Department of Mechanical Engineering

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Re: Flywheel Displacement Through Varying Rotational Frequencies

Acceleration readings were collected through a tri-axial accelerometer and a piezoelectric accelerometer through varying rotational frequencies of a flywheel in a balanced and unbalanced state. The rotational frequency was collected through an induction sensor providing peak amperage at each 1/6th of a rotation. The accuracy of the inductive apparatus was confirmed through a stroboscope. It was found that the piezoelectric accelerometer was interchangeable with the tri-axial accelerometer at rotational frequencies ranging from 14.98 ± 0.09 Hz [95%] to 32.05 ± 0.09 Hz [95%] on the unbalanced flywheel. The maximum vertical displacement was 35.00 ± 4.86mm [95%] for the balanced system and 14.52 ± 2.16 mm [95%] for the unbalanced system. Exponents of the experimental lines had an error of 36.8% for the balanced system and 59.2% for the unbalanced system on average compared to the expected value of -2. Axial displacement of the flywheel during the balanced and unbalanced tests are shown in Figure 1. It is predicted that the initially high YZ displacement is due to motor stutter since the axis' are perpendicular to the axis of rotation. The region of measured frequential overlap between the two accelerometers is shown in Figure 2.

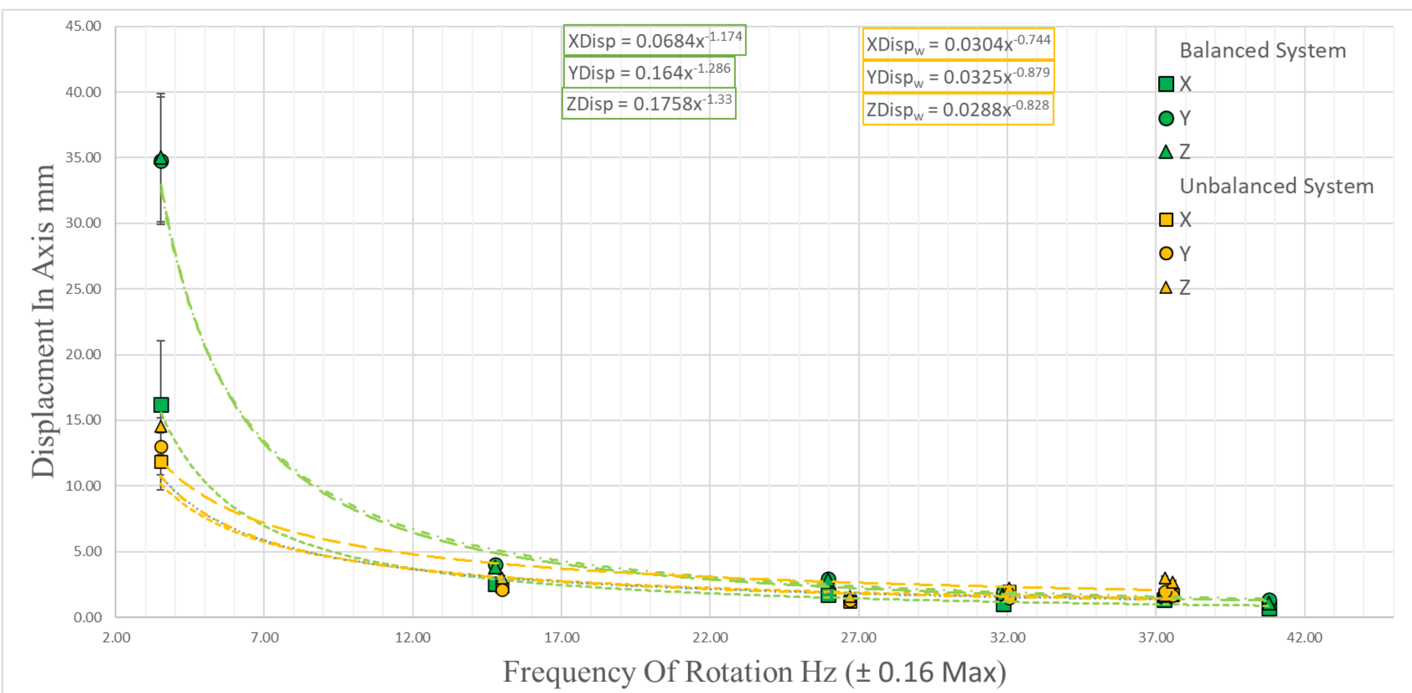

Figure 1. Maximum vertical displacement was 35.00 ± 4.86mm [95%] for the balanced system and 14.52 ± 2.16 mm [95%] for the unbalanced. Exponents of the experimental lines had errors of 36.8% and 59.2% respectively.

