Constants

Geometry

$$A_{circle} = \pi R^2 \mid V_{cylinder} = \pi R^2 h \mid V_{sphere} = \frac{4}{3}\pi R^3 \mid A_{sphere} = 4\pi R^2$$

Vectors

$$|\vec{A} \times \vec{B}| = AB\sin(\theta) \quad |\vec{A} \cdot \vec{B}| = AB\cos(\theta) \quad |\vec{A} \cdot \vec{B}| = A_x B_x + A_y B_y + A_z B_z$$
$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

Integrals

$$\int \frac{xdx}{\sqrt{x^2 + y^2}} = \sqrt{x^2 + y^2} \left| \int \frac{dx}{(x^2 + y^2)^{3/2}} = \frac{x}{y^2 \sqrt{x^2 + y^2}} \right| \int \frac{xdx}{(x^2 + y^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + y^2}}$$
$$\int \frac{dx}{\sqrt{x^2 + y^2}} = \ln\left(x + \sqrt{x^2 + y^2}\right)$$

Electrostatics and Magnetostatics

$$\vec{F}_e = \frac{k_e Qq}{r^2} \hat{r}$$

$$V = \frac{k_e Q}{r}$$

$$\Delta V_{A \to B} = -\int_A^B \vec{E} \cdot d\vec{l}$$

$$\vec{E}_{\perp} = \frac{-k_e}{\left[r^2 + \left(\frac{s}{2}\right)^2\right]^{3/2}} \vec{p} \approx -\frac{k_e}{r^3} \vec{p}$$

$$E_{plane} = \frac{\eta}{2\varepsilon_0}$$

$$|E_{ring_z}| = \frac{k_e z Q}{(z^2 + R^2)^{3/2}}$$

$$\Delta V_{\infty line} = 2k_e \lambda \ln \left| \frac{r_A}{r_B} \right|$$
Inside C: $\Delta V = 0$

$$\vec{E}_{ins} = \frac{\vec{E}_{app}}{\kappa}$$

$$\frac{\Delta U}{\Delta Volume} = \frac{1}{2}\varepsilon_0 E^2$$

$$\vec{E} = \frac{k_e Q}{r^2} \hat{r}$$

$$\vec{F}_e = q\vec{E}$$

$$\Delta U = q\Delta V$$

$$|\vec{p}| = qs (- \text{ to } +)$$

$$E_{capacitor} = \frac{\eta}{\varepsilon_0}$$

$$|E_{disk_z}| = \frac{\eta}{2\varepsilon_0} \left[1 - \frac{z}{\sqrt{R^2 + z^2}} \right]$$

$$V_{ring_z} = \frac{k_e Q}{\sqrt{R^2 + z^2}}$$

$$V_{inside} = \frac{k_e Q}{R} \text{ (C sphere)}$$

$$\Delta V_{ins} = \frac{\Delta V_{vac}}{\kappa}$$

$$E = \frac{\eta}{\varepsilon_0}$$

$$U = \frac{k_e Qq}{r}$$

$$\vec{E} = -\nabla V$$

$$\vec{E}_{axis} = \frac{2k_e r}{\left(r - \frac{s}{2}\right)^2 \left(r + \frac{s}{2}\right)^2} \vec{p} \approx \frac{2k_e \vec{p}}{r^3}$$

$$E_{\infty line} = \frac{2k_e |\lambda|}{r}$$

$$E_{insSphere} = \frac{k_e Q}{R^3} r, r < R$$

$$V_{disk_z} = \frac{k_e Q}{R} \left[\sqrt{R^2 + z^2} - |z| \right]$$

$$V_{inside} = \frac{k_e Q}{2R} \left[3 - \frac{r_A^2}{R^2} \right] \text{ (I sphere)}$$

$$\kappa = \text{dielectric constant}$$
 Energy density - Capacitor

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \hat{r}}{r^2}$$

$$B_{loop\ axis} = \frac{\mu_0 IR^2}{2(R^2 + z^2)^{3/2}}$$

$$\vec{B}_{loopaxis} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{r^3}$$

$$\mu_{electron} = \frac{ev}{2\pi R} \pi R^2 = \frac{1}{2} eRv$$

$$ec{F}_B = q ec{v} imes ec{B}$$
 Relativistic: $\left| \frac{d ec{p}}{d t} \right| = |q| v B \sin(\phi)$ $ec{F}_{Lor} = q (ec{E} + ec{v} imes ec{B})$ $ec{F}_{wire} = \int I d ec{L} imes ec{B}$ Hall:
$$ext{EMF: } \Delta V = \epsilon = v L B$$

