

## Constants

$k_e = 8.99 \times 10^9 \frac{Nm^2}{C^2}$	$e = 1.602 \times 10^{-19} C$	$m_p = 1.67 \times 10^{-27} kg$	$m_e = 9.11 \times 10^{-31} kg$
$\mu_0 = 4\pi \times 10^{-7} \frac{Tm}{A}$	$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}$		$k_e = \frac{1}{4\pi\epsilon_0}$
$milli (m) = 10^{-3}$	$micro (\mu) = 10^{-6}$	$nano (n) = 10^{-9}$	$pico (p) = 10^{-12}$

## Geometry

$$A_{circle} = \pi R^2 \quad V_{cylinder} = \pi R^2 h \quad V_{sphere} = \frac{4}{3}\pi R^3 \quad A_{sphere} = 4\pi R^2$$

## Vectors

$$|\vec{A} \times \vec{B}| = AB \sin(\theta) \quad \vec{A} \cdot \vec{B} = AB \cos(\theta) \quad \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

## Integrals

$$\int \frac{xdx}{\sqrt{x^2+y^2}} = \sqrt{x^2+y^2} \quad \int \frac{dx}{(x^2+y^2)^{3/2}} = \frac{x}{y^2 \sqrt{x^2+y^2}} \quad \int \frac{xdx}{(x^2+y^2)^{3/2}} = -\frac{1}{\sqrt{x^2+y^2}}$$

