The Production Sector in EURACE

EURACE Winter School 2009

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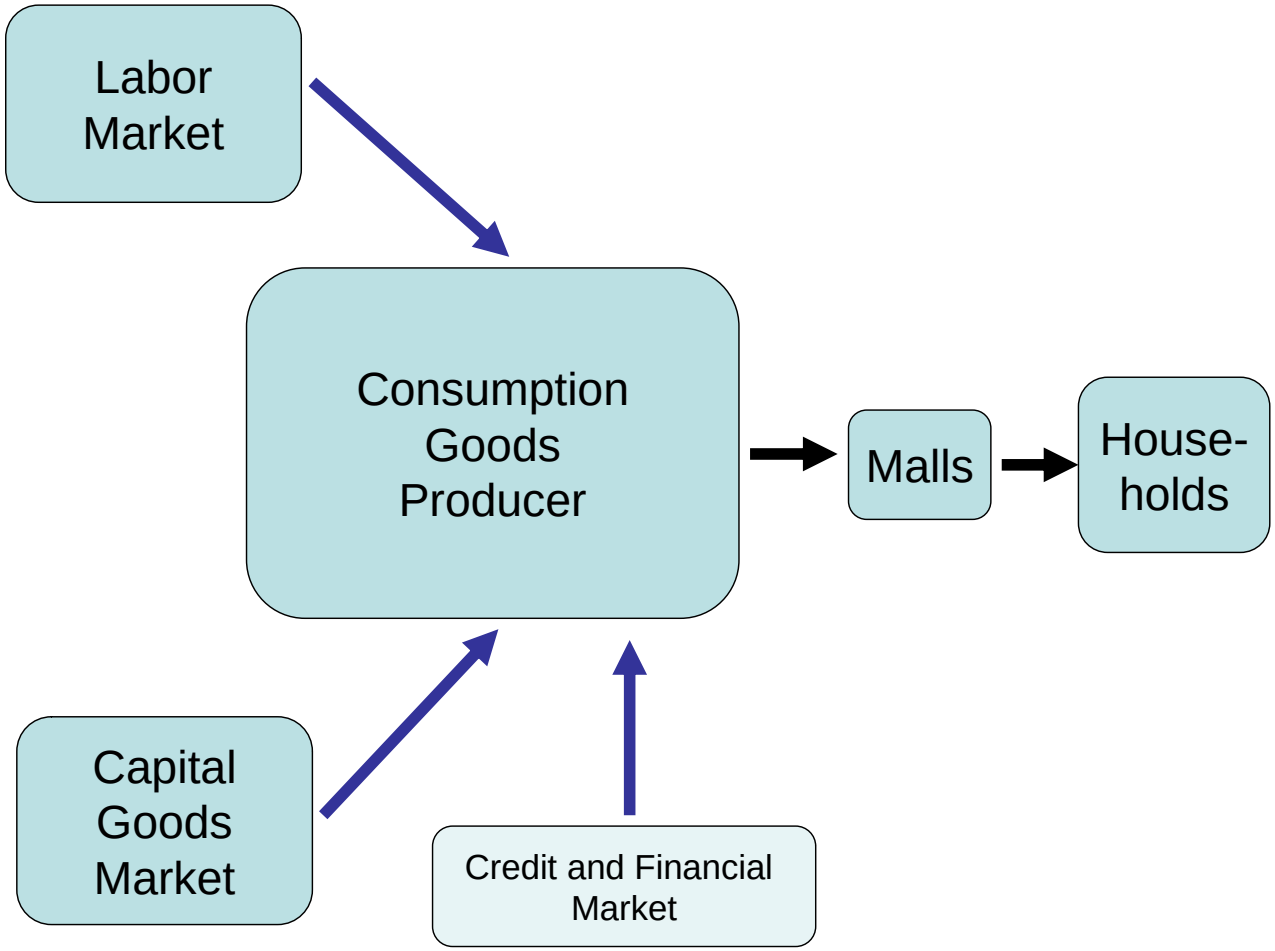
13. November 2009

The real side of EURACE

- There are 3 real markets in EURACE:
  - Consumption Goods Market.
    - Production and selling of a homogeneous consumption good.
    - Semi local market.
  - Capital Goods Market.
    - Capital goods are vertically differentiated among their productivity.
    - Global market.
  - Labor Market.
    - Labor differentiated among general and specific skills.
    - Semi local market.
  - Labor and capital goods are input factors for the production of the consumption good.