$$ext{$\Phi_B = \oiint ec{B} \cdot d ec{A}$}$$
 Law of Induction:
$$ext{Torque:}$$

$$\overline{v}=uE_{net}$$

$$i_e=n_eAv_d=n_eA\frac{e\tau}{m}E$$

$$n_e=\mathrm{e}^- \text{ density}$$

$$|q|=\text{magnitude of single charge}$$

$$\lambda=\frac{Q}{L}$$

 $\Delta U_m = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} \mu B \sin(\theta) d\theta$

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

$$B_{solenoid} = \frac{\mu_0 NI}{L}$$

$$\vec{\mu} = I\vec{A}$$

$$L_{electron} = Rmv$$

Magnetic Force & Flux
$$R = \frac{mv \sin(\phi)}{qB}$$

$$\omega = \frac{|q|B}{\gamma m}$$
Cyclo: $E_K = \Delta V q N$

$$\vec{F}_{straight} = I\vec{L} \times \vec{B}$$

$$V_H = wv_d B = \frac{IB}{dne}$$

$$F_B = ILB = \frac{\Delta V}{R} LB = \frac{(LB)^2 v}{R}$$

$$\Phi_{B,flat} = \vec{B} \cdot \vec{A}$$

$$\epsilon = -N \frac{d\Phi_B}{dt}$$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\Delta U_m = -\mu B \cos(\theta)|_{\theta_i}^{\theta_f}$$

$$u = \frac{e}{m_e} \overline{\Delta t}$$

$$v_d = \overline{v_{xi}} + \frac{eE}{m} \overline{\Delta t} = \frac{e\tau}{m} E$$

$$N_e = \text{total } \# \text{ of e}^-\text{s}$$

$$n = \text{density of charges}$$

$$\eta = \frac{Q}{A}$$

$$B_{\infty wire} = \frac{\mu_0 I}{2\pi d}$$

$$B_{wire} = \frac{\mu_0 I L}{2\pi d\sqrt{L^2 + d^2}}$$

$$\hat{A} = \hat{B}_{loopaxis}$$

$$\mu_{electron} = \frac{1}{2} \frac{e}{m} L$$

$$T = \frac{2\pi m}{qB}$$
$$T = \frac{2\pi m\gamma}{|q|B}$$

$$I = v_d A n e = v_d w d n e$$

$$P = F v = \frac{(LBv)^2}{R} = \frac{\Delta V^2}{R} = I \Delta V$$
 Unit: $1 \ W b = 1 \ T m^2$
$$E = -\frac{dB}{dt} \frac{r}{2}$$

$$\vec{\mu} = I \vec{A}$$

$$U_m = -\vec{\mu} \cdot \vec{B}$$

$$ec{p}=lphaec{E}$$

$$i_e=rac{N_e}{\Delta t}$$

$$I=|q|nAv_d$$

$$A= ext{cross-sectional area}$$

$$ho=rac{Q}{V}$$

Maxwell's Equations

$$\Phi_E = \int \int \vec{E} \cdot d\vec{A}$$
Gauss' Law:
Gauss' Law, Mag:
Ampere's Law:

$$\varepsilon_0 \oiint \vec{E} \cdot d\vec{A} = q_{enc}$$

$$\oiint \vec{B} \cdot d\vec{A} = 0$$

$$\oiint \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}$$

only steady currents

$$\begin{split} \nabla \cdot \vec{E} &= \frac{\rho}{\varepsilon_0} \\ \oint \vec{B} \cdot d\vec{r} &= \mu_0 i_{enc} + \mu_o \varepsilon_0 \frac{d\Phi_E}{dt} \end{split} \quad \begin{split} \nabla^2 V &= \Delta V = -\frac{\rho}{\varepsilon_0} \\ \oint \vec{E} \cdot d\vec{s} &= emf = -N \frac{d\Phi_B}{dt} \end{split} \quad \mathcal{N} \times \vec{B} = \mu_o \vec{J} \end{split}$$

Remark 0.1. If \vec{E} and $d\vec{A}$ are either parallel, perpendicular, or constant, one can use Gauss' Law to determine \vec{E} . If similar conditions apply to \vec{B} and $d\vec{r}$ we can determine \vec{B} by Ampere's Law.