Data was collected by two different types of accelerometers mounted on the flywheel system. A B&K type 4371 accelerometer was mounted to the location that produced the most displacement. The outputs were subjected to a B&K Charge Amplifier Type 2635. A tri axial 356A16 accelerometer was mounted directly below the flywheel where the X axis was the axis of rotation. The acceleration values provided by the 356A16 were collected through an NI9234. Rotational frequency was calculated through an existing inductive sensor placed beside a rotating magnetic component along the flywheels driveshaft. The displacements in each axis for both the balanced and unbalanced system was calculated as shown in the 'Calculations Attachment'. The data collection system is shown in the 'Experimental Setup Attachment'. Rotational frequency recorded by the inductive sensor was verified through stroboscope measurements at equivalent speeds. Collected frequency values for the verification can be found in the 'Frequency Verification Attachment'.

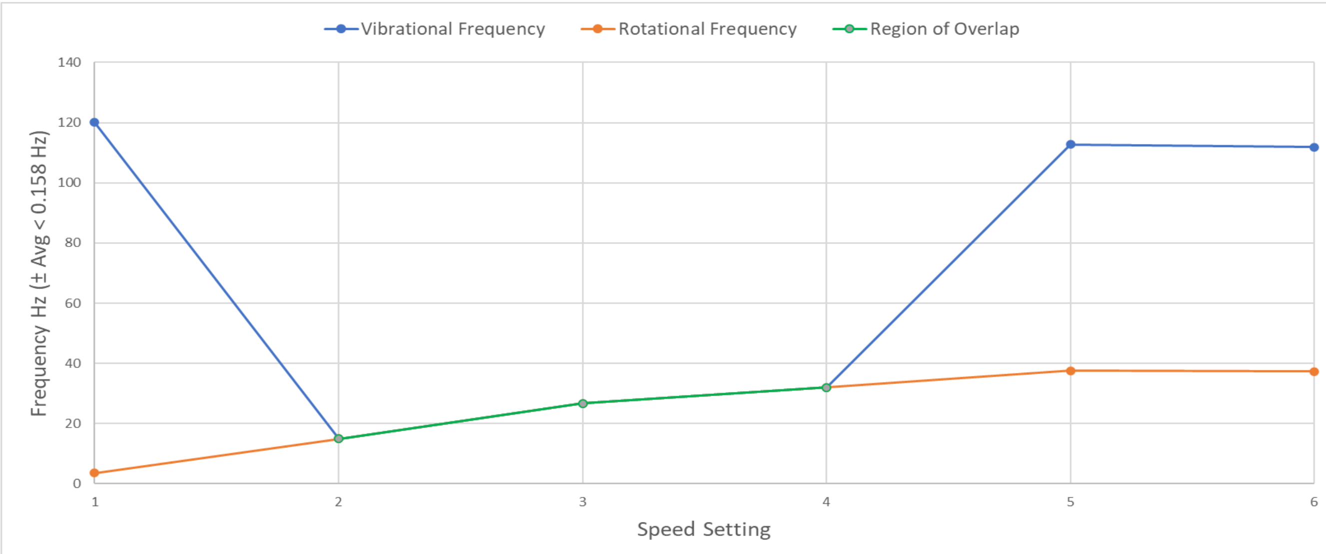

Figure 2. The region of frequential overlap between the piezoelectric and tri-axial accelerometer ranged from 14.98 ± 0.09 Hz [95%] to 32.05 ± 0.09 Hz [95%] on the unbalanced flywheel.

The frequential overlap region in Figure 2. shows the frequency range where both the tri-axis and piezo accelerometers provide similar displacement readings in the unbalanced system. This range alludes to the natural frequency of the system.

Uncertainty was calculated through the formulas shown in the 'Uncertainty Attachment'. The inductive sensor had no bias uncertainty. Due to single sample trials with the stroboscope, it only had a bias uncertainty of ±5 fpm (±0.08Hz). The bias uncertainty of the stroboscope was combined with the precision uncertainty of the inductive sensor to obtain the total uncertainty in the frequency of rotation at each point. The worst case uncertainty is denoted in Figure 1 due to display issues with their miniscule size. Displacement uncertanties are shown as the precision uncertainty calculated through Z axis displacement at each reading. Note that displacement uncertanties are too small to be visible in Figure 1. beyond initial readings. Individual uncertainty results can be found in the 'Uncertainty Charts Attachment'.

Maximum vertical displacement was 35.00 ± 4.86mm [95%] for the balanced system and 14.52 ± 2.16 mm [95%] for the unbalanced system. Exponents of the experimental lines had errors of 36.8% and 59.2% respectively. The large error in the results are attributed to an inaccurate stiffness value and the lack of damping in the calculations for the total axial displacement. The unexpectedly high initial displacement values for the balanced experiment are assumed to be related to unknown natural properties of the provided flywheel system. It is not advised to use provided displacement results for future needs. The region of frequential overlap between the piezoelectric and tri-axial accelerometer ranged from 14.98 ± 0.09 Hz [95%] to 32.05 ± 0.09 Hz [95%] on the unbalanced flywheel. The region is deemed to be valid as the results are not related to the aforementioned calculations and rely solely upon data obtained from experimentation.

Attachments: Frequency Verification, Calculations, Uncertainty, Uncertainty Charts , Experimental Setup.