Agents and their roles

- Consumption goods producer:
  - Producer and seller of consumption goods.
  - Buyer of labor and capital goods.
- Households:
  - Consumer on the Consumption Goods Market.
  - Supplier on the Labor Market.
- Investment Goods Producer:
  - Supplier on the Investment Goods Market.
- Malls:
  - Local market platforms where consumption goods producers store and offer their commodities.
  - Transfer of information and goods from producers to consumers.


Labor Market
Consumption Goods Producer
Malls
House-holds
Capital Goods Market
Credit and Financial Market

General modeling philosophy

- Strong micro-foundation of decision rules: firms and households act rule-based using backward looking expectations.
- Operational decisions of firms are modeled using standard decision rules from the Operations Management literature:
  - Pricing (markup)
  - Inventory and production planing (newsboy problem)
- Savings/consumption decisions of HHs are simplified versions of empirically confirmed rules.

General features

- The firm uses capital and labor to produce consumption goods.
- Firms are located in regions.
  - The commodities are sold at geographically distributed outlet malls.
  - Goods can be frictionless transfered to all regions/malls.
  - Firms have access to the (global) Investment Goods Market.
  - There are barriers to hire workers from outside regions (commuting costs).
- The firm can finance the production internally and externally.

Technology

- The production technology of firm i is embedded in firm's capital stock $K_{i,t}$ and is characterized by its technical productivity $A_{i,t}$.
- Average productivity $A_{i,t}$ depends on past investments.
- $A_{i,t}$ is updated by depreciation and new physical investments.
- The firm depreciates its capital stock $K_{i,t}$ at a rate δ, it follows $K_{i,t} = (1 - \delta)K_{i,t-1} + I_{i,t}$.

The impact of Skills

- A worker w has two skill dimensions:
  - General skills: Education and general abilities measured in 5 discrete skill groups $b_w^{gen} = \{1, ...5\}$.
  - Specific skills: $b_{w,t}$ are experiences and know how obtained on the job.
  - Specific skills of a worker w employed in firm i evolve through learning by doing etc. according to

$$b_{w,t} = b_{w,t-1} + \chi(b_w^{gen}) \cdot (A_{i,t} - b_{w,t-1}).$$

    - Building up specific skills depends on educational level.
    - Function χ increasing in the general skill level of worker w, $f'(b_w^{gen}) > 0$.

Interaction of Technology and Skills

- Complementarity between mean specific skills $B_{i,t}$ and technical productivity $A_{i,t}$.
- Effective productivity $A_{i,t}^{eff} = \min\left[A_{i,t}, B_{i,t}\right]$.
- Productivity of a given technology level is only fully exploited if workers in the firm have sufficiently high specific skills.

Production Function

- Production Function of a Consumption Goods Producer:
  - Cobb-Douglas production function

$$Q_{i,t} = \min\left[A_{i,t}, B_{i,t}\right] L_{i,t}^{\alpha} K_{i,t}^{\beta}.$$

  - $L_{i,t}$ current labor stock, $K_{i,t}$ capital stock, α, β input factor intensity with constant returns to scale, $\alpha + \beta = 1$.

Sequence of activities

- The sequence of decisions and actions
  - Production planning.
  - Tentative input factor planning.
  - Financial planning.
  - Final production and input factor determination.
  - Labor and Capital Market transactions.
  - Production and delivery.
  - Periodic earnings statement.

Timing

- Timing of production
  - Length of the production cycle: 1 month.
  - At the monthly activation day (first day of the cycle): Production planing, financing, production, and delivery to the malls.
  - Selling during the whole of the month.
  - Earnings statement at the last day of the production cycle.

