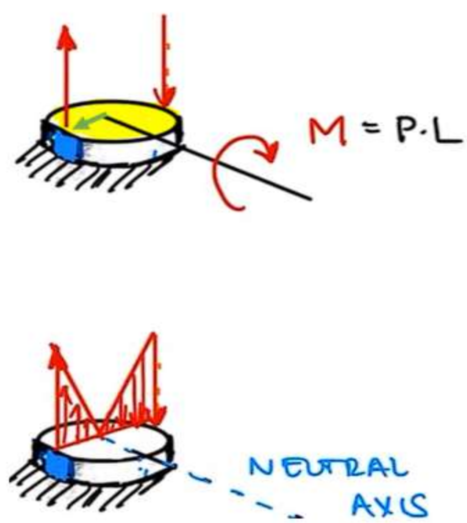


$$\tau = \frac{Tc}{J}$$

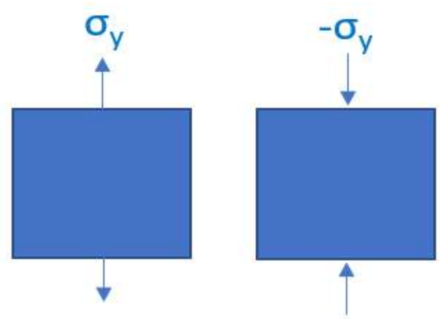


Bending

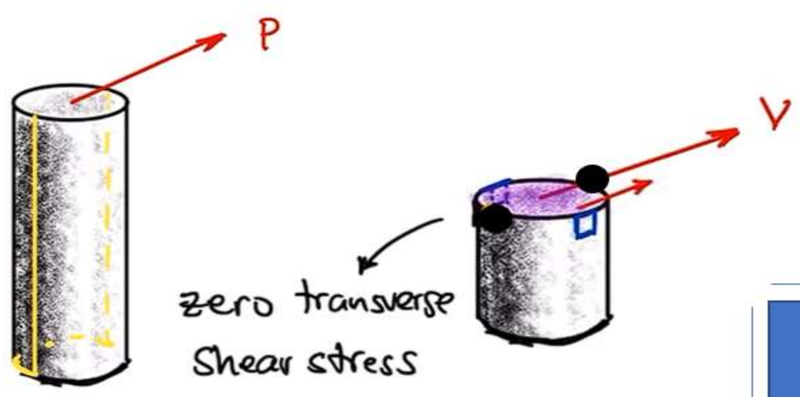


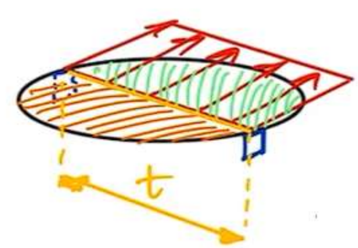


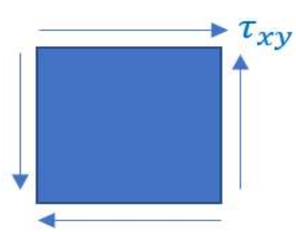
$$\sigma_b = \sigma_y = \frac{My}{I}$$



Transverse Shear







$$\tau = \tau_{xy} = \frac{VQ}{It}$$

Full Scale Dimension Sheet