Circuits				
$\sum I_{in} = \sum I_{out}$	$\sum \Delta V = 0$			
$ \Delta V_{battery} = E_C s = \frac{F_{NC} s}{e}$	$ \Delta V_{resistor} = EL$	$i = nA\overline{v} = nAuE$		
$i(t) = rac{dq}{dt}$	$q = \int_{0}^{t} i(t)dt$	$i = \int \vec{J} \cdot d\vec{A}$		
$q_{tot} = (nAL)e$	$i = q/t = (nAL)e/(L/v_d) = nAev_d$			
For uniform current:	i = JA	$ \vec{J} = (ne) \vec{v}_d $		
$n = density \times \frac{N_{AZ}}{Molar\ Mass}$				
$R = \frac{V}{i}$	$ ho = rac{E}{J}$	$\sigma = \frac{1}{\rho}$		
$R = \rho \frac{L}{A}$	$\rho = \frac{m}{e^2 n \tau}$	$\rho = \rho_0 (1 + \alpha (T - T_0))$		
$\frac{dU}{dt} = P = iV$				
$R_{eq,s} = \sum_{i=1}^{n} R_i$	$\frac{1}{R_{eq,p}} = \sum_{i=1}^{n} \frac{1}{R_i}$			

Capacitors and Resistors				
$q = C\Delta V$		a inner, b outer:		
$C_{plate} = \frac{\varepsilon_0 A}{d}$	$C_{cylinder} = \frac{2\pi L \varepsilon_0}{\ln\left \frac{b}{a}\right }$	$C_{sphere} = \frac{4\pi\varepsilon_0}{\frac{1}{a} - \frac{1}{b}}$		
$rac{1}{C_{eq,s}} = \sum_{i=1}^n rac{1}{C_i}$	$C_{eq,p} = \sum_{i=1}^{n} C_i$			
External: $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$	Density: $u = \frac{U}{Ad} = \frac{CV^2}{2Ad}$			

Charging:
$$q(t) = \varepsilon C \left(1 - e^{-t/RC}\right) \quad V_C(t) = \varepsilon \left(1 - e^{-t/RC}\right)$$
 Discharging:
$$q(t) = \varepsilon C e^{-t/RC} \qquad V_C(t) = \varepsilon e^{-t/RC}$$

	Inductors	
$L = \frac{N\Phi_B}{i}$	$L = \mu_0 n^2 l A$	n = N/l
$\varepsilon_L = -N \frac{d\Phi_B}{dt} = -L \frac{di}{dt}$		
$\frac{dU_B}{dt} = Li\frac{di}{dt}$	$U_B = \frac{1}{2}Li^2$	$u_B = \frac{U_B}{Al} = \frac{Li^2}{2lA} = \frac{1}{2}\mu_0 n^2 i^2$
Solenoid:	$B = \mu_0 i n$	$u_B = \frac{B^2}{2\mu_0}$
$q(t) = Q_0 \cos(\omega t + \phi_0)$ $\omega = \frac{1}{\sqrt{LC}}$	LC Circuit $i(t) = i_{max} \sin(\omega t + \phi_0)$ $\phi_0 = 0, q(0) = Q_0$	$i_{max} = Q_0 \omega$
	RLC Circuit	
$i(t) = i_0 \sin(\omega t + \phi_0)$	$\omega = \frac{1}{\sqrt{LC}}$	
$V_R(t) = Ri(t)$	$V_L = -Li_0\omega\cos(\omega t + \phi_0)$	$V_C = \frac{i_0}{\omega} \cos(\omega t + \phi_0)$

Definition 0.2 (Accelerated Charges). If a charge q is accelerated briefly by \vec{a} , then at an angle θ around the particle relative to the acceleration vector, we have electric radiation of

$$\vec{E}_{rad} = k_e \frac{-q\vec{a}_{\perp}}{c^2 r} \tag{0.1}$$

where $a_{\perp} = a \sin(\theta)$ and points in the direction $\vec{a} - proj_{\vec{r}}(\vec{a})$ (making a right triangle with one angle equal to θ).