Production planning

- Standard inventory rule with stochastic demand (see e.g. Nahmias (2008)): The firms compute different delivery volumes for all served malls.
- $Y_{i,r,t}$ is the critical stock of firm i in mall r, $SL_{i,r,t}$ is the current mall stock at the activation day.
- Desired replenishment quantity:

$$\tilde{D}_{i,r,t} = \begin{cases} 0 & SL_{i,r,t} \geq Y_{i,r,t}, \\ Y_{i,r,t} - SL_{i,r,t} & else. \end{cases}$$

Production planning

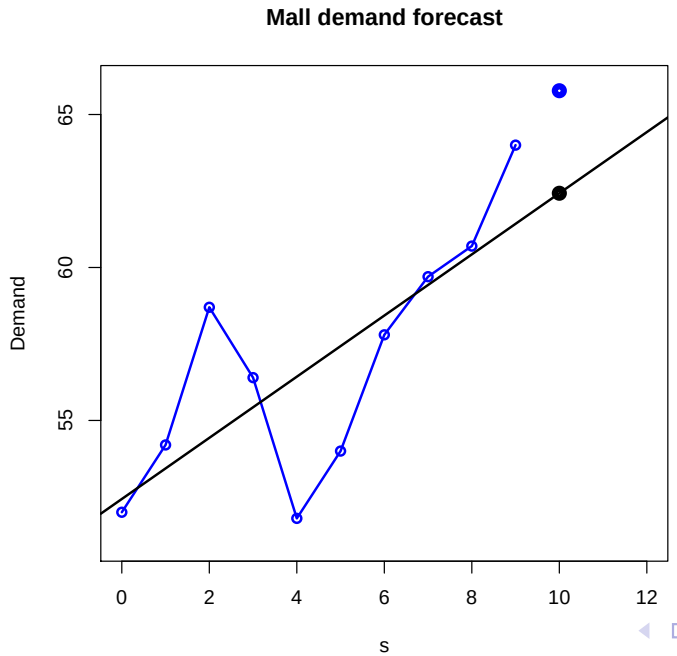
- Estimated demand of the last τ periods for mall r: $\left\{\hat{D}_{i,r,t-\tau}, ..., \hat{D}_{i,r,t-1}\right\}$.
- $Y_{i,r,t}$ is chosen such that the firm expects to be able to satisfy the market demand with probability $1-\chi$ (χ, stock-out probability).
- Linear regression model based on previous demands:

$$Y_{i,r,t} = \hat{a}_{i,r,t} + \tau \cdot \hat{b}_{i,r,t} + \bar{q}_{1-\chi} \cdot \sqrt{\hat{\delta}_{i,r,t}}.$$

- $\hat{a}, \hat{b}$ linear regression coefficients, $\hat{\delta}$ estimated variance, and $\bar{q}_{1-\chi}$ the $1-\chi$-quantile of the standard normal distribution.

Production planning

- Illustration: Linear regression of $\hat{D}_{i,r,t-\tau+s}$ with regressor s, and $\tau = 10$ past observations.
- Estimation of demand for $s = 10$: $\hat{D}_{i,r,t} = Y_{i,r,t}$



Production planning

- Desired replenishment quantity:

$$\tilde{D}_{i,r,t} = \begin{cases} 0 & SL_{i,r,t} \geq Y_{i,r,t}, \\ Y_{i,r,t} - SL_{i,r,t} & else. \end{cases}$$

- Sum of all replenishment quantities is $\tilde{D}_{i,t} = \sum_{r=1}^{R} \tilde{D}_{i,r,t}$.
- Smoothing of (planned) production quantity:

$$\tilde{Q}_{i,t} = \xi \tilde{D}_{i,t} + (1 - \xi) \frac{1}{T} \sum_{k=t-T}^{t-1} Q_{i,k}.$$

- Proportional adjustment of delivery volumes: $D_{i,r,t} = \frac{\tilde{D}_{i,r,t}}{\tilde{D}_{i,t}} \tilde{Q}_{i,t}$.