$$\int \frac{dx}{\sqrt{x^2+y^2}} = \ln \left( x + \sqrt{x^2+y^2} \right)$$

## Electrostatics and Magnetostatics

$\vec{F}_e = \frac{k_e Qq}{r^2} \hat{r}$ $V = \frac{k_e Q}{r}$ $\Delta V_{A \rightarrow B} = - \int_A^B \vec{E} \cdot d\vec{l}$ $\vec{E}_\perp = \frac{-k_e}{\left[r^2 + \left(\frac{s}{2}\right)^2\right]^{3/2}} \vec{p} \approx -\frac{k_e}{r^3} \vec{p}$  $E_{plane} = \frac{\eta}{2\epsilon_0}$ $ E_{ringz}  = \frac{k_e z Q}{(z^2 + R^2)^{3/2}}$ $\Delta V_{\infty line} = 2k_e \lambda \ln \left  \frac{r_A}{r_B} \right $ Inside C: $\Delta V = 0$ $\vec{E}_{ins} = \frac{\vec{E}_{app}}{\kappa}$	$\vec{E} = \frac{k_e Q}{r^2} \hat{r}$ $\vec{F}_e = q\vec{E}$ $\Delta U = q\Delta V$ $ \vec{p}  = qs \text{ (- to +)}$  $E_{capacitor} = \frac{\eta}{\epsilon_0}$ $ E_{diskz}  = \frac{\eta}{2\epsilon_0} \left[ 1 - \frac{z}{\sqrt{R^2 + z^2}} \right]$ $V_{ringz} = \frac{k_e Q}{\sqrt{R^2 + z^2}}$ $V_{inside} = \frac{k_e Q}{R} \text{ (C sphere)}$ $\Delta V_{ins} = \frac{\Delta V_{vac}}{\kappa}$	$U = \frac{k_e Qq}{r}$ $\vec{E} = -\nabla V$ $\vec{E}_{axis} = \frac{2k_e r}{\left(r - \frac{s}{2}\right)^2 \left(r + \frac{s}{2}\right)^2} \vec{p} \approx \frac{2k_e \vec{p}}{r^3}$  $E_{\infty line} = \frac{2k_e  \lambda }{r}$ $E_{insSphere} = \frac{k_e Q}{R^3} r, r < R$ $V_{diskz} = \frac{k_e Q}{R} \left[ \sqrt{R^2 + z^2} -  z  \right]$ $V_{inside} = \frac{k_e Q}{2R} \left[ 3 - \frac{r_A^2}{R^2} \right] \text{ (I sphere)}$ $\kappa = \text{dielectric constant}$
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$\frac{\Delta U}{\Delta V_{\text{volume}}} = \frac{1}{2} \epsilon_0 E^2$ $d\vec{B} = \frac{\mu_0}{4\pi} \frac{Id\vec{s} \times \hat{r}}{r^2}$ $B_{\text{loop axis}} = \frac{\mu_0 IR^2}{2(R^2+z^2)^{3/2}}$ $\vec{B}_{\text{loop axis}} = \frac{\mu_0}{4\pi} \frac{2\vec{\mu}}{r^3}$ $\mu_{\text{electron}} = \frac{ev}{2\pi R} \pi R^2 = \frac{1}{2} e R v$	$E = \frac{\eta}{\epsilon_0}$ $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$ $B_{\text{solenoid}} = \frac{\mu_0 NI}{L}$ $\vec{\mu} = I\vec{A}$ $L_{\text{electron}} = R m v$	<p>Energy density - Capacitor</p> $B_{\infty \text{wire}} = \frac{\mu_0 I}{2\pi d}$ $B_{\text{wire}} = \frac{\mu_0 IL}{2\pi d \sqrt{L^2+d^2}}$ $\hat{A} = \hat{B}_{\text{loop axis}}$ $\mu_{\text{electron}} = \frac{1}{2} \frac{e}{m} L$
$\vec{F}_B = q\vec{v} \times \vec{B}$ <p>Relativistic: <math>\left  \frac{d\vec{p}}{dt} \right  =  q vB \sin(\phi)</math></p> $\vec{F}_{\text{Lor}} = q(\vec{E} + \vec{v} \times \vec{B})$ $\vec{F}_{\text{wire}} = \int Id\vec{L} \times \vec{B}$ <p>Hall:</p> <p>EMF: <math>\Delta V = \epsilon = vLB</math></p> $\Phi_B = \oint \vec{B} \cdot d\vec{A}$ <p>Law of Induction:</p> <p>Torque:</p> $\Delta U_m = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} \mu B \sin(\theta) d\theta$	<p>Magnetic Force &amp; Flux</p> $R = \frac{mv \sin(\phi)}{qB}$ $\omega = \frac{ q B}{\gamma m}$ <p>Cyclo: <math>E_K = \Delta V q N</math></p> $\vec{F}_{\text{straight}} = I\vec{L} \times \vec{B}$ $V_H = w v_d B = \frac{IB}{dne}$ $F_B = ILB = \frac{\Delta V}{R} LB = \frac{(LB)^2 v}{R}$ $\Phi_{B, \text{flat}} = \vec{B} \cdot \vec{A}$ $\epsilon = -N \frac{d\Phi_B}{dt}$ $\vec{\tau} = \vec{\mu} \times \vec{B}$ $\Delta U_m = -\mu B \cos(\theta) \Big _{\theta_i}^{\theta_f}$	$T = \frac{2\pi m}{qB}$ $T = \frac{2\pi m \gamma}{ q B}$ $I = v_d A n e = v_d w d n e$ $P = Fv = \frac{(LBv)^2}{R} = \frac{\Delta V^2}{R} = I \Delta V$ <p>Unit: <math>1 \text{ Wb} = 1 \text{ T m}^2</math></p> $E = -\frac{dB}{dt} \frac{r}{2}$ $\vec{\mu} = I\vec{A}$ $U_m = -\vec{\mu} \cdot \vec{B}$
$\bar{v} = u E_{\text{net}}$ $i_e = n_e A v_d = n_e A \frac{e\tau}{m} E$ <p><math>n_e = e^-</math> density</p> <p><math> q </math> = magnitude of single charge</p> $\lambda = \frac{Q}{L}$	$u = \frac{e}{m_e} \overline{\Delta t}$ $v_d = \overline{v_{xi}} + \frac{eE}{m} \overline{\Delta t} = \frac{e\tau}{m} E$ <p><math>N_e</math> = total # of <math>e^-</math>s</p> <p><math>n</math> = density of charges</p> $\eta = \frac{Q}{A}$	$\vec{p} = \alpha \vec{E}$ $i_e = \frac{N_e}{\Delta t}$ $I =  q  n A v_d$ <p><math>A</math> = cross-sectional area</p> $\rho = \frac{Q}{V}$

## Maxwell's Equations

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

Gauss' Law:

Gauss' Law, Mag:

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

Ampere's Law:	$\oint \vec{B} \cdot d\vec{r} = \mu_0 I_{enc}$	only steady currents
$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$	$\nabla^2 V = \Delta V = -\frac{\rho}{\epsilon_0}$	$\nabla \times \vec{B} = \mu_0 \vec{J}$
$\oint \vec{B} \cdot d\vec{r} = \mu_0 i_{enc} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$	$\oint \vec{E} \cdot d\vec{s} = emf = -N \frac{d\Phi_B}{dt}$	$N = 1$ if free space

*Remark 0.1.* If  $\vec{E}$  and  $d\vec{A}$  are either parallel, perpendicular, or constant, one can use Gauss' Law to determine  $\vec{E}$ . If similar conditions apply to  $\vec{B}$  and  $d\vec{r}$  we can determine  $\vec{B}$  by Ampere's Law.

