Question 1 (4 POINTS).
Prove that any natural number $n > 1$ is either prime or composite.

Question 2 (3 POINTS).
Let $p > 2$ be a prime. Find all solutions to the congruence equation $x^2 \equiv x \mod p$.

Question 3 (4 POINTS).
Define a sequence by $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Prove by induction that

$$F_n = \frac{(\alpha_+)^n - (\alpha_-)^n}{\alpha_+ - \alpha_-}$$

where $\alpha_+ = \frac{1+\sqrt{5}}{2}$ and $\alpha_- = \frac{1-\sqrt{5}}{2}$.

Question 4 (3 POINTS).
Prove that for all sets A and B, $(A \backslash B) \cup (B \backslash A) = (A \cup B) \backslash (A \cap B)$.

Question 5 (4 POINTS).
Prove that for all $n \in \mathbb{N}$, given sets $A_1, ..., A_n$, we have that:

$$\left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^{n} A_i^c$$

Question 6 (3 POINTS).
Prove that a finite union of countable sets is countable.

Question 7 (6 POINTS).
Prove that if A is an infinite set, then A is countable if and only if there exists a surjection from $\mathbb{N}$ to A or an injection from A to $\mathbb{N}$

Question 8 (5 POINTS).
Prove that if $x, y \in \mathbb{Z}$ and $xy = 0$, then $x = 0$ or $y = 0$.

Question 9 (5 POINTS).
Prove that the rationals are countable.

Question 10 (6 POINTS).
Find a Cauchy sequence $\{a_n\}$ of rational numbers such that $\{(a_n)^2\}$ converges to 2.

Question 11 (6 POINTS).
Prove that the Nested Interval Property and the Least Upper Bound Property are equivalent

Question 12 (4 POINTS).
Suppose that $n \in \mathbb{N}$ and $z \in \mathbb{C}$ so that $z^n = 1$. Find and prove a formula for

$$\prod_{k=1}^{n-1} z^k$$