Input factor planning

- The firm aims to realize a capital to labor ratio according to the standard rule for CES production functions:

$$\frac{\tilde{K}_{i,t}}{p^{inv}} \Big/ \frac{\tilde{L}_{i,t}}{w_t^e} = \frac{\beta}{\alpha}.$$

- w_t^e expected mean wage, and p^{inv} a calculatory capital goods price.

Input factor planning: Capital

- Optimal capital stock

$$\tilde{\tilde{K}}_{i,t} = \frac{(\beta w_t^e)^\alpha \tilde{Q}_{i,t}}{(\alpha p^{inv})^\alpha \min[A_{i,t-1}, B_{i,t-1}]}.$$

- In two cases the desired capital stock $\tilde{K}$ deviates from the optimal value $\tilde{\tilde{K}}$:
  1. If $\tilde{\tilde{K}}_{i,t} < (1-\delta)K_{i,t-1} \Rightarrow \tilde{K}_{i,t} = (1-\delta)K_{i,t-1}$
     - No exceptional depreciation.
  2. If $\tilde{\tilde{K}}_{i,t} \geq (1+\kappa) \cdot K_{i,t-1}, \kappa > 0 \Rightarrow \tilde{K}_{i,t} = (1+\kappa) \cdot K_{i,t-1}$.
     - Inertia of the capital stock.
     - The monthly gross investments are limited: $I_{i,t} = (\delta + \kappa)K_{i,t-1}$.

Input factor planning: Labor

- After determining the desired capital stock $\tilde{K}_{i,t}$, the firm computes the required labor input:

$$\tilde{L}_{i,t} = \left(\frac{\tilde{Q}_{i,t}}{(\tilde{K}_{i,t}^{\beta} \min[A_{i,t-1}, B_{i,t-1}]} \right)^{1/\alpha} .$$

Financial planning

- Financial needs of a firm :
  - Expected expenditures for the production:

$$\hat{Exp}_{i,t}^{Prod} = w_t^e \tilde{L}_{i,t} + (\tilde{K}_{i,t} - K_{i,t-1})\bar{p}_t^{Inv}.$$

  - Financial obligations: $\hat{Exp}_{i,t}^{Fin}$ (Dividends, taxes, interest payments, debt installment payments).
  - Total financial needs:

$$\hat{Exp}_{i,t}^{Tot} = \hat{Exp}_{i,t}^{Prod} + \hat{Exp}_{i,t}^{Fin}.$$

- The firm checks how much can be financed internally.
  - The amount that cannot be financed by internal resources has to be obtained on the Credit or Financial Market.

Financial planning

- If the firm has to raise external liquidity:
  - Firm checks if it obtains the complete external financial needs.
  - If this is not the case: The firm has to adapt its expenditures.
    - Priority to serve the financial obligations.
    - Firm reduces the output quantity as longs as the recalculated expected expenditures are covered by the resources.
    - If the firm is still not able to serve its financial obligations: Bankruptcy.

Production

- Firm enters the Capital Market and the Labor Market.
- The capital stock after realized investments is $K_{i,t}$ with a technical productivity of $A_{i,t}$.
- The number of workers is $L_{i,t}$ with a mean specific skill level of $B_{i,t}$.
- The realized production quantity $Q_{i,t}$ is then:

$$Q_{i,t} = \min\left[A_{i,t}, B_{i,t}\right] L_{i,t}^{\alpha} K_{i,t}^{\beta}.$$

- Delivery volumes to malls: $D_{i,r,t} = \tilde{D}_{i,r,t} \frac{Q_{i,t}}{\tilde{Q}_{i,t}}$

Cost accounting

- The production expenditures are:

$$Exp_{i,t}^{Prod} = w_{i,t} \cdot L_{i,t} + I_{i,t} \cdot \bar{p_t}^{Inv}.$$

  - $w_{i,t}$ is the mean wage firm i pays in period t.
- The production costs are:

$$Cost_{i,t}^{Prod} = w_{i,t} \cdot L_{i,t} + Cost_{i,t}^{Cap} + Int_{i,t}.$$

  - $Cost_{i,t}^{Cap}$ calculatory capital costs, $Int_{i,t}$ interest payments.