Circuits		
$\sum I_{in} = \sum I_{out}$	$\sum_{loop} \Delta V = 0$	
$ \Delta V_{battery}  = E_C S = \frac{F_{NC} s}{e}$	$ \Delta V_{resistor}  = EL$	$i = nA\bar{v} = nAuE$
$i(t) = \frac{dq}{dt}$	$q = \int_0^t i(t) dt$	$i = \int \vec{J} \cdot d\vec{A}$
$q_{tot} = (nAL)e$	$i = q/t = (nAL)e/(L/v_d) = nAev_d$	
For uniform current:	$i = JA$	$ \vec{J}  = (ne) \vec{v}_d $
$n = density \times \frac{N_A z}{Molar Mass}$		
$R = \frac{V}{i}$	$\rho = \frac{E}{J}$	$\sigma = \frac{1}{\rho}$
$R = \rho \frac{L}{A}$	$\rho = \frac{m}{e^2 n \tau}$	$\rho = \rho_0(1 + \alpha(T - T_0))$
$\frac{dU}{dt} = P = iV$		
$R_{eq,s} = \sum_{i=1}^n R_i$	$\frac{1}{R_{eq,p}} = \sum_{i=1}^n \frac{1}{R_i}$	

Capacitors and Resistors		
$q = C\Delta V$		a inner, b outer:
$C_{plate} = \frac{\epsilon_0 A}{d}$	$C_{cylinder} = \frac{2\pi L \epsilon_0}{\ln \frac{b}{a} }$	$C_{sphere} = \frac{4\pi \epsilon_0}{\frac{1}{a} - \frac{1}{b}}$
$\frac{1}{C_{eq,s}} = \sum_{i=1}^n \frac{1}{C_i}$	$C_{eq,p} = \sum_{i=1}^n C_i$	

External: $U = \frac{q^2}{2C} = \frac{1}{2}CV^2$	Density: $u = \frac{U}{Ad} = \frac{CV^2}{2Ad}$	
Charging:	$q(t) = \varepsilon C (1 - e^{-t/RC})$	$V_C(t) = \varepsilon (1 - e^{-t/RC})$
Discharging:	$q(t) = \varepsilon C e^{-t/RC}$	$V_C(t) = \varepsilon e^{-t/RC}$

Inductors		
$L = \frac{N\Phi_B}{i}$	$L = \mu_0 n^2 l A$	$n = N/l$
$\varepsilon_L = -N \frac{d\Phi_B}{dt} = -L \frac{di}{dt}$		
$\frac{dU_B}{dt} = L i \frac{di}{dt}$	$U_B = \frac{1}{2} L i^2$	$u_B = \frac{U_B}{Al} = \frac{L i^2}{2lA} = \frac{1}{2} \mu_0 n^2 i^2$
Solenoid:	$B = \mu_0 i n$	$u_B = \frac{B^2}{2\mu_0}$
LC Circuit		
$q(t) = Q_0 \cos(\omega t + \phi_0)$	$i(t) = i_{max} \sin(\omega t + \phi_0)$	
$\omega = \frac{1}{\sqrt{LC}}$	$\phi_0 = 0, q(0) = Q_0$	$i_{max} = Q_0 \omega$
RLC Circuit		
$i(t) = i_0 \sin(\omega t + \phi_0)$	$\omega = \frac{1}{\sqrt{LC}}$	
$V_R(t) = R i(t)$	$V_L = -L i_0 \omega \cos(\omega t + \phi_0)$	$V_C = \frac{i_0}{\omega} \cos(\omega t + \phi_0)$

**Definition 0.2** (Accelerated Charges). If a charge  $q$  is accelerated briefly by  $\vec{a}$ , then at an angle  $\theta$  around the particle relative to the acceleration vector, we have electric radiation of

$$\vec{E}_{rad} = k_e \frac{-q \vec{a}_{\perp}}{c^2 r} \quad (0.1)$$

where  $a_{\perp} = a \sin(\theta)$  and points in the direction  $\vec{a} - \text{proj}_{\vec{r}}(\vec{a})$  (making a right triangle with one angle equal to  $\theta$ ).