### Question 1 (4 POINTS).

Prove that any natural number n > 1 is either prime or composite.

### Question 2 (3 POINTS).

Let p > 2 be a prime. Find all solutions to the congruence equation  $x^2 \equiv x \mod p$ .

### Question 3 (4 POINTS).

Define a sequence by  $F_0 = 0$ ,  $F_1 = 1$ , and  $F_n = F_{n-1} + F_{n-2}$  for  $n \ge 2$ . Prove by induction that

$$F_n = \frac{(\alpha_+)^n - (\alpha_-)^n}{\alpha_+ - \alpha_-}$$

where  $\alpha_+ = \frac{1+\sqrt{5}}{2}$  and  $\alpha_- = \frac{1-\sqrt{5}}{2}$ .

### Question 4 (3 POINTS).

Prove that for all sets A and B,  $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$ .

#### Question 5 (4 POINTS).

Prove that for all  $n \in \mathbb{N}$ , given sets  $A_1, ..., A_n$ , we have that:

$$\left(\bigcup_{i=1}^{n} A_i\right)^c = \bigcap_{i=1}^{n} A_i^c$$

# Question 6 (3 POINTS).

Prove that a finite union of countable sets is countable.

## Question 7 (6 POINTS).

Prove that if A is an infinite set, then A is countable if and only if there exists a surjection from  $\mathbb N$  to A or an injection from A to  $\mathbb N$ 

# Question 8 (5 Points).

Prove that if  $x, y \in \mathbb{Z}$  and xy = 0, then x = 0 or y = 0.

#### Question 9 (5 POINTS).

Prove that the rationals are countable.

# Question 10 (6 Points).

Find a Cauchy sequence  $\{a_n\}$  of rational numbers such that  $\{(a_n)^2\}$  converges to 2.

### Question 11 (6 POINTS).

Prove that the Nested Interval Property and the Least Upper Bound Property are equivalent

# Question 12 (4 POINTS).

Suppose that  $n \in \mathbb{N}$  and  $z \in \mathbb{C}$  so that  $z^n = 1$ . Find and prove a formula for

$$\prod_{k=1}^{n-1} z^k$$