Cost accounting

- Unit costs:

$$Cost_{i,t}^{Unit} = \frac{Cost_{i,t}^{Prod}}{Q_{i,t}}.$$

- Mark up pricing:

$$p_{i,t} = Cost_{i,t}^{Unit} \cdot \frac{1}{1 + 1/\epsilon_i^e}.$$

  - ϵ_i^e expected demand elasticity.

Earnings statement

- At the end of the selling period the mall informs the firm about the sold items $S_{i,r,t}$ and the current mall stock $SL_{i,r,t+1}$.
- The firm computes the EBIT:

$$EBIT_{i,t} = \sum_{R} S_{i,r,t} p_{i,t} - Cost_{i,t}^{Prod}.$$

- Determination of taxes, dividends, interests and debt installment payments to be paid in the next production cycle.

Earnings statement

- Determination of the (estimated) demand in all malls over the selling period.
  - Used for the determination of the production quantity in the following periods.

$$\hat{D}_{i,r,t} = \begin{cases} S_{i,r,t} & if SL_{i,r,t} > 0 \\ S_{i,r,t} \cdot (1+\nu) & if SL_{i,r,t} = 0, \end{cases}$$

where $0 < \nu < 1$.

General features

- The IG sector is in the current model version simplified.
- One IG firm offers its capital good on a global market.
- Consumption goods producers order the capital good and get the required amount without rationing.
- The IG firm produces without input factors.
- Net earnings (revenues minus taxes) are paid out as dividends.

Technological progress

- The IG firm carries out R&D activities to improve the technology.
- With a probability $prob^{Inno}$ the R&D is successful.
  - The productivity increases: $q_t^{Inv} = (1+\theta)q_{t-1}^{Inv}$.
  - The price of the investment good increases at the same rate: $p_t^{Inv} = (1+\theta)p_{t-1}^{Inv}$.
- Technological progress is driven by innovations in the investment goods sector, diffusion of the technology is induced by consumption good firms' investments in their capital stocks.

Malls

- The malls are local market platforms where consumption goods producer offer their goods and households purchase consumption goods.
- One mall per region.
- The mall informs the consumers about the range of provided goods and the corresponding prices.
- It receives orders from the household, if the sum of order quantities for a particular good exceeds the local inventory, consumers of that good are rationed.
- Sales are collected at the mall and transfered to firms.

Households purchasing decision

- Once a week a household goes to the closest located mall.
- $CB_{k,week_t}$ is the weekly consumption budget which a household can spend.
- $G_{k,week_t}$ is the set of goods, which is available at that day.

Households purchasing decision

- The household decides about one good to purchase using a standard discrete choice model.
- Logit model based on price differences.
- The value of consumption good i is given by $v_k(p_{i,t}) = -\ln(p_{i,t})$.
- Consumer k selects one good where the probability for good i to be selected is

$$Prob_{k,i,t} = \frac{\text{Exp}[\lambda^{cons} v_k(p_{i,t})]}{\sum_{i' \in G_{k,week_t}} \text{Exp}[\lambda_{cons} v_k(p_{i',t})]}$$

- The intensity of (price) competition is λ^{cons}

Households purchasing decision

- The household orders $\frac{CB_{k,week_t}}{p_{i,t}}$ units of the selected good i.
- If the household cannot spend the complete budget $CB_{k,week_t}$ due to rationing, the household enters a second loop in order to spend the remaining budget for another good.
- If the household is rationed once again, the remaining budget is rolled over to the following week.

- In this lecture presented:
  - Producer role of consumption goods producers.
  - IG firm.
  - Malls.
  - Consumer role of Households.
- Interactions with other EURACE components:
  - Labor Market.
  - Credit and Financial market.
  - Public sector.

Nahmias, S. (2008). *Production and operations analysis*. Mcgraw-Hill.