FREQUENCY VERIFICATION

The attachment shows the method used to verify the frequency of the inductive apparatus and the stroboscope. The goal is to ensure that the inductive sensor achieved an accurate collection of data that is necessary to calculate displacement. The table below shows the values collected from both measurement devices and the percent error of the inductive sensor.

# of Trials	Frequency Measurment Verification				
	Frequency of Magnetic Peak (Hz)	Complete Flywheel Rotations (rps)	Stroboscope (flash-pm)	Stroboscope Frequency	Percent Error of Inductive Sensor
Trial #1	66.4	11.07	665 ± 5	11.08 ± 0.08	0.09
Trial #2	81	13.5	823 ± 5	13.72 ± 0.08	1.60

CALCULATIONS

Magnetic Frequency Full Rotation

$$\boldsymbol{M_{FR} = \frac{M_{avg}}{n}}$$

where

n is the number of magnetic nubs on drive shaft

M_{avg} The average frequency collected from the inductive sensor for each set of trials

M_{FR} is the magnetic frequency for one full rotation

Period of Magnet

$$\boldsymbol{Period\ of\ Magnet = \left(\frac{1}{M_{FR}}\right)}$$

where

$Period\ of\ Magnet$ is the time interval for each rotation

M_{FR} is the frequency of the magnet after each full rotation

Angular Frequency

$$\boldsymbol{AF = \frac{2\pi}{M_{FR}}}$$

where

AF is the angular frequency at a specific period of rotation

M_{FR} is the period of the magnet

Complete Flywheel Rotation

$$\boldsymbol{RPS = \left(\frac{F_{mp}}{N}\right)}$$

where

F_{mp} is the frequency collected by magnetic pickup

N is the number of magnetic nubs

RPS The number of flywheel rotations per second

Stroboscope Frequency

$$\boldsymbol{Flash_{ps} = \left(\frac{Flash_{pm}}{60s}\right)}$$

where

$Flash_{pm}$ is the frequency collected by stroboscope

$Flash_{ps}$ are the flashes per minute

$60s$ is the number of seconds per minute

UNCERTAINTY

Standard Deviation - (Sample)

$$\boldsymbol{\sigma = \sqrt{\frac{\sum(x_i - \overline{x})^2}{N - 1}}}$$

where

x_i is each measurement in individual groups of seven points

$\overline{x}$ is the mean of each group of seven measurements

N is the number of data points in each data set

Standard Error of the Mean

$$\boldsymbol{\sigma_M = \frac{\sigma}{\sqrt{N}}}$$

where

σ is the standard deviation

N is the number of data points

Uncertainty - Precision

$$\boldsymbol{U_{xP} = \frac{Z * \sigma_M}{\sqrt{N}}}$$

where

Z is the z-score used for the desired confidence interval (1.96 for 95% CI)

σ_M is the standard error of the data set

N is the number of data points in each data set

Uncertainty - Combined

$$\boldsymbol{U_x = \sqrt{u_{xB}^2 + u_{xP}^2}}$$

where

U_{xB} is the bias uncertainty of the individual sensors

U_{xP} is the precision uncertainty of the individual sensors

UNCERTAINTY CHARTS

Frequency of Rotation Uncertainties					
StdDev (s)	StdError	Prec. Uncertainty	Bias Uncertainty	Combined Uncertainty	+ Bias Unc. Of Stroboscope
0.038469025	0.017203871	0.015079858	0	0.015079858	0.095079858
0.070521316	0.031538091	0.027644356	0	0.027644356	0.107644356
0.078383113	0.035053994	0.03072618	0	0.03072618	0.11072618
0.195408748	0.087389449	0.076600229	0	0.076600229	0.156600229
0.1741575	0.077885602	0.06826974	0	0.06826974	0.14826974
0.036445177	0.016298779	0.014286509	0	0.014286509	0.094286509

Vertical Displacement Uncertainties for Balanced System				
StdDev (s)	StdError	Prec. Uncertainty	Bias Uncertainty	Combined Uncertainty
12.39422194	5.542864559	4.858535002	0	4.858535002
0.528160913	0.236200741	0.207039078	0	0.207039078
0.821192112	0.367248277	0.321907308	0	0.321907308
0.216517401	0.096829525	0.084874821	0	0.084874821
0.657769349	0.294163396	0.257845585	0	0.257845585
0.315768647	0.141216032	0.12378131	0	0.12378131

Vertical Displacement Uncertainties for Unbalanced System				
StdDev (s)	StdError	Prec. Uncertainty	Bias Uncertainty	Combined Uncertainty
5.515286806	2.466511243	2.161992428	0	2.161992428
0.550905231	0.246372309	0.21595485	0	0.21595485
0.214086833	0.095742542	0.083922038	0	0.083922038
0.222396169	0.09945859	0.087179298	0	0.087179298
0.264222528	0.118163907	0.103575231	0	0.103575231
0.197077117	0.088135566	0.07725423	0	0.07725423

EXPERIMENTAL SETUP